Mapping of the two-channel Kondo problem to a resonant-level model

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Abelian bosonization is used to map the two-channel Kondo problem into a resonant-level Hamiltonian, which is equivalent to noninteracting fermions for a particular value of the longitudinal exchange coupling. This solvable point is analogous to the Toulouse limit of the ordinary Kondo problem. The impurity Green's function, susceptibility, and thermodynamic properties, together with certain conduction-electron correlation functions, are evaluated at the solvable point. In particular, it is shown that the pairing resonance of the two-channel Kondo problem may be characterized as an enhancement of superconducting pairing of the conduction electrons at the impurity site, correlated with the impurity pseudospin (or equivalently odd-time pairing of the conduction electrons).

I. INTRODUCTION

The general multichannel Kondo model, introduced by Blandin and Nozières[1] consists of a magnetic impurity of spin S in a sea of conduction electrons, which have spin-1/2 and n-degenerate orbital channels or flavors. Since its inception, the model has been solved by renormalization-group,[1,2] Bethe Ansatz,[3] and conformal invariance[4–8] techniques and, by now, it is well understood. At the present time, it seems that the most likely physical realizations of the model do not involve magnetic impurities, but rather inelastic impurity scattering. A number of years ago, it was pointed out[9] that the inelastic scattering of conduction electrons from a nonmagnetic two-level impurity is equivalent to a single-channel Kondo problem. In this mapping, the two levels are represented by an impurity pseudospin, but the conduction electrons do not have a corresponding degree of freedom. Subsequently, it was emphasized by Zawadowski,[10] in the context of the scattering of conduction electrons from two-level systems in metallic glasses, that inelastic scattering may change the parity of the angular momentum state of the conduction electrons, thereby giving them an internal degree of freedom coupled to that of the impurity. Indeed, Murumatsu and Guinea[11] used a renormalization-group analysis to show that, for strong electron-assisted tunneling between the states of the two-level system, the low-temperature behavior is dominated by the nontrivial fixed point of the multichannel Kondo problem. Physically, the spin of the conduction electrons plays the role of the channel or flavor index, so $n=2$.

Recently it has been discovered[12] that the specific heat, resistivity, and residual entropy of $Y_{1-x}U_xPd_3$ have the behavior expected for the two-channel Kondo problem. The latter is attributed to quadrupolar degrees of freedom on the uranium sites, and it has been proposed[13] that such a model may also be appropriate for the heavy fermion superconductors UBe_{13} UPt_3 and URu_2Si_2.

Another possible application is to high-temperature superconductors. We have found[14] that the competition between long-range Coulomb interactions and the tendency of holes in an antiferromagnet to separate into hole-rich and hole-poor phases, leads to low-energy, localized collective modes with internal degrees of freedom. The scattering of conduction electrons from the collective modes may be modeled by a two-channel Kondo problem with anisotropic exchange.[14] Previously, it had been noted by Cox[15] that the marginal Fermi-liquid phenomenology of high-temperature superconductors[16] is reminiscent of the behavior of a two-channel Kondo system.

As these examples make clear, the most important case from a physical point of view is the two-channel Kondo problem, $n=2$, $S=1/2$, for which the conduction electrons overscreen the impurity.[1] This offers the possibility of exploring interesting physical effects because the low-energy behavior may not be characterized as a Fermi liquid.[1–8] However, the consequences may be different in different realizations of the model.

The purpose of this paper is to describe another way of solving the two-channel Kondo problem, specifically by mapping it into a resonant-level model, which reduces to noninteracting fermions for a particular value of the z component of the exchange. This special case is analogous to the Toulouse limit[17] of the ordinary Kondo problem. The mapping will be derived by using an Abelian boson representation of the conduction-electron fields. It may also be obtained, but less obviously, from an expansion of the partition function and correlation functions in powers of the spin-flip part of the exchange interaction. Similar methods have been used for the single-channel case[18] and in the context of quantum dissipative systems.[19] We feel that the method is useful because it allows a very explicit evaluation of correlation functions, and gives a perspective on properties such as the ground-state entropy.[1–8] Moreover it may be generalized to more than one impurity, especially in a purely one-dimensional version of the model.

