Conductivity and magnetoresistance of magnetic multilayered structures

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We consider multilayered structures consisting of magnetic and nonmagnetic metals. We derive analytic expressions for the conductivities by treating the scattering at the interfaces between layers in the same way as that throughout the layers (bulk). The application of an external magnetic field reorients the magnetization in the magnetic layers, which in turn alter the mean free path of the conduction electrons. This is the origin of the giant magnetoresistance seen in iron-chromium (Fe/Cr) superlattices. We present analytic results on the magnetoresistance in limiting cases where either the mean free path is much greater or less than the layer thickness, as well as numerical results for the realistic situation found in Fe/Cr superlattices when they are comparable.

I. INTRODUCTION

Recently, giant magnetoresistance effects have been found in Fe/Cr magnetically layered structures[1–5] for current in the plane of the layers. One finds that as the antiferromagnetically coupled adjacent layers of iron are brought into parallel alignment by an external magnetic field, the resistance drops. In some cases this decrease is over 40%. This phenomenon is distinct from the conventional magnetoresistance due to the effect of magnetic fields directly on the conduction electrons or on the scattering of conduction electrons by local impurities. Rather the extraordinary magnetoresistance in multilayered structures comes from the *reorientation* of magnetic moments by the field.

The theoretical understanding of this effect has been based on the pioneering work of Fuchs and Sondheimer;[6] this was initially applied to the resistivity of thin films due to surface roughness, and extended to multilayers by Carcia and Suna.[7] The basic idea in these treatments is that the scattering at surfaces and interfaces can be represented by a reflection coefficient p (not to be confused with the p introduced in the article), which is zero for perfectly rough surfaces and one for perfectly smooth surfaces. The former corresponds to diffuse reflection (all loss of directionality) and the latter to specular reflection, i.e., not producing any resistance. A discussion of the physics behind this approach is found in Ziman's book; see Ref. 6. Recently the giant magnetoresistance of iron-chromium multilayered structures was analyzed by using this quasiclassical approach by including the spin-dependent interface scattering as well as a spin-dependent bulk scattering.[8] This approach does not treat the interface roughness scattering on an equal footing with that coming from the bulk; therefore, *inter alia,* for Fe/Cr *superlattices* with high resistivities (about 80 $\mu\Omega$ cm) we find it underestimates the contributions to the resistivity from the interfaces when the layer thickness is smaller than the mean free path.[9]

The main drawback of the semiclassical approach is that one treats the scattering of conduction electrons due to the roughness of the interfaces (surfaces) differently from scattering in the layers (bulk). That is, the interfacial scattering is treated phenomenologically by introducing the proportions of electrons transmitted, reflected and scattered from the interfaces. What is needed is a *unified* treatment of the scattering from the interfaces and bulk, e.g., the approach used by Tešanovic, Jarić, and Maekawa to discuss the resistivity of *thin* films coming from the roughness of their surface.[10] The extension of their treatment to multilayered structures is complicated because (1) the spatial inhomogeneities in layered structures, which produce the resistivity, cannot be reduced to a homogeneous, translationally invariant, scattering potential as is usually done by averaging over randomly situated impurities and (2) the conduction electrons are scattered by magnetic impurities whose spin direction vary from one layer to another. The combination of these spatial and spin inhomogeneities leads to a *position-dependent* conductivity that depends on the orientation of the magnetization of the individual layers. To determine the electrical transport properties of these extended periodic structures we have extended the approach of Tešanovic, Jarić, and Maekawa in three ways: (1) we consider the surface roughness scattering from a periodic array of interfaces, not just one, (2) we make the scattering dependent on spin, and (3) we explicitly solve for the conductivity to all orders of the surface scattering, i.e., we do not limit ourselves to lowest order. By using the Kubo formalism, we have succeeded in deriving the position-dependent conductivity for layered structures.

While our primary intention is to apply our result to periodic superlattices, with dimensions L much greater than the mean free path of the electrons λ, our formalism is equally applicable to cases where there is no periodicity and to sandwich structures of finite thickness, $L \lesssim \lambda$, e.g., Fe/Cr/Fe (Refs. 3, 4, and 8) We find, *inter alia,* that, while the translationally invariant homogeneous trans-

port properties of these inhomogeneous structures are characterized by a mean free path that is related to the *diagonal* part of the conduction-electron Green's function, their position dependence comes from the *off-diagonal* parts of the Green's function. Also we find interfacial scattering contributions to the mean free path, while in the quasiclassical approach, the mean free path is due solely to bulk scattering.

In the next section we introduce the local position-dependent conductivity $\sigma(z)$ and relate it to the conductivities measured for currents parallel to the layers of these structures. Then we derive $\sigma(z)$ for magnetic multilayered structures. In Sec. IV we consider various limiting cases of our formalism and show that our results reduce to the conventional wisdom. Next we apply our formalism to the effect of magnetic fields on the transport in magnetic superlattices. In Sec. VI we apply our result to analyze the magnetoresistance of iron-chromium superlattices in terms of the spin-dependent bulk (in the layer) and interfacial scattering present in these superlattices. We close by discussing our results and indicating how our formalism applies to multilayered structures other than the superlattices we have explicitly worked out in detail.

II. LONGITUDINAL CONDUCTIVITY

Due to the inhomogeneous nature of multilayered structures, the application of a uniform electric field does not ensure that the internal field is uniform. For a field *parallel* to the layers uniformity is maintained; however, perpendicular to the layers the internal field varies from one layer to the next. Therefore, even when the local conductivity tensor is isotropic, as it would be for cubic metals such as iron and chromium, one finds that the global or measured conductivity is anisotropic, i.e., different for electric fields parallel and perpendicular to the layers.

We consider the layers to be the x-y plane and to be stacked along the z direction. In general the current is given as

$$\mathbf{J}(\mathbf{q})=\int d\mathbf{q}'\vec{\sigma}(\mathbf{q},\mathbf{q}')\cdot\mathbf{E}(\mathbf{q}') \ . \tag{2.1}$$

For a uniform electric field *parallel* to the layers, e.g., along the $\hat{\mathbf{x}}$ direction, the current in this direction is

$$J_x(\mathbf{q})=\sigma_{xx}(\mathbf{q},0)E \ . \tag{2.2}$$

In the x-y plane the system is homogeneous, and

$$\sigma_{xx}(\mathbf{q},0)=\sigma(\bar{\nu})\delta_{q0} \ , \tag{2.3}$$

where $\sigma(\bar{\nu})\equiv\sigma_{xx}(q_z,0)$, $q\equiv q_x q_y$, and $\bar{\nu}\equiv q_z$. The longitudinal conductivity for a sample of macroscopic dimensions is $\sigma(\bar{\nu}=0)$. As we presently show one cannot directly calculate $\sigma(\bar{\nu}=0)$, but we can find its Fourier transform,

$$\sigma(z)=\frac{L}{2\pi}\int_{-\infty}^{\infty}e^{i\bar{\nu}z}\sigma(\bar{\nu})d\bar{\nu} \ , \tag{2.4}$$

where L is the length of the sample in the z direction. We define the current in the layer planes (CIP) conductivity as

$$\sigma_{\parallel}=\frac{1}{L}\int_{L}\sigma(z)dz \ . \tag{2.5}$$

This average is synonomous with $\sigma(\bar{\nu}=0)$; as we are unable to directly evaluate $\sigma(\bar{\nu}=0)$ in closed form, we proceed as indicated above to find $\sigma_{\parallel}$.

When we include the spin, each spin direction contributes independently, at least at low enough temperature so that there is no spin-flip scattering by spin waves; then the total conductivity is the sum from each direction, i.e.,

$$\sigma=\sum_{s}\sigma(s) \ . \tag{2.6}$$

As long as we neglect spin-orbit coupling this result (in our model) is independent of the direction of the current because then the direction of the spin is decoupled from the direction of the electron motion.

III. CALCULATIONS OF CONDUCTIVITY

To calculate the position-dependent local conductivity $\sigma(z)$, see Eq. (2.4), we model the transport properties of magnetic multilayered structures by considering conduction-electron subject to *bulk* scattering, which occurs throughout the sample, and *interfacial roughness* scattering, which occurs at the interfaces between the iron and chromium layers. We confine ourselves to low temperatures and neglect phonon and magnon scattering; the latter should determine the temperature dependence of the magnetoresistance. Here we explicitly refer to Fe/Cr superlattices; the main idea is to consider multilayered structures with alternating magnetic and nonmagnetic layers. The extension to aperiodic layered structures is straightforward. The scattering at the interfaces comes from their roughness; iron atoms find themselves in the chromium layer and vice versa. As the iron layers are magnetized this produces *spin-dependent interfacial roughness* scattering.[1] The bulk scattering comes from imperfections and impurities in the iron and chromium layers.

By following Tešanovic, Jarić, and Maekawa[11] we represent the interface roughness scattering by a potential that is random in the plane of the interface and a δ function in the third dimension. We consider the iron and chromium layers to be parallel to the x-y plane and to be stacked along the z direction. We denote the thickness of the iron (magnetic) and chromium (nominally nonmagnetic) layers a and b, and in order to consider antiferromagnetic ordering we take the period of the superlattice (along z) to be $T\equiv 2(a+b)$. The scattering potential giving rise to resistivity is

$$V(\mathbf{r},\hat{\boldsymbol{\sigma}})=\sum_{i}V_i^b(\hat{\boldsymbol{\sigma}})\delta(\mathbf{r}-\mathbf{R}_i)+\sum_{l}V_l^s(\hat{\boldsymbol{\sigma}})f_l(\rho)\delta(z-z_l) \ , \tag{3.1}$$

where $\mathbf{r}\equiv\rho,z$ (z is normal to the layers), $\mathbf{R}_i$ is the position of an impurity or defect, z_l is the position of the lth Fe/Cr interface, $f(\rho=x,y)$ represents interface roughness, the sum over i is *per unit volume,* and sum over l is *per unit length.* The spin-dependent potentials are written as

$$V(\hat{\sigma}) \equiv v + j\hat{\mathbf{M}}_{\mathrm{Fe}} \cdot \hat{\sigma} \ , \tag{3.2}$$

where the operator $\hat{\sigma}$ represents the Pauli spin matrix, and $\hat{\mathbf{M}}_{\mathrm{Fe}}$ is a unit vector in the direction of the magnetization of an iron layer. For the interfaces we have v^s and j^s, while for the bulk the parameters are v^b_{Fe}, v^b_{Cr}, and j^b_{Fe}. The chromium layers are nominally nonmagnetic; therefore we set $j^b_{\mathrm{Cr}}=0$ in these layers. While the *bulk* scattering is randomly distributed throughout the layers, the *interface* scattering is confined to a plane $z=z_l$ representing an Fe/Cr interface.

