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Quasiparticle effective mass and enhanced g factor for a two-dimensional electron gas at intermediate magnetic fields

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The single-particle self-energy in a Landau quantized two-dimensional electron gas has been evaluated in magnetic fields corresponding to arbitrarily large integer filling factors using the random-phase approximation. Quasiparticle energy separations have been determined for Landau levels near the Fermi level and comparisons are made with values determined from recent activation energy measurements on integral quantum Hall plateaus. Conclusions are drawn concerning the field strengths required for the validity of commonly adopted strong-field approximations. A series of transitions characterized by jumps in the spin polarization is predicted to occur near odd filling factors for sufficiently low densities.

The effect of electron-electron interactions on many properties of an electron-gas system can be expressed in terms of the shifts produced in the positions of poles in the one-particle Green's function. Usually it is only the relative changes in the positions of poles near the Fermi level which are physically relevant. In the absence of a magnetic field these may be expressed in terms of two parameters:

$$m/m^* = \left.\frac{dE_\sigma(k)/dk}{d\varepsilon_\sigma(k)dk}\right|_{k=k_F} \tag{1}$$

and

$$g^*/g = \frac{E_+(k_F) - E_-(k_F)}{\varepsilon_+(k_F) - \varepsilon_-(k_F)}. \tag{2}$$

Here $E_\sigma(k)$ is the quasiparicle energy at which the Green's function poles occur and $\varepsilon_\sigma(k)$ is the electron energy in the absence of interactions. m and g are the bare electron mass and g factor, respectively, and m/m^* and g^*/g measure the enhancement of the wave vector and spin dependence of the quasiparticle energy, respectively. (k_F is the Fermi wave vector.) Note that the cyclotron effective mass measured by infrared spectroscopy is, like the zero-field plasma frequency, a property of the whole electron gas at long wavelengths, and is not directly related to the the quasiparticle effective mass calculated here. In particular, Kohn's theorem[1] implies that the cyclotron mass, unlike the quasiparticle mass, is not altered by electron-electron interactions.

The evaluation of these two quantities has long[2] been a central problem of the application of many-body perturbative techniques to the electron gas. In the two-dimensional (2D) case there have been numerous[3–9] calculations of m/m^* and g^*/g at zero and weak magnetic fields following the early work of Janak[10] showing a large exchange enhancement of g^* at low density. At stronger magnetic fields, in 2D, Landau quantization becomes important and it is known experimentally[11–15] that the g-factor enhancement can become extremely large whenever the Fermi level lies between spin-split Landau levels. The large enhancements are explained qualitatively by the calculation of Ando and Uemura[16] who evaluated the self-energy including Landau quantization in a static screening approximation. In the extreme strong-field limit, at least for integer filling factors,[17] the importance of interaction effects is reduced again because of the suppression of density fluctuations by kinetic-energy quantization which creates an energetic separatation between occupied and empty Landau levels. In this paper we present the first calculations of quasiparticle energy differences near the Fermi level using a dynamic screening approximation which is expected to be reliable at arbitrary field strength. We find that quantitatively important corrections to the commonly adopted strong-magnetic-field approximation, in which Landau-level mixing is neglected, occur at typical densities for all integral filling factors larger than 1. At weak fields we recover results obtained from zero-field calculations. At stronger fields good agreement with values determined from recent activation energy measurements[15] on integral quantum Hall plateaus is obtained.

We consider a 2D electron gas in a perpendicular magnetic field, with electrons interacting via a Coulomb interaction and calculate the corrections to the quasiparticle energy of electrons in each Landau level resulting from many-body effects. At zero temperature and integer filling factor, $\nu = n$, the ground state in the absence of interactions is a single Slater determinant and the system has a finite excitation gap equal to the Landau-level separation. To lowest order the self-energy has only the exchange contribution of the Hartree-Fock (HF) approximation which can be evaluated exactly.[18] We approximate the correlation contributions to the self-energy using the random-phase approximation (RPA) in which the interaction in the exchange term is dynamically screened. The frequency dependence is quantitatively important at all fields, although qualitatively similar results can be obtained in a static screening approximation. For the frequency-dependence we use a single-pole approximation as was done at zero field for 2D by Vinter.[6]