The layout of the paper is as follows. The mapping to the resonant-level model is described in Sec. II where it is shown that an essential difference between the single-

channel and two-channel versions is that in the latter case, only "half" of the impurity degrees of freedom are coupled to the conduction electrons. The impurity Green's function, susceptibility, and thermodynamic properties are evaluated in Sec. III and conduction-electron correlation functions in Sec. IV. In particular, it is shown that superconductive pairing of the conduction electrons at the impurity site is enhanced provided it is correlated with the impurity pseudospin. This behavior is a reflection of the pairing resonance that characterizes the low-energy state of the two-channel Kondo problem.

In applications to problems such as inelastic impurity scattering, the physical meaning of the variables may not correspond to the usual Kondo nomenclature. "Flavor" in the Kondo language may represent the spin of the conduction electrons, and the Kondo "spin" may correspond to some other property such as parity. In an attempt to minimize the potential confusion we shall use flavor in the usual Kondo sense of a channel degeneracy, but the internal degrees of freedom of the impurity will be called "pseudospin."

II. REDUCTION TO FREE-FERMION FORM

It is well known that the critical behavior of a Kondo problem is dominated by a single angular momentum state of the conduction electrons, and therefore it is sufficient to consider only the radial motion. This is equivalent to a one-dimensional problem in half-space $x \geq 0$ (since the radial coordinate is positive). The critical behavior may be obtained by taking the continuum limit, in which the kinetic energy of the conduction electrons is given by a Dirac equation involving right- and left-going particles. An equivalent representation, to be adopted here, is to retain only the left-going fermions and to allow x to range over all space $-\infty < x < \infty$. A derivation of this representation was given by Affleck and Ludwig.[7]

The Hamiltonian is given by

$$\mathcal{H}=\mathcal{H}_0+\mathcal{H}_1 , \tag{2.1}$$

where

$$\mathcal{H}_0=iv_F \sum_{i,\alpha=1}^{2} \int_{-\infty}^{\infty} dx\, \psi_{i\alpha}^{+}(x)\frac{\partial \psi_{i\alpha}(x)}{\partial x} \tag{2.2}$$

is the kinetic energy of the conduction electrons, v_F is the Fermi velocity, and $\psi_{i\alpha}(x)$ annihilates a left-going fermion with pseudospin α and flavor i at position x. The impurity part of $\mathcal{H}$ is given by

$$\mathcal{H}_1=\tfrac{1}{2}\sum_{\alpha,\beta i=1}^{2} \sum_{\lambda=x,y,z} J_\lambda \tau^\lambda \sigma_{\alpha\beta}^{\lambda}\psi_{i\alpha}^{\dagger}(0)\psi_{i\beta}(0)+H\tau^z , \tag{2.3}$$

where τ^λ are the three components of the impurity pseudospin operator, σ^λ are Pauli matrices, and H is the magnetic field acting on the impurity. It is assumed that the impurity is at the origin, $x=0$.

The boson representation of the fermion fields is given by[20]

$$\psi_{i\alpha}(x)=\frac{e^{-i\Phi_{i\alpha}(x)}}{\sqrt{2\pi a}} , \tag{2.4}$$

where

$$\Phi_{i\alpha}(x)\equiv\sqrt{\pi}\left\{\int_{-\infty}^{x} dx'\hat{\Pi}_{i\alpha}(x')-\phi_{i\alpha}(x)\right\} . \tag{2.5}$$

Here $\phi_{i\alpha}(x)$ are Bose fields and $\hat{\Pi}_{i\alpha}(x)$ their conjugate momenta, satisfying commutation relations

$$\left[\phi_{i\alpha}(x),\hat{\Pi}_{j\beta}(x')\right]=i\delta_{ij}\delta_{\alpha\beta}\delta(x-x') . \tag{2.6}$$