Our model Hamiltonian is

$$H = \sum_{\mathbf{k}\sigma} \varepsilon_k c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \sum_{\mathbf{k}\sigma\mathbf{k}'\sigma'} V_{\mathbf{k}\sigma,\mathbf{k}'\sigma'} c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{k}'\sigma'} \ . \tag{3.3}$$

The conduction electrons are subject to different potentials in the iron and chromium layers, and, in principle, one should determine their wave functions for a Kronig-Penney-like potential representing the Fe/Cr superlattice. However, when (1) the Fermi level is far from the bottom of the potential for both metals, (2) there are no gaps at Fermi level, e.g., in the chromium layers due to the possibility of spin density waves, and (3) the size of the sample in the z direction L and the lattice periodicity $(a+b)$ (if there would be significant scattering from the Kronig-Penney-like potential) are much greater than the mean free path λ (so that there are no quantum well effects), it is reasonable to use plane waves to represent conduction electrons in calculating the transport properties of Fe/Cr superlattices. Therefore we take ε_k for free electrons which is, *inter alia,* spherical.

The position-dependent scattering [Eq. (3.1)] breaks the translational invariance of the system. However, by taking an average over the impurities that are situated at random *within a layer* and by using a random function $f(\rho)$ to represent the roughness of the interfaces, one restores this symmetry in the plane $(x$-$y)$ of the layers.

The self-energy or t matrix to first order in the scattering is real, and we will subsume it by shifting the energy. To second order in the scattering the t matrix near the Fermi surface is

$$t_{\bar{\nu}}(k\omega,\hat{\sigma})|_{\omega\approx\varepsilon_F} \equiv -i\pi \left[\sum_t \Delta^b_t(\hat{\sigma}) e^{-i\bar{\nu}z_t} + \sum_l \Delta^s_l(\hat{\sigma}) e^{-i\bar{\nu}z_l} \right] , \tag{3.4}$$

where

$$\begin{aligned} \Delta^b_t(\hat{\sigma}) &= \rho(\varepsilon_F)\langle [V^b_t(\hat{\sigma})]^2 \rangle_t \ , \\ \Delta^s_l(\hat{\sigma}) &= \rho(\varepsilon_F)[V^s_l(\hat{\sigma})]^2 \langle f^2_l \rangle \ , \end{aligned} \tag{3.5}$$

$\mathbf{k} \equiv (k,\nu)$ has been used, i.e., $k \equiv k_x,k_y$ has the directions in reciprocal space parallel to the layers, $\nu \equiv k_z$ is normal to the layers, $\bar{\nu} \equiv \nu - \nu'$, and $\rho(\varepsilon_F)$ is the density of states of conduction electrons at the Fermi surface. To arrive at this form we averaged the bulk scattering potential $V^b_t(\hat{\sigma})$ over a random distribution of impurities in a plane t parallel to the layers, and we used a "white noise" surface profile for the uncorrelated atomically rough Fe/Cr interface, i.e., we take $\langle |f_l(k)|^2 \rangle$ a constant independent of k (Ref. 11). For the matrix elements of the *scattering* potential to exist for all vectors $\bar{\nu}$, and not just those of the reciprocal lattice of the superlattice, it is necessary that the parameters Δ^s_l (Δ^b_t) are random. As the real parts of these parameters have the periodicity of the superlattice, we must make these parameters complex and take their imaginary parts to be random. In addition, we make the assumption that the Δ's are independent of ν near the Fermi surface. In normal metal, i.e., no Kondo effect or resonant scattering at Fermi surface, this is an entirely reasonable approximation.

The loss of translational invariance requires us to calculate the current at finite $\mathbf{q}$ even though we apply a *uniform* electric field. We calculate the conductivity $\sigma(z)$ rather than its Fourier transform, by using the Kubo formalism[12] in which the current response is given in terms of Matsubara Green's functions,

$$\sigma(\bar{\nu}) = -\lim_{\omega\to 0} \left[\frac{1}{\omega} \mathrm{Im}\Pi(\bar{\nu},\omega) \right] , \tag{3.6}$$

where

$$\Pi(\bar{\nu},\omega) = \frac{1}{2} \left[\frac{e}{m} \right]^2 \sum_{k\sigma,\sigma'} k^2 \sum_{\nu\nu'} \frac{1}{\beta} \sum_{i\nu_m} G^{\sigma\sigma'}_{\nu+\bar{\nu}\nu'}(k, i\nu_m + i\omega_m) G^{\sigma'\sigma}_{\nu'\nu}(k, i\nu_m)|_{i\omega_m = \omega + i0} \ ,$$

for the CIP conductivity $\sigma_{\parallel}(\bar{\nu})$. $G^{\sigma\sigma'}_{\nu\nu'}(k,i\omega)$ is a Matsubara Green's function, which is diagonal in the indices $k=k_x,k_y$ but has off-diagonal elements $\nu\nu'(k_z k'_z)$. This property follows directly from the translational invariance of the superlattices in a direction parallel to the layers. To arrive at the electron-hole correlation function $\Pi(\bar{\nu},\omega)$ we *assumed* there are no vertex corrections, i.e., that the function can be written as the product of electron and hole correlation functions.

While it is not possible for us to find an exact solution for the Green's function, we have made a number of plausible assumptions (key approximations) so that we do find the Fourier transforms of $\sigma(\bar{\nu})$. Before we derive our result, we list them: (i) we assume the concentration of scattering centers is low (the dilute limit)—this is plausible in the given context, (ii) in our self-energy [Eqs. (3.16) and (3.17)] we neglect interference terms from scattering at different sites—these are quite small, (iii) to derive Eq. (3.19) we assumed the averages are independent of the momentum ν near the Fermi surface, (iv) we use a spherical Fermi surface to evaluate the sum [Eq. (3.29)] over the Fermi sea, and finally (v) as we are considering only plane-wave states in our present calculation the mean free paths must be small compared to the thickness of the

sandwich or superlattice structure, i.e., $\lambda \ll L$, or if there is scattering due to the Kronig-Penney potential, compared to the period of the lattice $(a+b)$; otherwise, one must consider the confinement potential in the growth direction (z), and the attendant effects due to the quantization of the states.[10,11,13]

It remains for us to find the off-diagonal Green's function $G^{\sigma\sigma'}_{\nu\nu'}$. These functions satisfy the equation

$$G^{\sigma\sigma'}_{\nu\nu'}(k,\omega)=G^0_\nu(k,\omega)\delta_{\nu\nu'}\delta_{\sigma\sigma'} + G^0_\nu(k,\omega)T^{\sigma\sigma'}_{\nu\nu'}(k,\omega)G^0_{\nu'}(k,\omega) , \quad (3.7)$$

where we have used $G^{\sigma\sigma'}_{\nu\nu'}=G^0_\nu\delta_{\nu\nu'}\delta_{\sigma\sigma'}$ for *free* electrons, i.e., *inter alia*, we neglect the effect of the magnetic field on the conduction electrons; only the effect of the field on the local moments is considered in $T^{\sigma\sigma'}_{\nu\nu'}$. While similar in some aspects to the surface scattering problem solved by Tešanovic, Jarić, and Maekawa,[10] see their Eq. (5), the presence of several scattering interfaces in this multilayer problem does not allow us to write the scattering potential Eq. (3.1) in a separable form,

$$V_{\nu\nu'}(k-k')=\sum_l e^{-i(\nu-\nu')z_l}V_l(k-k') \neq f_\nu f^*_{\nu'}V(k-k') . \quad (3.8)$$

Only when there is a single scattering surface can one write $f_\nu=e^{-i\nu z_l}$ and proceed as in Ref. 10. Therefore, to obtain a solution to Eq. (3.7) we separate the self-energy into *diagonal* and *off-diagonal* parts (reverting to operator form instead of matrix elements),

$$\Sigma=\hat{\Sigma}+\Sigma' . \quad (3.9)$$

Then the Green's function Eq. (3.7) is written as

$$G=\frac{\hat{G}}{1-\hat{G}\Sigma'} , \quad (3.10)$$

where

$$\hat{G}\equiv[(G^0)^{-1}-\hat{\Sigma}]^{-1} . \quad (3.11)$$

The Green's function $\hat{G}$ has only diagonal matrix elements which are written as

$$\hat{G}^\sigma_\nu=\frac{1}{\omega-\varepsilon_{k\nu}+i\Delta^\sigma_{k\nu}(\omega)} , \quad (3.12)$$

where

$$\Delta^\sigma_{k\nu}=-\frac{1}{\pi}\mathrm{Im}\hat{\Sigma}^\sigma_\nu(k,\omega) . \quad (3.13)$$

By placing the Green's functions Eq. (3.10) in the expression for the conductivity [Eq. (3.6)] and carrying out the indicated operations, we find

$$\sigma(\bar{\nu})=\frac{1}{2}\left[\frac{e}{m}\right]^2\sum_{k\nu,\sigma}\frac{k^2}{2\Delta^\sigma(\varepsilon_{k\nu})}\left[-\frac{\partial f(\varepsilon_{k\nu})}{\partial\varepsilon_{k\nu}}\right] \times\left\langle k,\nu+\bar{\nu};\sigma\left|\left[\frac{1}{1-\mathrm{Re}\hat{G}\Sigma'}\right]^2\right|k\nu;\sigma\right\rangle . \quad (3.14)$$

To obtain this expression we assumed that the spectral density of the conduction electrons is represented by a δ function,[12] i.e.,

$$A_\nu(k,\varepsilon)=-\mathrm{Im}\hat{G}_\nu(k,\varepsilon)=\pi\delta(\varepsilon-\varepsilon_{k\nu}) , \quad (3.15)$$

and

$$A^2_\nu=\pi\delta(\varepsilon-\varepsilon_{k\nu})/2\Delta(\varepsilon_{k\nu}) .$$

The self-energy can be written as

$$\Sigma=\langle VGV\rangle=\left\langle\frac{V\hat{G}V}{1-\hat{G}\Sigma'}\right\rangle \quad (3.16)$$

as we have taken the average of the scattering potential V to be zero. The large brackets refer to the average over the random distribution in the planes parallel to the layers. Equation (3.16) is similar but not identical to Eq. (4) of Ref. 10. *One* subset of all contribution to the self-energy can be rewritten in terms of the t matrix, Eq. (3.4), as

$$\Sigma=\frac{t}{1-\hat{G}\Sigma'} . \quad (3.17)$$

By multiplying both sides by the Green's function $\hat{G}$ and then by $(1-\hat{G}\Sigma')$ we eventually find

$$\left\langle k,\nu+\bar{\nu};\sigma\left|\left[\frac{1}{1-\mathrm{Re}\hat{G}\Sigma'}\right]^2\right|k,\nu;\sigma\right\rangle =\left\langle k,\nu+\bar{\nu};\sigma\left|\frac{1}{1-\mathrm{Re}\hat{G}t'}\right|k,\nu;\sigma\right\rangle , \quad (3.18)$$

where t' is the off-diagonal ($\bar{\nu}\neq 0$) part of the t-matrix Eq. (3.4).

