Quantum corrections to the conductivity in icosahedral Al-Cu-Fe alloys

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We report measurements of the temperature and magnetic-field dependence of the resistivity in single-phase icosahedral Al-Cu-Fe alloys between 0.15 and 41 K and fields up to 8.8 T. The data are consistent with the predictions of weak localization and electron-electron-interaction theories, even though the disorder parameter $(k_F l_e)^{-1}$ is larger than one. The spin-orbit scattering and the electron-wave-function dephasing rates are found from a fit of the magnetoresistance. The dephasing rate is found to vary as AT^p with $p \sim 1.5$, characteristic of electron-electron scattering in the strong disorder regime. Independent determination of the dephasing rate from the temperature dependence of the resistivity, however, gives significantly different values of A and p. This discrepancy is discussed in terms of electron-electron and electron-phonon scattering rates. We also observe directly the antilocalization effect due to spin-orbit scattering on the temperature dependence of the resistivity in one of the samples.

I. INTRODUCTION

The quasicrystalline alloy system Al-Cu-Fe is a stable peritectic between 700 °C and the melting point at 872 °C.[1–3] As a consequence it is one of the most attractive quasicrystals for study since a high level of structural and compositional order may be obtained by appropriate annealing, whereas samples of metastable quasicrystals, such as Al-Mn, tend to be contaminated with amorphous or proximant crystal phases. This characteristic is of particular relevance to transport studies since it has been found that the intrinsic resistivity of many quasicrystals is extremely high and contamination by a few percent even of amorphous phases can lead to the erroneous interpretation of results.

One very interesting feature of quasicrystals is that the already high resistivity tends to further increase with annealing[4,5]—i.e., with increasing atomic order—in contrast to conventional metals where removing defects always lowers the resistivity. This has led to the view that high resistivity is not caused by disorder but is intrinsic to the phase, and reflects a very low electron density[6,7] at the Fermi level due to a close matching of the first Brillouin zone with the Fermi sphere. This view is supported both by specific-heat measurements, which show a low density of states at the Fermi level,[6,7] and by recent optical conductivity measurements which show a linear frequency dependence of the conductivity, a characteristic of marginally metallic systems.[8] To set against this, however, are the recent results of Klein *et al.*[7,9] where the temperature dependence of the resistivity was found to be well described by the theories of quantum corrections to the conductivity (QCC), known as weak localization (WL),[10] and enhanced electron-electron interactions (EEI),[11] which are only valid in the limit of high disorder. Furthermore, the measured Hall coefficient is apparently always negative,[6] whereas in a nearly filled zone one could expect at least some samples with a positive value due to unfilled hole states.

In the present article we present a detailed quantitative study of the temperature and magnetic-field dependence of the resistivity of four single-phase quasicrystalline alloys over the range 0.15–41 K and 0–8.8 T. The alloys are all close to the composition $Al_{63}Cu_{25}Fe_{12}$ (see Table I) where the formation of thermodynamically stable single-phase icosahedral structure has been established. The data are *completely* analyzed within the framework of WL and EEI theories (in contrast to earlier studies where only the temperature dependence was so analyzed[7,9]) and estimates given of the spin-orbit and dephasing scattering rates. We show that it is important to consider the full fitting to obtain a clear picture of dephasing. When

TABLE I. Physical parameters of icosahedral Al-Cu-Fe alloys. Errors: ρ, ±10%; $1/\tau_{so}$, $1/\tau_\phi$, ±10%.

Alloy	$\rho_{4.2\,K}$ ($\mu\Omega$ cm)	$\rho_{4.2\,K}/\rho_{300\,K}$	D (cm^2 s^{-1})	$\tilde{F}_\sigma$ [a]	$\tilde{F}_\sigma$ [b]	$1/\tau_{so}$ (10^{12} s^{-1})	$1/\tau_\phi$ (s^{-1})[a]	$1/\tau_\phi$ (s^{-1})[b]
$Al_{63.5}Cu_{24.5}Fe_{12}$	4622	1.66	0.25	0.25	0.56	1.4	9.9×10^9 T$^{1.47}$	8.4×10^8 T^2
$Al_{63}Cu_{25}Fe_{12}$	5330	1.72	0.25	0.2	0.60	1.4	1.03×10^{10} T$^{1.45}$	8.6×10^8 T^2
$Al_{62.5}Cu_{25.5}Fe_{12}$	6700	1.73	0.25	0.1	0.64	1.4	1.7×10^{10} T$^{1.38}$	9.7×10^8 T^2
$Al_{62}Cu_{25.5}Fe_{12.5}$	9730	2.21	0.25	1.06	1.08	1.4	1.8×10^{10} T$^{1.43}$	1.97×10^{10} T$^{1.34}$

[a]From the magnetoresistance.
[b]From the temperature dependence of the resistivity.

