

Order-from-disorder phenomena in Heisenberg antiferromagnets on a triangular lattice

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We study the Heisenberg antiferromagnet on a triangular lattice with nearest- (J) and next-nearest- (αJ) neighbor exchange interactions. For $\alpha > \frac{1}{8}$, this system is known to have an accidental degeneracy at the classical level, which is lifted by quantum fluctuations ("order from disorder" phenomena). We use large- S perturbation theory and confirm previous spin-wave and numerical observations that quantum fluctuations always select planar arrangement: There is thus no chiral symmetry breaking. When α increases, the conventional 120° Néel state first undergoes a first-order transition to a commensurate metamagnet at $\alpha \approx \frac{1}{8}$. This state then transforms by a continuous transition into an incommensurate state. We show that the fluctuation corrections do not diverge at the transition point. There is thus no disordered intermediate phase around the classical transition point.

The discovery of the remarkable magnetic properties of high- T_c superconductors¹ has led to an intense activity in the subject of antiferromagnetic quantum spin systems mainly in two dimensions. Of great interest is the possibility of unconventional spin ordering in simple magnetic models.² In particular, the breaking of a chiral spin symmetry may lead to a well-studied chiral spin liquid state³ if quantum fluctuations restore rotational symmetry. Baskaran proposed a few years ago⁴ that the $S = \frac{1}{2}$ Heisenberg antiferromagnet on a triangular lattice has such a ground state, provided one adds antiferromagnetic exchange between next-nearest neighbors.

Chiral symmetry breaking means that there is a nonzero expectation value in the ground state for the operator

$$\chi = \mathbf{S}_i \cdot (\mathbf{S}_j \times \mathbf{S}_k) . \quad (1)$$

The site indices in this formula belong to an elementary plaquette of the triangular lattice. In a chiral state, the parity P as well as the time reversal T symmetry are broken. However, the combined PT operation remains a good symmetry of the ground state. Note that the order parameter χ is zero for any planar spin arrangement.

The copper spins of the high- T_c materials have focused attention¹ on the extreme quantum case of $S = \frac{1}{2}$. Certainly, the quantum fluctuations are then the largest and this should be the best situation to find a phase with chiral symmetry breaking, but without long-range order in the ground state. However, a controlled perturbative expansion can be performed only in the large-spin (S) limit for $SU(2)$ magnets or in the large- N limit⁵ for $SU(N)$ generalizations of the spin model. In two dimensions, zero-point quantum fluctuations do not diverge, and in cases where the expansion parameter is small, quantum fluctuations do not wash out long-range order. The only known exception⁶⁻¹³ happens when the system is close to

a second-order transition where spin-wave excitations undergo additional softening: dispersion $\varepsilon \propto k^2$ instead of $\varepsilon \propto k$. In this case, perturbative corrections diverge for arbitrary S . Nonzero chirality is certainly not related to the presence or absence of long-range order. The only requirement that chiral ordering imposes on the spin structure is a nonplanar arrangement of the spins on an elementary plaquette. With this condition, one may expect to test the possibility of a chiral spin liquid phase in the $S = \frac{1}{2}$ case by studying the large- S version of the same model. Finding a nonplanar arrangement in the large- S limit would be then an argument in favor of a chiral spin liquid state in a situation where long-range magnetic ordering is no longer present.

In this Brief Report, we perform a large- S study of a triangular antiferromagnet with nearest-^{14,15} (J) and next-nearest-¹⁶ (αJ) neighbor exchange interaction. The corresponding Hamiltonian is

$$H = J \sum_{NN} \mathbf{S}_i \cdot \mathbf{S}_j + \alpha J \sum_{NNN} \mathbf{S}_i \cdot \mathbf{S}_k . \quad (2)$$