Equation (3.18) is a formal solution to our problem and, in principle, one only need $\bar{\nu}=0$ for $\sigma_\parallel$, see Eq. (2.5), however it is not possible to evaluate it. It represents an infinite series containing off-diagonal t matrices. We do not know how to decouple the repeated products of $\hat{G}t'$ in momentum space. However, we can decouple these products by taking their Fourier transform with respect to the variable $\bar{\nu}$ and by averaging over the variable ν. We find

$$\frac{1}{k_F}\int_0^{k_F}d\nu\frac{L}{2\pi}\int_{-\infty}^{\infty}d\bar{\nu}\,e^{-i\bar{\nu}z}\left\langle\nu+\bar{\nu};\sigma\left|\frac{1}{1-\mathrm{Re}\hat{G}t'}\right|\nu;\sigma\right\rangle \approx\left\langle\sigma\left|\frac{1}{1-\mathrm{Re}\langle\hat{G}t'\rangle(z)}\right|\sigma\right\rangle , \quad (3.19)$$

where

$$\langle\hat{G}t'\rangle(z)\equiv\frac{1}{k_F}\int_0^{k_F}d\nu\frac{L}{2\pi}\int_{-\infty}^{\infty}d\nu' e^{-i(\nu'-\nu)z} \times\hat{G}_{\nu'}t_{\nu'\nu}(1-\delta_{\nu\nu'}) , \quad (3.20)$$

and we consider $\hat{G}_{\nu'}$ and $t_{\nu'\nu}$ as *operators* in spin space, i.e., they represent 2×2 matrices with elements $\hat{G}^\sigma_{\nu'}\delta_{\sigma\sigma'}$ and $t^{\sigma\sigma'}_{\nu'\nu}$. To arrive at this approximate solution we have made the approximation

$$\langle(\hat{G}t')^n\rangle(z)\approx[\langle\hat{G}t'\rangle(z)]^n . \quad (3.21)$$

By using the t matrix Eq. (3.4), we find the spin matrix element in Eq. (3.19) can be written as

$$\frac{1}{\Delta^\sigma}\left\langle\sigma\left|\frac{1}{1-\mathrm{Re}\langle\hat{\mathbf{G}}t'\rangle(z)}\right|\sigma\right\rangle=\frac{a^{-\sigma}}{a^\sigma a^{-\sigma}-b^\sigma b^{-\sigma}} \quad (3.22)$$

which represents the inversion of a 2×2 matrix where we have defined the matrix elements of the spin operator as

$$\Delta^\sigma(\delta_{\sigma\sigma'}+\mathrm{Re}\langle\hat{\mathbf{G}}t'\rangle^{\sigma\sigma'})\equiv a^\sigma\delta_{\sigma\sigma'}+b^\sigma(1-\delta_{\sigma\sigma'}) \quad (3.23)$$

where we have temporarily suppressed the indices $k\nu$ and z for clarity; the coefficients a^σ and b^σ are,

$$a^\sigma=\frac{1}{\lambda^\sigma}\left[\sum_t \mathrm{Re}\Delta_t^{\sigma\sigma}E_1(|z-z_t|/\lambda^\sigma)+\sum_l \mathrm{Re}\Delta_l^{\sigma\sigma}E_1(|z-z_l|/\lambda^\sigma)\right], \quad (3.24)$$

$$b^\sigma=\frac{1}{\lambda^\sigma}\left[\sum_t \mathrm{Re}\Delta_t^{\sigma-\sigma}E_1(|z-z_t|/\lambda^\sigma)+\sum_l \mathrm{Re}\Delta_l^{\sigma-\sigma}E_1(|z-z_l|/\lambda^\sigma)\right], \quad (3.25)$$

and $E_1(x)=\int_x^\infty e^{-y}/y\,dy$ is the exponential integral of the first order. From the t matrix [see Eqs. (3.2), (3.4), and (3.5)], we find

$$\mathrm{Re}\Delta_t^{\sigma\sigma'}(\varepsilon_F)=w_t\langle\sigma|(1+p_t^2+2p_t\hat{\boldsymbol{\sigma}}\cdot\hat{\mathbf{M}}_t)|\sigma'\rangle\ ,$$
$$\mathrm{Re}\Delta_l^{\sigma\sigma'}(\varepsilon_F)=w_l\langle\sigma|(1+p_l^2+2p_l\hat{\boldsymbol{\sigma}}\cdot\hat{\mathbf{M}}_l)|\sigma'\rangle\ , \quad (3.26)$$

where $p\equiv j/v, w_t\equiv\langle v_t^2\rangle\rho(\varepsilon_F), w_l\equiv\langle v_l^2\rangle\rho(\varepsilon_F)\langle f_l^2\rangle$, and the mean free path is derived from the *diagonal* part of the self-energy

$$\lambda^\sigma\equiv\frac{k_F}{m\Delta^\sigma(\varepsilon_{k\nu}=\varepsilon_F)}\ , \quad (3.27)$$

where

$$\Delta^\sigma=\frac{1}{L}\left[\sum_{t\varepsilon L}\mathrm{Re}\Delta_t^{\sigma\sigma}+\sum_{l\varepsilon L}\mathrm{Re}\Delta_l^{\sigma\sigma}\right]\equiv\Delta_b^\sigma+\Delta_s^\sigma \quad (3.28)$$

which represents the bulk and surface scattering contributions to λ^σ.

By placing Eq. (3.22) into Eq. (3.14), and performing the sum over the two-dimensional k and ν, we find

$$\sum_{k\nu}k^2\delta(\varepsilon_{k\nu}-\varepsilon_F)=\tfrac{2}{3}k_F^2\rho(\varepsilon_F)\ , \quad (3.29)$$

where $k_F=(3\pi^2n)^{1/3}$. The position-dependent conductivity is given as

$$\sigma(z)\approx\frac{ne^2}{2m}\sum_\sigma\frac{a^{-\sigma}(z)}{a^\sigma(z)a^{-\sigma}(z)-b^\sigma(z)b^{-\sigma}(z)}\ . \quad (3.30)$$

Another decoupling scheme which is less accurate but leads to a simpler result is to evaluate the ν dependence of the matrix element Eq. (3.18) at the Fermi level instead of averaging it as in Eq. (3.20); this yields a simple exponential expression for a^σ and b^σ

$$a^\sigma(z)=\frac{1}{\lambda^\sigma}\left[\sum_t \mathrm{Re}\Delta_t^{\sigma\sigma}e^{-|z-z_t|/\lambda^\sigma}+\sum_l \mathrm{Re}\Delta_l^{\sigma\sigma}e^{-|z-z_l|/\lambda^\sigma}\right], \quad (3.31)$$

and

$$b^\sigma(z)=\frac{1}{\lambda^\sigma}\left[\sum_t \mathrm{Re}\Delta_t^{\sigma-\sigma}e^{-|z-z_t|/\lambda^\sigma}+\sum_l \mathrm{Re}\Delta_l^{\sigma-\sigma}e^{-|z-z_l|/\lambda^\sigma}\right]. \quad (3.32)$$

The conductivity Eq. (3.30) is our main result. We note that both bulk Δ_t^b and surface Δ_l^s scattering contribute on *equal* footing. From Eqs. (3.27) and (3.28) we note they both contribute to the mean free path; this is determined only from the diagonal part of the self-energy. The *off-diagonal* part of the self-energy controls the *position dependence* of the conductivity and resistivity. For layered structures it is this off-diagonality which describes their inhomogeneities. In Appendix A we discuss the conductivities in limiting cases where simple conclusions can be drawn.

IV. MAGNETORESISTANCE OF MULTILAYERED STRUCTURES

As mentioned in the Introduction, the extraordinary magnetoresistance found in multilayered structures comes from the *reorientation* of the magnetization of the layers. For this effect to occur for currents in the layer planes it is necessary that the mean free path of the electrons be larger than the spacing between the magnetic layers, i.e., the thickness of the nominally nonmagnetic layers $\lambda>d_0$. This result can be understood when one relates the mean free path to the spatial extent over which the velocity or momentum of the plane waves, which represent the electrons, remain well defined, i.e., λ is a measure of the size of the wave packet that represents an electron. For currents parallel to the layers, these wave packets sample in the *transverse* direction those layers within a distance λ from their center. Therefore, if $\lambda\gg d_{\mathrm{in}}$ (the characteristic length scale for the inhomogeneities in the multilayered structure), it is irrelevant where one places the center of the wave packet because one always samples the same distribution of inhomogeneities; the CIP conductivity is independent of position z in this limit. When the mean free path is less than the spacing between magnetic layers, a wave packet does not see more than one magnetized layer. As one is free to choose the axis of spin quantization for the conduction electrons' spin parallel to the magnetization of that individual layer, which the wave packet samples, the application of an external magnetic field, which merely rotates the magnetization, will not alter the conductivity of a superlattice in this limit. In other words, a wave packet must sample two or more magnetized layers, which reorient themselves relative to one another, for there to be a magnetoresistive effect of the type we are discussing;

i.e., it is necessary for the electron to see an *internal* reorientation of the magnetized layers. A *uniform* rotation of all the magnetized layers, without any internal rearrangement, does not produce magnetoresistance.

