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Absence of weak antilocalization for spin-1 particle waves

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Localization in disordered two-dimensional systems in the presence of spin-orbit scattering (SOS) still presents a challenging problem. In perturbation one obtains for spin-$\frac{1}{2}$ particle waves an increase of the conductance due to a quantum interference effect, the so-called weak antilocalization. We extend the perturbation calculation to arbitrary quantum spins. For integer-spin-s particle waves we find no weak antilocalization but the SOS reduces the quantum correction by the factor $1/(2s+1)$. For odd multiples of spin $\frac{1}{2}$ the SOS reverses the sign of the quantum correction to the conductance and reduces it by $1/(2s+1)$. In the limit of infinite spin SOS destroys weak antilocalization, similar to magnetic impurities in the unitary limit.

The problem of localization in two dimensions has been intensively studied since the late 1970s.[1–5] It is now generally believed that spin-zero particles (or waves) are localized in a disordered two-dimensional system. The situation is much less understood for spin-$\frac{1}{2}$ particles in the presence of spin-orbit scattering (SOS).[6–11] In perturbation theory spin-$\frac{1}{2}$ particles experience even a reduction of their resistance below the classical resistance in the presence of SOS (Ref. 12) (see, for example, the review articles in Refs. 13–16). In the presence of large SOS localization is suppressed (except at the band edges where it could even be enhanced[10]). If the SOS is finite but less than the normal elastic scattering the question of localization is still under investigation.

In this connection we raised the question of what the localization behavior of particles with a higher spin might be. Before one starts an extensive numerical calculation it is worthwhile to study this question by perturbation. In perturbation one calculates the Langer-Neal diagram[17] for particles with spin s in the presence of SOS. This diagram is sketched in Fig. 1. The indices α, β, γ, and δ are spin indices. In this particle-particle diagram one particle is scattered from the state $\alpha \rightarrow \beta$ and the other from $\gamma \rightarrow \delta$. Since the Langer-Neal diagram is originally a particle-hole fan diagram one has the conditions that $\alpha = \delta$ and $\beta = \gamma$. If we denote the diagrams with $\Gamma^{\alpha\beta,\gamma\delta}$ then only the diagrams $\Gamma^{\alpha\beta,\beta\alpha}$ contribute to the conductance correction and one has

$$\Delta\sigma = \frac{1}{2s+1}\sum_{\alpha,\beta}\Gamma^{\alpha\beta,\beta\alpha}, \tag{1}$$

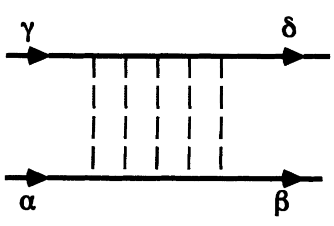

FIG. 1. The spin-dependent pair propagator $\Gamma^{\alpha\beta,\gamma\delta}$.

where s is the spin of the particles. For spin-1 particles, α and β can take the values $+1, 0, -1$, which we denote as $+, 0, -$. Therefore, we have nine Γ terms with the superscripts $(++,++), (+0,0+), (+-,-+)$, etc.

In real space, $\Gamma^{\alpha\beta,\beta\alpha}$ corresponds to a product of two amplitudes of an electron with spin α which moves from the position O on a closed loop C back to the position O. One amplitude results from the propagation along the loop C and the other is the conjugate complex of the amplitude resulting from the propagation along C^*, i.e., along the curve C in the opposite direction. If $A'^{\alpha\beta}$ is the β (spin) component of the amplitude along C and $A''^{\alpha\beta}$ the amplitude along the the path C^* then $\Gamma^{\alpha\beta,\beta\alpha}$ is the product of the first amplitude times the conjugated complex of the second, i.e., $\Gamma^{\alpha\beta,\beta\alpha} = A'^{\alpha\beta}A''^{\alpha\beta*}$. Since C^* is the time-reversed path for the orbital motion of the electron, $\Gamma^{\alpha\beta,\beta\alpha}$ can also be interpreted as the pair amplitude of an electron pair $(\alpha\beta)$ to be scattered along the path C into the state $(\beta\alpha)$.

For spin $\frac{1}{2}$ one often uses the trick to replace the sum of the pair propagators $\Gamma^{\alpha\beta,\beta\alpha}$ by the pair propagators with constant total spin S and total M, i.e., by Γ_M^S. We use the same approach for spin 1 and find