In the following we use quasiatomic units with $a_0^* = \hbar^2\epsilon_r/m_b e^2$ as the length unit, and the unit of energy is $2R_0^* = e^2/\epsilon_r a_0^*$, with m_b the band mass and ϵ_r the bulk dielectric constant. The bare one-particle energies of the states in Landau level n with spin σ $(= \pm\frac{1}{2})$ are given by

$$\omega_{n,\sigma} = (2n + g_{\text{eff}}\sigma)\gamma, \tag{3}$$

where $g_{\text{eff}} = gm_b/m$, and $\gamma = \hbar\omega_c/2R_0^*$ is the dimensionless magnetic field. The RPA self-energy approximation is summarized diagramatically in Fig. 1. Using finite-temperature perturbation theory,

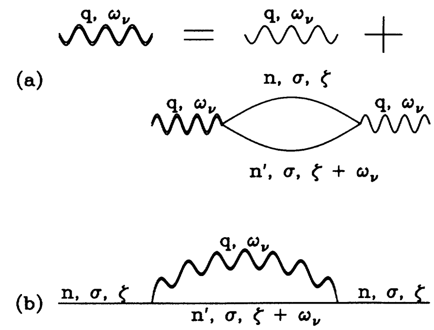

FIG. 1. Feynman diagrams. (a) is the bubble diagram in the density-density response, and (b) is the diagram used for the self-energy calculation.

$$\Sigma_{n,\sigma}^{RPA}(i\zeta_\nu) = -\frac{1}{\beta}\int\frac{d^2q}{(2\pi)^2}\sum_{n'}P_{n,n'}(q^2l^2/2)\cdot\sum_{\omega_\mu}\frac{V_c(q)}{\epsilon(q,i\omega_\mu)}\frac{1}{i(\zeta_\nu-\omega_\mu)-(\omega_{n',\sigma}-\mu)}, \tag{4}$$

where ω_μ is a Boson Matsubara frequency, ζ_ν is a Fermion Matsubara frequency, $V_c(q) = 2\pi/q$ is the two-dimensional Coulomb interaction, and $\epsilon(q,\omega) = 1 - V_c(q)\Pi^0(q,\omega)$ is the frequency-dependent dielectric function. Here $\Pi^0(q,\omega)$ is the polarization function for a noninteracting electron system,

$$2\pi l^2\Pi^0(q,i\omega_\nu) = \sum_\sigma\sum_{n,n'}P_{n,n'}(q^2l^2/2)\frac{f(\omega_{n,\sigma})-f(\omega_{n',\sigma})}{i\omega_\nu+\omega_{n,\sigma}-\omega_{n',\sigma}}. \tag{5}$$

$P_{n,n'}$ is proportional to the total oscillator strength for transitions from any state in Landau level n to any state in Landau level n':

$$\begin{aligned}P_{n,n'}(x) &= |<n'|e^{i\mathbf{q}\cdot\mathbf{r}}|n>|^2\\ &= (-1)^{n+n'}e^{-x}\mathrm{L}_{n'}^{n-n'}(x)\mathrm{L}_n^{n'-n}(x),\end{aligned} \tag{6}$$

where $x = q^2l^2/2$, and $f(\varepsilon)$ is the Fermi function. Details on the efficient evaluation of $\epsilon(q,i\omega_\mu)$ and other numerical aspects of this work will appear elsewhere.

In the HF approximation ($\Pi^0 \equiv 0$) the q integral for each n' can be evaluated analytically[19] and takes the form

$$\Sigma_{n,\sigma}^{ex} = -\frac{1}{l}\sum_{n'}X_{n,n'}f(\omega_{n',\sigma}), \tag{7}$$

where $X_{n,n'}$ is a real number depending only on n and n'. To evaluate the remaining contribution to the self-energy, $\Sigma_{n,\sigma}^{\text{corr}}(\omega)$, we must perform a sum over Matsubara frequencies. Because we ultimately wish to analytically continue the result to real frequencies it is desirable to perform the sum analytically. This is possible, using the standard contour integration method, provided we know the locations and residues of the poles of the summand. Aside from the pole coming from the Green's function this means finding the poles in the complex frequency plane of $1/\epsilon(q,\omega) - 1$. Fortunately, the most important of these is that at the real frequency corresponding to the magnetoplasmon. We will, in the following, use a "magnetoplasmon-pole" approximation, in analogy to the plasmon-pole approximation used in the zero-field calculation of Vinter.[6] Thus we approximate

$$\frac{1}{\epsilon(q,\omega)} - 1 \approx -\frac{\alpha}{\pi}\left(\frac{1}{\omega-\omega_q+i\delta} - \frac{1}{\omega+\omega_q+i\delta}\right). \tag{8}$$