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spin-spin scattering is absent, the destruction of the phase coherence of the electron's wave function is controlled by either inelastic electron-phonon scattering or electron-electron scattering; the only way to distinguish between these two processes is from their dependence on temperature. Both follow a power law (T^p) with $p=2$–4 for the former and $p=\frac{3}{2}$ or 2 for the latter. Hence the exponent p can in principle identify the dominant cause behind the dephasing.

II. EXPERIMENTAL TECHNIQUES

The Al-Cu-Fe alloys were prepared by arc-melting appropriate amounts of the constituents under an argon atmosphere. The purity of the starting elements was: Al 99.999%, Cu 99.999%, Fe 99.9%. In order to ensure homogeneity the alloys were melted several times. The samples were made by melt spinning buttons of about 1 g onto a copper wheel rotating at a tangential velocity of 30 m/s under 15 kPa of He. The resulting ribbons were typically 1–2 mm wide, 20 μm thick and up to 30 mm long. The as-made ribbons are found to contain both the icosahedral phase and a crystalline phase of Al-Fe as has been generally seen.[3] To obtain a pure icosahedral phase, the ribbons were annealed in vacuum at 750 °C for 3 h. The icosahedral structure was confirmed by x-ray diffraction and transmission electron microscopy (TEM) and no traces of contaminant phases were observed in the final samples by either technique. High-resolution TEM also revealed a high degree of order as shown in Fig. 1 for one of the alloys.

Resistance changes with temperature and magnetic field were measured by a four-terminal ac bridge (LR400 from Linear Research, California, USA). Above 1.4 K the temperature was varied either by pumped helium or by a servo-controlled heater. Below 1 K the measurements were performed on a dilution refrigerator. During field sweeps the temperature was kept stable to within 1% or better. A magnetic field of up to 8.8 T was provided by a superconducting solenoid.

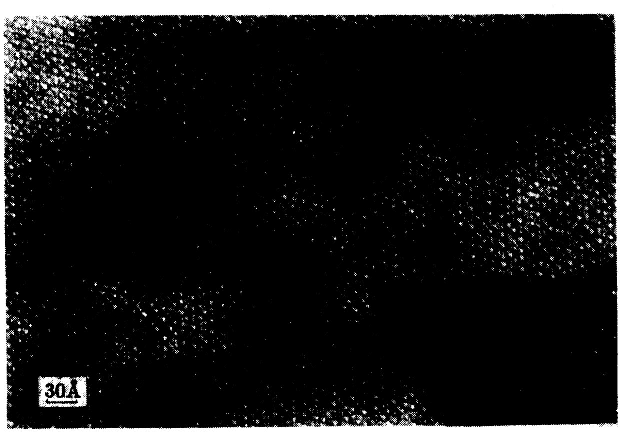

FIG. 1. High-resolution transmission electron micrograph of $Al_{63.5}Cu_{24.5}Fe_{12}$. The inset shows the corresponding electron-diffraction pattern along the fivefold axis.