$$\begin{aligned}
\Gamma_2^2 &= \Gamma^{++,++},\\
\Gamma_1^2 &= \tfrac{1}{2}[\Gamma^{+0,+0} + \Gamma^{0+,0+} + \Gamma^{+0,0+} + \Gamma^{0+,+0}],\\
\Gamma_1^1 &= \tfrac{1}{2}[\Gamma^{+0,+0} + \Gamma^{0+,0+} - \Gamma^{+0,0+} - \Gamma^{0+,+0}],\\
\Gamma_0^2 &= \tfrac{1}{6}[\Gamma^{+-,+-} + 2\Gamma^{+-,00} + \Gamma^{+-,-+} + 2\Gamma^{00,+-}\\
&\quad +4\Gamma^{00,00} + 2\Gamma^{00,-+} + \Gamma^{-+,+-} + 2\Gamma^{-+,00}\\
&\quad +\Gamma^{-+,-+}],\\
\Gamma_0^1 &= \tfrac{1}{2}[\Gamma^{+-,+-} + \Gamma^{-+,-+} - \Gamma^{+-,-+} - \Gamma^{-+,+-}],\\
\Gamma_0^0 &= \tfrac{1}{3}[\Gamma^{+-,+-} - \Gamma^{+-,00} + \Gamma^{+-,-+} - \Gamma^{00,+-}\\
&\quad +\Gamma^{00,00} - \Gamma^{00,-+} + \Gamma^{-+,+-} - \Gamma^{-+,00} + \Gamma^{-+,-+}].
\end{aligned} \tag{2}$$

Therefore we obtain

$$\sum_{\alpha,\beta}\Gamma^{\alpha\beta,\beta\alpha} = [\Gamma_2^2+\Gamma_1^2+\Gamma_0^2+\Gamma_{-1}^2+\Gamma_{-2}^2] - [\Gamma_1^1+\Gamma_0^1+\Gamma_{-1}^1]+\Gamma_0^0. \tag{3}$$

The sum of the (spin-dependent) pair amplitudes can be expressed as sums and differences of the total spin pair amplitudes.

The SOS affects the individual electron amplitudes by the scattering Hamiltonian

$$H_1' = i\lambda V_{\mathbf{k},\mathbf{k}'}(\mathbf{k}\times\mathbf{k}')\mathbf{s}_1 = i\mathbf{K}\mathbf{s}_1,$$
$$H_2' = \{i\lambda V_{\mathbf{k},\mathbf{k}'}[(-\mathbf{k}')\times(-\mathbf{k})]\mathbf{s}_2\}^* = i\mathbf{K}^*\mathbf{s}_2. \tag{4}$$

The product is $H'' = -(\mathbf{K}\mathbf{s}_1)(\mathbf{K}^*\mathbf{s}_2)$.

When the SOS is (on the average) isotropic, i.e., $\langle|K_x|^2\rangle = \langle|K_y|^2\rangle = \langle|K_z|^2\rangle$ and when the three components are uncorrelated, one obtains on the average for the product

$$H'' = \tfrac{1}{3}\langle K^2\rangle \mathbf{s}_1\mathbf{s}_2 = \tfrac{1}{3}\langle K^2\rangle[\tfrac{1}{2}(\mathbf{s}_1^2+\mathbf{s}_2)^2 - \tfrac{1}{2}\mathbf{s}_1^2 - \tfrac{1}{2}\mathbf{s}_2^2]. \tag{5}$$

Each individual electron amplitude is damped according to the SOS Hamiltonian H' [Eq. (4)] and decays (besides the elastic decay which is not relevant for the decay of electron pairs) with the rate $\frac{1}{2\tau_{so}}$, where

$$\frac{1}{\tau_{so}} = \frac{\hbar}{2\pi}\frac{1}{3}\langle K^2\rangle N_0 s(s+1) \tag{6}$$

and N_0 is the density of states. Therefore, the pair amplitude is reduced with the rate $1/\tau_{so}$ because of the decay of the individual electron amplitudes. However, the negative terms in H'', i.e., $(\mathbf{s}_1^2+\mathbf{s}_2^2)/2$ compensate this damping. As a consequence the total decay of the pair amplitude is given by the Hamiltonian $\langle K^2\rangle S^2/6$ where $S = s_1 + s_2$ is the total spin. The resulting decay rate of the pairs with total spin S is

$$\frac{1}{\tau_{so}(S)} = \frac{\hbar}{2\pi}\frac{1}{3}\langle K^2\rangle N_0 S(S+1) = \frac{S(S+1)}{2s(s+1)}\frac{1}{\tau_{so}}. \tag{7}$$

The orbital, i.e., spin-independent part of the Langer-Neal diagram decays in time as $t^{-d/2}\exp[-t/\tau_i]$, where d is the dimension of the system. The SOS in the case of a finite spin s yields an additional factor for the decay as a function of time which we denote as $\Gamma_{SOS}(t)$. We obtain for spin $s = 1$ the following result:

$$\Gamma_{SOS}(t) = \tfrac{1}{3}\ \{5\exp[-t/\tau_{so}(2)] - 3\exp[-t/\tau_{so}(1)] +1\exp[-t/\tau_{so}(0)]\}. \tag{8}$$