Following Ref. 6 we choose α and ω_q in order to satisfy the zero-frequency Kramers-Kronig relation and the f sum rule. We find

$$\alpha(q) = -\omega_p(q)^2 = -\pi nq, \tag{9}$$

where n is the density of the 2D electron gas, and

$$\omega_q^2 = \frac{\alpha(q)}{[1/\epsilon(q,0)] - 1}. \tag{10}$$

The resulting pole frequency is shown in Fig. 2. This approximation for the frequency dependence can be systematically improved by finding higher-order Padé approximants to the frequency dependence of $1/\epsilon - 1$ and the frequency sums can still be analytically evaluated. We have found that this improved approximation does not significantly change our results.

The correlation correction to the HF self-energy in the plasmon pole approximation is

$$\Sigma_{n,\sigma}^{\text{corr}}(\epsilon) = \sum_{n'}\int_0^\infty dqP_{n,n'}(q^2l^2/2)\cdot\times\left(\frac{f_1(q,\omega_{n',\sigma})}{\epsilon+\mu-\omega_q-\omega_{n',\sigma}} - \frac{f_2(q,\omega_{n',\sigma})}{\epsilon+\mu+\omega_q-\omega_{n',\sigma}}\right), \tag{11}$$

where

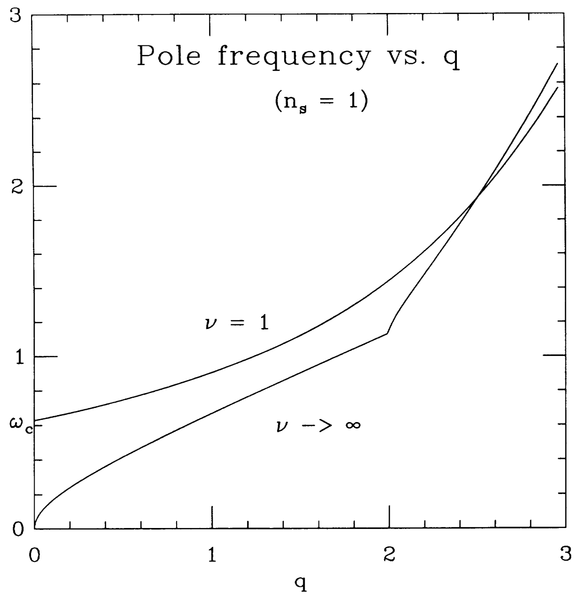

FIG. 2. The plasmon pole frequency of Eq. (8). In this figure and in the following figures the units are quasiatomic. For GaAs the density $n_s = 1$ corresponds to $9.7 \times 10^{11}\text{cm}^{-2}$, temperature $T = 0.1$ to 12.8 K, and field $\gamma = 1$ to 6.4 T.

$$f_{1,2}(q,E) = \frac{\omega_p^2}{2\omega_q}\left(\frac{1}{\exp[-\beta(E-\mu)]+1} + \frac{1}{\exp(\pm\beta\omega_q)-1}\right). \tag{12}$$

At zero field the quasiparticle mass is defined in terms of the rate of change of quasiparticle energy with momentum. At finite field we no longer have a continuous excitation spectrum, and it is natural to define an effective mass in terms of the rate of change of quasiparticle energy with the Landau-level index. $[m/m^* \equiv (E_{N+1} - E_N)/\hbar\omega_c]$, where N is the highest filled Landau level. (This mass will approach the zero-field mass in the limit $N \to \infty$.) The Dyson equation for the one-particle Green's function implies that[20]

$$E_n + \mu = n\omega_c + \text{Re}\{\Sigma_n^{\text{ex}} + \Sigma_n^{\text{corr}}(E_n)\}. \tag{13}$$

