Power laws, flicker noise, and the Barkhausen effect

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The Barkhausen effect was studied in three ferromagnetic metals: an amorphous alloy, iron, and alumel. The data exhibit all the characteristics of self-organized critical behavior enumerated by Bak, Tang, and Wiesenfeld: The distributions of pulse durations, areas, and energies have the form of power laws, which have been modified to account for finite-size effects as suggested by Kadanoff, Nagel, Wu, and Zhou, and the power spectral densities have the form of flicker noise. Furthermore, the parameters describing the Barkhausen noise pulse distributions are consistent with those characterizing the power spectral density in the light of the results of Jensen, Christensen, and Fogedby. The data are also consistent with a model based on an inherent static fractal structure, independent of a self-organizing principle.

I. INTRODUCTION

Bak, Tang, and Weisenfeld¹ (BTW) introduced the concept of self-organized criticality (SOC) to provide a consistent explanation for the fractal spatial structures, power-law distributions, and flicker noise commonly observed in spatially extended, dissipative, dynamical systems. Jensen, Christensen, and Fogedby² (JCF) clarified the ideas in BTW and established the connection between the power-law dependencies and the power spectral densities. The consequences of the SOC concept have been extensively explored through computer simulations of sandpile dynamics¹⁻⁴ and earthquake phenomena.^{5,6}

The most striking application of the theory describes the state of the Earth's crust as a self-organized critical system, and provides a basis for understanding the Gutenberg-Richter law for the distribution of earthquake magnitudes.⁵ There is also a growing body of experimental investigations of SOC related phenomena including studies of actual sandpile dynamics,^{7,8} motion of magnetic domains in garnet films,⁹ sliding friction,¹⁰ and the Barkhausen effect in glassy metals.¹¹

The Barkhausen effect is a natural candidate for the SOC concept. The magnetic response of a ferromagnet is determined by its initial magnetic domain distribution and its response to applied magnetic fields.

For small fields, kinetic barriers permit only small reversible domain-wall motions and the system remains magnetically elastic in analogy with elastic-plastic deformation or slip-stick SOC earthquake models.⁵ As the applied field approaches and exceeds the magnitude of the coercive force, the specimen magnetization increases rapidly. The transition region is characterized by random irreversible domain-wall jumps and clusters of jumps analogous to slip in solids (e.g., in the Earth's crust) when the applied force exceeds the yield strength and produces rapid, steplike increases in strain.

The stochastic character of domain-wall jumps, which is analogous to a mechanical slip in solids, arises from the complex patterns of local magnetic fields, internal stresses, and bulk and surface defects in ferromagnetic alloys. The fingerprint of SOC behavior is detected in the Barkhausen noise spectrum, which generally exhibits the f^{-E} dependence¹³ characteristic of SOC phenomena.

The present study refines the analysis of Barkhausen effect in glassy metals¹¹ and extends it to the Barkhausen effect in crystalline ferromagnets. In particular, the role of composition and microstructure on the Barkhausen effect was investigated by analysis of three different ferromagnetic materials: a glassy metal, iron, and alumel.

Measurements in the three ferromagnetic materials are consistent with the behavior of a self-organized critical system. The distributions of pulse energy releases for the three materials are remarkably well described by a power-law analogous to the Gutenberg-Richter law. Further, the Barkhausen pulse distributions from the three specimens exhibit a cutoff on pulse duration (size effect) that correlates with the presence of internal defects such as grain boundaries.

II. SAMPLE DESCRIPTION AND PREPARATION

We report on Barkhausen noise in three ferromagnetic metals.

(i) Metglas 2605S ($Fe_{78}B_{13}Si_{19}$). The nominally 2 milthick Metglas ribbon, supplied by Allied-Signal, was cut into 2-mm by 20-mm samples, which were subsequently annealed for 15 min at 350 °C to relieve residual stresses from the cutting process.

In some cases, highly reproducible peaks uncharacteristic of the general random behavior of the pulse trains were observed. The presence of the large reproducible peaks, however, did not significantly affect the results of the pulse analysis in those trials where they were detected. Annealing at $T \ge 400$ °C, which completely relieves the internal stresses, was impractical since the Barkhausen peak amplitudes were reduced to such an extent that they could not be reliably distinguished.

