Spatial dispersion effects in polariton solitons

K. T. Stoychev and M. T. Primatarowa
Institute of Solid State Physics, Bulgarian Academy of Sciences, Sofia 1784, Bulgaria
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Effects of the exciton dispersion on the properties of polariton solitons in gyrotropic and nongyrotropic crystals are investigated. Bell- and kink-type solutions are obtained, and the conditions for their existence are determined. For excitons with a positive effective mass, a change of the type of the solution is shown to take place at a critical point on the lower polariton branch, accompanied by a critical behavior of the soliton's width and amplitude.

I. INTRODUCTION

Since the works of McCall and Hahn[1,2] on self-induced transparency (SIT), intensive investigations of optical solitons formed in the process of interaction of intensive light pulses with elementary excitations in condensed media have been carried out. The theory of SIT is based on the interaction of an electromagnetic pulse with a system of two-level atoms, where the electric field couples to the dipole moments of the transitions through the population inversion and the nonlinearity of this interaction forms the steady shape of the pulse. The influence of the medium on the dispersion of the carrier wave was neglected in the earlier works, and the dispersion relation of light was used instead. A generalization of the SIT theory for electronic excitons in solids has been carried out in Refs. 3–7.

When the linear exciton-photon coupling is taken into account, it leads to the formation of mixed states known as polaritons.[8–11] The role of the polariton effect on the properties of optical solitons in the exciton region has been investigated in Refs. 12–15 where the dispersion relation for the carrier wave has been obtained in a self-consistent way. It has been shown to depend on the nonlinearity and the soliton parameters such as the pulse width or amplitude. For short pulses it approaches the dispersion relation of light while for long pulses it is similar to that of the linear polaritons.

The formation of polariton solitons in nonlinear optical processes has been studied in Refs. 16–20, transmission and reflection of polariton solitons at a boundary has been considered in Refs. 20 and 21, and surface and guided-wave polariton solitons have been investigated in Refs. 22 and 23.

Due to their mixed character, polariton spectra exhibit a strong intrinsic dispersion in the resonance region, where the frequency and the wave vector of light are close to those of the excitons. Polariton spectra in this region change in a continuous manner from a photon to an exciton type on the lower polariton branch and from an exciton to a photon type on the upper branch. For larger values of the wave vector ($k \gg \omega_{ex}/c$), the lower polariton branch may show a dispersion associated with the wave vector dependence of the exciton energy. This kind of spatial dispersion has been studied extensively for linear excitons and polaritons (see, for example, the review articles[24–26] and the references therein). The effects of the exciton dispersion on polariton solitons however have not been investigated in such detail. In Refs. 13–15 a quadratic dependence on the wave vector has been considered within the effective-mass approximation and is shown to play a minor role for the soliton properties in the polariton region ($k \sim \omega_{ex}/c$).

The investigation of polariton solitons in this paper is extended to larger values of the wave vector, where effects associated with the exciton dispersion may become important. Besides nondegenerate excitations, the treatment includes degenerate excitations in gyrotropic crystals, which are characterized by a linear in k dependence of the exciton energy.[24–26] Moderate intensities are considered, when the induced polarization is proportional to the electric field and the only nonlinearity in the system is associated with the exciton-exciton interaction. In addition to Refs. 13–15 where fixed amplitude or fixed temporal-width solitons have been studied, solitons with a fixed spatial-width or a fixed exciton number are investigated in this paper. Bell- and kink-type solutions are obtained corresponding to bright and dark optical solitons, and the conditions for their existence are determined.

II. BASIC EQUATIONS

The formation of polariton solitons in gyrotropic media may be looked upon in the following way. Consider a circularly polarized light pulse propagating along the optical axis of a gyrotropic crystal. The electric field of the pulse couples to the dipole-active excitations of the media (vibrational or electronic excitons) having the same frequency, wave vector, and polarization to form polariton states. For nondegenerate excitations as well as for degenerate excitations in nongyrotropic crystals there is no linear in k splitting and the results for circular and linear polarizations coincide. The presence of a nonlinear exciton-exciton interaction leads to an interaction between the corresponding polaritons and causes the formation of polariton solitons. Obviously such a picture is valid for long enough pulses, whose temporal width is greater than the inverse polariton gap frequency.[12] As shown in Refs. 12 and 15, short light pulses are not very

sensitive to the polariton effect and are more of a photon rather than of a polariton type.

