Ginzburg-Landau theory of the spin-charge-separated system

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The phenomenological Ginzburg-Landau theory is developed for the resonating-valence-bond state where the spin and charge degrees of freedom are separated. We have the two order parameters corresponding to the fermion pairing and the Bose condensation, respectively, which are coupled with the gauge field. We find only one transition temperature which is the superconducting T_c. Above T_c, a crossover occurs from the spin-charge-separated phase to the Fermi liquid as the concentration of holes is increased. Below T_c, the penetration length λ is given by $\lambda^2=\lambda_F^2+\lambda_B^2$. The coherence length ξ, on the other hand, is complicated but is predicted to increase rapidly as the concentration approaches the overdoped region. There are two types of the vortex structure with the flux quantization $hc/2e$ and hc/e, respectively. In the mean-field theory and the type-II limit, the former is stable in almost all the cases, but the latter becomes stable near T_c in the low-hole-concentration region.

Since the proposal of the resonating-valence-bond (RVB) states and the spin-charge separation in these states as a model of the high-T_c superconductors,[1,2] extensive studies have been done to clarify the nature of this exotic state. Soon it was found that there are many kinds of RVB states, and the statistic of the spinons and holons depends on the specific RVB state. The scenario of the normal state as well as the superconductivity is quite different for each RVB state. This situation is best described by the Ioffe-Larkin formula[3] for the response function Π of the total system to the electromagnetic field which is written as

$$\Pi(q,\omega)=\frac{\Pi_F(q,\omega)\Pi_B(q,\omega)}{\Pi_F(q,\omega)+\Pi_B(q,\omega)} ,$$

where Π_F and Π_B are the response functions of the fermion and boson systems to the gauge field, respectively. This means that Π_F or Π_B whichever is smaller dominates the response of the total system. In the extreme case one of the systems is frozen to show the rigidity, which results in the finite value of the corresponding static transverse response function in the uniform limit $\omega=0, q\rightarrow 0$ (Meissner effect). The response Π is then determined by that of the other system. Some of the RVB theories assume that this is the case in the normal state and ignore the system which is frozen and shows the Meissner effect, while retaining only the other system. When both Π_F and Π_B show Meissner effect the total system is superconducting because the physical transverse Π remains finite in the limit $\omega=0, q\rightarrow 0$ and shows the Meissner effect. Recently, the present authors have investigated the normal state properties of the uniform RVB state.[4] In this state the spinon is a fermion while the holon is a boson, and it is assumed that neither spinons nor holons are frozen in the normal phase. We expect many unusual properties because both the fermions and bosons contribute to the physical processes. In the mean-field approximation[5] we have the two transition temperatures $T_D^{(0)}$ and $T_{\rm BE}^{(0)}$ according to which we have four distinct phases shown in Fig. 1(a). $T_D^{(0)}$ is the transi-

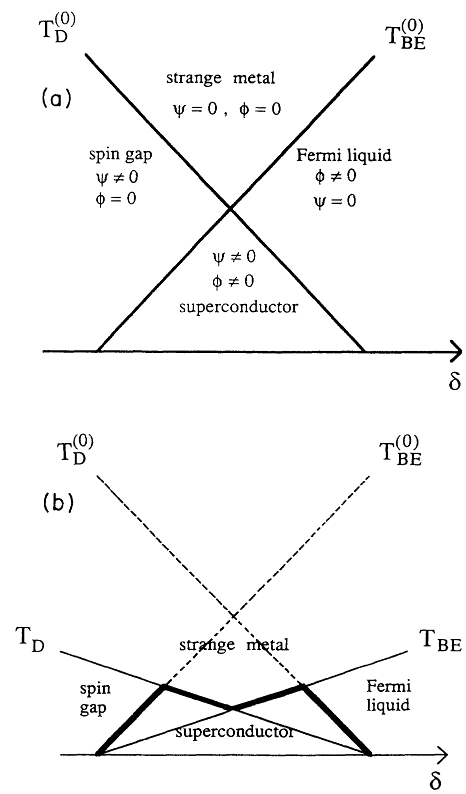

FIG. 1. (a) The mean-field phase diagram. (b) Modified phase diagram when inelastic lifetime effects are taken into account. The only true phase transition is across the heavy line into the superconducting phase.