These observations are consistent with the behavior of domain structures in 2605S studied by Livingston and

Work of the U. S. Government Not subject to U. S. copyright Morris.¹⁴ In the as-cast state, there is a wide distribution of domain sizes and types (in the unsaturated condition) generated by stresses from a variety of defects. At the other extreme, the fully annealed state, the stresses are greatly reduced, producing only a few large domains which tend to orient along the long axis of the specimen. In this condition, the permeability is at a maximum because the domain walls encounter much smaller resistances and the corresponding Barkhausen pulses are smaller and more difficult to detect. It should be noted that, except for the highly reproducible large-amplitude pulses described above, no major differences were observed in the original¹¹ and current studies of distributions of Barkhausen pulses from as-cast or partially annealed specimens.

(ii) Iron. Polycrystalline 5-mil iron thermocouple wire, which was provided by Omega Engineering, was cut into approximately 5-cm-long specimens. Grain size was approximately 10μ .

(iii). Alumel (Ni₉₅Al₃Mn₂). Polycrystalline 5-mil alumel thermocouple wire, which was provided by Omega Engineering, was cut into approximately 5-cm-long specimens. Mean grain size was approximately 10μ but, in contrast to the iron specimen, the grains showed evidence of annealing twins in a light microscope, which is effectively a further subdivision of the grains. The density of annealing twins generally increases with the amount of prior deformation, which was necessarily large with the drawing of 5-mil wire.

Unlike Metglas, which has no grain boundaries in the amorphous state, the grain boundaries in polycrystalline metals represent a limit on the maximum size of the domains. In contrast to Metglas, where cutting or deformation can generate large-amplitude Barkhausen pulses, mechanical deformation and surface abrasion of the polycrystalline specimens produced no dramatic changes in the character of the trains.

It also should be noted that the number of measurable Barkhausen pulses in the alumel specimen is relatively low and is sensitive to position along the wire so that specimen has to be carefully positioned within the pickup coil to register a sufficiently large number of observable pulses within a given train for analysis.

III. THEORY

The salient idea in the theory of self-organized criticality as conceived by BTW is that extended dissipative dynamical systems tend to organize themselves into a critical state where chain reactions of *all* sizes in time and space propagate through the system. The absence of characteristic length and time scales in the SOC state imply fractal structure and "1/f" (or flicker) noise power spectral density. A collateral consequence of SOC is power-law dependencies (hyperbolic distributions) of energies released, chain reaction lifetimes, and cluster sizes. Thus, SOC provides a coherent basis for the common observation of power laws, 1/f noise, and fractal structure in nature.

Jensen, Christensen, and Fogedby² clarified the results in BTW and explicitly revealed the important connection between power-law dependencies and power spectral density described by BTW. Kadanoff, Nagel, Wu, and Zhou³ extended the SOC concept to systems of finite extent.

JCF derive an expression for the power spectral density of a train of pulses, which represent the chain reactions in an SOC system. JCF demonstrate that the power spectral density can be expressed as

$$S(f) = \frac{r}{(\pi f)^2} \int_0^\infty d\tau \, G(\tau) \sin^2(\pi f \tau) , \qquad (1)$$

where r is the pulse repetition rate and G(T) is a weighted distribution of lifetimes, defined as

$$G(T) = \int_0^\infty dA P(A,T) [A/T]^2 , \qquad (2)$$

where P(A,T) is the joint probability for a pulse to have area A and length T.

JCF argue that, if

$$G(T) \propto T^{\alpha} \exp(-T/T_0)$$
 for $t_0 \leq T \leq \infty$, (3)

then

(i) for
$$1/T_0 \le f \le 1/t_0$$
, $S(f) \approx \text{const}$ when $\alpha < -3$,

$$\propto f^{-(3+\alpha)}$$
 when $-3 < \alpha < -1$, (4b)

$$\propto f^{-2}$$
 when $\alpha > -1$, (4c)

(ii) for $f \leq 1/T_0$, $S(f) \approx \text{const}$,

and

(iii) for
$$1/t_0 \le f$$
, $S(f) \propto f^{-2}$. (4e)

(4d)

(4a)

We find that essentially the same results are obtained if

$$G(T) \propto T^{\alpha} \Theta(T_c - T) , \qquad (3')$$

where $\Theta(x)$ is the Heaviside function and the "cutoff pulse length" T_c plays the role of T_0 . We refer to $1/T_c$ or $1/T_0$, which defines the break in Eq. (4), as the "transition frequency."

IV. APPROACH

The two parts to our approach are as follows.

(a) Measure the joint distribution functions $P_L[\ln(A), \ln(T)]$ for logarithmically spaced bins. Linearly spaced bins are not employed because the resulting histogram would contain only a few percent of the data outside the first bin. Thus, for linearly spaced bins, the parameters of interest are determined by only a small fraction of the data, which represent ranges of pulse lengths where "size effects" are dominant and statistical significance is low.