We shall consider in some detail the formation of polariton solitons associated with intramolecular vibrational excitons (optical phonons). A similar approach can be applied also to electronic (Frenkel) excitons. The intramolecular vibrations have narrow energy bands and the Hamiltonian for them can be written in the Heitler-London approximation as[24,27]

$$H=\hbar\omega_0\sum_n B_n^\dagger B_n-\sum_{n,m}(V_{nm}^s+V_{nm}^{as})B_n^\dagger B_m -\frac{A}{2}\sum_n B_n^\dagger B_n^\dagger B_n B_n \ , \tag{1}$$

where $\hbar\omega_0$ is the molecular excitation energy and $B_n^\dagger$ (B_n) are the corresponding creation (annihilation) Bose operators (only one molecule per unit cell and only one vibrational mode of the molecule are taken into account). $V_{nm}^s=V_{mn}^s$ and $V_{nm}^{as}=-V_{mn}^{as}$ are the symmetric and antisymmetric parts of the matrix elements of the intermolecular interaction operator. V_{nm}^s characterizes the resonant dipole-dipole interaction between the molecules n and m, while V_{nm}^{as} as associated with the dipole-quadrupole interaction which is different for intermolecular exchange of left and right circularly polarized excitations.[27,28] It is the nonvanishing V_{nm}^{as} which causes the optical activity of the system. A is the quartic anharmonicity constant. The terms associated with the cubic anharmonicity do not conserve the number of particles and their contribution to the energy of narrow-band excitations is negligible. The nonlinear interaction between left and right circularly polarized excitations has also been neglected.

For moderate intensities and low exciton concentrations the exciton-photon interaction can be considered to be linear in both the exciton operators and the electric field. The Hamiltonian of this interaction can be described in a semiclassical way by

$$H_{\rm int}=-d\sum_n (B_n^\dagger E_n^+ + B_n E_n^-) \ , \tag{2}$$

where d is the dipole moment matrix element for the transition from the ground state to the excited state of the molecule and E_n^+ (E_n^-) is the positive (negative) -frequency part of the macroscopic electric field. The latter is related to the macroscopic polarization of the medium through Maxwell's wave equation which can be represented as

$$\left[\Delta_n-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right]E_n^+=\frac{4\pi d}{c^2a^3}\frac{\partial^2 B_n}{\partial t^2} \ , \tag{3}$$

where c is the velocity of light and a is the lattice constant.

The equation of motion for the operator B_n yields

$$i\hbar\frac{\partial}{\partial t}B_n=\hbar\omega_0 B_n-\sum_m (V_{nm}^s+V_{nm}^{as})B_m-AB_n^\dagger B_n B_n -dE_n^+ \ . \tag{4}$$

The quantities V_{nm}^s and V_{nm}^{as} depend in general on the molecule displacements from the equilibrium positions but in what follows we shall consider fixed molecules only, thus neglecting the influence of lattice phonons on the properties of polariton solitons.

In writing the interaction term (2) we have assumed that the number of the excitons N_B is small compared with the total number of the molecules N in the crystal ($N_B \ll N$). Now we shall take into account that a macroscopic number of exciton states are occupied ($N_B \gg 1$). Apparently, as $N\sim 10^{22}$, the inequality $1\ll N_B \ll N$ holds for a wide range of exciton concentrations.

We can calculate the average exciton amplitudes using a wave function of the form

$$|\tilde{\beta}\rangle=\prod_n |\beta_n\rangle \ , \quad B_n|\beta_n\rangle=\beta_n|\beta_n\rangle \ , \tag{5}$$

where $|\beta_n\rangle$ are Glauber's coherent states.[29] Turning to a continuum approximation which is valid for excitations localized in a region L much larger than the lattice constant, the following equation for the complex wave amplitudes $\beta(x,t)$ is readily obtained:

$$i\hbar\frac{\partial\beta}{\partial t}=\hbar\omega_1\beta-Va^2\frac{\partial^2\beta}{\partial x^2}-i\gamma a\frac{\partial\beta}{\partial x}-A|\beta|^2\beta-dE^+ \ , \tag{6}$$

where

$$\hbar\omega_1=\hbar\omega_0-\frac{1}{a}\int V^s(x-x')dx' \ , \tag{7a}$$

$$V=\frac{1}{2a^3}\int V^s(x-x')(x-x')^2dx' \ , \tag{7b}$$

$$i\gamma=\frac{1}{a^2}\int V^{as}(x-x')(x-x')dx' \ . \tag{7c}$$

The coefficient V is related to the exciton effective mass

$$m_{\rm ex}=\hbar^2/(2Va^2) \tag{8}$$

and γ is the gyration constant.