tion temperature for the pairing of the spinons below which Π_F shows Meissner effect, and $T_{\rm BE}^{(0)}$ is the Bose-Einstein condensation temperature of the holons below which Π_B shows the Meissner effect. $T_D^{(0)}$ is a decreasing function of the hole concentration δ and becomes zero when $t\delta \sim J$ where t is the hopping energy of the electron and J is the exchange interaction between spins. $T_{\rm BE}^{(0)}$, on the other hand, is an increasing function of δ. When $T_D^{(0)} < T$ and $T_{\rm BE}^{(0)} < T$, neither of the orders is present and the system is a "strange metal" which has been discussed in Ref. 4. When $T_D^{(0)} < T_{\rm BE}^{(0)}$ we have the Fermi liquid between these two temperatures and $T_D^{(0)}$ is the superconducting transition temperature $T_c^{(0)}$. When $T_{\rm BE}^{(0)} < T < T_D^{(0)}$ the spin gap opens due to the pairing of the fermions ("spin gap state") and the response Π of the total system is Π_B between these two temperatures. It behaves like a nondegenerate semiconductor instead of a metal, and $T_{\rm BE}^{(0)}$ becomes $T_c^{(0)}$. When $T_D^{(0)} > T$ and $T_{\rm BE}^{(0)} > T$ the system is a superconductor. In any case, we have the two successive phase transitions when $T_D^{(0)} \neq T_{\rm BE}^{(0)}$.

The inelastic scattering by the gauge field gives rise to the pair breaking effect and hence reduces both transition temperatures from $T_D^{(0)}, T_{\rm BE}^{(0)}$ (dashed lines) to $T_D, T_{\rm BE}$ (solid lines) as shown in Fig. 1(b). When one of Π_F and Π_B shows Meissner effect the gauge field is stiffened up and the pair-breaking effect is reduced. Therefore, at $T_c = \max(T_D, T_{\rm BE})$ both Π_F and Π_B show Meissner effect (superconductivity) as long as both T_D and $T_{\rm BE}$ are below $\min(T_D^{(0)}, T_{\rm BE}^{(0)})$ [the thick line in Fig. 1(b)]. This discussion modifies the mean-field picture shown in Fig. 1(a) in that a single transition between "strange metal" and superconductor now occurs over a finite range of doping concentration δ. When δ is further increased and $T_{\rm BE}$ exceeds $T_D^{(0)}$ however, we have two successive transitions between which we have the Fermi liquid. For free Bose gas in two dimensional (2D), $T_{\rm BE}$ is just a crossover temperature below which the correlation length grows exponentially. When repulsive interaction between bosons is taken into account, we expect a finite mean-field transition temperature which one might naively expect to turn into a Kosterlitz-Thouless[6] transition when vortex excitations are taken into account. In addition, the interlayer coupling can make the phase transition three dimensional.

In this paper we give a more concrete description of the above scenario. We investigate the possibility of these two successive transitions by using a Ginzburg-Landau theory taking into account the interlayer coupling. We also study the superconducting phase of the spin-charge-separated system. In addition, the value of the flux quantization is a nontrivial problem because both the boson condensation and the pairing of the fermions are involved. The structure of the vortex is clarified and the lower and upper critical fields are obtained. For both of these two problems, the gauge field plays an essential role without which we obtain unphysical results.

We start with the following Ginzburg-Landau (GL) functional for the free energy F:[7]

$$F = \sum_i F_i + \sum_i F_{i,i+1} + F_{\rm em} . \quad (1)$$

The free energy F_i for each layer i is

$$F_i = F_F[\psi_i, \mathbf{a}_i, \mathbf{A}] + F_B[\phi_i, \mathbf{a}_i] + F_{\rm gauge}[\mathbf{a}_i, \mathbf{A}] , \quad (2)$$

where

$$F_F[\psi, \mathbf{a}, \mathbf{A}] = \frac{H_{cF}^2}{8\pi} \int d^2r \left[2\xi_F^2 \left| \left(\nabla - i2\mathbf{a} - i\frac{2e}{c}\mathbf{A} \right) \psi \right|^2 + 2\,\mathrm{sgn}(T - T_D^{(0)}) |\psi|^2 + |\psi|^4 \right] \quad (3a)$$