The joint distribution functions P(A,T), which appear in Eq. (2), are related to the measured logarithmic joint distribution functions as follows:

$$P(A,T)dA dT = P_L[\ln(A),\ln(T)]d[\ln(A)]d[\ln(T)]$$

= { $P_L[\ln(A),\ln(T)]/(AT)$ } $dA dT$,

which implies that

$$P(A,T) = P_L[\ln(A),\ln(T)]/(AT) .$$
(5)

(i) Integrate $P_L[\ln(A), \ln(T)]$ to obtain the log-spaced distribution functions $n_L[\ln(T)]$ and $n_L[\ln(A)]$. Investigate the suitability of hyperbolic distribution functions to describe $n_L[\ln(T)]$ and $n_L[\ln(A)]$. (The same notation is employed for different functions. Hopefully, no confusion will arise.)

Relationships analogous to Eq. (5) can be established between the log-spaced distribution functions and their linear-spaced counterparts n(T) and n(A). It is apparent that hyperbolic distribution of $n_L[\ln(x)]$ implies a hyperbolic distribution of n(x) with the exponent reduced by unity.

(ii) Compute G(T) via Eq. (2). Investigate the suitability of hyperbolic distribution functions and "size effect" forms to describe G(T). Use the JCF rules, i.e., Eq. (4), to deduce the exponent and transition frequency in the power spectral density.

(b) Employ fast Fourier transform (FFT) and the Wiener-Khintchine theorem to deduce the power spectral density directly from Barkhausen noise signals. Determine exponents and transition frequencies and compare with the JCF computed results.

V. EXPERIMENTAL PROCEDURES

The sample is positioned within a pickup coil and the assembly placed within an air core solenoid. A signal generator provides a sinusoidal current to the solenoid, which produces a field that continuously drives the specimen through a B-H loop. The pickup coil detects the random, steplike changes in sample magnetization and

the data are amplified and recorded on a digital oscilloscope. The magnitude and frequency of the driving field are adjusted to minimize pulse overlap while maintaining a sufficiently large number of pulses for meaningful analysis.

The air coil solenoid, which is comprised of numerous turns of 10-mil copper wire, is 5 cm long and 1 cm in diameter. The pickup coil, which comprises 200 turns of 5-mil copper wire, is 10 mm long and 3 mm in diameter. Generally, the driving field magnitude was maintained at less than 1 Oe and its frequency less than 1 Hz. The signal from the pickup coil was amplified with a gain of 5000 through a Stanford Research Systems low-noise preamplifier with bandwidth set at 0.3-30 kHz.

Typically trains of several hundred Barkhausen pulses were selected at random and captured on a Nicolet digital oscilloscope, which recorded 4000 values per train. Sampling rates were selected to minimize aliasing effects on power spectra calculated via FFT; typical sampling rates were $5-10 \mu$ sec per point.

VI. NUMERICAL PROCEDURES

Procedures for defining individual peaks and determining their distributions in Metglas 2605S, iron, and alumel were as follows.

(i) A polynomial baseline, generally a cubic, is fit through minima on 20 equally spaced intervals on each train. The signal level is then defined as the measured level minus the baseline.

(ii) The median signal level in a train is selected as a pulse-defining critical signal level. The critical signal level el defined in this way lies just above the background noise and passes through the apparent base of the pulses and generally yields results consistent with a less quantitative intuitive selection of peaks on the digital scope.

A typical train and its cubic baseline are shown in Fig. 1. An expanded section from the same train, which is shown in Fig. 2, illustrates the separation of major peaks.

(iii) The joint distribution function for logarithmically spaced bins in pulse area A and length T, $P_L[\ln(A), \ln(T)]$ is measured for pulses defined as described above.

(iv) The measured $P_L[\ln(A), \ln(T)]$ were numerically integrated to obtain the distribution functions $n_L[\ln(A)]$ and $n_L[\ln(T)]$ and the JCF weighted density of states G(T).



FIG. 1. A typical Barkhausen pulse train for the iron specimen and a third-order polynomial baseline.





FIG. 2. An expanded section of the Barkhausen pulse train of Fig. 1, illustrating the separation of the major peaks.

(v) Power spectral densities S(f) were computed by numerical integration based on the deduced G(T).

(vi) Power spectral densities were determined directly from the pulse train data by means of FFT and the Wiener-Khintchine theorem.