An equation similar to (6) can be obtained for molecular electronic excitons, in which case A has to be identified with the sum of the dynamic and the kinematic exciton-exciton interaction, the latter emerging from the nonboson character of the electronic excitons.

We shall consider the case of a weak phase modulation and look for solutions in the form of amplitude-modulated waves:

$$\beta(x,t)=e^{i(kx-\omega t)}\varphi(\xi) \ , \qquad E^+(x,t)=e^{i(kx-\omega t)}\mathcal{E}(\xi) \ , \quad \xi\equiv x-vt \ , \tag{9}$$

where the amplitudes φ and $\mathcal{E}$ are real slowly varying functions of position and time and v is the velocity of the solitary wave.

With the help of (9) Eqs. (6) and (3) are transformed into

$$(\omega_k-\omega)\varphi-i(b_k-v)\frac{\partial\varphi}{\partial\xi}-\frac{\hbar}{2m_{\rm ex}}\frac{\partial^2\varphi}{\partial\xi^2}-\frac{A}{\hbar}\varphi^3-\frac{d}{\hbar}\mathcal{E}=0 \ , \tag{10}$$

$$\left[\frac{\omega^2}{c^2}-k^2\right]\mathcal{E}+2i\left[k-\frac{\omega v}{c^2}\right]\frac{\partial\mathcal{E}}{\partial\xi}+\left[1-\frac{v^2}{c^2}\right]\frac{\partial^2\mathcal{E}}{\partial\xi^2}$$
$$=\frac{4\pi d}{c^2a^3}\left[-\omega^2\varphi+2i\omega v\frac{\partial\varphi}{\partial\xi}-v^2\frac{\partial^2\varphi}{\partial\xi^2}\right], \quad (11)$$

where

$$\hbar\omega_k=\hbar\omega_1+V(ak)^2+\gamma ak\ ,$$
$$b_k=a(\gamma+2Vak)/\hbar\ . \quad (12)$$

From (10) and (11) the following equation for φ is obtained:

$$\chi_k\varphi-M_k\frac{\partial^2\varphi}{\partial\xi^2}-\frac{A}{\hbar}\varphi^3-i(P_k-Q_k\varphi^2)\frac{\partial\varphi}{\partial\xi}=0\ , \quad (13)$$

where

$$\chi_k=\omega_k-\omega-\frac{\Omega_0\omega^2}{c^2k^2-\omega^2}\ , \quad (14a)$$
$$M_k=a^2(V+W)/\hbar\ , \quad (14b)$$
$$W\equiv[2(c^2k-\omega v)(b_k-v)+(c^2-v^2)(\omega_k-\omega)$$
$$-\Omega_0v^2](c^2k^2-\omega^2)^{-1}\hbar/a^2\ , \quad (14c)$$
$$P_k=(b_k-v)+[2(c^2k-\omega v)(\omega_k-\omega)$$
$$-2\Omega_0\omega v](c^2k^2-\omega^2)^{-1}\ , \quad (14d)$$
$$Q_k=6A(c^2k-\omega v)(c^2k^2-\omega^2)^{-1}/\hbar\ , \quad (14e)$$
$$\Omega_0=4\pi d^2/\hbar a^3\ . \quad (14f)$$

The coefficients (14) are complicated functions of the frequency and the wave vector. The expression for M_k derived in Ref. 13 corresponds to the first term in the square brackets in (14c) for $\gamma=0$. Thus the results in Ref. 13 are valid in the resonance region only ($\omega\approx\omega_1$, $k\approx\omega_1/c$) while our expression holds in a much wider range of frequencies and wave vectors.

Equation (13) decomposes into two equations for the real and the imaginary parts. Neglecting the term $Q_k\varphi^2$ in comparison to P_k, we get

$$\chi_k\varphi-M_k\frac{\partial^2\varphi}{\partial\xi^2}-\frac{A}{\hbar}\varphi^3=0\ , \quad (15)$$
$$P_k=0\ . \quad (16)$$

These are the basic equations which describe the propagation of pulses of coupled nonlinear excitons and photons.