is the GL free energy for the spinon pairing and

$$F_B[\phi, \mathbf{a}] = \frac{H_{cB}^2}{8\pi} \int d^2r [2\xi_B^2 |(\nabla - i\mathbf{a})\phi|^2 + 2\,\mathrm{sgn}(T - T_{\rm BE}^{(0)}) |\phi|^2 + |\phi|^4] \quad (3b)$$

is that for the holon condensation. $\mathbf{A}$ is the vector potential for the electromagnetic field, c is the velocity of light, and $\hbar$ is put to be 1. In the above expressions all the quantities are scaled appropriately. In particular below the transition temperatures ξ_F and ξ_B, H_{cF} and H_{cB} have the usual meaning of the coherence length and the thermodynamic critical field. By using the penetration depth λ_F or λ_B, H_{cF} and H_{cB} are written as

$$H_{cF} = \frac{\phi_0}{2\sqrt{2}\pi\xi_F\lambda_F}, \quad H_{cB} = \frac{\phi_0}{\sqrt{2}\pi\xi_B\lambda_B} ,$$

where the difference in the factor of 2 comes from the fact that the pairs of the spinons condense while the holons themselves condense. Above the transition temperatures ξ_F and λ_F, ξ_B and λ_B are symmetrically defined and are proportional $|T - T_D^{(0)}|^{-1/2}$ and $|T - T_{\rm BE}^{(0)}|^{-1/2}$, respectively. The transition temperatures $T_D^{(0)}$ and $T_{\rm BE}^{(0)}$ are assumed to be different in general. We now give a rough estimate of these quantities for the uniform RVB state at zero temperature. The kinetic energy of the fermions and bosons are assumed to be of the order of the exchange interaction J.[8] We take the lattice constant as the unit of the length. Then $H_{cB}(0)^2/8\pi \sim J\delta^2$, $\xi_B(0) \sim \delta^{-1/2}$, and $\lambda_B(0) \sim \phi_0 (J\delta)^{-1/2}$ for the boson system, and $H_{cF}(0)^2/8\pi \sim \Delta^2/J$, $\xi_F(0) \sim J/\Delta$, and $\lambda_F(0) \sim \phi_0 [J(1-\delta)]^{-1/2}$ for the fermion system where Δ is the gap in the spinon spectrum due to the pairing. It can be seen that the $\xi_B(0)$ is extremely short while $\xi_F(0)$ depends on the ratio Δ/J.

The free energy $F_{\rm gauge}$ is obtained by integrating out the fermionic and bosonic degrees of freedom with high $|\mathbf{q}|$ and ω components and is given by[9]

$$F_{\rm gauge}[\mathbf{a}_i, \mathbf{A}] = \int \left\{ \chi_F \left[\nabla \times \left(\mathbf{a} + \frac{e}{c}\mathbf{A} \right) \right]^2 + \chi_B (\nabla \times \mathbf{a})^2 \right\} d^2r , \quad (4)$$

where χ_F and χ_B are the diamagnetic susceptibilities of the fermions and bosons in the normal phase. These do not include the contribution from the fluctuation of the spinon pairing or holon condensation which are described by the GL theory. χ_F and χ_B are of the order of J. By taking the functional derivative of the free energy with respect to $\mathbf{a}$, the sum of the currents J_F+J_B is obtained. $\delta(F_F+F_B)/\delta\mathbf{a}$ is the supercurrents $J_F^S+J_B^S$ while $\delta F_{\text{gauge}}/\delta\mathbf{a}$ is the normal currents, the sum of which vanishes.

As has been pointed out by Wheatley, Hsu, and Anderson[10] it is the physical electron and not the individual spinon or holon which can hop between the layers. When the temperature T is larger than the hopping t_c perpendicular to the layer, to which we restrict ourselves in this paper, we can treat t_c as a perturbation to obtain the interlayer Josephson coupling as

$$-J_1\sum_i\sum_\sigma\int d^2r\, C_{i,\sigma}^\dagger(r)C_{i,-\sigma}^\dagger(r)C_{i+1,-\sigma}(r)C_{i+1,\sigma}(r) + \text{H.c.}\ , \quad (5)$$

where $J_1\sim t_c^2/T$. When T is lower than t_c we have to consider the coherent motion of the electron between the layers. Using the slave boson expression of $C_{i,\sigma}^\dagger(r), C_{i,\sigma}(r)$ we translate Eq. (5) into the GL theory to obtain the interlayer coupling $F_{i,i+1}$ as

$$F_{i,i+1}=-J_1\int d^2r\, \psi_i^*(r)\psi_{i+1}(r)[\phi_i(r)]^2[\phi_{i+1}^*(r)]^2+\text{c.c.} \quad (6)$$

The last term F_{em} in Eq. (1) is the energy for the magnetic field and is given by

$$F_{\text{em}}=\int\frac{1}{8\pi}(\nabla\times\mathbf{A})^2d^2r\ , \quad (7)$$

which completes the description of our free energy.