This yields the factor 1 for zero SOS and the factor $\frac{1}{3}$ for very large SOS. In contrast to the case of spin $\frac{1}{2}$ one does not find a reversal of the sign for infinite SOS but only a reduction by a factor of $3 = 2s + 1$. If the spin-1 particles have the charge e then one finds for conductance correction as a function of temperature and magnetic field

$$\frac{\Delta G(H,T)}{G_{00}} = \frac{1}{3}\left[5f\left(\frac{H}{H(2)}\right) - 3f\left(\frac{H}{H(1)}\right) +1f\left(\frac{H}{H(0)}\right)\right], \tag{9}$$

where

$$G_{00} = \frac{e^2}{2\pi^2\hbar}, \tag{10}$$

$$H(S) = H_i(T) + \frac{S(S+1)}{2s(s+1)}H_{so}, \tag{11}$$

$$H_n\tau_n = \hbar e\rho N_0/4, \tag{12}$$

where the rates $1/\tau_n$ have been replaced by the corresponding magnetic fields, ρ is the resistivity of the two-dimensional system, and N_0 is the density of states.

In two dimensions $f(H/H(S))$ has the usual form (Ref. 12)

$$f\left(\frac{H}{H(S)}\right) = \Psi\left(\frac{1}{2}+\frac{H(S)}{H}\right) - \Psi\left(\frac{1}{2}+\frac{H_0}{H}\right), \tag{13}$$

$$H_0 = \hbar e\rho N_0/4\tau_0, \tag{14}$$

and $\Psi(x)$ is the digamma function.[18]

The main result is that the magnetoresistance is negative for zero SOS as well as for the strong SOS limit. There is no reversal of sign as in the spin-$\frac{1}{2}$ case, only a reduction of the magnetoresistance amplitude by a factor of 3. In other words, we do not observe weak antilocalization.

This result for the quantum correction to the conductance can be easily generalized for arbitrary spin

$$\frac{\Delta G(H,T)}{G_{00}} = \sum_{S=0}^{2s}(-1)^{2s-S}\frac{S(S+1)}{2s+1}f\left(\frac{H}{H(S)}\right), \tag{15}$$

where $H(S)$ is defined as in Eq. (11).

One easily realizes that the correction to the conductance in the case of strong SOS is reduced by the factor $1/(2s+1)$ with respect to the case of zero SOS. In addi-

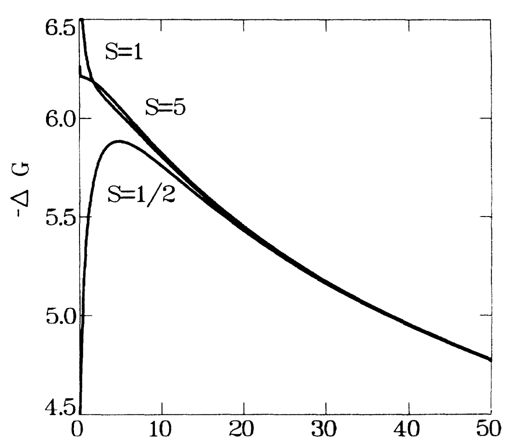

FIG. 2. The magnetoresistance for different spins s. The spin-orbit scattering strength is represented by $H_{so} = 1$.

tion one finds that only for s equal to an odd multiple of $\frac{1}{2}$ will the strong SOS yield weak antilocalization. (The experimental finding, that electrons with small SOS weak localization changes into weak antilocalization with half the amplitude if one introduces strong SOS, is a direct experimental proof that electrons in a metal have spin $\frac{1}{2}$.)

In Fig. 2 we have plotted the resistance, i.e., the negative conductance in units of G_{00} as a function of the magnetic field for different spin values. The curves differ only in their low-field behavior. Particle waves with integer spin show always a negative magnetoresistance, while those with an odd multiple of spin $\frac{1}{2}$ show a positive magnetoresistance at small fields. The larger the spin the smaller the field range of positive magnetoresistance. The field range, in which the finite spin deviates from the limiting case of infinite spin shrinks with increasing spin.

In the limit of infinite spin s one can replace the sums by an integration (after combining each pair of terms with opposite sign). This yields for the time-dependent echo a decay with the rate $\exp[-2t/\tau_{so}]$. As a result one finds for infinite spin that the conductance correction due to weak localization contains only one term:

$$\frac{\Delta G(H,T)}{G_{00}} = f\left(\frac{H}{H_t}\right), \tag{16}$$

where

$$H_t = H_i(T) + 2H_{so}. \tag{17}$$

For infinite spin s the SOS has a similar effect as magnetic scattering, which is known as the unitary limit. In the β function, as applied by Ref. 3 the linear term in $1/g$ vanishes for infinite spin and large SOS. If the spin s is finite but large and an odd multiple of $\frac{1}{2}$ then a small residual of weak antilocalization survives. It is an interesting question whether this residual antilocalization is strong enough to suppress localization in a large sample.

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