III. SOLITON SOLUTIONS

The form of the solution of the nonlinear equation (15) depends on the sign of the ratio M_k/A. When $M_k/A>0$ Eq. (15) has the well-known hyperbolic-secant solution

$$\varphi(x,t)=\varphi_0\mathrm{sech}[(x-vt)/L]\ , \quad (17)$$

which corresponds to a bell-type envelope function with an amplitude φ_0 and a spatial width $2L$. The substitution of (17) in (15) leads to two algebraic equations, which together with (16) give a set of three equations for the five quantities ω, k, v, L, and φ_0:

$$\omega_k-\frac{A\varphi_0^2}{2\hbar}-\omega-\frac{\Omega_0\omega^2}{c^2k^2-\omega^2}=0\ , \quad (18a)$$
$$L^2=2M_k\hbar/A\varphi_0^2\ , \quad (18b)$$
$$v=\frac{2c^2k(\omega_k-\omega)+(c^2k^2-\omega^2)b_k}{2\omega(\omega_k-\omega+\Omega_0)+(c^2k^2-\omega^2)}\ . \quad (18c)$$

It is convenient to choose the wave vector or the frequency as an independent variable and one of the other three unknown quantities (v, L, or φ_0) as an external parameter. For solitons having a fixed amplitude φ_0, Eq. (18a) gives the dispersion relation of the carrier wave. This is the well-known dispersion relation for polaritons, in which the exciton frequency ω_k contains a nonlinear shift $A\varphi_0^2/2\hbar$. The velocity v of the soliton can be obtained from (18c); and the width L, from (18b). In all other cases (solitons with a fixed spatial or temporal width or a fixed exciton number) Eqs. (18a)–(18c) have to be solved simultaneously.

When $M_k/A<0$ Eq. (15) has a solution with the shape of a kink

$$\varphi(x,t)=\varphi_0\tanh[(x-vt)/L]\ . \quad (19)$$

The amplitude $\varphi(x,t)$ of the kind has a finite value at infinity

$$|\varphi(x,t)|\longrightarrow\varphi_0>0\ ,\quad \text{as } x\to\pm\infty\ . \quad (20)$$

As the exciton density is proportional to φ^2, kink-type solutions correspond to a region with a decreased exciton density (dark soliton), while bell-type solutions correspond to a region with an increased exciton density (bright soliton).

In the case of kinks, Eqs. (15) and (16) lead to

$$\omega_k-\frac{A\varphi_0^2}{\hbar}-\omega-\frac{\Omega_0\omega^2}{c^2k^2-\omega^2}=0\ , \quad (21a)$$
$$L^2=-2M_k\hbar/A\varphi_0^2\ , \quad (21b)$$
$$v=\frac{2c^2k(\omega_k-\omega)+(c^2k^2-\omega^2)b_k}{2\omega(\omega_k-\omega+\Omega_0)+(c^2k^2-\omega^2)}\ . \quad (21c)$$

The nonlinear terms in (18a) and (21a) are usually small compared to all other terms and the dispersion curve $\omega(k)$ of the carrier wave for both bell- and kink-type solitons practically coincides with the dispersion curve of the linear polaritons (Fig. 1). It consists of a lower and an upper branch, separated in the resonance region by a gap of the order of Ω_0. In some cases however, as we shall see below, the nonlinear terms may become large which leads to considerable changes in the dispersion curves.

The conditions for a slowly varying envelope function are

$$|\partial\varphi/\partial x|\ll k\varphi,\quad |\partial\varphi/\partial t|\ll\omega\varphi\ . \quad (22a)$$

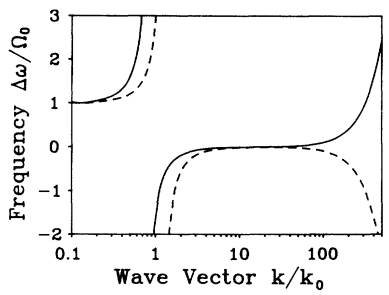

FIG. 1. Polariton dispersion curves for $\gamma=0$ and $V=0.1\hbar\omega_0$ (solid line) or $V=-0.1\hbar\omega_0$ (dashed line). $k_0=\omega_0/c$ and $\Delta\omega=\omega-\omega_1$ [note that ω_1 is different for $V>0$ and $V<0$, Eq. (7)].

Taking into account that $|\partial\varphi/\partial x|\sim 1/L$ and $|\partial\varphi/\partial t|\sim 1/T$ where $T=L/v$ is the temporal width of the soliton, the conditions (22a) turn into

$$L\gg\lambda \text{ and } T\gg\tau\ , \tag{22b}$$

where $\lambda=2\pi/k$ and $\tau=2\pi/\omega$ are the wavelength and the period of the carrier wave. For frequencies far from the exciton resonance, $\lambda(\omega)$ approaches the wavelength of light and the solitons are of an optical type. In the large wave-vector region of the lower branch $\lambda\ll c/\omega_1$ and the soliton's spatial width L may become of the order or even smaller than the corresponding wavelength of light but still (22) may hold. These solitons are of a mechanical type and their properties are similar to the properties of exciton solitary waves.[30,31] In the resonance region ($\omega\sim\omega_1$ and $k\sim\omega_1/c$) the solitons are of a mixed polariton type.