In Eq. (2.4), a is a length, which $\to 0$ in the continuum limit. Physically it corresponds to a lattice spacing. In terms of the Bose fields, the two parts of H become[20]

$$\mathcal{H}_0=\tfrac{1}{2}v_F\sum_{i,\alpha=1}^{2}\int_{-\infty}^{\infty}dx\left[\hat{\Pi}_{i\alpha}^2+\left[\frac{\partial\phi_{i\alpha}}{\partial x}\right]^2\right] , \tag{2.7}$$

$$\begin{aligned}\mathcal{H}_1=&\frac{J_z}{2\pi}\tau^z\sum_{i,\alpha=1}^{2}\sigma_{\alpha\alpha}^{z}\left[\frac{\partial\Phi_{i\alpha}}{\partial x}\right]_{x=0}\\&+\frac{1}{4\pi a}\sum_{i,\alpha,\beta=1}^{2}\sum_{\lambda=y,z}J_\lambda\tau^\lambda\sigma_{\alpha\beta}^{\lambda}\\&\times\exp\left\{i\Phi_{i\alpha}(0)-i\Phi_{i\beta}(0)\right\} .\end{aligned} \tag{2.8}$$

The form of $\mathcal{H}_1$ may be simplified by introducing Bose fields corresponding to collective modes for charge $\phi_c(x)$, pseudospin $\phi_s(x)$, flavor $\phi_f(x)$, and pseudospin flavor $\phi_{sf}(x)$:

$$\begin{aligned}\phi_c&=\tfrac{1}{2}\sum_{i,\alpha=1}^{2}\phi_{i\alpha} ,\\ \phi_s&=\tfrac{1}{2}\sum_{i,\alpha=1}^{2}\sigma_{\alpha\alpha}^{z}\phi_{i\alpha} ,\\ \phi_f&=\tfrac{1}{2}\sum_{i,\alpha=1}^{2}\sigma_{ii}^{z}\phi_{i\alpha} ,\\ \phi_{sf}&=\tfrac{1}{2}\sum_{i,\alpha=1}^{2}\sigma_{ii}^{z}\sigma_{\alpha\alpha}^{z}\phi_{i\alpha} .\end{aligned} \tag{2.9}$$

This is a canonical transformation, and the $\hat{\Pi}_c$, $\hat{\Pi}_s$, etc., and Φ_c, Φ_s, etc., are defined by the same set of linear equations. In terms of the new variables, $\mathcal{H}_0$ remains a diagonal quadratic form, corresponding to free bosons and

$$\begin{aligned}\mathcal{H}_1=&\frac{J_z}{\pi}\tau^z\left[\frac{\partial\Phi_s}{\partial x}\right]_{x=0}\\&+\frac{1}{\pi a}\left\{J_x\tau^x\cos\Phi_s(0)+J_y\tau^y\sin\Phi_s(0)\right\}\\&\times\cos\Phi_{sf}(0)+H\tau^z .\end{aligned} \tag{2.10}$$

Note that the charge- and pseudospin-flavor fields, $\phi_c(x)$ and $\phi_{sf}(x)$ do not enter into $\mathcal{H}_1$.

At this point, the discussion will be restricted to the antiferromagnetic xxz model ($J_x=J_y$, $J_z>0$). In the Appendix, it will be shown that the same critical behavior is

obtained for general anisotropic coupling, whatever the sign of J_z. Then $\Phi_s(0)$ may be eliminated from the J_x, J_y part of $\mathcal{H}_1$ in Eq. (2.10) by carrying out a rotation in τ-pseudospin space through an angle $-\Phi_s(0)$ about the z axis. This may be accomplished via a unitary transformation $U\mathcal{H}U^{-1}$ with $U=\exp[\frac{1}{2}i\tau^z\Phi_s(0)]$, which also transforms the kinetic energy $\mathcal{H}_0$ to

$$U\mathcal{H}_0U^{-1}=\mathcal{H}_0-v_F\tau^z\left[\frac{\partial\Phi_s}{\partial x}\right]_{(x=0)} . \tag{2.11}$$

We shall mainly be concerned with a special value of the z component of the exchange, $J_z=\pi v_F$, for which the Hamiltonian turns out to be particularly simple: Later we shall consider the general case. With this assumption, the τ^z terms in $U\mathcal{H}_0U^{-1}$ and $U\mathcal{H}_1U^{-1}$ cancel, and the fully transformed Hamiltonian becomes

$$U\mathcal{H}U^{-1}=\mathcal{H}_0+\frac{J_x}{\pi a}\tau^x\cos\Phi_{\rm sf}(0)+H\tau^z . \tag{2.12}$$

We shall now show that Eq. (2.12) is equivalent to a sum of free-fermion and free-boson Hamiltonians, which is exactly solvable.