In this equation, the first sum runs over nearest neighbors (NN) and the second sum over next-nearest neighbors (NNN). The classical ground states of this model have been studied in Ref. 16, where a leading-order spin-wave calculation gave the phase diagram. In this Brief Report, we extend these results through a self-consistent spin-wave calculation. We elucidate the nature of the transition points and show that there is no evidence for disordered phases. For α small enough, the classical ground state is the Néel state with planar 120° structure. When $\alpha = 0$, there is strong evidence for conventional long-range order of this type.^{14,15} This state has an energy per site which is $E = -\frac{3}{2}J + 3\alpha J$. The spin-wave spectrum corresponding to this classical state contains three Goldstone modes, as required by the complete breaking of the

SO(3) rotational symmetry. The dispersion relation is given by the formula

$$\begin{aligned} \varepsilon_k &= J\sqrt{C \times D} \ , \\ C &= 3 - v_x - 2v_{x/2}v_{\bar{y}} - 2\alpha(3 - 2v_{\bar{y}}v_{3x/2} - v_{2\bar{y}}) \ , \\ D &= 3 + 2(v_x + 2v_{x/2}v_{\bar{y}}) - 2\alpha(3 - 2v_{\bar{y}}v_{3x/2} - v_{2\bar{y}}) \ . \end{aligned} \quad (3)$$

We note that $v_k = \cos k$ and x, y stand for the momenta K_x, K_y . The Brillouin zone is chosen to be hexagonal and is shown in Fig. 1. We have also used $\bar{y} = \sqrt{3}K_y/2$.

The zero modes in the dispersion relation (3) are located at $K = (0,0)$, as well as at the corners of the Brillouin zone, labeled A and B in Fig. 1. All B points are equivalent as well as A points. Hence the number of zero modes is exactly what is required by the symmetry-breaking pattern. Consequently, fluctuations are not expected to play any role, apart from renormalizing the spin-wave velocity.

At $\alpha = \frac{1}{8}$, the spin-wave spectrum (3) softens at the points labeled C, D , and E in Fig. 1. This signals a transition to a different spin configuration.¹⁶ The selection of the ground state when $\alpha > \frac{1}{8}$ is somewhat peculiar. At the classical level, the sole requirement on a ground-state spin configuration is zero total spin on each elementary cell made of two adjacent triangles.¹⁷ Indeed, the energy of any four-sublattice configuration shown in Fig. 2 is given by

$$E = -J - \alpha J \ , \quad (4)$$

independently of the relative orientation of the spins as long as the total spin around the plaquette is equal to zero. One of these configurations is the two-sublattice metamagnet discussed in Ref. 16. It has ferromagnetic ordering along one direction and antiferromagnetic ordering along the two other principal directions of the tri-

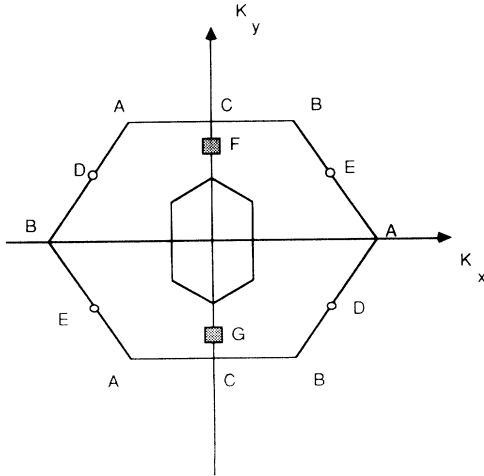


FIG. 1. Brillouin zone for the triangular lattice. Conventional Néel order is described by the wave vector at point A or B . The metamagnetic order corresponds to points C, D , and E . If one chooses ferromagnetic ordering along the x axis, then the corresponding wave vector is C and the extra unphysical zero modes are at D and E . For $\alpha > 1$, the incommensurate ordering is described by points F and G smoothly starting from C and reaching the reduced Brillouin-zone boundary as $\alpha \rightarrow \infty$.