It is important to note, that there are two types of transport mechanisms in the system under consideration. One is associated with the propagation of the electromagnetic wave and the exciton-photon coupling and the other with the intermolecular interaction and the delocalization of the excitons. These two mechanisms lead to characteristic spatial dispersion effects in the polariton spectra and in the properties of the corresponding solitary waves.

As mentioned above, the type of the solution depends on the sign of the ratio M_k/A. The coefficient M_k [(13)–(15)] plays the role of an inverse effective mass of the polariton and contains contributions from both the polariton and the exciton dispersion. On the photonlike parts of the dispersion curves and in the resonance region the polariton-type dispersion dominates $|W|\gg|V|$ and M_k is negative on the lower branch ($k>\omega/c$) and positive on the upper one ($k<\omega/c$). Hence for $A>0$, kink-type solutions are possible on the lower branch and bell-type solutions on the upper one. For $A<0$ the solutions change places.

For large values of the wave vector on the lower branch the exciton-type dispersion dominates and the quantity M_k reduces to $Va^2/\hbar$. The type of the solution in this region is governed by the sign of V/A and bell-type solutions exist for $V/A>0$, while kink-type solutions exist for $V/A<0$. Usually A is positive and corresponds to exciton attraction, while V is positive for electronic excitons and negative for vibrational excitons (optical phonons).

The competition between the two dispersion mechanisms may lead to a change of the type of the solution along one and the same polariton branch when W and V have different signs. On the lower branch $|W|$ decreases with the increase of the wave vector and when $V>0$ the coefficient M_k changes sign at a critical value k_c of the wave vector. Thus for $A>0$, kink solutions exist in the region $k<k_c$ and bell solutions in the region $k>k_c$. For $A<0$ the solutions on the lower branch change places. A similar spatial dispersion effect (although with a different nature) has been obtained for surface polariton solitons in layered systems by Boardman *et al.*[23] At the critical point, M_k goes through zero, the polariton effective mass ($\sim M_k^{-1}$) becomes infinite, and soliton solutions do not exist. In the close vicinity of k_c, as we shall see in the next section, a critical behavior of the soliton parameters is observed.

The polariton-type dispersion usually dominates along the whole upper branch and a change of the solution may occur only for $V<0$ and very weak exciton-photon coupling ($\hbar\Omega_0/|V|<10^{-3}$). When the sign of V coincides with the sign of W, the type of the solution does not change along the corresponding polariton branch and no critical behavior of the soliton parameters is to be expected.

IV. NUMERICAL RESULTS

A. Fixed amplitude solutions

In this part we present the results of numerical calculations of the parameters of solitons with a fixed amplitude $\varphi_0=10^{-3}$. The other parameters have been given the values $A=\pm 0.01\hbar\omega_0$, $\Omega_0=0.01\omega_0$, and $V=\pm 0.1\hbar\omega_0$, where $\hbar\omega_0$ is the molecular excitation energy. These values are appropriate to both vibrational and electronic excitons.[24] The lattice constant is chosen $a=10^{-3}k_0^{-1}$, where $k_0=\omega_0/c$ is the wave vector of light at resonance with the molecular excitation.

The dispersion curves for the carrier wave are shown in Fig. 2. For the chosen value of the amplitude, the nonlinear term in the dispersion equation is very small and the dispersion curves for bells with $A<0$ and for kinks with $A>0$ practically coincide. The type of solution on the upper branch and on the small wave-vector part of the lower branch ($k<k_c$) depends exclusively on the sign of A. The type of solution on the large wave-vector part of the lower branch ($k>k_c$) depends also on the sign of V. Thus for $A<0$ and $V>0$ a transition from a bell- to a kink-type solution takes place at $k_c\approx 21k_0$, while for $A<0$ and $V<0$ the whole lower branch corresponds to a kink-type solution. For $A>0$ the bell- and kink-type solutions change places. As in Ref. 13, only the resonance region $k\approx\omega_1/c$ has been investigated; such a critical behavior has not been obtained.