First we assume that the spinons remain normal and the order parameter ψ is fluctuating around $\psi=0$ [the "Fermi-liquid" region in Fig. 1(a)]. Integrating over the ψ field, we obtain the following effective Lagrangian for the bose system.

$$F_{\text{eff}}=\int d^2r\left[\frac{H_{cB}^2}{8\pi}[2\xi_B^2|(\nabla-i\mathbf{a})\phi|^2-2|\phi|^2+|\phi|^4] + \tilde{\chi}_F(h+H)^2+\chi_B h^2\right]\ , \quad (8)$$

where $\tilde{\chi}_F$ is the Landau's diamagnetic susceptibility of the fermion system including the contribution from the fluctuating ψ field. The gauge flux h is given by $h=\partial_x a_y-\partial_y a_x$, and the magnetic field $\mathbf{H}=\nabla\times\mathbf{A}$ is assumed to be perpendicular to the plane with the magnitude H. Because only the gauge invariant product $\psi\phi^{*2}$ and its complex conjugate can appear in the Josephson coupling, the random phase average of the ψ field makes the interlayer coupling ineffective and each layer becomes an independent 2D system.

In the absence of the external magnetic field ($\mathbf{H}=\mathbf{0}$) the boson order parameter is coupled only with the fluctuating gauge field $\mathbf{a}$. This model has been discussed to show a first-order phase transition in $d=4-\varepsilon$ dimensions in terms of the renormalization-group method.[11] In the present 2D case however, topological excitations, i.e., vortices can be thermally activated even when $\mathbf{H}=0$, which alters the situation completely as is discussed below. If there is no gauge field the excitation energy of a single vortex is logarithmically divergent and there is a Kosterlitz-Thouless transition at T_{KT}. Below T_{KT} the system shows Meissner effect while the true long-range ordering is still absent. If we take into account the gauge field, however, the logarithmic interaction between the vortices is screened to be short ranged. Therefore, the correlation length ξ remains always finite of the order of the intervortex distance. The excitation energy $E_V(T)$ of the vortex for Eq. (8) is given roughly by

$$E_V(T)\sim\frac{\phi_0^2}{\lambda_B(T)^2}\sim J\delta(1-T/T_{\text{BE}}^{(0)})\sim C(T_{\text{BE}}^{(0)}-T)\ , \quad (9)$$

where C is a constant of the order of unity. The density n_V of the vortices is dominantly determined by the Boltzmann factor, i.e., $n_V\sim\exp[-E_V(T)/T]$. When $E_V(T)<T$, i.e., $T>T_{\text{BE}}=[C/(1+C)]T_{\text{BE}}^{(0)}$ the thermal population of the vortices is large and the amplitude of the order parameter $\langle|\phi|^2\rangle$ itself is considerably reduced from unity. [It should be noted that $|\psi|=1(|\phi|=1)$ means that the order parameter takes the temperature-dependent mean-field value which is proportional to $|T-T_D^{(0)}|^{1/2}(|T-T_{\text{BE}}^{(0)}|^{1/2})$ near the transition temperature $T_D^{(0)}(T_{\text{BE}}^{(0)})$.] When $E_V(T)\gg T$, i.e., $T\ll T_{\text{BE}}$, on the other hand, the vortices are dilutely populated and can be regarded as phase defects with $\langle|\phi|^2\rangle$ remaining nearly unity. The role of ψ and ϕ can be interchanged in the above discussion, and the following conclusions are obtained. It is impossible that only one of the bose condensation and the fermion pairing occurs without accompanying the other. The phase transition is replaced by a crossover with the gradual development of the amplitude of the order parameter. The only true phase transition is the superconducting one at which both the bose condensation and the fermion pairing occur simultaneously. When $T_D>T_{\text{BE}}^{(0)}$ and $T_{\text{BE}}>T_D^{(0)}$ are satisfied the amplitudes of ϕ and ψ grow almost simultaneously at $T_c=\max(T_D,T_{\text{BE}})$. This situation corresponds to the concentration region with high-T_c ($0.05<\delta<0.3$). For higher δ (overdoped case) the amplitude of ϕ develops while its phase remains disordered with the correlation length $\xi\sim n_V^{-1/2}\sim e^{E_V(T)/2T}$ in the temperature region $T_D^{(0)}=T_c<T<T_{\text{BE}}$. This exponentially large ξ makes it difficult to distinguish the system from the ordinary Fermi liquid. Actually, ξ can easily exceed the thermal length $\hbar v_F/k_BT$ of the fermions and the bosons can be regarded as a condensate within the thermal length.