We have found that near the Fermi energy $\Sigma_n^{\text{corr}}(E)$ tends to be very close to linear, and therefore that it can be approximated as

$$\Sigma_n^{\text{RPA}}(E) - \Sigma_n^{\text{RPA}}(0) \approx E\left.\frac{\partial\Sigma_n^{\text{RPA}}}{\partial E}\right|_{E=0}. \tag{14}$$

The effective mass can then be immediately obtained:

$$\frac{m^*}{m_b} = \frac{1-\left\langle\frac{\partial\Sigma^{\text{RPA}}}{\partial E}\right\rangle}{1+[\Delta\Sigma^{\text{ex}}+\Delta\Sigma^{\text{corr}}(0)]/\omega_c}, \tag{15}$$

where $\Delta\Sigma = \text{Re}\{\Sigma_{N+1} - \Sigma_N\}$ and the frequency derivative is a weighted average of the (real part of the) derivatives at N and $N+1$.

Figure 3(a) shows our results for the dependence of the effective mass on density and filling factor. At strong fields where the HF approximation is accurate the Landau-level separation is enhanced by interactions by $\sim e^2/\epsilon l \sim \sqrt{B}$ so that $m/m^* - 1 \sim 1/\sqrt{B}$. The correlation correction to this quantity increases as the field is lowered. In the weak-field limit it is known [and can be seen from Fig. 3(a)] that because of cancellations between contributions from the wave vector and frequency dependence of the self-energy m/m^* is always quite close to and, for typical densities, smaller than 1. Typical GaAs 2D electron densities are 0.01–1.0 in our units; we see from Fig. 3(a) that in this density region the strong-field HF behavior holds only for $\nu = 1$.

Our results for odd filling factors for the effective g factor are shown in Fig. 3(b), where g_{eff}^* is defined by $E_{N,\uparrow} - E_{N,\downarrow} = \omega_c g_{\text{eff}}^*/2$, with N the highest filled Landau level. At low temperatures when the chemical potential lies between spin-split Landau levels (i.e., for ν odd), the system is partially polarized, and the g factor becomes very strongly enhanced due to exchange effects. (The bare value of g_{eff} is only 0.029 in GaAs and this remains nearly unchanged for even ν.) We again see from Fig.

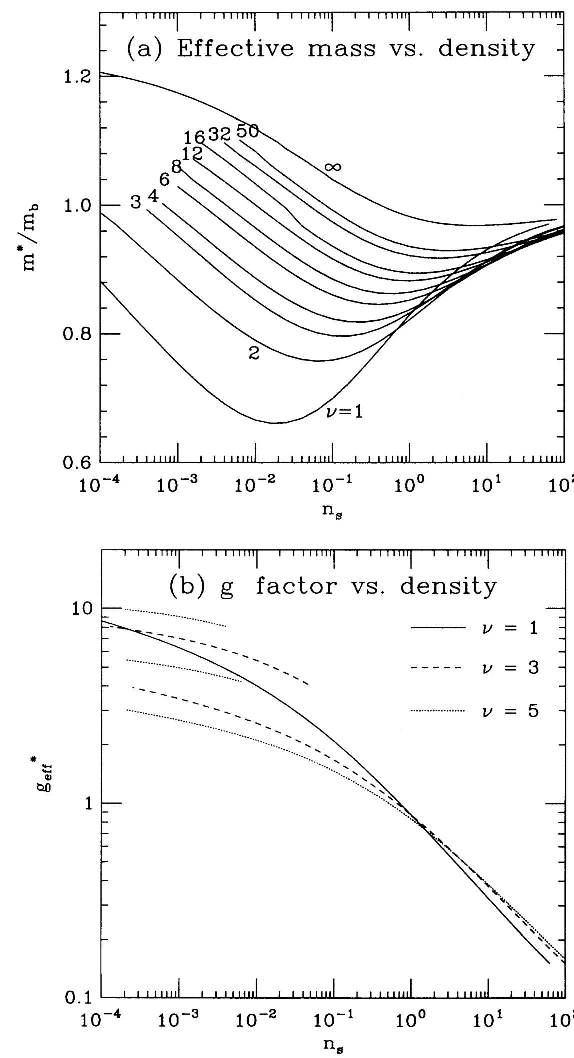

FIG. 3. (a) The effective mass and (b) g factor vs density for a range of filling factors ν. Note that there are several possible g_{eff}^* values for odd filling factors greater than 1 at low density, corresponding to different polarization states.