The presence of gyrotropy (Fig. 2, dashed curves) leads to a characteristic linear in k dependence of the lower polariton branch frequency in the region $5\le k/k_0\le 100$, where γ is positive or negative for right or left circular

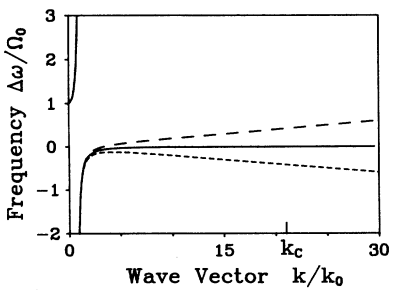

FIG. 2. Dispersion curves of the carrier wave of fixed amplitude solitons with $V=0.1\hbar\omega_0$ and $\gamma=0$ (solid line), $\gamma=0.2\hbar\omega_0$ (long-dashed line), or $\gamma=-0.2\hbar\omega_0$ (short-dashed line).

polarization, respectively. The type of the solution changes in the same manner along the lower branch and the gyrotropy changes the frequency of the transition but not the critical value k_c of the wave vector. It might seem curious that, although the effect of the finite exciton effective mass on the polariton spectrum in this region is negligible ($k_c^2a^2\sim10^{-4}$), it should play such a crucial role for the properties of polariton solitons. This behavior can be understood if we take into account that the coefficient M_k is essentially the second derivative of the dispersion relation ($M_k\sim d^2\omega/dk^2$) and the linear term γka in the exciton energy gives negligible contribution. The type of the solution changes at the inflex point of the dispersion curve where the curvature changes sign.

The velocity of the solitons (Fig. 3) is close to the group velocity of the corresponding linear polaritons ($v\sim d\omega/dk$). On the lower branch, it decreases rapidly with the increase of the wave vector and for $V>0$ has a shallow minimum at $k=k_c$. The presence of gyrotropy changes the large wave-vector values to $v\approx\gamma a/\hbar$. It is interesting to note, that for $\gamma<0$ as well as for $V<0$ the velocity may become negative and the soliton will move in a direction opposite to this of the carrier wave.

The relative width of the soliton L/λ is shown in Fig. 4. For $V>0$ (solid curves) when a change of the type of the solution along the lower branch takes place, L/λ exhibits a critical behavior with a narrow deep minimum at $k=k_c$. In the close vicinity of the minimum L becomes of the order of the wavelength λ of the carrier wave and

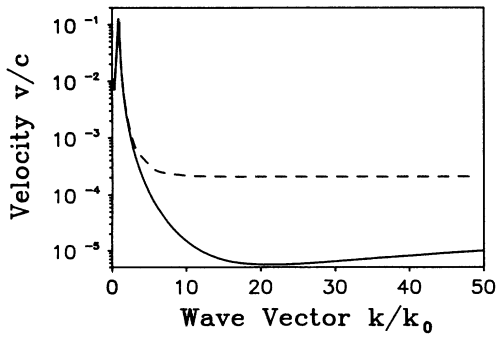

FIG. 3. Soliton velocity for $V=0.1\hbar\omega_0$ and $\gamma=0$ (solid line) or $\gamma=0.2\hbar\omega_0$ (dashed line). The curve for $k/k_0<1$ corresponds to the upper polariton branch.

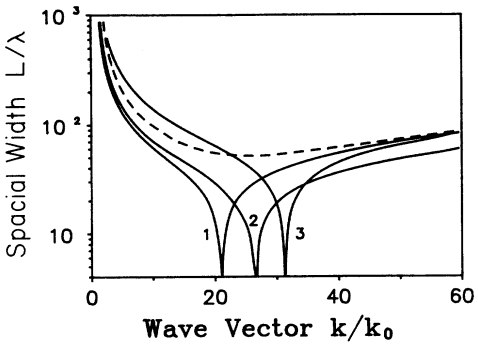

FIG. 4. Relative spatial widths of the solitons for $V<0$ (dashed curve) and for $V>0$ (solid curves); curve 1: $V=0.1\hbar\omega_0$ and $\Omega_0=0.01\hbar\omega_0$, curve 2: $V=0.05\hbar\omega_0$ and $\Omega_0=0.01\hbar\omega_0$, and curve 3: $V=0.1\hbar\omega_0$ and $\Omega_0=0.05\hbar\omega_0$.

the assumption of slowly varying amplitudes (22) fails. As could be expected, a decrease of V (curve 2) or an increase of Ω_0 (curve 3) leads to an increase of k_c. For $V<0$ no change of the type of the solution takes place and the soliton width is a smooth function (dashed curve). The soliton width corresponding to the upper branch has a maximum in the resonance region.

We would like to point out, that the change of the type of the soliton and the critical behavior of its parameters take place in a region which is accessible in optical experiments like stimulated Raman scattering and frequency mixing.