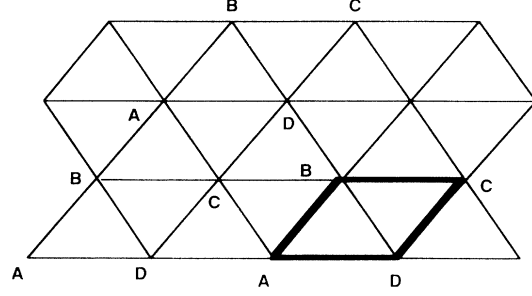


FIG. 2. Ground-state configuration in the intermediate regime $\frac{1}{8} < \alpha < 1$. The most general configuration contains four sublattices since the only requirement to the classical ground state is that the spins at points A, B, C , and D should obey the constraint $\mathbf{S}_A + \mathbf{S}_B + \mathbf{S}_C + \mathbf{S}_D = 0$.

angular lattice. As was shown in Ref. 16, this state is energetically preferred among the planar states once one takes into account the first quantum corrections to the ground-state energy produced by noninteracting spin waves.

In fact, classical degeneracy allows not only various planar configurations, but also nonplanar spin arrangement. The most symmetrical nonplanar configuration which satisfies the constraint of zero total spin on each square plaquette is a tetrahedron constructed from the four spins A, B, C , and D of Fig. 2. This configuration has nonzero chirality, and if selected by quantum fluctuations, it would support the idea of a chiral spin liquid for $S = \frac{1}{2}$. This tetrahedral configuration was recently considered by Korshunov.¹⁷ He computed numerically the first quantum correction to the classical ground-state energy and found that the tetrahedral state has always a higher energy than the metamagnetic state. We have also obtained similar results, confirming the irrelevance of the tetrahedral state. This rules out the possibility of chiral symmetry breaking in a model such as (2), at least in the large- S regime.

Let us now study in more details the spin-wave spectrum above the metamagnetic configuration. In the metamagnetic phase, there are two kinds of sites. This bipartite structure naturally leads to a description in terms of two bosonic fields. The dispersion relation is obtained through a standard procedure:

$$\begin{aligned} \varepsilon_k &= (A_k^2 - B_k^2)^{1/2} \ , \\ A_k &= JS [1 + \alpha + v_x + \alpha v_{2\bar{y}}] \ , \\ B_k &= 2JS v_{\bar{y}} [v_{x/2} + \alpha v_{3x/2}] \ . \end{aligned} \quad (5)$$

In this computation, we have assumed that the ferromagnetic arrangement of the spins holds along the x axis. As required by the symmetry-breaking pattern $\text{SO}(3) \rightarrow \text{SO}(2)$, the spectrum (5) has two zero modes. One is at $K = (0,0)$, and the other one is located at the Brillouin-zone boundary $Q_0 = (0, 2\pi/\sqrt{3})$ (point C in Fig. 1). In the neighborhood of these zero modes, the spectrum is relativistic:

$$\varepsilon_k^2 = JS(1 + \alpha)[3(1 - \alpha)k_y^2 + (9\alpha - 1)k_x^2] \ , \quad (6)$$

where the momenta k is measured from $K=(0,0)$ or $K=Q_0$. These spin-wave excitations are stable up to $\alpha=1$, where the stiffness in the y direction goes to zero and the system undergoes a transition to an incommensurate phase (discussed below). However, the spectrum of Eq. (5) has two additional zero modes which are not related to any kind of broken symmetry, but rather reflect an accidental degeneracy of the classical ground state. Indeed, both A_k and B_k vanish at two other points in the Brillouin zone which were symmetry related to C at $\alpha=\frac{1}{8}$ (these are D and E points in Fig. 1). The expansion around these points leads to a spectrum which scales quadratically with the momentum as one approaches D or E , contrary to the linear behavior (6) around the true zero modes. Such a quadratic spectrum, if taken seriously, would lead to logarithmically divergent corrections to the sublattice magnetization in two dimensions. This would lead to a disappearance of the long-range order. However, the existence of an accidental degeneracy is a peculiarity of the classical system and has no reason to survive in the case of quantum spins. This means that quantum fluctuations should normally lift the accidental degeneracy and restore long-range magnetic order at zero temperature. This is the typical "order-from-disorder" phenomenon which has been recently observed in many frustrated magnetic systems.¹⁸