(vii) Pulse energies were computed by numerical integration of the square of the measured voltages. We assume that these computed energies are approximately proportional to the energy released in a Barkhausen pulse.

MATLABTM was used for all computations.

VII. RESULTS

Figure 3 is a log-log plot of the pulse area vs pulse duration for a 297-pulse train of Barkhausen pulses for the iron specimen. Similar results are obtained with Metglas and alumel. Linear regression through all the points gives a slope of 1.4. Essentially the same value is obtained in Metglas and alumel and was measured in a previous study¹¹ of Metglas 2605S.

Figures 4-6 are log-log plots of the distribution of the 297-pulse durations, areas, and energies for iron. The data reflect power-law distributions with exponents -1.64, -1.88, and -1.44, respectively.

Figure 7 shows a log-log plot of the computed values of the JCF weighted distribution of pulse lifetimes G(T) for the iron data. The slope of the linear portion of the graph is -1.0. Essentially the same exponent is found for alumel and Metglas.

Figure 8 shows a log-log plot of S(f) computed via



FIG. 3. Pulse area vs pulse duration for the train of 297 Barkhausen pulses in the iron specimen.



FIG. 4. Distribution of pulse durations for the train of Fig. 3.



FIG. 5. Distribution of areas for the train of Fig. 3.



FIG. 6. Distribution of energies for the train of Fig. 3.



FIG. 7. Computed values of the JCF weighted distribution of pulse lifetimes for the train of Fig. 3.



FIG. 8. Power spectral density computed from the results of pulse analysis and via FFT of the train of Fig. 3.



FIG. 9. Distribution of durations for a train of 137 Barkhausen pulses in the alumel specimen.



FIG. 10. Distribution of areas for the train of Fig. 9.



FIG. 11. Distribution of energies for the train of Fig. 9.



FIG. 12. Power spectral density computed from the results of pulse analysis and via FFT of the train of Fig. 9.

pulse analysis and FFT for the iron specimen. Both curves exhibit a transition to approximately f^{-2} for f > 500 Hz. The f^{-2} dependence is consistent with the $T^{-1}H(T-T_c)$ form for G(T).

Figures 9-11 are log-log plots of the distributions of the 137-pulse durations, areas, and energies for the alumel specimen. The corresponding power-law exponents are -2.1, -1.78, and -1.58, respectively.

Figure 12 shows the computed pulse and FFT results for alumel. Both show a transition to approximately f^{-2} for f > 2000 Hz. The difference in transition frequencies for iron and alumel are consistent with the differences in cutoff values for pulse durations as in Figs. 4 and 9.

Figures 13–15 are log-log plots of the distributions of durations, areas, and energies of 218 pulses for the Metglas sample. The corresponding power-law exponents are -1.82, -1.74, and -1.60, respectively. Earlier results¹¹ on Metglas gave -2.26 rather than -1.81 for the duration exponent. The discrepancy (which is less than 1.3σ from Ref. 11) results from fitting to the sharp cutoff functional form with logarithmic binning rather than to the exponential cutoff form of JCF with linear binning as in Ref. 11. The new result is believed to be more reliable.

Figure 16 shows the agreement between the computed pulse and FFT results for S(f) for the Metglas sample. Both show a transition to approximately f^{-2} for f > 1000 Hz. The earlier result for the FFT computed S(f) exponent was -1.244.¹¹ We now believe that aliasing biased those data to produce the smaller exponent.



FIG. 13. Distribution of durations for a train of 218 Barkhausen pulses in the Metglas specimen.



FIG. 14. Distribution of areas for the train of Fig. 13.

VIII. DISCUSSION

A. Pulse distribution analysis

Since hyperbolically distributed data tend to be intensely concentrated at small sizes, logarithmically spaced binning has proved to be better suited to fitting the data than conventional linearly spaced binning. The problem with linear binning is that the fitting parameters (i.e., the exponents) are largely determined by the occupation values for low probability bins. Employing the techniques described in Sec. VI, i.e., log-spaced binning, the situation is dramatically improved. The problems in fitting reported in Ref. 11 could have been largely resolved by employing logarithmically spaced binning.

B. The functional form of the pulse distribution data

1. The exponential cutoff pulse distribution form of JCF

As noted in our earlier report on Metglas, which fit the Barkhausen pulse distributions to a power law with exponential cutoff, the data appear to cut off faster than $\exp(-T/T_0)$.