The above discussion can be generalized to the case of nonzero external magnetic field H. In the T-H plane, we concentrate on the small region near $(T,H)=(T_c,0)$ and discuss the slope $s=dH_{c2}(T)/dT|_{T=T_c}$. When δ is small enough and $T_{\text{BE}}^{(0)}<T_D$, the spinon pairing is already developed in amplitude at $T=T_c=T_{\text{BE}}^{(0)}$, and the superconducting properties are determined by the Bose condensation. The slope s is very steep determined by $\xi_B(T)$,

and the coherence length $\xi(0)$ estimated from s becomes $\xi_B(0)\sim\delta^{-1/2}$, which is of order 10 Å. When δ is large enough and $T_D^{(0)}<T_{\rm BE}$, the superconducting properties are determined by the fermion pairing. The slope s is smaller than the above case, and $\xi(0)$ estimated from it is $\xi_F(0)\sim J/\Delta$ which is of the order 100 Å. In the intermediate region of δ where $T_D>T_{\rm BE}^{(0)}$ and $T_{\rm BE}>T_D^{(0)}$, the situation is complicated, but we expect that the slope s is larger than that expected from the BCS theory using a reasonable value of the bandwidth because the boson character remains important in this region. The screening of the magnetic field by the gauge field occurs, and the fermions and bosons feel reduced fields the sum of which is the external magnetic field H. This effect also increases $H_{c2}(T)$ and hence s, and the coherence length $\xi(0)$ estimated from the slope s is very short in this intermediate concentration region compared with the overdoped region. As a result, a more rapid increase of ξ is predicted as the concentration δ is increased towards the overdoped region than is expected from the relation $\xi(0)\propto T_c^{-1}$. The scenario of the single superconducting T_c at which both $|\psi|$ and $|\phi|$ develop substantially can be applied also in the case of nonzero magnetic field by regarding the transition temperatures $T_D^{(0)}(H), T_{\rm BE}^{(0)}(H)$ and $T_D(H), T_{\rm BE}(H)$ as functions of H. This means that in the normal state realized by applying magnetic field below T_c, both ψ and ϕ are destroyed as in the normal phase above T_c in the intermediate concentration region.

Next we discuss the structure of a single vortex in the ordered phase assuming that the system is in the type-II limit which is reasonable for the high-T_c superconductors. The GL equations are obtained by putting the functional derivative of the free energy F with respect to ψ, ϕ, **a**, and **A** to be zero.

$$-\xi_F^2\left[\nabla-2i\mathbf{a}-\frac{2ie}{c}\mathbf{A}\right]^2\psi+(|\psi|^2-1)\psi=0\ , \tag{10a}$$

$$-\xi_B^2(\nabla-i\mathbf{a})^2\phi+(|\phi|^2-1)\phi=0\ , \tag{10b}$$

$$\chi_F\nabla^2\left[\mathbf{a}+\frac{e}{c}\mathbf{A}\right]+\chi_B\nabla^2\mathbf{a}=-\frac{1}{2e}(\mathbf{J}_F^S+\mathbf{J}_B^S)\ , \tag{10c}$$

$$\nabla^2\mathbf{A}+\frac{8\pi e\chi_F}{c}\nabla^2\left[\mathbf{a}+\frac{e}{c}\mathbf{A}\right]=-\frac{4\pi}{c}\mathbf{J}_F^S\ , \tag{10d}$$

where the supercurrents $\mathbf{J}_F^S$ and $\mathbf{J}_B^S$ are given by

$$\mathbf{J}_F^S=\frac{c^2}{16\pi e\lambda_F^2 i}\left[\psi^*\left[\nabla-2i\mathbf{a}-\frac{2ie}{c}\mathbf{A}\right]\psi-\text{c.c.}\right]\ , \tag{11a}$$

$$\mathbf{J}_B^S=\frac{c^2}{8\pi e\lambda_B^2 i}[\phi^*(\nabla-i\mathbf{a})\phi-\text{c.c.}]\ . \tag{11b}$$