B. Fixed spatial-width solutions

Equations (18a) and (18b) describing bell-type solitons may be rearranged in the following way:

$$\omega_k-\omega-\frac{\Omega_0\omega^2}{c^2k^2-\omega^2}-\frac{M_k}{L^2}=0\ , \tag{23a}$$

$$\varphi_0^2=2M_k\hbar/AL^2\ , \tag{23b}$$

while these for kinks (21a) and (21b) change into

$$\omega_k-\omega-\frac{\Omega_0\omega^2}{c^2k^2-\omega^2}+2\frac{M_k}{L^2}=0\ , \tag{24a}$$

$$\varphi_0^2=-2M_k\hbar/AL^2\ . \tag{24b}$$

For sufficiently long solitons ($L\gg10^2k_0^{-1}$), the nonlinear terms in (23a) and (24a) give negligible contribution in the dispersion curves of the carrier waves. The conditions for the existence of bell- and kink-type solitons is the same as for fixed amplitude solitons and the change of the sign of M_k leads to a change of the type of the solution at a critical value k_c of the wave vector which is close to the corresponding value for the fixed-amplitude case. The dependence of the amplitude of the soliton on the wave vector (Fig. 5) has a critical behavior for $V>0$ (solid curve) and changes in a monotonous way for $V<0$ (dashed curve).

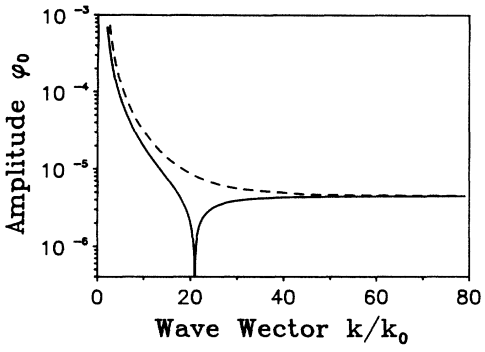

FIG. 5. Amplitude of fixed spatial-width solitons with $\gamma=0$ and $V>0$ (solid curve) or $V<0$ (dashed curve).

C. Fixed temporal-width solutions

Solutions with a given temporal-width T have been investigated by Goll and Haken.[15] For short pulses they obtain a dispersion curve of the carrier wave which consists of two polaritonlike branches, a backbent branch with anomalous dispersion and an excitonlike branch. For long enough pulses only the two polariton branches remain.

Our results for fixed temporal-width solitons are consistent with these in Ref. 15. We calculated the dispersion curve of the carrier wave, the velocity, and the amplitude of the solitons using (18c), (21c), (23a), (23b), (24a), and (24b) and taking into account that $L=Tv$. For short pulses ($T=10\,000\omega_0^{-1}$, Fig. 6) the dispersion curves for bell- and kink-type solutions differ considerably. In the case of bells (solid curve), the lower polariton branch exhibits a backbending at $k\approx 6k_0$ and the excitonlike branch has a weak dispersion. The solutions with $k<21k_0$ correspond to $A<0$ and those with $k>21k_0$ to $A>0$. For kink-type solutions (dashed curve) the backbending of the lower polariton branch takes place at $k\approx 24k_0$, and the excitonlike branch shows a stronger dispersion. The polaritonlike and the backbent branches correspond to $A>0$, while the excitonlike branch corresponds to $A<0$. For long enough pulses ($T>100\,000\omega_0^{-1}$) the backbent and the excitonlike branches disappear and the dispersion curve contains only the two polariton branches. It is interesting to note

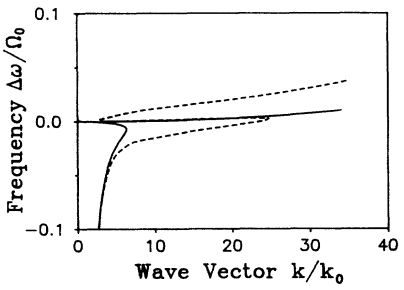

FIG. 6. Dispersion curves for the carrier waves of solitons with a temporal width $T=10\,000\omega_0^{-1}$, $V=0.1\hbar\omega_0$, and $\gamma=0$. Solid curves —bell solutions, dashed curves—kink solutions.

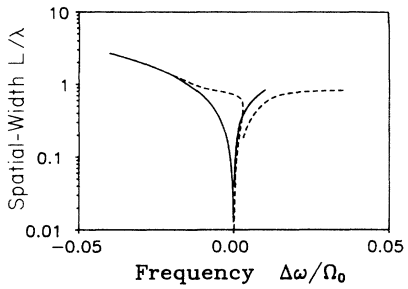

FIG. 7. Frequency dependence of the relative spatial width for the values used in Fig. 6. Solid curves—bell solutions, dashed curves—kink solutions.

that we did not obtain a backbending of the dispersion curve in the case of solitons with fixed short spatial widths ($L<\lambda$).