The kinetic energy part of (2.12), $\mathcal{H}_0$, has contributions from ϕ_c, ϕ_s, and ϕ_f, which separate because they do not appear in $\mathcal{H}_1$. Therefore, we shall focus on the part involving $\phi_{\rm sf}$, which may be written in terms of fermion fields by again using a representation similar to (2.4), but in reverse

$$\psi_{\rm sf}(x)=\frac{e^{-i\Phi_{\rm sf}(x)}}{\sqrt{2\pi a}} . \tag{2.13}$$

In order to have the correct signature for the J_x term of (2.12), the τ-pseudospin operators must be written in terms of a fermion variable d to give the Hamiltonian for the pseudospin-flavor degrees of freedom:

$$\mathcal{H}_{\rm sf}=iv_F\int_{-\infty}^{\infty}dx\,\psi_{\rm sf}^\dagger(x)\frac{\partial\psi_{\rm sf}(x)}{\partial x}+\frac{J_x}{\sqrt{2\pi a}}[\psi_{\rm sf}^\dagger(0)+\psi_{\rm sf}(0)][d^\dagger-d]+H(d^\dagger d-1/2) . \tag{2.14}$$

Now $\mathcal{H}_{sf}$ is a quadratic form and therefore is exactly solvable. It is analogous to the Toulouse limit of the single-channel Kondo problem, where the Hamiltonian may also be mapped onto free fermions but in that case the coupling to the impurity has the form $\psi_{\rm sf}^\dagger(0)d+d^\dagger\psi_{\rm sf}(0)$. This difference has profound consequences for the correlation functions and thermodynamic properties of the model, which is evident if we transform to Majorana (real) fermions

$$\hat{a}=\frac{d^\dagger+d}{\sqrt{2}} , \qquad \hat{b}=\frac{d^\dagger-d}{i\sqrt{2}} . \tag{2.15}$$

Then the two-channel Hamiltonian $\mathcal{H}_{\rm sf}$ does not depend on $\hat{a}$ or $[\psi_{\rm sf}^\dagger(0)-\psi_{\rm sf}(0)]$. In a sense only half of the impurity and conduction-electron pseudospin-flavor variables at the impurity site are coupled.

Eq. (2.14) is the central result of this section. The free-fermion form of $\mathcal{H}_{\rm sf}$ enables us to work out all impurity Green's functions and correlation functions. Also it is possible to evaluate many of the correlation functions of the conduction electrons, provided they may be expressed as functions of the fermion field $\psi_{\rm sf}$ and the boson fields ϕ_c, ϕ_s, and ϕ_f.

If $J_z/\pi v_F\neq 1$, the term proportional to $(\partial\Phi_s/\partial x)_{x=0}$ in $\mathcal{H}$ will not be cancelled. Then the pseudospin-wave part of the Hamiltonian may be rewritten in terms of a fermion field $\psi_s(x)$ [defined by Eq. (2.14) with $\Phi_{\rm sf}$ replaced by Φ_s] as

$$\mathcal{H}_s=iv_F\int_{-\infty}^{\infty}dx\,\psi_s^\dagger(x)\frac{\partial\psi_s(x)}{\partial x}+2(J_z-\pi v_F)(d^\dagger d-\tfrac{1}{2})\psi_s^\dagger(0)\psi_s(0) . \tag{2.16}$$

The coupling between the conduction electrons and the impurity is now fully described by $\mathcal{H}_s+\mathcal{H}_{\rm sf}$, which is the two-channel version of the resonant-level model. Departures from exactly solvable limit may be considered using perturbation theory in the interaction term in Eq. (2.16).

III. IMPURITY PROPERTIES

It is straightforward to obtain the correlation functions and thermodynamic properties of the impurity from the free-fermion Hamiltonian (2.14). Since the total number of fermions is not conserved, there are anomalous Green's functions, as in the theory of superconductivity, and it is convenient to use the Nambu notation[21]

$$\Delta=\begin{bmatrix} d \\ d^\dagger \end{bmatrix} \tag{3.1}$$

and obtain the impurity Green's function as a 2×2 matrix

$$\hat{G}_d(t)=-i\langle T\Delta(t)\Delta^\dagger\rangle , \tag{3.2}$$

where T is the time ordering operator and $\langle\hat{O}\rangle$ denotes thermal average of an operator $\hat{O}$. Then it is straightforward to solve the equations of motion and evaluate the Fourier transform $G_d(\omega)$ of $\hat{G}_d(t)$ to find