In the present treatment the effect of an external magnetic field on the conductivity enters through the orientation of the magnetizations $\hat{\mathbf{M}}_l, \hat{\mathbf{M}}_t$ in the scattering terms [Eqs. (3.26)]; i.e., we do not consider any effect of the field on the orbits of the conduction electrons, or in altering the magnetization of the layers. So that the mean free path [Eq. (3.27)] is diagonal in spin space, it is necessary to choose the axis of quantization for the spin so that

$$\mathrm{Re}\Delta^{\sigma\sigma'}=\left\langle\sigma\left|\sum_t w_t(1+p_t^2+2p_t\hat{\mathbf{M}}_t\cdot\hat{\boldsymbol{\sigma}})+\sum_l w_l(1+p_l^2+2p_l\hat{\mathbf{M}}_l\cdot\hat{\boldsymbol{\sigma}})\right|\sigma'\right\rangle , \tag{4.1}$$

is diagonal. As can be seen from Eqs. (3.26) and (3.28), this requires us to choose the $\hat{\boldsymbol{\sigma}}$ parallel to

$$\sum_t p_t w_t\hat{\mathbf{M}}_t+\sum_l p_l w_l\hat{\mathbf{M}}_l . \tag{4.2}$$

In general, this is quite different from the resultant of the individual magnetizations. However, for a *bipartite* superlattice where only *one* layer is magnetic, i.e., Fe/Cr, this reduces to

$$(p_b w_b+p_s w_s)(\hat{\mathbf{M}}_A+\hat{\mathbf{M}}_B) , \tag{4.3}$$

where w_b and w_s represent the bulk and interfacial scatterings. In other words, one must choose the axis parallel to the resultant of the magnetizations; in the absence of crystalline anisotropy this is parallel to the direction of the external field $\hat{\boldsymbol{\sigma}}\|\hat{\mathbf{H}}$.

With this choice of quantization the matrix elements [Eqs. (3.26)] entering Eqs. (3.31) and (3.32) for a bipartite superlattice with alternating magnetic and nonmagnetic layers are

$$\mathrm{Re}\Delta_i^{\sigma\sigma}(\theta)=w_i(1+p_i^2+2p_i\sigma\cos\theta_i)$$

and

$$\mathrm{Re}\Delta_i^{\sigma-\sigma}(\theta)=2w_i p_i\sin\theta_i \tag{4.4}$$

where $i=t,l$ and $\theta_{t,l}$ are the angles the magnetization in the layers make with respect to the axis $\hat{\boldsymbol{\sigma}}\|\hat{\mathbf{H}}$, i.e.,

$$\cos\theta_i\equiv\hat{\boldsymbol{\sigma}}\cdot\hat{M}_i$$

and

$$\sin\theta_i\equiv|\hat{\boldsymbol{\sigma}}\times\hat{\mathbf{M}}_i| . \tag{4.5}$$

As the spin-dependent scattering depends on θ_i and this in turn depends on the field strength, one obtains a magnetoresistive effect. A check on our results for the spin-dependent conductivity expression [Eq. (3.30)] is given in Appendix B, where we show that in the case $\hat{\mathbf{M}}_A+\hat{\mathbf{M}}_b=0$, i.e., antiferromagnetic ordering between layers, the conductivity is *independent* of our choice of the axis of quantization for spin.

The expressions for the position dependent conductivity [Eq. (3.30)] for arbitrary angles θ and for all ranges of layer thickness compared to the mean free path are unwieldy; in the next section we present our numerical results. However, in the limiting cases treated in Appendix A, $\lambda\gg d_{\mathrm{in}}$ and $\lambda\ll d_0$, there are relatively simple results. When $\lambda\ll d_0$ the magnetoresistance is *zero,* as the spin scattering in each region can be considered independently of the others. For $\lambda\gg d_{\mathrm{in}}$ the z dependence of the conductivity disappears, as the structure looks homogeneous for electrons with extremely long mean free paths. Then, as we have shown in the Appendix A,

$$a^\sigma(z)\to\Delta^\sigma,\quad b^\sigma(z)\to 0 ,$$

and

$$\sigma_\parallel(z)\to\frac{ne^2}{2m}\sum_\sigma\frac{1}{\Delta^\sigma} . \tag{4.6}$$

In this limit the conductivity is "self-averaging."

In this limit ($\lambda\gg d_{\mathrm{in}}$) the scattering is given by Δ^σ [Eq. (3.28)]; for a *bipartite* lattice as Fe/Cr we can replace the sum over the entire lattice by summing over one period of the magnetic unit cell $T\equiv 2(a+b)$,

$$\Delta^\sigma=\frac{1}{T}\sum_{t\in T}\Delta_b^t(\sigma)+\frac{1}{T}\sum_{l\in T}\Delta_s^l(\sigma)=\Delta_b^\sigma+\Delta_s^\sigma, \tag{4.7}$$

where for iron layers,

$$\Delta_b^{\mathrm{Fe}}=w_{\mathrm{Fe}}(1+p_{\mathrm{Fe}}^2+2\sigma p_{\mathrm{Fe}}\cos\theta) , \tag{4.8}$$

for chromium layers, where $p_{\mathrm{Cr}}=0$,

$$\Delta_b^{\mathrm{Cr}}=w_{\mathrm{Cr}} ,$$

and for the interfaces

$$\Delta_s^l=w_s(1+p_s^2+2\sigma p_s\cos\theta) .$$

Note we used the fact that $\cos\theta_l=\cos\theta_t=\cos\theta$ for the axis of spin quantization chosen in the way described earlier. By placing these expressions in Eq. (4.7) we find

$$\Delta_b^\sigma=\frac{2a/a_0}{2(a+b)}w_{\mathrm{Fe}}(1+p_{\mathrm{Fe}}^2+2\sigma p_{\mathrm{Fe}}\cos\theta)+\frac{2b/b_0}{2(a+b)}w_{\mathrm{Cr}}, \tag{4.9}$$

and

$$\Delta_s^\sigma=\frac{4}{2(a+b)}w_s(1+p_s^2+2\sigma p_s\cos\theta) ,$$

where a_0,b_0 are the distance between atomic planes [for the bcc structure of iron and chromium in the (100) direction this is $\frac{1}{2}$ the lattice constant], so that $a/a_0,b/b_0$ are the number of the atomic planes or monolayers in the layers a and b. For the conductivity [Eq. (4.6)] we find

$$\sigma(H)=\frac{ne^2}{m}\frac{\alpha}{\alpha^2-\beta^2\cos^2\theta(H)} , \tag{4.10}$$

where

$$\alpha\equiv\frac{1}{a+b}[(a/a_0)(1+p_{\mathrm{Fe}}^2)w_{\mathrm{Fe}}+(b/b_0)w_{\mathrm{Cr}}+2(1+p_s^2)w_s] ,$$

and

$$\beta \equiv \frac{2}{a+b}[(a/a_0)p_{\mathrm{Fe}}w_{\mathrm{Fe}}+2p_s w_s] .$$

Even for this relatively restrictive example there are many parameters that enter. After choosing the thickness of the layers a and b, there are *five* "adjustable" parameters: p_{Fe}, p_s, w_{Fe}, w_{Cr}, and w_s. As the origin of the potential scattering (v) in the layers is similar we take $w_{\mathrm{Fe}}=w_{\mathrm{Cr}}\equiv w_b$; furthermore as the spin-dependent scattering at the interfaces as well as in the iron layers is related to chromium or other impurities, we set $p_{\mathrm{Fe}}=p_s$ in order to reduce the number of unknown parameters to *three*. Thus

$$\alpha=\frac{1}{a_0}w_b+\frac{2}{a+b}w_s+\frac{p^2}{a+b}\left[\frac{a}{a_0}w_b+2w_s\right] , \tag{4.11}$$

and

$$\beta=\frac{2p}{a+b}\left[\frac{a}{a_0}w_b+2w_s\right] .$$

From Eq. (4.10) we note the resistivity is

$$\rho(H)=\frac{m}{ne^2}\left[\alpha-\frac{\beta^2}{\alpha}\cos^2\theta(H)\right] . \tag{4.12}$$

In the absence of anisotropy, one finds $\cos\theta$ is proportional to H, and one would predict $\rho(H)\propto A-BH^2$. However, from the existing data on $\rho(H)$,[1-5] this is by and large not observed; we conclude that one should not use the simple relation $\cos\theta\propto H$. If one models the magnetic layers as uniformly and rigidly ordered, one can use the experimentally observed magnetization versus field curves and from the relation

$$M(H)=2M_0\cos\theta(H) , \tag{4.13}$$

obtain $\theta(H)$. In the next section we present some results on $\rho(H)$ for realistic cases when λ is comparable to d_{Cr}.

A measure of the magnetoresistive effect is the quantity that is related to the amplitude of the effect,

$$R\equiv\frac{\rho(H=0)-\rho(H=H_s)}{\rho(H=0)}=\frac{\rho(\theta=\pi/2)-\rho(\theta=0)}{\rho(\theta=\pi/2)} . \tag{4.14}$$

In the limit that we are considering $\lambda\gg d_{\mathrm{in}}$, we find from Eq. (4.12),

$$R=\frac{\beta^2}{\alpha^2} . \tag{4.15}$$

The magnitude of R is governed by the parameter p; see Eqs. (4.11). It is related to the ratio $\alpha\equiv\rho_\uparrow/\rho_\downarrow$ of the resistivities for electrons with spin parallel to the magnetization to that for electrons with spin antiparallel

$$p=\frac{\sqrt{\alpha}-1}{\sqrt{\alpha}+1} . \tag{4.16}$$

As $\alpha\geq 1$, the minimum $p=0$ gives $R=0$, while the maximum $p=1$ (as $\alpha\rightarrow\infty$) yields

$$R(p=1)=\left[1+\frac{(b/a_0)w_b}{2[(a/a_0)w_b+2w_s]}\right]^{-2} . \tag{4.17}$$

As $b\rightarrow 0$, or $w_b\rightarrow 0$ we obtain $R=1$. Other limiting cases are (1) if $w_b=0$, i.e., only interfacial roughness scattering,

$$R(w_b=0)=\frac{4p^2}{(1+p^2)^2}=\left[\frac{\alpha-1}{\alpha+1}\right]^2=\frac{\rho_\uparrow-\rho_\downarrow}{\rho_\uparrow+\rho_\downarrow} , \tag{4.18}$$

and (2) if $w_s=0$, i.e., only bulk scattering,

$$R(w_s=0)=\frac{4p^2}{(1+p^2+b/a)^2} . \tag{4.19}$$

The presence of the ratio b/a is understandable as it represents the proportion of the multilayered structure that is *magnetically inert*, i.e., the Cr layers are nonmagnetic $p_{\mathrm{Cr}}=0$, and they do not contribute to the magnetoresistance. This ratio does not appear in Eq. (4.18) because all the interfacial scattering is spin dependent.