We are now interested in the cylindrically symmetric solution, and assume

$$\psi(\mathbf{r})=f(r)e^{iq\theta}\ , \tag{12a}$$

$$\phi(\mathbf{r})=b(r)e^{ip\theta}\ , \tag{12b}$$

$$\mathbf{a}(\mathbf{r})=a(r)\hat{\boldsymbol{\theta}}\ , \tag{12c}$$

$$\mathbf{A}(\mathbf{r})=A(r)\hat{\boldsymbol{\theta}}\ , \tag{12d}$$

where p,q are integers and $\hat{\boldsymbol{\theta}}$ is the unit vector $(-\sin\theta, \cos\theta)$. Putting Eqs. (12) into Eqs. (10) and (11), we obtain

$$-\frac{1}{r}\frac{d}{dr}\left[r\frac{df}{dr}\right]+Q^2f=\xi_F^{-2}(1-f^2)f\ , \tag{13a}$$

$$-\frac{1}{r}\frac{d}{dr}\left[r\frac{db}{dr}\right]+P^2b=\xi_B^{-2}(1-b^2)b\ , \tag{13b}$$

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left\{r\left[\chi_F\left[a+\frac{e}{c}A\right]+\chi_B a\right]\right\}\right]=\frac{c^2}{16\pi e^2}\left[\frac{Qf^2}{\lambda_F^2}+\frac{2Pb^2}{\lambda_B^2}\right]\ , \tag{13c}$$

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left\{r\left[A+\frac{8\pi e\chi_F}{e}\left[a+\frac{e}{c}A\right]\right]\right\}\right]=\frac{c}{2e\lambda_F^2}Qf^2\ , \tag{13d}$$

where

$$Q(r)=\frac{q}{r}-2a(r)-\frac{2e}{c}A(r)\ , \tag{14a}$$

$$P(r)=\frac{p}{r}-a(r)\ . \tag{14b}$$

Now we investigate the behavior of the solution to Eqs. (13). In the limit $r\to\infty$, both f and b are unity and both Q and P should go to zero faster than r^{-1} because otherwise the energy diverges at least logarithmically. Therefore, we obtain from Eqs. (14)

$$q-2Ra(R)-\frac{2e}{c}RA(R)\to 0\ , \tag{15a}$$

$$p-Ra(R)\to 0\ , \tag{15b}$$

as $R\to\infty$. Recognizing

$$2\pi RA(R)=\oint_{|\mathbf{r}|=R}\mathbf{A}\cdot d\mathbf{r}=\int\int_{|\mathbf{r}|<R}\nabla\times\mathbf{A}\cdot d\mathbf{S}\ , \tag{16}$$

we conclude that the total magnetic flux Φ penetrating the system is

$$\Phi=\lim_{R\to\infty}2\pi RA(R)=\frac{\pi c}{e}(q-2p)\ , \tag{17}$$

which is the multiple of $\phi_0=\pi c/e$ (which is $hc/2e$ when we recover h) because $q-2p$ is an integer. From Eq. (10c) the penetration depth λ_a for the gauge field is estimated as

$$\frac{1}{\lambda_a^2}=\frac{c^2}{16\pi e^2(\chi_F+\chi_B)}\left[\frac{1}{\lambda_F^2}+\frac{1}{\lambda_B^2}\right]\ , \tag{18}$$

and $\lambda_a\ll\lambda_F,\lambda_B$ because $c^2/e^2J\gg 1$. For r larger than ξ_F,ξ_B, and λ_a, f and b can be regarded as unity, and we can neglect the left-hand side of Eq. (10c). In this region the London equation (13d) is transformed with the help of Eqs. (13c) and (14) as