$$G_d(\omega)=\left[\omega+2H\tau^z\pm\frac{i\Gamma}{2}(1-2\tau^x)\right]^{-1} , \tag{3.3}$$

where $\Gamma=J_x^2/(\pi v_F a)$ and the upper or lower sign corresponds to ω in the upper or lower half plane, respectively. For $H=0$, the corresponding impurity spectral function is

$$A(\omega)=\pi\delta(\omega)(1+2\tau^x)+\frac{\Gamma}{\omega^2+\Gamma^2}(1-2\tau^x) . \tag{3.4}$$

This expression brings out the essential feature of the two-channel version of the Kondo problem. Half of the spectral weight of the impurity is decoupled from the electron gas [the $\delta(\omega)$ term in Eq. (3.4)] and half has the

usual Lorentzian form, with the width Γ, which is another way of showing that only half of the impurity degrees of freedom are coupled to the conduction electrons. It will be seen that the interplay between the two contributions to $A(\omega)$ is responsible for the characteristic behavior of the correlation functions at low temperature.

The impurity contribution to the free energy F may be calculated in the usual way by evaluating the thermodynamic average of

$$[\psi_{\rm sf}^{\dagger}(0)+\psi_{\rm sf}(0)](d^{\dagger}-d) ,$$

using Eq. (3.3), and integrating over the coupling constant. The result is

$$F=F_0+\frac{1}{2\pi}\int_{-v_F/a}^{v_F/a} d\omega\, f(\omega)\tan^{-1}\left[\frac{\Gamma\omega}{\omega^2-H^2}\right] , \tag{3.5}$$

where F_0 is the free energy of the impurity with $J_x=0$, and $f(\omega)=(e^{\beta\omega}+1)^{-1}$. It is necessary to retain the short-distance cutoff a, here, in order to obtain a finite expression for F, but a may be set equal to zero in evaluating universal quantities obtained by differentiating F. From Eq. (3.5) the impurity contribution to the entropy $(-\partial F/\partial T)$ may easily be evaluated, to find

$$\lim_{T\to 0}\lim_{H\to 0} S=\tfrac{1}{2}\ln 2 , \tag{3.6}$$

a result obtained previously.[3] There are two contributions to Eq. (3.6). For $J_x=0$, $S_0\equiv-\partial F_0/\partial T=\ln 2$, because an isolated impurity has a two-fold degenerate ground state. But the integral in Eq. (3.5) contributes $-\frac{1}{2}\ln 2$ to S; once again only half of the impurity is coupled to the conduction electrons. If the order of limits in Eq. (3.6) is reversed, the value of the entropy is zero, which agrees with the conclusion of Tsvelick[4] but is different from the result obtained by Sacramento and Schlottmann[3] from the Bethe Ansatz solution for isotropic coupling.

It is known[3,4,7] that the specific heat at low temperatures has an impurity contribution C_I, which varies as $T\ln T$. However, using Eq. (3.5), we find that the leading term in C_I is $\pi T/6\Gamma$. This is a feature of the exactly solvable limit: if $J_z\neq\pi v_F$, the last term in Eq. (2.16) gives a $T\ln T$ contribution to C_I in second-order perturbation theory. On the other hand, we do find that the magnetic susceptibility of the impurity has logarithmic divergences

$$\chi=\frac{1}{\pi\Gamma}\ln(u) , \tag{3.7}$$

where $\mu=\Gamma/T$ for $H=0$ and $u=(\Gamma/H)^2$ for $T=0$. The coefficient of the logarithm in Eq. (3.7) agrees with that obtained from the Bethe Ansatz solution[3] if we identify $\Gamma=2\pi k_B T_K$, where T_K is the Kondo temperature. Then the coefficient of $\ln H$ agrees exactly with the result of Tsvelick and Wiegmann,[3] and the coefficient of $(\ln T)/T_k$ is $(2\pi^2)^{-1}$, whereas Sacramento and Schlottmann[3] obtain 0.05 numerically. Note that H and T have different scale dimensions $T\sim H^2$.