V. Fe/Cr SUPERLATTICES

From the above results, we note that even in the limiting cases the expressions for $\sigma(H)$ or $\rho(H)$ are rather complicated and depend on at least three parameters p, w_b, and w_s, which characterize the ratio of spin to potential scattering, and set the scale of the bulk and interfacial scattering. These two limits represent the minimum and maximum values of the magnetoresistive effect, $0<R<1$. For realistic values of the mean free path relative to the layer thicknesses, one is in between these limits. Then the conductivity [Eq. (3.30)] is position dependent, and the off-diagonal spin matrix elements [Eq. (3.32)] enter. As general expressions for $\sigma(z)$ or $\sigma_\parallel$ would be unwieldly, we now present some numerical results for the cases that have been studied in recent experiments on Fe/Cr superlattices.[1,2]

In cases where λ is about the same order as d_{in}, the expression for the mean free path λ^σ defined by Eqs. (3.27) and (3.28) is identical to that for the limit considered above. The reason for this is that the mean free path is determined from the *diagonal* part of the Green's function [Eq. (3.12)]. The self-energy Δ^σ_{kv} is an average of the scattering t matrix over the entire sample (or one period if it is periodic); it does not depend on the details of the inhomogeneities present in multilayered structures. It must be the same for $\lambda\approx d_n$ as for $\lambda\gg d_{\mathrm{in}}$. Therefore, from [Eq. (4.9)] with $p_{\mathrm{Fe}}=p_s=p$, $w_{\mathrm{Cr}}=w_{\mathrm{Fe}}$, and $a_0=b_0$ we can immediately write

$$\frac{1}{\lambda^\sigma}=\frac{1}{\lambda^\sigma_b}+\frac{1}{\lambda^\sigma_s} \tag{5.1}$$

with

$$\lambda^\sigma_b=\frac{\lambda_b}{1+\frac{a}{a+b}(p^2+2p\sigma\cos\theta)} , \tag{5.2}$$

$$\lambda^\sigma_s=\frac{(a+b)\lambda'_s}{2(1+p^2+2p\sigma\cos\theta)} , \tag{5.3}$$

where we defined $\lambda_b \equiv a_0 k_f / m w_b$, and $\lambda_s' \equiv k_F / m w_s$. As before the magnitude of λ^σ is determined primarily by λ_b, λ_s' and its spin dependence by p, and $\theta(H)$. We note that the mean free path from interfacial scattering depends on the *distances* between interface $(a+b)$ as well as the *strength* of the scattering w_s. On the contrary the bulk mean free path depends only *weakly* on distances $a/(a+b)$ because for the cases of interest $b \ll a$.

The mean free path [Eq. (5.1)] sets the length scale for the position dependence in the conductivity, see Eq. (3.30). In Fig. 1 we show the CIP conductivity $\sigma_\parallel(z)$ for an Fe/Cr superlattice with different amounts of scattering, and therefore different mean free paths. For Fig. 1(a), we used the parameters λ_b, λ_s' and p, see Eqs. (5.2) and (5.3), which best fit the magnetoresistance data of Fert and co-workers[1,2,9] (see Fig. 4). The average of the mean free paths in the ferromagnetic (F) configuration is longer than in the antiferromagnetic (AF). This attenuates the variation with position of the conductivity $\sigma(z)$ in the F configuration relative to the AF, i.e., the *shorter* the mean free paths, the larger the amplitude of the position dependence of $\sigma(z)$. Thus we note that when we *reduce* the mean free paths, see Fig. 1(b), the amplitudes of the variation *increase;* increasing the mean free paths diminishes the amplitudes, Fig. 1(c).

In general, the conductivity is larger in the F state than the AF; however, this is not universal. In the preceding section we found for $\lambda \gg d_{\text{in}}$ that the ratio R [Eq. (4.15)] is always positive, i.e., $\sigma_F > \sigma_{\text{AF}}$. This is attributable to the "short-circuit" effect present in the F state, in which the mean free path for the minority spin direction $\lambda_F^\downarrow$ is much larger than that for the majority spin direction $\lambda_F^\uparrow$. For the AF state both directions of spin have the same mean free path with $\lambda_F^\uparrow < \lambda_{\text{AF}} < \lambda_F^\downarrow$; this readily yields $\sigma_F > \sigma_{\text{AF}}$. However, when $\lambda \approx d_{\text{in}}$ we have found $R < 0$ for some values of p_s, p_b, λ_s', and λ_b. The values that produce $R < 0$ are not realistic in the context of the Fe/Cr superlattices that have been studied to date. Therefore, while one can reasonably expect $R > 0$ for Fe/Cr superlattices, it should be kept in mind that for other multilayered structures it is possible to have situations where $R < 0$, e.g., see Figs. 2 and 3.

In Fig. 2, we present the CIP magnetoresistance ratio $R_\parallel$ as a function of the chromium layer with the iron layer fixed at $a = 30$ Å. We present results for $\lambda_s' = 1.1$ and $\lambda_b = 19$ Å (solid lines), as well as for different values of λ_b, $\lambda_b = 10$ and 30 Å (dashed lines). In Fig. 2(a) only the bulk scattering is spin dependent ($p_b = 0.55$), and in Fig. 2(b) only the interface scattering is spin dependent ($p_s = 0.55$). In case b (only spin-dependent scattering at interfaces), $R_\parallel$ decreases steeply with the thickness of chromium. The length scale of the variation is λ_b; this is

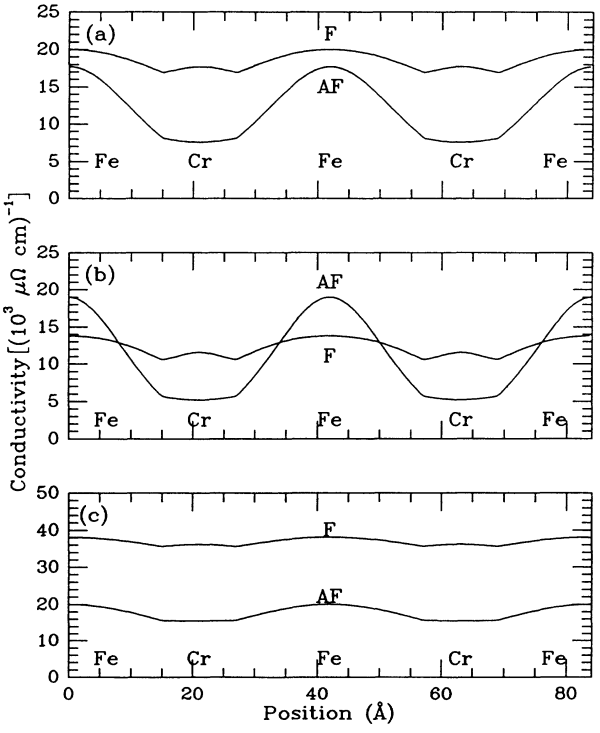

FIG. 1. The position-dependent CIP conductivity of an Fe/Cr superlattice with $a = 30$ Å and $b = 12$ Å for one magnetic unit cell, $T = 2(a+b) = 84$ Å. (a) For the parameters $\lambda_s' = 1.1$, $\lambda_b = 19$ Å, and $p = 0.55$, which produce the best fit to the magnetoresistance, see Fig. 4. The parameters yield $\lambda_{\text{AF}} = 8.5$ Å, $\lambda_F^\uparrow = 5.3$ Å, and $\lambda_F^\downarrow = 44$ Å. (b) For the parameters $\lambda_s' = 0.8$, $\lambda_b = 12$ Å, and $p = 0.55$. (c) For $\lambda_s' = 2$, $\lambda_b = 40$ Å, and $p = 0.55$.

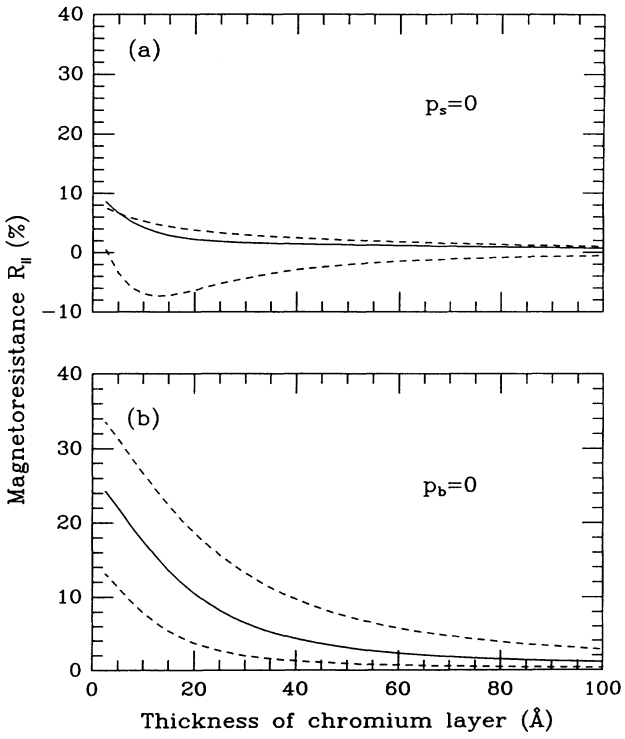

FIG. 2. The CIP magnetoresistance ratio $R_\parallel$ as a function of the thickness of the chromium layers b for $a = 30$ Å, with $\lambda_s' = 1.1$, and $\lambda_b = 19$ Å (solid lines), $\lambda_b = 10$ Å (lower dashed lines), and $\lambda_b = 30$ Å (upper dashed lines). (a) Only the bulk scattering is spin-dependent; $p_s = 0$ and $p_b = 0.55$. (b) Only the interface scattering is spin-dependent; $p_b = 0$ and $p_s = 0.55$.

seen by the steep decrease for $\lambda_b = 10$ Å, and the gradual decrease for $\lambda_b = 30$ Å. Also, $R_\parallel$ is larger when λ_b is larger because the proportion of spin-dependent to spin-independent scattering is larger. In case *a* (only spin-dependent scattering in the bulk), the dependence of $R_\parallel$ on the thickness of chromium is more complicated and difficult to explain by simple arguments. Note the *negative* $R_\parallel(\rho_F > \rho_{AF})$ for $p_s = 0$ and $\lambda_b = 10$ Å. This exception underscores the fact that there is *no proof* that $R > 0$, for all values of the parameters that enter our expressions, i.e., in the limiting case considered in Sec. IV, we found $R > 0$; however, when the position-dependent factors entering the conductivity [Eq. (3.30)] are important, it is not possible to prove $\sigma_F > \sigma_{AF}$.