We would like to point out that, due to the dramatic decrease of the soliton velocity away from the lightline $\omega=kc$, the soliton spatial-width on the backbending part of the dispersion curve and on the lower-frequency part of the excitonlike branch becomes smaller than the wavelength of the carrier wave (Fig. 7). The assumption for slowly varying in space amplitude fails in these regions and the corresponding solutions may not be valid. Reliable short-width solutions can be obtained outside the slowly varying amplitudes approximation (22) if the higher derivatives in Eq. (13) are taken into account. Unfortunately, the corresponding nonlinear equation can be solved only numerically. The short pulses investigated in Ref. 15 have a temporal width of the order of $10^3\omega_0^{-1}$ and they correspond to solitons with very short relative spatial widths ($L/\lambda<1$) in the resonance region $\omega\approx\omega_k$. Our calculations show, that the presence of gyrotropy ($\gamma\neq 0$) increases the absolute value of the velocity and the spatial width of the solitons and in such a way improves the conditions for the existence of solutions with slowly varying amplitudes.

It might be interesting to note that a similar backbending of the dispersion curve $\omega(k)$ of spatially damped linear polaritons has been discussed in Ref. 32 and is shown to have a dubious physical meaning. The polariton response functions derived in Refs. 32–35 show no traces of such a backbending.

D. Fixed exciton-number solutions

We can consider (at least in principle) solutions corresponding to a fixed exciton number N_B. The exciton amplitude in this case has to be normalized as

$$k_0\int_{-\infty}^{\infty}\varphi^2(x,t)dx=2\varphi_0^2Lk_0=N_B/N\ , \tag{25a}$$

for bells and

$$k_0\int_{-\infty}^{\infty}[\varphi_0^2-\varphi^2(x,t)]dx=2\varphi_0^2Lk_0=N_B/N \tag{25b}$$

for kinks, where N is the total number of molecules in the crystal. Equations (25) are similar to the condition for a fixed area under the envelope function and provide an additional relation between the amplitude and the spatial

width of the solitons, which together with (18b) or (21b) allow for both φ_0 and L to be determined as functions of the wave vector for a given exciton number. For small enough number of the excitons ($N_B \leq 10^{-4}N$) the spatial width of the solitons is large compared with the wavelength of the carrier wave and the dispersion curves for bells and kinks practically coincide. For $V>0$ there is a change of the type of the solution at a critical point k_c on the lower polariton branch and a corresponding critical behavior of both the soliton width and amplitude (Fig. 8, solid curves). The spatial width of the soliton (curve 1) exhibits a minimum for $k=k_c$, while the amplitude φ_0 has a maximum (curve 2). For $V<0$ there is no change of the type of the solution and hence no critical behavior of the soliton parameters (dashed curves).

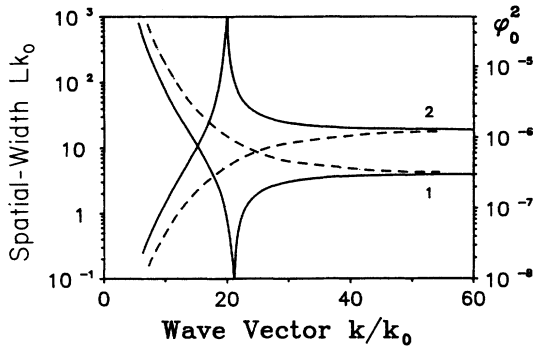

FIG. 8. Normalized spatial width (curves 1) and squared amplitude (curves 2) of solitons with a fixed exciton number. Solid lines—$V>0$, dashed lines—$V<0$.

V. CONCLUSION

We have investigated the effects of the polariton- and the exciton-type dispersion on the properties of polariton solitary waves. We have shown that the competition between the two dispersion mechanisms in the case of excitons with a positive effective mass leads to a change from a bell- to a kink-type solution or vice versa at a critical point on the lower polariton branch. The critical wave vector depends on the exciton effective mass and the exciton-photon coupling constant, but not on the gyration constant. The change of the type of the solution is accompanied by a critical behavior of some soliton parameters such as the width or the amplitude. An encouraging feature of this dispersion-induced critical behavior is that it takes place in a region which is accessible for optical experiments.

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