The most interesting impurity correlation function for our purposes is the longitudinal pseudospin susceptibility, which may be expressed in terms of the fermion variables as

$$\chi(\omega,T)=-i\int_{-\infty}^{\infty} dt\, e^{-i\omega t}\langle[\hat{n},\hat{n}(t)]\rangle , \tag{3.8}$$

where $\hat{n}\equiv d^{\dagger}d$. Since the fermions are noninteracting, the thermal average in Eq. (3.8) factors into averages of pairs of $d^{\dagger},d$, which may then be obtained from the matrix elements of $A(\omega)$. The result of these manipulations is

$$\chi(\omega,T)=\frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega_1\frac{\tanh(\frac{1}{2}\beta\omega_1)}{\omega_1-\omega}\frac{\Gamma}{\omega_1^2+\Gamma^2} . \tag{3.9}$$

This expression involves one frequency integral instead of the usual two because of the $\delta(\omega)$ contribution to $A(\omega)$ in Eq. (3.4). For $\omega=0$, Eq. (3.9) agrees with Eq. (3.7) at low temperatures. For $T=0$, the real part of χ is given by Eq. (3.7) with T replaced by ω. The imaginary part of $\chi(\omega,T)$, for any ω, T is given by

$$\mathrm{Im}\chi(\omega,T)=\tfrac{1}{2}\tanh(\tfrac{1}{2}\beta\omega)\frac{\Gamma}{\omega^2+\Gamma^2} . \tag{3.10}$$

For small ω, this agrees with the result obtained by Tsvelick,[4] apart from a factor of 4.

IV. CONDUCTION ELECTRON PROPERTIES

Some of the correlation functions of the conduction electrons in the two-channel Kondo problem have been derived by means of conformal invariance techniques.[4–8] In this section, we shall evaluate quantities that may be of interest for high-temperature and heavy-fermion superconductors, in particular pseudospin density and pairing correlations. The procedure for evaluating the correlation functions is the same as in Sec. II: (a) introduce the boson representation (2.4), (b) change to collective variables via Eq. (2.9), (c) carry out the rotation in τ-pseudospin space generated by U, and (d) change back to the fermion field $\psi_{\rm sf}(x)$. Then the correlation functions are expressed in terms of free-boson or free-fermion variables and may be evaluated exactly. This procedure is straightforward for two-particle properties. However, we shall not consider single-particle correlation functions because they involve, formally $\sqrt{\psi_{\rm sf}(x)}$, which is difficult to work with.

The z component of the conduction-electron pseudospin density is given by

$$S^z(x)=\tfrac{1}{2}\sum_{i,\alpha,\beta}\sigma^z_{\alpha\beta}\psi^{\dagger}_{i\alpha}(x)\psi_{i\beta}(x) . \tag{4.1}$$

Then following the steps outlined above we arrive at

$$US^z(x)U^{-1}=\frac{1}{\pi}\frac{\partial\Phi_s}{\partial x}+\tau^z\delta(x) . \tag{4.2}$$

Here, only the second term on the right involves the impurity because the pseudospin density is decoupled from the resonant-level Hamiltonian $\mathcal{H}_{\rm sf}$. Since $\tau^z=\hat{n}-1/2$, the impurity contribution to the correlation function for the total pseudospin density is given by $\chi(\omega,T)$, defined by Eq. (3.8) and evaluated in Eqs. (3.9) and (3.10). This result shows that the time evolution of the impurity pseu-

dospin feeds back into conduction-electron properties. It is of interest for calculating the optical conductivity in a particular realization of the two-channel Kondo problem, proposed in the context of high-temperature superconductivity.[14] In that realization, the flavor variables correspond to the spin of the conduction electrons and the pseudospin is an internal degree of freedom of a local collective excitation represented by the "impurity." Since there are four collective degrees of freedom, it is possible to construct four current-density operators for the one-dimensional version of the model. They are the charge-, pseudospin-, flavor-, and pseudospin-flavor densities of the left-going fermions. In the boson representation,[20] they are proportional to $\partial\Phi_c/\partial x$, $\partial\Phi_f/\partial x$, $\partial\Phi_s/\partial x$, and $\partial\Phi_{sf}/\partial x$. Of these, only the last two couple to the impurity: $\partial\Phi_{sf}/\partial x$ [because $\Phi_{sf}(0)$ occurs in Eq. (2.12)] and $\partial\Phi_s/\partial x$ [because the canonical transformation generated by U gives it a contribution $\tau^z\delta(x)$, as in Eq. (4.2)]. Then, evaluating the corresponding current-current correlation functions, we find a Drude contribution to the pseudospin-flavor conductivity. But the impurity contribution to the pseudospin conductivity is proportional to $\chi(\omega,T)$ and we find for its real part

$$\mathrm{Re}\sigma_s(\omega,T)\approx\frac{\tanh(\frac{1}{2}\beta\omega)}{\omega}\frac{\Gamma}{\omega^2+\Gamma^2}\ . \tag{4.3}$$