III. RESULTS AND DISCUSSION

Figure 2 shows the magnetoresistance

$$\frac{\Delta\rho}{\rho}=\frac{\rho(B,T)-\rho(0,T)]}{\rho(0,T)}$$

of $Al_{63.5}Cu_{24.5}Fe_{12}$, $Al_{63}Cu_{25}Fe_{12}$, $Al_{62.5}Cu_{25}Fe_{12.5}$, and $Al_{62}Cu_{25.5}Fe_{12.5}$ as a function of temperature from 1.5 to 41 K. The dots represent the experimental data and the solid line the fit to the theoretical expressions, as explained below. It is at least an order of magnitude larger than the observed magnetoresistance in amorphous systems and rises to about 12% in $Al_{62}Cu_{25.5}Fe_{12.5}$ at 1.5 K and 8.8 T. This behavior is consistent with QCC theories which predict $\Delta\rho/\rho \propto \rho$. The magnetoresistance is positive over the whole range of temperature and field, reflecting the presence of spin-orbit scattering. As the temperature increases its magnitude decreases due to the destruction of phase coherence by inelastic scattering events. The data are fitted to the predictions from WL and EEI, which are given in the Appendix. Before we describe the fitting procedure we should mention that no negative magnetoresistance was observed in any of the samples at any temperature considered here, in contrast to Ref. 7 where a negative magnetoresistance is reported in $Al_{63}Cu_{25}Fe_{12}$ at 30 K. Details of the numerical methods used in evaluating the different terms are given by Baxter *et al.*[12] The diffusion constant in the expressions is calculated from specific-heat data,[6] using the Einstein relation for $Al_{63.5}Cu_{24.5}Fe_{12}$ and was assumed to be the same for the remaining alloys. In other words, we assume that the change in ρ is only due to the varying density of states at the Fermi level. As a first step, the fitting is restricted to low fields ($B/T \leq 1\ \mathrm{T\,K^{-1}}$) with the resistivity ρ, the dephasing field B_ϕ and the spin-orbit scattering field B_{so} as free parameters using WL expression only, the EEI contribution being important only at high fields. B_{so} is temperature independent and is therefore the same for all temperatures. A similar fitting procedure was successfully used for amorphous alloys.[13] Using ρ as a free parameter in the WL expression allows us to determine the resistivity in a way that is independent of the sample geometry and microcracks that might exist in these very brittle samples. As a second step, the fitting is extended to the entire field range, including the EEI term with the screened Coulomb interaction parameter $\tilde{F}_\sigma$, which is also temperature independent, as the only free variable. Thus each family of curves in Fig. 2 is fitted with common values of B_{so} and $\tilde{F}_\sigma$ and one value of B_ϕ at each temperature. Agreement is very good over the entire range of field and temperature. In all cases, the EEI contribution is important at high fields and even exceeds that of WL in $Al_{62}Cu_{25.5}Fe_{12.5}$ in contrast to amorphous alloys and thin films where the magnetoresistance is always dominated by WL. This is consistent with the increasing role of interaction effects when the resistivity becomes very large.[14] The contribution is positive and comes from the diffusion channel only; the contribution from the Cooper channel was found to be very small and therefore was neglected. The values of the resistivity obtained from the fits at low temperatures are consistent with measurements made at room temperature with conventional four-terminal measurements and are in excellent agreement with the values reported by Biggs, Li, and Poon[6] and Klein *et al.*[7] The spin-orbit scattering field is found to be about 11 T ($1/\tau_{so} \sim 1.4\times10^{12}$ s^{-1}) and is the same in all samples. This is expected since their chemical compositions are very close and so provides a consistency check on the fitting procedure. It is worth noting that because of the large value of B_{so} we are not able to see a maximum in the magnetoresistance which is normally seen when $B \geq B_{so}$. This maximum is further delayed by the large positive EEI contribution at high fields, as mentioned earlier and supported by the results of Klein *et al.*[7,9] where they observed only a slight decrease in the slope of the magnetoresistance up to 35 T. In $Al_{62}Cu_{25.5}Fe_{12.5}$ the best fit is obtained for $\tilde{F}_\sigma = 1.06\pm0.03$ corresponding to a value of $F = 1.15$, where F is the average of the screened Coulomb potential over the Fermi surface. The large value of F may be an indication of the breakdown of the Thomas-Fermi theory when the density of electrons is very low, as is the case in these alloys. In fact measurements of the Hall constant revealed that the density of electrons is only 6.3×10^{20} cm^{-3} in $Al_{63.5}Cu_{24.5}Fe_{12}$,[6] and may be even less in $Al_{62}Cu_{25.5}Fe_{12.5}$. In such a situation the screening of the Coulomb potential is decreased and the screening length itself tends to diverge.[15] Another possibility for the large value of F may be band-structure effects, as suggested by Thomas *et al.*,[16] where F is replaced by λF; λ being a constant that takes into account intervalley scattering and/or mass anisotropy ($\lambda > 1$).

At this stage a remark about the use of WL and EEI theories for this system is in order. WL and EEI expressions are first-order terms in the disorder parameter