2. Sharp cutoff pulse distribution

The present pulse distributions and the earlier data are well approximated by hyperbolic distributions with a *sharp cutoff*, i.e.,

$$n(T) = FT^a \Theta(T_c - T) , \qquad (6)$$



FIG. 15. Distribution of energies for the train of Fig. 13.



FIG. 16. Power spectral density computed from the results of pulse analysis and via FFT of the train of Fig. 13.

where $\Theta(x)$ is the Heaviside function and F is a constant as in Eq. (3'). The cutoff pulse duration T_c plays the role that T_0 plays in the exponential cutoff form in determining the frequency at which the transition to f^{-E} behavior in the power spectral density occurs. The differences in T_c as determined from the analysis of the Barkhausen pulse distributions in Metglas 2605S, iron, and alumel are reflected in the crossover frequencies in their power spectral densities.

3. Physical interpretation of T_0 or T_c

The origin of cutoffs in SOC avalanche models is determined by the size of the model which imposes an upper limit on the size of avalanches. However, the present experimental data are consistent with the view that internal defects rather than external dimensions determine the cutoff on the size of the Barkhausen avalanches.

(a) Polycrystalline samples. The cutoff pulse size T_c correlates with the grain size in the polycrystalline samples. The metallographically determined grain structure is finer in the alumel wire (considering the subgrain annealing twins) than in the iron wire samples. Thus, the maximum domain size and largest avalanche would be expected to be smaller in the alumel wire, as observed.

(b) Amorphous samples. The situation is different in the Metglas 2605S samples. There are no grain boundaries. Furthermore, the domains in the Metglas ribbons are essentially two dimensional and extend from the top to bottom surfaces.¹⁴ (The ribbon thickness was nonuniform and ranged from 1 to 2 mils in our samples.) The defects that limit the maximum size of domain jumps in the amorphous ribbons are expected to be internal stresses and surface imperfections produced during the rapid solidification process or subsequent treatment. The cutoffs are also consistent with the existence of selforganized subcritical states as in the magnetic domain pattern studies of Bak and Flyvbjerg.¹⁵

C. The Korcak law

The "area-number relations" are discussed by Mandelbrot¹² in terms of the number of pulses whose area A exceeds a, denoted N(A > a), which is related to the pulse distribution by

$$N(A > a) \equiv \int_{a}^{\infty} n(A) dA.$$
⁽⁷⁾

For hyperbolic distribution functions, $n(A) \approx n(1)A^{-q}$ with q a positive constant,

$$N(A > a) \approx n(1) \int_{a}^{\infty} A^{-q} dA = n(1) a^{-(q-1)} / (q-1)$$

= Fa^{-B}, (8)

i.e., the number of pulses whose area exceeds a will also be hyperbolically distributed if B = q - 1 > 0. Mandelbrot refers to the hyperbolic form of area-number relations as the Korcak law.

D. Universality

Despite the substantial differences in domain sizes and configurations, the pulse distributions and power spectra measured in Metglas 2605S, iron, and alumel are strikingly similar. The only substantive differences in the Barkhausen data for the three materials are manifested in variations in the pulse cutoff values T_c and the corresponding limiting frequencies for the f^{-E} behavior in the power spectral density.

E. Barkhausen effect as a manifestation of SOC

The theory of self-organized critical behavior described in BTW provides a fresh perspective on Barkhausen phenomena. The power-law distributions (hyperbolic distributions) and flicker noise, which are the hallmark of SOC, are conspicuous in Barkhausen phenomena. Furthermore, the quantitative link between the exponents characterizing the pulse distribution and the flicker noise in the power spectral density established by JCF for SOC systems holds for the present Barkhausen results.

The BTW theory of SOC phenomena provides a dynamical basis for the occurrence of hyperbolic distributions and the concomitant flicker noise. Therefore, the present Barkhausen data are consistent with SOC concepts concerning power-law distributions in dissipative dynamical systems.

F. Barkhausen effect reflecting scale invariant static structure

As Mandelbrot has stressed,¹² hyperbolic distributions are found in diverse scale invariant systems. Thus, inherent static structural features in Barkhausen systems, e.g., fragmentation or fractal grain or domain structure, independent of a self-organizing principle could yield the observed hyperbolic distributions of Barkhausen avalanche sizes.

A similar interpretation has been proposed for earthquake data. There is some evidence^{16,17} that the powerlaw distributions of earthquake magnitudes are a consequence of power-law distributions of individual fault dimensions rather than a power-law distribution of earthquake sizes on a given fault. We plan to explore this possibility for the Barkhausen effect.

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