This quantity is proportional to ω^{-1} for $T\ll\omega\ll\Gamma$ and to T^{-1} for $\omega\ll T\ll\Gamma$, which is the behavior characteristic of the normal state of high-temperature superconductors.[22] Of course, it remains to be shown that, in a particular model, the optical conductivity maps into the pseudospin conductivity of the equivalent two-channel Kondo problem. Moreover it is clear that the z component of the impurity pseudospin operator must have the same transformation properties as a current operator, i.e., it must be odd under parity and time reversal. These issues will be discussed more fully elsewhere.[14]

Another issue of importance for applications is the behavior of the conduction-electron pairing susceptibility at the impurity site. In particular, since the low-temperature state is a pairing resonance, rather than a single-particle resonance as in the single-channel Kondo problem, it is conceivable that pairing correlations might be enhanced.[8] The correlation functions of the relevant pairing operators $\psi_{i\alpha}(0)\psi_{j\beta}(0)$ may be evaluated by the method outlined above and it is found that they vary as t^{-2} for long times.[23] This behavior is the same as for free fermions[23] and we find, in agreement with Affleck and Ludwig,[8] that there is no enhancement of ordinary pairing at the impurity site. But this is not the whole story, for the pairing resonance also involves the impurity pseudospin. Therefore, it is perhaps not unreasonable that there is no enhancement of simple pairing of the conduction electrons.

We may however construct an operator to probe the nature of the pairing resonance, by considering the solution for zero hopping $\mathcal{H}_0=0$. Then two electrons of opposite flavor are exchange coupled to the impurity. It is easy to show that resultant state has a z component of pseudospin equal to $+1/2$ or $-1/2$, and that it is a superposition of conduction-electron triplets. It may be shown that there is no enhancement for the triplet $S=\pm1$ components, so we consider the operator

$$P_\pm=(\tfrac{1}{2}\pm\tau^z)(\psi_{11}\psi_{22}-\psi_{21}\psi_{12})\ , \tag{4.4}$$

which is a flavor singlet, pseudospin triplet $S=0$ and projects onto impurity pseudospin $\pm1/2$. Expressed in terms of the collective modes and the d fermions

$$P_+=\frac{1}{2\pi a}d^\dagger d(\psi_{sf}^\dagger+\psi_{sf})\psi_c\ , \tag{4.5}$$

where ψ_c is defined by Eq. (2.14) with Φ_{sf} replaced by Φ_c. Using Eq. (2.15), $d^\dagger,d$ may be replaced by the Majorana fermions $\hat{a},\hat{b}$, to give

$$P_+=(1-i\hat{a}\hat{b})(\psi_{sf}^\dagger+\psi_{sf})\psi_c\ . \tag{4.6}$$

Now, when $H=0$, $\hat{a}$ is a constant of the motion with $\hat{a}^2=1/2$. Also, since $\hat{b}$ appears in $\mathcal{H}_{sf}$, $\langle b(\psi_{sf}^\dagger+\psi_{sf})\rangle$ is finite. Then there is a contribution to the P_+ susceptibility equal to a constant multiplied by the ψ_c susceptibility, which is proportional to t^{-1}. This is a slower falloff than for free fermions and hence the frequency-dependent susceptibility is enhanced. The same behavior is obtained for P_-.