We have thus computed explicitly the first quantum corrections to the quasiparticle spectrum. These corrections come from one-loop diagrams and renormalize both A_k and B_k in Eq. (5):

$$\delta A_k = \frac{J}{N} \sum_p \left[\frac{\bar{A}_p - \bar{\epsilon}_p}{2\bar{\epsilon}_p} F(k,p) + \frac{\bar{B}_p^2}{2\bar{\epsilon}_p} \right],$$

$$F(k,p) = (1 - v_{k_x})(1 - v_{p_x})$$

$$+ \alpha(1 - v_{2\bar{k}_y})(1 - v_{2\bar{p}_y}) - 2(1 + \alpha);$$

$$\delta B_k = -\frac{J}{N} \sum_p \left[\frac{\bar{A}_p - \bar{\epsilon}_p}{2\bar{\epsilon}_p} \bar{B}_k - \frac{\bar{B}_p}{2\bar{\epsilon}_p} G(k,p) \right],$$

$$G(k,p) = v_{\bar{k}_y + k_x/2} v_{\bar{p}_y + p_x/2} + v_{\bar{k}_y - k_x/2} v_{\bar{p}_y - p_x/2}$$

$$+ \alpha[v_{\bar{k}_y + 3k_x/2} v_{\bar{p}_y + 3p_x/2} + v_{\bar{k}_y - 3k_x/2} v_{\bar{p}_y - 3p_x/2}],$$

where $\bar{Z} = Z/JS$ ($Z = A, B, \epsilon$).

The corrections to the spectrum satisfy the condition

$$\delta A_{(0,0)} - \delta B_{(0,0)} = \delta A_{Q_0} + \delta B_{Q_0} = 0. \quad (8)$$

A simple inspection of Eq. (5) then shows that the fluctuation corrections do not destroy the gapless spectrum corresponding to the physical zero modes. On the contrary, at the two other classically soft points, $\delta B=0$, while the correction to A_k remains finite:

$$\delta A = \frac{J}{N} \sum_p \frac{2\bar{A}_p(1+\alpha) + \bar{B}_p^2 - 2\bar{A}_p^2}{2\bar{\epsilon}_p}. \quad (9)$$

As a result, the excitation spectrum near D and E points in the Brillouin zone acquires a finite (nonzero) gap $\Delta \equiv \delta A \approx (JS) \times 1/S$. This gap leads to a finite correction

to the sublattice magnetization, which in the large- S limit differs from a conventional result only by a logarithmical factor $\langle \Delta S \rangle / S \approx 1/S \ln(1/S)$.

The metamagnetic configuration remains stable up to $\alpha=1$ when, according to Eq. (5), the stiffness in the y direction vanishes. The classical spectrum near D and E is also sensitive to this criticality: Along particular directions, the dispersion becomes cubic in momenta at $\alpha=1$. However, we have checked that this has no dramatic consequences: The gap produced by quantum fluctuations remains finite and of the order of $(JS) \times 1/S$ also at $\alpha=1$.

For $\alpha > 1$, the ordering wave vector moves into the Brillouin zone and the metamagnetic configuration smoothly transforms into an incommensurate spiral defined by $(0, \pm Q)$, where

$$\cos \left[\frac{\sqrt{3}}{2} Q \right] = -\frac{1}{2} - \frac{1}{2\alpha}. \quad (10)$$

The corresponding points in Fig. 1 are labeled F and G . In this phase, the spins remain ferromagnetically ordered along the x axis, while the antiferromagnetic arrangements along the other two principal axis of the triangular lattice are distorted into a spiral state. The classical spin-wave spectrum above the spiral state contains three physical zero modes at $K=(0,0)$ and $(0, \pm Q)$, but also four additional zero modes which reflect the accidental degeneracy at the classical level. These zero modes are located inside the Brillouin zone at the lines connecting D and E with the origin. They can be deduced from the F point by use of a discrete symmetry. Obviously, this classical degeneracy cannot survive at the quantum level, and one-loop quantum corrections produce a gap $\Delta \approx (JS) \times 1/S$ for all the accidental soft modes except for $\alpha = \infty$ when the system decouples into three independent sublattices. In this limiting case, one has to consider the reduced Brillouin zone, which is the small hexagon in Fig. 1. The four additional soft points are then equivalent either to F or G .