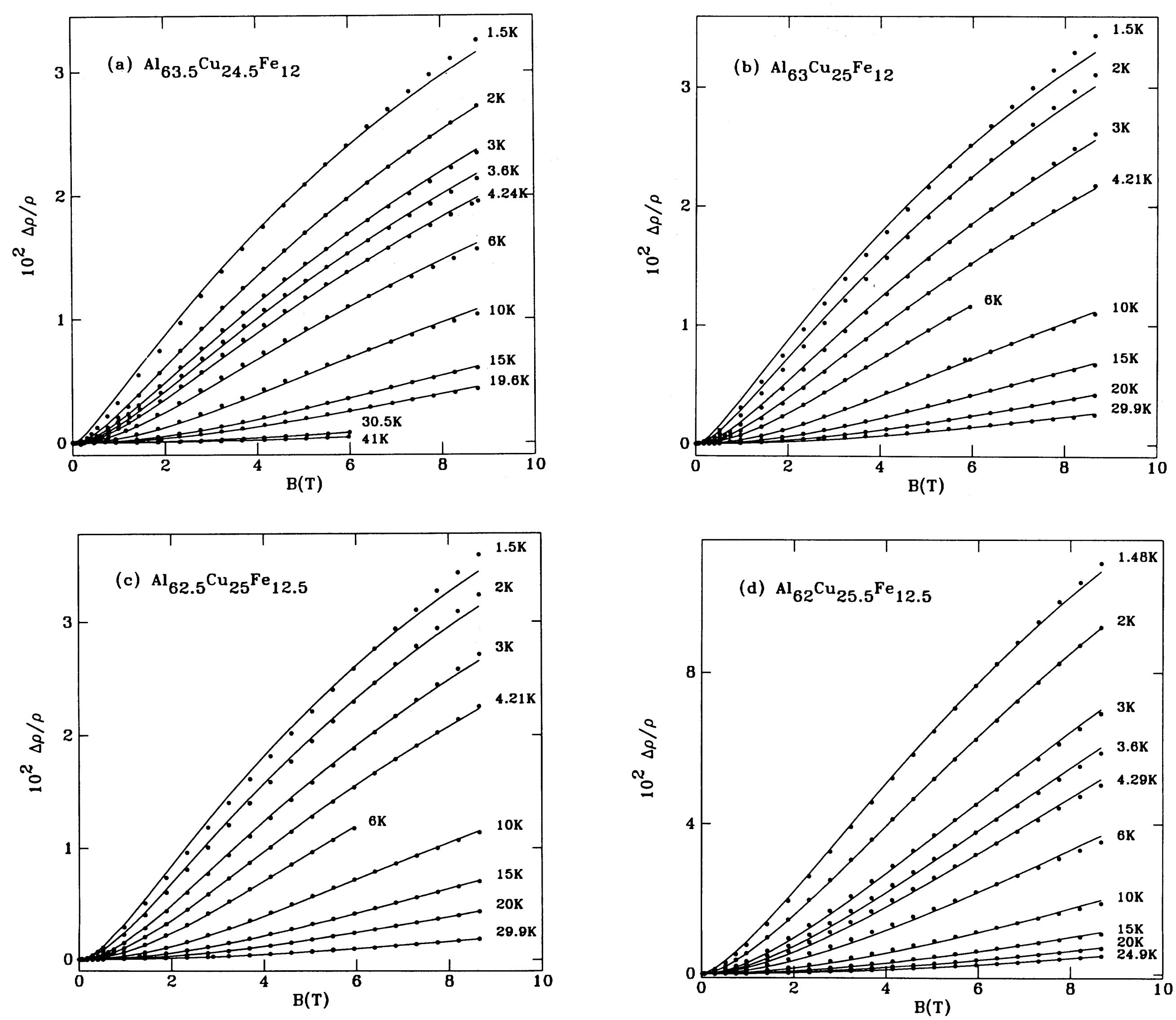

FIG. 2. Magnetoresistance of icosahedral Al-Cu-Fe alloys. The points are the experimental data and the solid line a fit as described in the text. Temperatures are indicated in the figure. (a) $Al_{63.5}Cu_{24.5}Fe_{12}$, (b) $Al_{63}Cu_{25}Fe_{12}$, (c) $Al_{62.5}Cu_{25.5}Fe_{12}$, and (d) $Al_{62}Cu_{25.5}Fe_{12.5}$.

$(k_F l_e)^{-1}$ (k_F is the Fermi wave vector and l_e the electron mean free path) of a perturbation treatment of the disorder and should be used, in principle, only when $(k_F l_e)^{-1} \ll 1$. In icosahedral Al-Cu-Fe alloys $(k_F l_e)^{-1}$ is estimated from the free-electron formula, $(k_F l_e)^{-1} = \hbar/(3m_e D)$, to be ~ 1.5. However, theoretical considerations suggest that higher-order terms are small and these expressions may not be restricted to the weak disorder limit.[11,17] It has also been found experimentally that these expressions account accurately for the data in high-resistivity amorphous metals when $(k_F l_e)^{-1}$ is of order unity.[13,18]

The temperature dependence of the dephasing rate as deduced from the magnetoresistance analysis is shown in Fig. 3 and may be described by an expression of the form: $1/\tau_\phi = AT^p$. In all samples, a best fit is obtained for $p \sim 1.5$. It is well known[11,19,20] that an exponent $\frac{3}{2}$ is expected for electron-electron scattering in the so-called dirty limit when $(k_F l_e)^{-1} > 1$. In fact Schmid[20] has given a general expression for the electron-electron scattering rate in disordered metals:

$$\frac{1}{\tau_{ee}} = \frac{\pi}{8}\frac{(k_B T)^2}{\hbar E_F} + \frac{\sqrt{3}}{2}(k_F l_e)^{-3/2}\frac{(k_B T)^{3/2}}{\hbar\sqrt{E_F}} , \tag{1}$$

where E_F is the Fermi energy. A similar expression has also been derived by Al'tshuler and Aronov.[11]