The pairing correlations that are enhanced at the impurity site may also be written as singlet, odd-time pairing of the type considered by A. Balatsky and E. Abrahams, Phys. Rev. B **45**, 13 125 (1992). This may be seen as follows: Introduce the Heisenberg operators $\psi_{\alpha,\beta}(t)$ and define the flavor-singlet operator:

$$O(t)=\psi_{11}(t)\psi_{22}(0)-\psi_{21}(t)\psi_{12}(0)\ .$$

Then, using Eq. (2.3), it is straightforward to show that all odd-time derivatives $[d^nO(t)/dt^n]$, evaluated at $t=0$, contain (among others) a contribution from the J_z term of the Hamiltonian, which is proportional to P_+-P_-. The operators P_+ and P_- were defined in Eq. (4.4), and immediately afterwards it was shown that their susceptibilities are enhanced at the impurity site. It follows that the susceptibility of any odd-time derivative of $O(t)$, evaluated at $t=0$, is similarly enhanced, but not the susceptibilities of even-time derivatives. Consequently, a superconducting state emerging from a collection of two-channel Kondo impurities should have pairing that is odd in time. Fermi statistics imply that the anomalous Green's function $\langle TO(t)\rangle$ is even under time reversal combined with pseudospin exchange: Thus the order parameter should be a pseudospin singlet as well as a flavor singlet.

V. CONCLUSIONS

It has been shown that the anisotropic two-channel Kondo problem may be solved exactly in the continuum limit for a particular value of J_z. Apart from the $T\ln T$ term in the impurity contribution to the specific heat, the behavior is the same as for the isotropic case. Moreover the $T\ln T$ term is restored, when J_z is away from the solvable point. This shows that exchange anisotropy is irrelevant at the low-temperature fixed point, in agreement

with the conclusion of Affleck.[2] *et al.* The Abelian bosonization method, used here, allows an explicit evaluation of the correlation functions for the two-channel Kondo problem. It has enabled us to define and calculate various conductivities in the model and to probe the nature of the pairing resonance. It may also be used to consider the behavior of more than one impurity, and to consider annealed as well as quenched averaging. These issues will be addressed in a future publication.

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APPENDIX

The discussion following Eq. (2.10) was presented for the special (but most important) case of an antiferromagnetic *xxz* exchange coupling between the conduction electrons and the impurity. Here, we consider the general anisotropic situation, $J_x \neq J_y \neq J_z$, following a similar procedure. Now, we start by carrying out a rotation in τ-pseudospin space through an angle $\pm\Phi_s(0)$ about the z axis, i.e., we allow rotations of either sense.

After the transformation, the coefficient of $\cos\Phi_{sf}(0)$ in Eq. (2.10) is proportional to

$$\frac{J_x \mp J_y}{2} + \left[\frac{J_x \pm J_y}{2}\right]\left[\tau^x \cos 2\Phi_s(0) + \tau^y \sin 2\Phi_s(0)\right] \tag{A1}$$

and the kinetic energy becomes

$$U\mathcal{H}_0 U^{-1} = \mathcal{H}_0 \mp v_F \tau^z \left[\frac{\partial \Phi_s}{\partial x}\right]_{(x=0)} . \tag{A2}$$

Now, provided $|J_z/\pi v_F| = 1$, we may arrange for the τ^z terms in $U\mathcal{H}_0 U^{-1}$ and $U\mathcal{H}_1 U^{-1}$ to cancel, whatever the sign of J_z, by choosing the sense of rotation appropriately. However, the part of the expression (A2) that depends on $\Phi_s(0)$ will not vanish. Nevertheless, we may argue that the first term in (A2), the constant, dominates the behavior in the continuum limit, unless it vanishes, and that Eqs. (2.12) or (2.16) characterize the singular low-energy behavior of the general two-channel Kondo problem. In order to make the point clear, consider ferromagnetic *xxz* exchange ($J_z > 0$, $J_x = -J_y$, or $J_z < 0$ $J_x = J_y$) for which the constant term in (A1) vanishes. Now it is known from renormalization-group arguments[1] that the coupling to the impurity is an irrelevant variable in this case. But this is just the $\Phi_s(0)$-dependent part of (A1). Therefore, we conclude that the constant part of (A1) dominates the critical behavior and that the second part may be neglected. The exceptional case for which the constant term in (A1) vanishes is precisely ferromagnetic *xxz* exchange. In the general, fully anisotropic, case the same singular low-energy behavior is obtained for both ferromagnetic coupling ($J_x J_y J_z < 0$) and antiferromagnetic coupling ($J_x J_y J_z > 0$). The same conclusion may be obtained from the renormalization-group equations of Murumatsu and Guinea.[11]

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