Finally, we discuss the nature of the phase transitions at $\alpha=\frac{1}{8}$ and 1. The transition at $\alpha=\frac{1}{8}$ is between two states with different symmetry-breaking patterns. It should be either of first order or involve an intermediate phase. In a classical description, the spin-wave excitations in 120° and metamagnetic configurations soften simultaneously at the points labeled in Fig. 1 as C, D, E and A, B respectively. However, we found that when the first quantum corrections are taken into account by Eqs. (7), the stability region of the metamagnetic state extends to a region $\alpha < \frac{1}{8}$. We did not perform the analogous calculations for the 120° phase, but it is most likely that the stability regions of the two states overlap in a finite interval around $\alpha=\frac{1}{8}$; i.e., the transition is of first order, as is the case for the square lattice.¹⁰

The situation at $\alpha=1$ is different. Here the transition is continuous and the spin-wave velocity for Goldstone excitations vanishes at the transition. In two dimensions, this may lead to a tiny disordered region around a classical second-order transition point. This is what happens

in the so-called J_1 - J_2 - J_3 model on a square lattice.⁹ For finite third-neighbor coupling $J_3 > J_{3cr}$, this model has a second-order transition on the boundary of the Néel state $J_1 \approx 2J_2 + 4J_3$. In the present case, the situation is different because the softening occurs only along one direction of the two-dimensional lattice. This implies that the spin-wave spectrum near the physical zero modes behaves as $\varepsilon_k \approx JS(k_x^2 + k_y^4)^{1/2}$ (omitting numerical constants) and fluctuations do not produce divergent corrections neither to the sublattice magnetization nor to the coupling constant. The only effect of quantum fluctuations is a shift of the transition point from a classical value $\alpha = 1$. We found

$$\alpha_c = 1 + \frac{1}{4SN} \sum_p \frac{\bar{B}_p^2 - 4\bar{A}_p v_{2\bar{p}_y}}{\bar{\varepsilon}_p}. \quad (11)$$

This shift is of order $1/S$ without logarithmic corrections since the integral in Eq. (11) is convergent.

To summarize, in this Brief Report we have studied spin-wave excitations in a triangular Heisenberg antifer-

romagnet with exchange interactions between nearest- and next-nearest neighbors. Our spin-wave calculation goes beyond the leading-order calculation of Ref. 16. We have shown that the order-from-disorder phenomenon pointed out in Ref. 16 extends to nonplanar configurations and selects only planar states. Finally, we have shown that quantum fluctuations do not lead to a disordered phase near the second-order transition point between two magnetically ordered phases. This is quite different from the case of the square lattice, where there is a disordered phase for some region of the exchange parameters.

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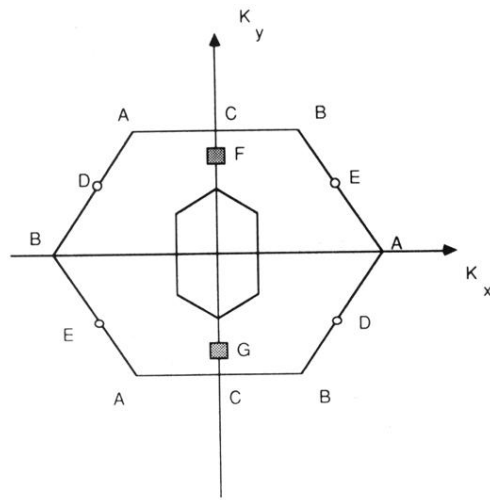


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