3(b) that for physical densities, particularly for $\nu > 1$, the strong-field HF value for the enhancement is substantially reduced by correlation contributions. Note also that at low densities solutions of the Dyson equations can be found for more than one spin polarization.

We note here that the correlation contribution in the RPA comes solely from inter-Landau-level excitations, since the bubble diagram of Fig. 1(a) is zero if $n = n'$, and at zero temperature the contribution is zero unless one of the levels $\{n, n'\}$ is below ϵ_F and one is above. Thus any approximation that neglects inter-Landau-level excitations is incorrect for filling factors where the correlation contribution is important, i.e., in particular, for all filling factors greater than 1 at typical physical densities.

Two important physical parameters have been neglected thus far, but must be included for comparison with experimental systems. These are the nonzero thickness of the "2D" layer, and the disorder broadening of the Landau levels. Modeling the first effect by a standard "form-factor"[16] multiplying the Coulomb interaction and the second by an effective temperature (scaling with $\omega_c^{1/2}$) the g-factor enhancements of Fig. 3(b) become substantially reduced, and we obtain good agreement with the experiments of Usher *et al.*[15] as shown in Fig. 4, for physically reasonable values of these two parameters. Many other experiments[11–14] have studied enhanced g factors at both weak and strong magnetic fields. These should be well described by our theory, but quantitative comparison is made difficult by the different samples used with different thicknesses and mobilities, and that these experiments used a tilted field geometry.

In conclusion, we find that the many-body effects associated with the screening of the Coulomb interaction due to inter-Landau-level excitations are important for $\nu \geq 2$ for physical ranges of electron density. The experiments of Usher *et al.* on activation energies in the quantum Hall effect are in the region of this crossover from low-field (and low-density) many-body behavior to high-field (high-density) HF behavior, as can be seen from Fig. 4, and we obtain good fits to their data by taking into account finite thickness and level broadening effects. Although these act to reduce the g-factor enhancement, it is possible that the polarization transitions to be expected from Fig. 3(b) may still be experimentally accessible.

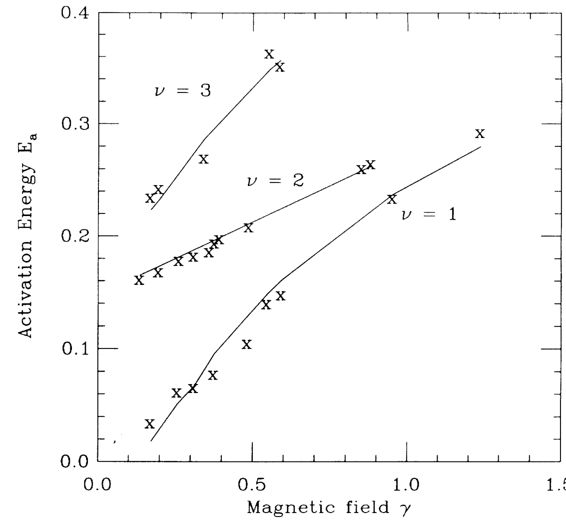

FIG. 4. Calculated Landau-level separations compared to the activation energies from Figs. 3 and 4 of Reference 15. The $\nu = 2$ curve is scaled by $\frac{1}{10}$ and shifted vertically 0.15 for clarity, and the $\nu = 3$ curve is shifted by 0.2. The X's are experimental points and the solid curves are fits using the 2 parameters of an effective thickness (31 Å, 10 Å, and 35 Å for $\nu = 1, 2, 3$ — note $a_0 = 101.5$ Å in GaAs) and broadening factor $[T = (C\omega_c)^{1/2}$ with $C = 0.33$ K, 0.2 K, and 0.08 K]. The $\nu = 2$ fit was not very sensitive to the two parameters, so these seem to give a consistent value for thickness and a broadening parameter that decreases with ν.

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