In Fig. 3, we present the magnetoresistance ratio $R_\parallel$ as a function of the thickness of the iron layer (a) with the chromium layer (b) fixed at 12 Å. The parameters are the same as in Fig. 2; the solid line is for $\lambda_b = 19$ Å, and the dashed lines are for $\lambda_b = 10$ and 30 Å. When only the interface scattering is spin dependent, Fig. 3(b), $R_\parallel$ decreases with a but more gradually than with b in Fig. 2. The length scale of this variation is λ_b; this is seen by comparing the variation of $R_\parallel(a)$ for the three values of λ_b. When only bulk scattering is spin dependent, Fig. 3(a), $R_\parallel$ starts at zero for $a = 0$, then increases with a, because the proportion of spin-dependent scattering increases, presents a maximum, and becomes negative at large values of a. As seen from the variation of $R_\parallel(a)$ for the different λ_b the length scale of this variation is λ_b.

To fit the experimental results on Fe/Cr superlattices[1,2] shown in Fig. 4 we proceed in the following way. First, we note that the large value of the experimental values of $R_\parallel$ and also the shape of the variation with a, which is intermediate between Figs. 3(a) and 3(b), are strongly in favor of spin dependence for *both* bulk *and* interface scattering. For simplicity we choose the same parameter p for interface and bulk scattering. The three parameters of the problem are thus p, λ_b, and λ_s'. First the parameter λ_b is fixed to fit the dependence on b; λ_b determine the length scale of this variation. Then, as the variation with a is very sensitive to the proportion of bulk to interface scattering, we fix the ratio of λ_s' to λ_b to obtain the $R_\parallel(a)$ that is experimentally observed with a broad maximum at small values of a and then a slow decrease. Third, we fix p to obtain the magnitude of the magnetoresistance. We have obtained the best fit with the following parameters: $p = 0.55$, $\lambda_s' = 1.1$, and $\lambda_b = 19$ Å. This fit is shown in Fig. 4, where we compare the experimental and calculated dependencies of $R_\parallel$ on the thickness of Cr and Fe.

In order to fit the data of the magnetoresistance of Fe/Cr superlattices[1,2] with a reasonable value of the parameter p we find the bulk and interface scattering are about equally important, e.g., for a superlattice with

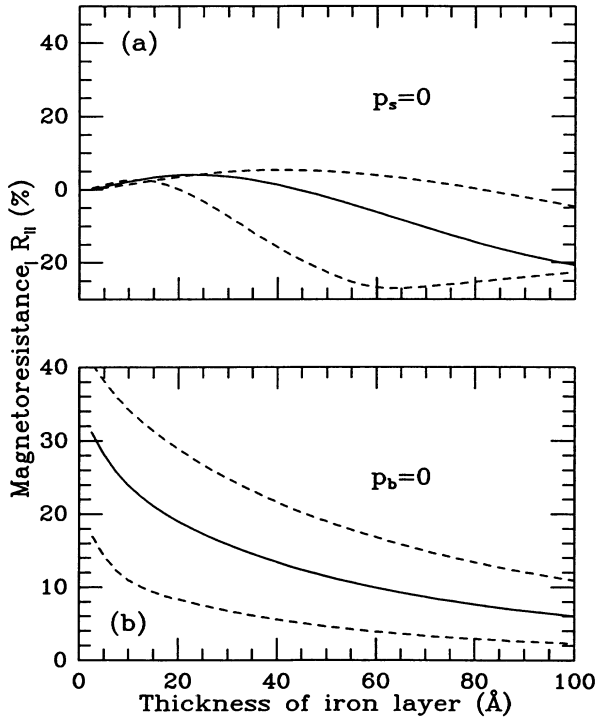

FIG. 3. The CIP magnetoresistance ratio $R_\parallel$ as a function of the thickness of the iron layers a for $b = 12$ Å, with $\lambda_s' = 1.1$, and $\lambda_b = 19$ Å (solid lines), $\lambda_b = 10$ Å (lower dashed lines), and $\lambda_b = 30$ Å (upper dashed lines). (a) Only the bulk scattering is spin-dependent; $p_s = 0$ and $p_b = 0.55$. (b) Only the interface scattering is spin-dependent; $p_b = 0$ and $p_s = 0.55$.

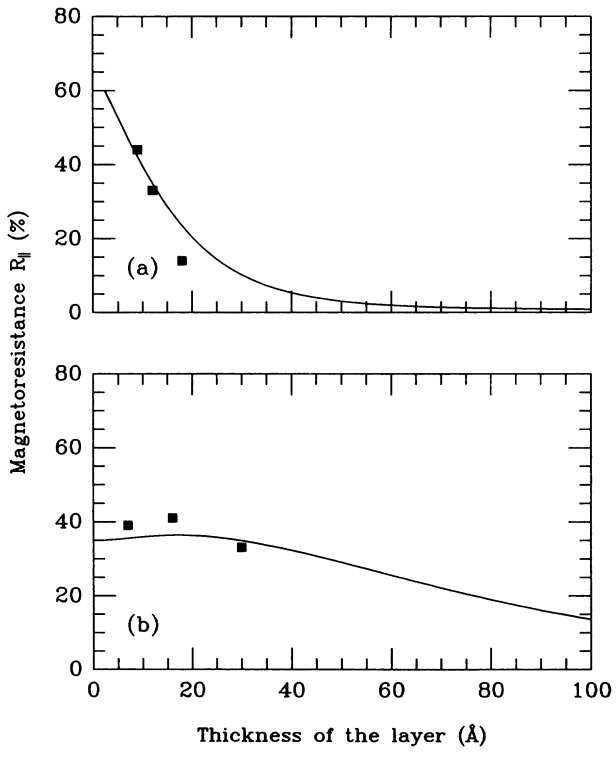

FIG. 4. The magnetoresistance ratio $R_\parallel$ with the parameters that produce the best fit to the experimental data at $T = 4.2$ K given in Refs. 1 and 2: $\lambda_s' = 1.1$, $\lambda_b = 19$ Å, and $p = 0.55$. (a) As a function of thickness of the chromium layers b for $a = 30$ Å. (b) As a function of thickness of the iron layers a for $b = 12$ Å. Lines are our calculations, and squares are experimental points from Refs. 1 and 2.

$a=30$ Å and $b=12$ Å the best fits are found with $\lambda_b=19$ Å and $\lambda_s=\frac{1}{2}(a+b)\lambda_s'=23$ Å. The value of $p=0.55$, which produces the best fits to the data, corresponds to $\alpha=12$. This value is larger than $\alpha=6$ found for chromium impurities in iron;[14] however, from recent band-structure calculations for iron impurities in chromium,[15] $\alpha=12$ at the *interfaces* looks quite reasonable.

A check on the parameters we use in our fits is the absolute value of the resistivity they yield. To calculate the absolute value of resistivity we need to determine the prefactor entering the conductivity, Eq. (3.30). We have estimated that

$$\frac{ne^2}{k_F}\approx 2\times 10^5\ (\mu\Omega\,\mathrm{cm}^2)^{-1}\ . \tag{5.4}$$

With this value, and by using the parameters that best fit the magnetoresistance data, we find the longitudinal resistivity for a sample with $a=30$ Å and $b=12$ Å for $H=0$ and $T=0$ K is

$$\rho_{AF}=83\ \mu\Omega\,\mathrm{cm}\ . \tag{5.5}$$

This is to be compared with the experimental values which vary between 40 and 80 $\mu\Omega$ cm.

A further check on the appropriateness of our fit comes from the parameter λ_s' which characterizes the strength of the surface roughness scattering, see Eq. (3.5). We may estimate the surface roughness $\langle f_l^2\rangle$ by using our best fit parameter λ_s'. By assuming that the impurity potential at the interface and in the bulk are the same, and by using the fact that the resistivity of iron with 1% of Cr impurities is 2.2 $\mu\Omega$ cm, see Table 2 of Ref. 14, we find from Eq. (3.5) that $(\langle f_l^2\rangle)^{1/2}\approx 2a_0/5$. This rms deviation of the interface, which is 40% of the distance between atomic planes, is quite reasonable.

Finally in Fig. 5 we show our results for $\rho_\parallel(H)$ for $a=30$ Å, and $b=12$ and 18 Å. We used for p, λ_s', and λ_b the values that produced the best fit to the data on $R_\parallel$.[1,2,9] On comparing our results for $\rho_\parallel(H)$ with the field dependent data of Baibich *et al.*[1] we find the agreement is excellent. As our expressions, Eqs. (2.5), (2.8), and (3.30), yield $\rho(\theta)$, we found $\theta(H)$ by using the method outlined in Sec. V. The data on the magnetization $M(H)$ is given in Ref. 2. To compare the *field dependence* of the resistivity for $b=18$ Å, we had to rescale the experimental resistivities so that $R_\parallel(a=30\ \text{Å}, b=18\ \text{Å})=0.23$, instead of the experimental value of 0.14.