To compare with the experimental results, we use the value of $(k_F l_e)^{-1}$ given above and take $E_F \sim 1$ eV for the Fermi energy. Equation (1) then yields

$$1/\tau_{ee} = 4.4\times 10^6 T^2 + 2\times 10^9 T^{3/2}\ \mathrm{s}^{-1} .$$

Although the calculated value is smaller than the one we find from the fits, it is consistent with the fact that the $T^{3/2}$ term dominates. The first term in Eq. (1) is expected to dominate when the disorder is not too strong. The difference in the magnitude between the estimated $1/\tau_\phi$ and the experimental one is not specific to the present case. It is consistent with the results of several systems where the theoretical expression of $1/\tau_{ee}$ always underestimates the observed dephasing rate (see Ref. 11, for example). We therefore conclude that dephasing is due to electron-electron scattering, in the strong disorder limit and that Eq. (1) needs to be reevaluated so that quantitative comparison with the experiment can be made.

The temperature dependence of the zero-field resistivity below 30 K is shown in Fig. 4. In all samples, the resistivity increases as the temperature is lowered except in $Al_{62}Cu_{25.5}Fe_{12.5}$ where, after reaching a maximum around 14 K, it decreases down to the lowest temperatures. As for the magnetoresistance, we fitted the zero-field resistivity data using the predictions of QCC. In the present high-resistivity system, WL contribution to the temperature dependence of the resistivity is important and must be included in the analysis. Moreover, it increases with increasing resistivity and dominates in $Al_{62}Cu_{25.5}Fe_{12.5}$. The values of ρ and B_{so} are the same as those extracted from the magnetoresistance fit. However, the dephasing field B_ϕ and the Coulomb interaction parameter $\tilde{F}_\sigma$ were allowed to vary in order to fit the data over a wide range of temperature. Here also good fits are found from the lowest temperatures to approximately 15 K. However, although the dephasing rate $1/\tau_\phi$ is found to follow a power law AT^p, the value of p is equal to 2.0 ± 0.1 and the coefficient $A \sim 9\times 10^8\ \mathrm{K}^{-2}\mathrm{s}^{-1}$, in the low-resistivity samples. Similar values of A and p were reported in Refs. 7 and 9 using simplified QCC expressions of the resistivity temperature dependence. The

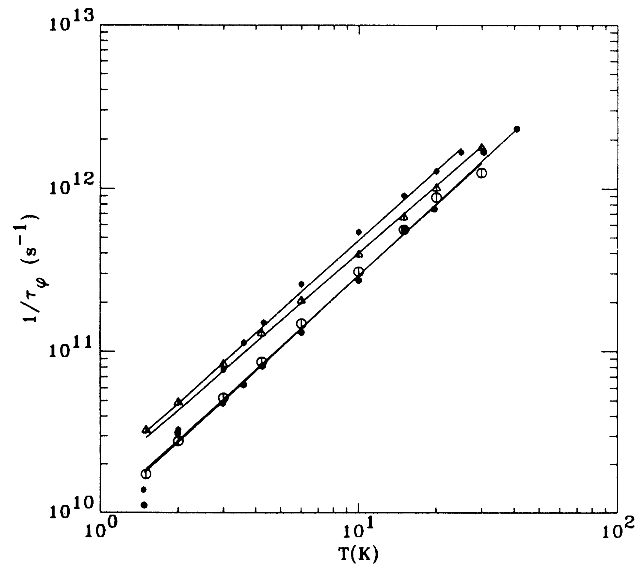

FIG. 3. Dephasing rate $1/\tau_\phi$ as a function of temperature in icosahedral Al-Cu-Fe alloys, as extracted from the magnetoresistance. The solid line is a fit to the data using $1/\tau_\phi = AT^p$. (●) $Al_{63.5}Cu_{24.5}Fe_{12}$, (○) $Al_{63}Cu_{25}Fe_{12}$, (△) $Al_{62.5}Cu_{25.5}Fe_{12}$ (∘) $Al_{62}Cu_{25.5}Fe_{12.5}$.