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}(r\hat{A})\right]=-\frac{1}{\lambda_F^2+\lambda_B^2}\hat{A}\ , \tag{19}$$

where

$$\hat{A}=A-\frac{c}{2e}\frac{q-2p}{r} .$$

Equation (19) is the same as the usual London equation with the penetration depth λ given by

$$\lambda^2=\lambda_F^2+\lambda_B^2 . \quad (20)$$

Therefore, the solution of Eq. (19) is given by

$$H(r)=\frac{(q-2p)\Phi_0}{2\pi\lambda^2}K_0\left[\frac{r}{\lambda}\right] , \quad (21)$$

where K_0 is the zero-order Bessel function of an imaginary argument, and the contribution E_1 to the excitation energy of a vortex from this region is given by

$$E_1=(q-2p)^2E_0 , \quad (22)$$

where

$$E_0=\left[\frac{\phi_0}{4\pi\lambda}\right]^2\ln\left[\frac{\lambda}{\max(\xi_F,\xi_B)}\right] .$$

In the normal core region, particularly in the limit $r\rightarrow 0$, the behavior of f and b is determined by $m_f=\lim_{r\rightarrow 0}rQ(r)$ and $m_b=\lim_{r\rightarrow 0}rP(r)$, respectively. From Eqs. (13a) and (13b) $f\sim r^{|m_f|}$ and $b\sim r^{|m_b|}$ for small r. In order for the magnetic energy F_{em} in Eq. (7) to be finite, the physical magnetic field H cannot be singular at $r=0$. The gauge field **a**, on the other hand, is defined on a lattice originally, and we have the natural short distance cutoff of the order of the lattice constant. Therefore, the singular gauge transformation as $a\sim 1/r$ is possible, and $m_f(m_b)$ is not necessarily equal to $q(p)$ although m_f-2m_b is equal to $q-2p$. However, this singular gauge will cost an energy of the order of $\chi_F+\chi_B\sim J$ which destabilizes the vortex with $m_f\neq q(m_b\neq p)$.[9] Therefore, we consider only the case where $m_f=q$ and $m_b=p$. The two possibilities for the lowest energy configuration are [1] $q=\pm 1$, $p=0$, and [2] $q=0$, $p=\pm 1$. In case [1] the spinon pairing is destroyed at the center while the holon condensation remains. Hence the normal state appearing in the core region is the usual Fermi liquid. The flux quantum is ϕ_0 and the energy $E_{[1]}$ of the vortex is estimated as

$$E_{[1]}=E_0+c_1\left[\frac{\phi_0}{4\pi\lambda_F}\right]^2 , \quad (23)$$

where c_1 is a constant of the order of 0.1.[7] In case [2] the holon condensation is destroyed at the center while the spinon pairing remains, which means that the "spin gap state" appears in the normal core region. The flux quantum is $2\phi_0$ and the energy $E_{[2]}$ of this vortex is given by

$$E_{[2]}=4E_0+c_1\left[\frac{\phi_0}{4\pi\lambda_B}\right]^2 . \quad (24)$$

Under the external magnetic field $2E_{[1]}$ and $E_{[2]}$ should be compared. In the type-II limit E_0 is larger than the core energy. Therefore, $E_{[2]}$ is larger than $2E_{[1]}$ and the vortex of type [1] is realized. The vortex of type [2] is stabilized only when $\lambda_F\ll\lambda_B$ which is realized near T_c in the low-concentration region. We wish to caution, however, that this result is obtained only in the mean-field theory, and inclusion of fluctuations may destabilize the type-[2] vortex. For example, the coupling to gauge-field fluctuations may make the transition first order,[11] in which case there may be no region of stability for the single flux quantum vortex.

In summary, we have developed the GL theory of the spin-charge-separated system. There are two order parameters corresponding to the spinon pairing and the holon condensation which are coupled through the gauge field. The Kosterlitz-Thouless type phase transition in 2D is replaced by a crossover when either the holon condensation or the spinon pairing occurs. Therefore, we have only the superconducting phase transition at which both of them occur simultaneously. The combination laws for the physical quantities discussed in the normal state thus far are extended to the superconducting phase, and we obtain $\lambda^2=\lambda_F^2+\lambda_B^2$. The coherence length ξ, on the other hand, is given by ξ_B and ξ_F for small and large δ limits, respectively, but is complicated in the intermediate high-T_c region. However, ξ is expected to be very short in this region so that the system is in the type-II limit. We find the two possible types of the vortex with the flux quantization $hc/2e$ and hc/e, respectively. In the type-II limit the former is stable in almost all the cases while the latter is stable near T_c in the low-concentration region.

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