Our procedure for extracting $\theta(H)$ is based on the hypotheses (1) that the magnetization in the iron layers rotate *rigidly* and (2) that the chromium layer does not change its state of magnetization as the antiferromagnetically aligned iron layers are rotated into ferromagnetic alignment by an external field. It is possible that the magnetization in the iron layers do not uniformly rotate, and our procedure is not entirely correct. Also from some band-structure calculations of Fe/Cr superlattices,[16] there is an indication that the magnetic configuration of the chromium layers is altered as the adjacent iron layers are reoriented by an external field from AF to F configurations; this would change the small spin-dependent scattering in the chromium layers, that we have neglected, as we set $p_{Cr}=0$. The good agreement we have obtained for $R_\parallel(a)$, $R_\parallel(b)$, and $\rho_\parallel(H)$ with entirely reasonable parameters, leads us to believe that the hypotheses on which we based our analysis are well founded; the putative corrections, if they exist, are small.

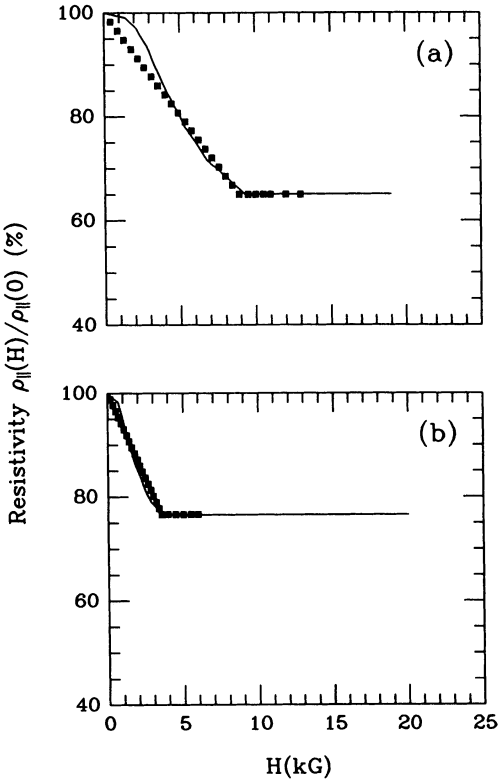

FIG. 5. The resistivity $\rho_\parallel(H)$ for $\lambda_s'=1.1$, $\lambda_b=19$ Å, and $p=0.55$. (a) For $a=30$ Å and $b=12$ Å ($\lambda_s=23$ Å). (b) For $a=30$ Å and $b=18$ Å ($\lambda_s=26$ Å). Lines are our calculations, and squares are experimental data from Refs. 1 and 2. To compare the field dependence for $b=18$ Å with experiment we rescaled the experimental data so $R_\parallel=0.23$.

VI. DISCUSSION

In summary, the position dependence of the conductivity, see Eq. (3.30), plays an important role in obtaining the proper dependence of the conductivity and magnetoresistance as a function of the thickness of the layers. While we have explicitly developed our formalism for an extended periodic lattice, it is equally applicable to an extended lattice with no periodicity. Then the averages and sums entering the conductivity are performed over the whole lattice L. For sandwich structures of finite thickness, e.g., Fe/Cr/Fe,[3,4] we replace the plane waves e^{ivz} by states appropriate to the confinement (square well) potential. Because of the translational invariance in the x-y plane (parallel to the layers) this leads to two-dimensional bands of states.[10,11] The effects of the confinement potential are felt when the thickness L is comparable or smaller than the mean free path of the electrons. While it has been possible to produce thin films with $\lambda>L$, e.g., $CoSi_2$,[10] in which the effects of the confinement potential

are felt, this does not seem to be the case with most of the Fe/Cr/Fe sandwich structures produced up till now.

Finally we have neglected the differences in the bottoms of the potential wells of the individual layers. To date, we have found good agreement with the experimental data on the CIP magnetoresistance by considering only plane waves, i.e., we have not considered the effects of a periodic potential on the electrons.

The salient difference between the approach presented here and the previous treatment of the magnetoresistance of multilayered structures is that we treat the bulk and interface scattering the *same* way. In the earlier Fuchs-Sondheimer-like approach the interface scattering was treated phenomenologically in terms of transmission, reflection, and scattering coefficients. We have found that applying this approach to Fe/Cr *superlattices* with high resistivities (in the range of 80 $\mu\Omega$ cm) leads to an underestimate of the interfacial contribution to the resistivity. To fit the magnetoresistance data in this approach one needs unrealistically large mean free paths, i.e., an aberrantly small bulk scattering. The drawback of assuming such an unrealistic value of λ, apart from the small absolute value of the resistivity, is the distortion of the length scale that determines the thickness dependence of the magnetoresistance.

ACKNOWLEDGMENTS

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APPENDIX A: LIMITING CASES

The length scale λ^σ inherent in $\sigma(z)$ plays a crucial role in the CIP conductivity $\sigma_\parallel$. It is very different in the limits $\lambda^\sigma \ll d_0$ (the minimum thickness of a layer) and $\lambda^\sigma \gg d_{\rm in}$ (the characteristic length scale for the inhomogeneities in the multilayered structure); we now consider these limiting cases.

(1) $\lambda \gg d_{\rm in}$. We consider a structure with m different kinds of layers with self-energies $\mathrm{Re}\Delta_b^n \equiv \delta_b^n$ $(n=1,2,\ldots,m)$. Then the characteristic length scale of the inhomogeneities is

$$d_{\rm in} = \sum_{n=1}^{m} d_n \ , \tag{A1}$$

where d_n is the thickness of each layer, and from Eq. (3.31)

$$a^\sigma(z)|_{\rm bulk} = \frac{1}{\lambda^\sigma} \sum_{n=1}^{m} \delta_b^n \sum_{\{t_n\}} e^{-|z-z_{t_n}|/\lambda^\sigma} \ , \tag{A2}$$

where $\{t_n\}$ labels the atomic planes in the n type layers *throughout* the structure. In the limit $\lambda^\sigma \gg d_{\rm in}$ we can replace the sum by an integral

$$\sum_{\{t_n\}} e^{-|z-z_{t_n}|/\lambda^\sigma} = \frac{\{N_n\}}{L} \int_0^L e^{-|z-z_{t_n}|/\lambda^\sigma} dz_{t_n} \ , \tag{A3}$$

where

$$\{N_n\} = \left[\frac{L}{d_{\rm in}} \right] \frac{d_n}{a_n} \ ,$$

that is, the number of n type layers in the structure multiplied by the number of atomic planes in a layer; a_n is the distance between planes and [ ] indicates the integral part of $L/d_{\rm in}$. In the limit that the sample thickness is large $L \to \infty$ one finds

$$\lim_{L\to\infty} \int_0^L e^{-|z-z_{t_n}|/\lambda^\sigma} dz_{t_n} = \lambda^\sigma \ ,$$

$$\lim_{L\to\infty} \frac{\{N_n\}}{L} = \frac{d_n}{d_{\rm in} a_n} \ , \tag{A4}$$

and

$$a^\sigma(z)|_{\rm bulk} = \sum_{n=1}^{m} \delta_b^n \frac{d_n}{d_{\rm in} a_n} \ .$$

As δ_b^n is the scattering per atomic plane and d_n/a_n is the number of planes per layer, it follows that $\delta_b^n (d_n/a_n) \equiv \Delta_b^n$ is the scattering from the nth layer. Thus

$$\frac{1}{d_{\rm in}} \sum_n \delta_b^n \frac{d_n}{a_n} = \frac{1}{d_{\rm in}} \sum_n \Delta_b^n \equiv \Delta_b \ . \tag{A5}$$

By comparing this result with Eq. (3.28) we find

$$\lim_{\lambda \gg d_{\rm in}} a^\sigma(z)|_{\rm bulk} = \Delta_b \ . \tag{A6}$$

Using a similar line of reasoning we find the same result for the scattering from interfaces.

Now we must also determine the form of the off-diagonal spin matrix elements $b^\sigma(z)$, Eq. (3.32). In the limit $\lambda \gg d_{\rm in}$, the sum over l and t may be treated the same way as $a^\sigma(z)$; i.e., the only difference between $a^\sigma(z)$ and b^σ is replacing $\Delta_{t(l)}^{\sigma\sigma}$ by $\Delta_{t(l)}^{\sigma-\sigma}$, and therefore,

$$\lim_{\lambda \gg d_{\rm in}} b^\sigma(z) \approx \frac{1}{L} \left[\sum_t \mathrm{Re}\Delta_t^{\sigma-\sigma} + \sum_l \mathrm{Re}\Delta_l^{\sigma-\sigma} \right] \equiv \Delta^{\sigma-\sigma} \ . \tag{A7}$$

According to our choice of quantization for the operator $\hat{\Delta}$ in spin space, see discussion after Eq. (3.11) and in Sec. IV.

$$\langle \sigma | \hat{\Delta} | \sigma' \rangle = \Delta^\sigma \delta_{\sigma\sigma'} \ . \tag{A8}$$

Therefore

$$\lim_{\lambda \gg d_{\rm in}} b^\sigma(z) = 0 \ . \tag{A9}$$

Also, this result could have been intuited from the fact that $b(z)$ only exists for scattering potentials [Eq. (3.4)]

with $\bar{v}\neq 0$; in the limit $\lambda \gg d_{\text{in}}$ only $\bar{v}=0$ exists and it follows that $b(z)=0$. By inserting the limiting expressions Eq. (A6) and (A9) in Eqs. (2.5) and (3.30), we find

$$\sigma_{\parallel}=\frac{ne^2}{2m}\sum_{\sigma}\frac{1}{\Delta^{\sigma}} . \tag{A10}$$

This is to be expected because the inhomogeneities of the system do not show up in this limit.