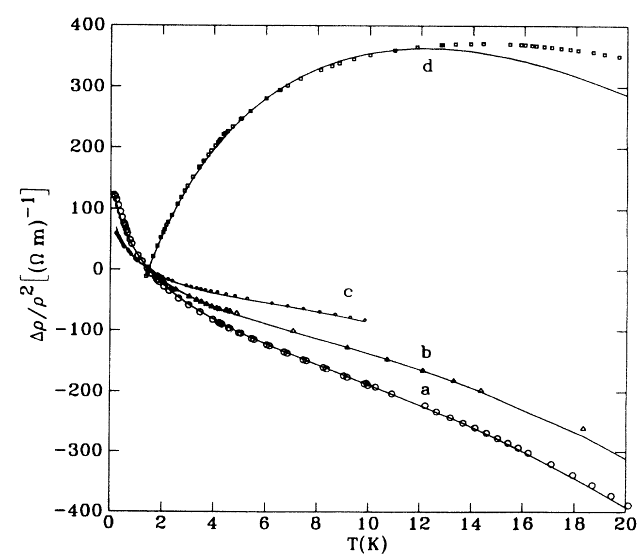

FIG. 4. Low-temperature resistivity of icosahedral Al-Cu-Fe alloys. The points are the experimental data and the solid line a fit taking into account both WL and EEI contributions. (a) $Al_{63.5}Cu_{24.5}Fe_{12}$, (b) $Al_{63}Cu_{25}Fe_{12}$, (c) $Al_{62.5}Cu_{25.5}Fe_{12}$, and (d) $Al_{62}Cu_{25.5}Fe_{12.5}$.

present results are significantly different from our findings from the magnetoresistance analysis. They suggest that perhaps only the first term of the dephasing rate in Eq. (1) is relevant to the temperature dependence of the resistivity in the relatively low-resistivity samples. However, as mentioned in the Introduction, an exponent $p=2$ can also be attributed to electron-phonon scattering. In the highest-resistivity alloy, i.e., $Al_{62}Cu_{25.5}Fe_{12.5}$, both the magnetoresistance and the resistivity temperature dependence give, within error, the same $1/\tau_\phi$ (see Table I) and suggest that dephasing is due to electron-electron scattering only. It is not clear why the other samples give different results and further investigation is needed in order to explain the observed discrepancy in the value of the exponent p.

Figure 5 shows the resistivity $Al_{62}Cu_{25.5}Fe_{12.5}$ in different magnetic fields. As mentioned above, the zero-field resistivity reaches a maximum then decreases below 14 K. Similar positive slope of the resistivity as a function of temperature is only seen in highly doped semiconductor of comparable resistivities (e.g., Si:P and Ge:Sb).[16,21,22] In their most recent work Klein *et al.*[9] attributed this peculiar behavior of the resistivity to a band-structure effect in this very high-resistivity sample. Here we show that it is consistent with antilocalization effects caused by spin-orbit scattering.[23] In fact, in the presence of spin-orbit scattering weak localization correction to the resistivity increases with temperature and as a function of B_ϕ its slope changes sign for $B_\phi \geq \frac{1}{6}B_{so}$, as inelastic scattering becomes more important at high temperatures. We should mention that for the other three samples, it is not possible to see such an effect because, as deduced from the fitting, $B_\phi \ll B_{so}$. Another way to destroy antilocalization effects on the resistivity is by applying a magnetic field. One can see from the figure that as the field increases the negative slope of the resistivity with temperature is progressively recovered. We also note that even a field as high as 8.4 T is not sufficient to suppress the temperature dependence of the localization contribution to the resistivity. The solid lines in Fig. 5 are a result of a combination of WL and EEI contributions using the full theoretical expressions with the same parameters as those extracted from the magnetoresistance and are therefore zero-parameter fits to the data. Since $\tilde{F}_\sigma = 1.08$ the EEI contribution to the zero-field resistivity in this sample is positive whereas it is negative for the other samples of lower resistivity. A similar sign change of the EEI contribution has also been observed by Klein *et al.*[9] in this alloy system and by Thomas *et al.*[16] and Rosenbaum *et al.*[22] in doped Ge:Sb and Si:P as a function of concentration, just above the metal-insulator transition. This observation together with the large effect of EEI mentioned earlier suggest that $Al_{62}Cu_{25.5}Fe_{25.5}$ is also very close to the transition and/or band-structure effects are very important in this alloy as proposed in Ref. 9.

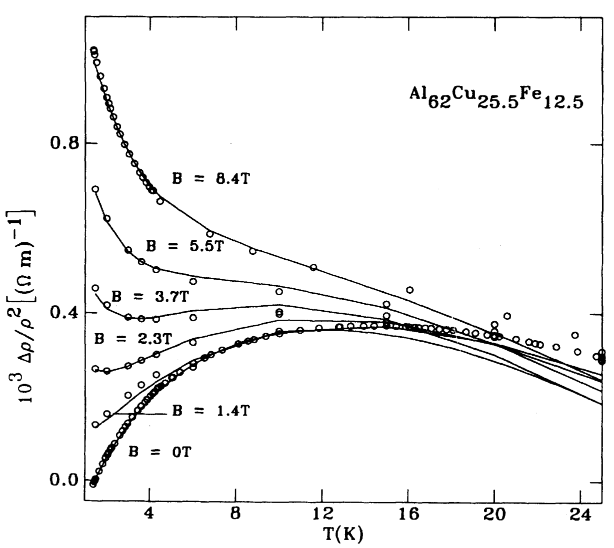

FIG. 5. Low-temperature resistivity of icosahedral $Al_{62}Cu_{25.5}Fe_{12.5}$ in a magnetic field. The points are the experimental data and the solid line a fit as described in the text.

IV. SUMMARY AND CONCLUSIONS

We have studied in detail the low-temperature resistivity and magnetoresistance of single-phase icosahedral Al-Cu-Fe alloys. The overall behavior is in general agreement with QCC theories even though $(k_F l_e)^{-1} > 1$. In this high-resistivity system WL and EEI are necessary for a full account of the magnetoresistance and the resistivity temperature-dependence data. The EEI contribution is very large and is dominated by the contribution from the diffusion channel. In spite of the good agreement, discrepancies are found in the magnitude and temperature dependence of the dephasing rate. As deduced from the magnetoresistance $1/\tau_\phi \propto AT^p$ with $p=\frac{3}{2}$, a characteristic of electron-electron scattering in the strong disorder limit. The zero-field resistivity data, on the other hand, suggest that dephasing is due to electron-phonon scattering or electron-electron scattering in the weak disorder limit ($p=2$). This difference is unexpected, but it opens the question of whether the magnetoresistance is more sensitive to disorder than the resistivity temperature dependence.

In $Al_{62}Cu_{25.5}Fe_{12.5}$ antilocalization was directly observed and was nicely accounted for by the theory. A sign change in the EEI contribution to the zero-field resistivity in this sample was found and is attributed to the proximity of the metal-insulator transition in similarity with doped Si:P and Ge:Sb systems.

The question of whether Al-Cu-Fe is ordered or disordered remains. The success of QCC in describing the magnetotransport properties at low temperatures implies immediately very intense scattering of the electrons at E_F. If we assume the electron mass $\sim m_e$, then the parameters obtained from the fitting give an elastic mean free path ~ 3 Å and an elastic scattering rate $\sim 10^{15}\ \mathrm{s}^{-1}$. These figures correspond to extremely high disorder and compare with the most resistive amorphous metals. Even if the bands are non-free-electron-like,[24] and the effective mass anomalously high (or the Fermi velocity anomalously low), the quantitative fitting of WL and EEI expressions require $\tau_e^{-1}/\tau_\phi^{-1}$, $\tau_e^{-1}/\tau_{so}^{-1} \gg 1$. If we assume a factor of 10 as the lower limit for these ratios, then the mean free path has an upper limit of only 30 Å, which still represent a very high degree of disorder; it remains to reconcile this with the high level of atomic order displayed in Fig. 1.

ACKNOWLEDGMENT

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APPENDIX

1. Weak localization

According to Fukuyama and Hoshino, the magnetoresistance due to localization of the conduction electrons, in the presence of spin-orbit and inelastic scattering and including splitting of the spin subbands, is given by[25]

$$\left[\frac{\Delta\rho}{\rho}\right]_{\rm WL}(B,T)=\rho\frac{e^2}{2\pi^2\hbar}\left[\frac{eB}{\hbar}\right]^{1/2}\left\{\frac{1}{2\sqrt{1-\gamma}}\left[f_3\left[\frac{B}{B_-}\right]-f_3\left[\frac{B}{B_+}\right]\right]-f_3\left[\frac{B}{B_2}\right]\right.$$
$$\left.-\left[\frac{4B_{\rm so}}{3B}\right]^{1/2}\left[\frac{1}{\sqrt{1-\gamma}}(\sqrt{t_+}-\sqrt{t_-})+\sqrt{t}-\sqrt{t+1}\right]\right\}, \tag{A1}$$

where

$$t=\frac{3(B_i+2B_s)}{4B_{\rm so}},$$

$$t_\pm=t+\tfrac{1}{2}(1\pm\sqrt{1-\gamma}),$$

$$B_\pm=B_i+2B_s+\frac{2(B_{\rm so}-B_s)}{3}(1\pm\sqrt{1-\gamma}),$$

$$B_2=B_i+\tfrac{2}{3}B_s+\tfrac{4}{3}B_{\rm so},$$

with

$$\gamma=\left[\frac{3g^*\mu_B B}{8eD(B_{\rm so}-B_s)}\right]^2.$$

g^* is the effective g factor and D the diffusion constant.

The characteristic fields are related to the electron-scattering times by the relaxation $B_x=\hbar/4eD\tau_x$, where τ_x is the inelastic scattering time τ_i, spin-orbit scattering time $\tau_{\rm so}$, and the magnetic impurity scattering time τ_s. The dephasing field B_ϕ is defined as $B_\phi=B_i+2B_s$, however, since spin-spin scattering is absent in Al-Cu-Fe,[9] $B_\phi\equiv B_i$.

The function $f_3(x)$ in Eq. (A1) has been derived by Kawabata[26] and is given by

$$f_3(x)=\sum_0^\infty\left[2\sqrt{n+1+1/x}-2\sqrt{n+1/x}-\frac{1}{\sqrt{n+\frac{1}{2}+1/x}}\right].$$

In zero magnetic field the correction to the resistivity from weak localization reduces to the following expression:[25]

$$\left[\frac{\Delta\rho}{\rho}\right]_{\rm WL}(B=0,T)=-\rho\frac{e^2}{2\pi^2\hbar}(3\sqrt{\tfrac{4}{3}B_{\rm so}+B_\phi}-\sqrt{B_\phi}). \tag{A2}$$

2. Electron-electron interaction

The magnetoresistance due to enhanced electron-electron interaction is a combination of two terms: one known as the diffusion-channel term and the other as the Cooper-channel term. For very low-diffusivity systems, the latter term is negligible as mentioned above and its expression is not given here.

The diffusion-channel magnetoresistance has been calculated by Lee and Ramakrishnan and is given by[19]

$$\left[\frac{\Delta\rho}{\rho}\right]_{\rm DC}(B,T)=\rho\frac{e^2}{2\pi^2\hbar}\left[\frac{eB}{\hbar}\right]^{1/2}\frac{\tilde F_\sigma}{2}\left[\frac{k_BT}{2eDB}\right]^{1/2}g_3\times\left[\frac{g\mu_BB}{k_BT}\right], \tag{A3}$$

where

$$\tilde F_\sigma=-\frac{32}{3F}\left[1+\frac{3F}{4}-\left[1+\frac{F}{2}\right]^{1/2}\right],$$

$$g_3(x)=\int_0^\infty d\omega\left[\frac{d^2}{d\omega^2}[\omega N(\omega)]\right]\times(\sqrt{\omega+x}+\sqrt{\omega-x}-2\sqrt{\omega}).$$

F is the screening factor whose value according to Thomas-Fermi theory[27] lies between zero, for no screening, and unity for complete screening of the Coulomb interaction and $N(\omega)=1/(e^\omega-1)$.

The temperature dependence of the correction to the resistivity from the diffusion channel in the absence of a magnetic field is given by[19]

$$\left[\frac{\Delta\rho}{\rho}\right]_{\rm DC}(B=0,T)=-\rho\frac{0.915e^2}{4\pi^2\hbar}\left[\frac{4}{3}-\frac{3}{2}\tilde F_\sigma\right]\times\left[\frac{k_BT}{\hbar D}\right]^{1/2}. \tag{A4}$$

In fitting the data in a magnetic field we simply combined the magnetoresistance and the zero-field expressions for the various terms.

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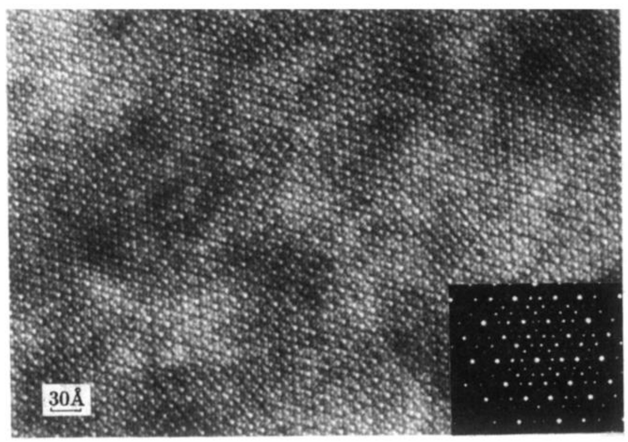

FIG. 1. High-resolution transmission electron micrograph of $Al_{63.5}Cu_{24.5}Fe_{12}$. The inset shows the corresponding electron-diffraction pattern along the fivefold axis.