(2) $\lambda \ll d_0$ In this limit the mean free path is very small compared to the thickness of each layer and as we now show the integral in Eq. (2.5) can be replaced by a sum over layers. In the limit of $\lambda \ll d_n$, where d_n is the thickness of the nth-type layer, we have

$$\lim_{\lambda^{\sigma}\to 0}\frac{1}{\lambda^{\sigma}}e^{-|z-z_t|/\lambda^{\sigma}}=\delta(z-z_t) . \tag{A11}$$

By placing this result in Eqs. (3.31) and (3.32), we find

$$a^{\sigma}(z)=\sum_{t_n}\delta_n^{\sigma\sigma}\delta(z-z_{t_n}) \tag{A12}$$

and

$$b_{\sigma}(z)=\sum_{t_n}\delta_n^{\sigma-\sigma}\delta(z-z_{t_n}) ,$$

where $\delta_n^{\sigma\sigma'}\equiv \mathrm{Re}\Delta_n^{\sigma\sigma'}$. Hence the average CIP conductivity Eq. (2.5) and (3.30) is

$$\sigma_{\parallel}=\frac{ne^2}{2m}\frac{1}{d_{\text{in}}}\sum_{\sigma}\int_0^{d_{\text{in}}}dz\frac{a^{-\sigma}(z)}{a^{\sigma}(z)a^{-\sigma}(z)-b^{\sigma}(z)b^{-\sigma}(z)} . \tag{A13}$$

When z is located in an nth-type layer, $a^{\sigma}(z)$, $b^{\sigma}(z)$ assume the values $\delta_n^{\sigma\sigma}, \delta_n^{\sigma-\sigma}$ respectively, and we write

$$\sigma_{\parallel}=\frac{ne^2}{2m}\frac{1}{d_{\text{in}}}\sum_{\sigma}\sum_{n=1}^{m}d_n\frac{\delta_n^{-\sigma-\sigma}}{\delta_n^{\sigma\sigma}\delta_n^{-\sigma-\sigma}-\delta_n^{\sigma-\sigma}\delta_n^{-\sigma\sigma}} ; \tag{A14}$$

i.e., the conductivity breaks up into the sum of conductivities of each layer. In the present case with $\lambda \ll d_0$ we are free to choose the axis of spin quantization for each layer *independent* of the others; we choose $\delta_n^{\sigma-\sigma}=0$, and the above expression reduces to

$$\sigma_{\parallel}=\frac{ne^2}{2m}\frac{1}{d_{\text{in}}}\sum_{\sigma,n=1}^{m}\frac{d_n}{\Delta_n^{\sigma}} , \tag{A15}$$

where Δ_n^{σ} denotes the self-energy of the nth-type layer. The physical interpretation of Eq. (A15) is that each layer can be treated as an independent conductor, and the total conductance is obtained by summing over the individual conductances of each layer.

Up till now we have not focused on the origin of the scattering, i.e., bulk versus interface. The bulk and interface scatterings produce quite different effects on the conductivity. Let us now consider some limiting cases of bulk versus interface scattering for all ranges of mean free path compared to layer thicknesses d_n.

(3) $\Delta_s \gg \Delta_b$. For simplicity, we consider only spin-independent scattering and assume that each layer has the *same* thickness a. If the bulk scattering is very weak compared to interface scattering, we set $\Delta_b^t=0$, and by evaluating the integral in Eqs. (2.5) and (3.30) we find

$$\sigma_{\parallel}=\frac{2ne^2}{m}a\xi^3(e^{1/2\xi}-e^{-1/2\xi})(\tan^{-1}e^{1/2\xi}-\tan^{-1}e^{-1/2\xi}) , \tag{A16}$$

where

$$\xi=\frac{k_F}{mw_l} \tag{A17}$$

represents the strength of interface scattering, which is *independent* of a.

For finite bulk scattering the expressions (4.18) and (4.19) are more complicated. Specifically, $\sigma_{\parallel}$ is not simply proportional to thickness a. In Fig. 6 we show the resistivity as a function of thickness of the layers, where we assume the bulk mean free paths are the same in each layer, i.e., the bulk is considered homogeneous. For the surface mean free path $\lambda_s=a\xi$ we used $\xi=0.2$ and for the bulk mean-free-path $\lambda_b=100$ Å; note that λ_b does not vary with a, while λ_s does. There are three regimes, I, II, and III, corresponding to surface scattering domination ($\lambda_s<\lambda_b$), intermediate domination ($\lambda_s\approx\lambda_b$), bulk scattering domination ($\lambda_s\gg\lambda_b$), respectively.

The other case of interest is

(4) $\Delta_b \gg \Delta_s$; i.e., the case when surface scatterings are negligible, while we have two *different* bulk mean free paths in the layered structure, e.g., λ_1 for odd layers and λ_2 for even. For $\lambda \gg a$ we find from Eq. (A10),

$$\sigma_{\parallel}=\frac{ne^2}{k_F}\frac{2\lambda_1\lambda_2}{\lambda_1+\lambda_2} , \tag{A18}$$

while for $\lambda \ll a$ we find from Eq. (A15)

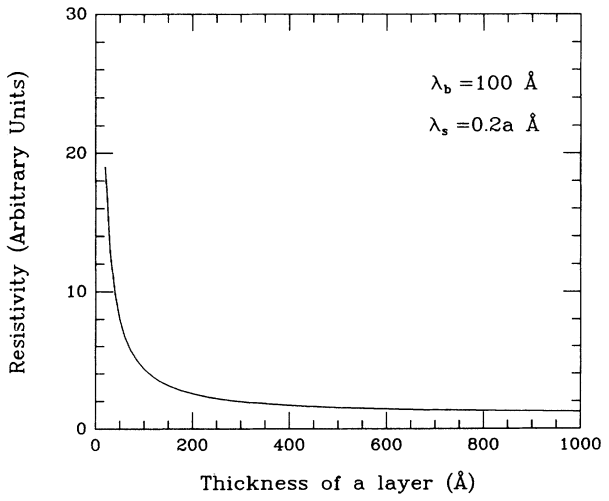

FIG. 6. The CIP resistivity of nonmagnetic superlattices of one metal with layers of equal thickness a dominated by interfacial scattering.

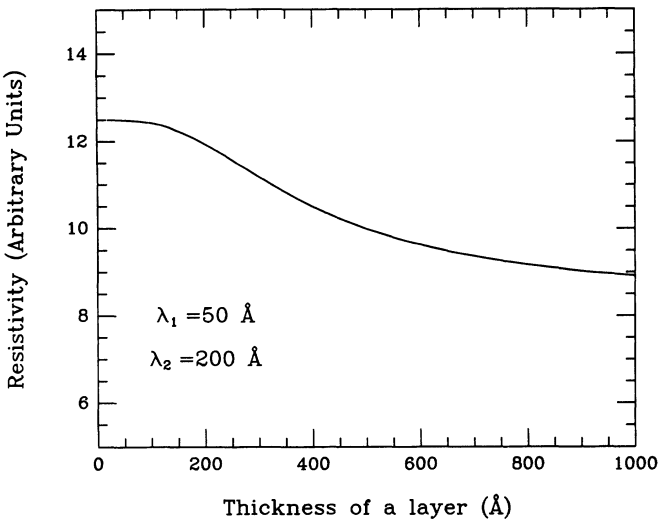

FIG. 7. The CIP resistivity of nonmagnetic superlattices made up of metals with different mean-free-paths λ_1 and λ_2. We take the interface scattering to be zero.

$$\sigma_{\parallel}=\frac{ne^2}{k_F}\frac{\lambda_1+\lambda_2}{2} . \tag{A19}$$

We note that the difference in $\sigma_{\parallel}$ in the two limits Eqs. (A18) and (A19) depends on the inhomogeneity of the layers ($\lambda_1 \neq \lambda_2$). In Fig. 7 we show the variation of $\rho_{\parallel}$ and $\rho_{\perp}$ for $\lambda_1=50$ Å and $\lambda_2=200$ Å. From Eqs. (A18) and (A19) we find the ratio of the resistivities is

$$\frac{\rho_{\parallel}(\lambda \gg a)}{\rho_{\parallel}(\lambda \ll a)}=\frac{(\lambda_1+\lambda_2)^2}{4\lambda_1\lambda_2} , \tag{A20}$$

which for our example yield 25/16. The three regimes in Fig. 7 correspond to case I $\lambda_1,\lambda_2 > a$, II $\lambda_1,\lambda_2 \approx a$, and III $\lambda_1,\lambda_2 < a$.

APPENDIX B: A CHECK

A check on our results for the spin-dependent conductivity expression Eq. (3.30) is to show that in the case $\hat{\mathbf{M}}_A+\hat{\mathbf{M}}_B=\mathbf{0}$, i.e., antiferromagnetic ordering between layers, the conductivity is independent of our choice of the axis of quantization for spin. First, if we choose the axis colinear with the antiparallel moments we have $\theta_A=0$ and $\theta_B=\pi$, so that $\cos\theta_{A,B}=\pm 1$ and $\sin\theta_{A,B}=0$. From Eqs. (3.31), (3.32), and (4.4) we find

$$a^{\sigma}(z;\theta=0,\pi) = \frac{1}{\lambda}\sum_i [1+p_i^2+2\sigma(-1)^i p_i] e^{-|z-z_i|/\lambda} , \tag{B1}$$

and

$$b^{\sigma}(z;\theta=0,\pi)=0 ,$$

where $(-1)^i=\cos\theta_i$ for $\theta_i=0,\pi$, and we used the fact that, for $\hat{\mathbf{M}}_A+\hat{\mathbf{M}}_B=\mathbf{0}$, $\lambda^{\sigma}=\lambda$ is independent of σ. For the second choice of quantization we choose an axis perpendicular to the moments so that $\theta_{A,B}=\pm\pi/2$. For this choice we find

$$a^{\sigma}(z;\theta=\pm\pi/2)=\frac{1}{\lambda}\sum_i (1+p_i^2) e^{-|z-z_i|/\lambda} \tag{B2}$$

and

$$b^{\sigma}(z;\theta=\pm\pi/2)=\frac{2}{\lambda}\sum_i (-1)^i p_i e^{-|z-z_i|/\lambda} .$$

By comparing the two expressions we find

$$a^{\sigma}(z;\theta=0,\pi) \equiv a^{\sigma}(z;\theta=\pm\pi/2)+\sigma b^{\sigma}(z;\theta=\pm\pi/2) . \tag{B3}$$

By placing this result in the expression for the conductivity Eq. (3.30), we find

$$\sigma(z;\theta=0,\pi)=\sigma(z;\theta=\pm\pi/2) , \tag{B4}$$

where we use the fact that a and b do not depend on the spin index σ for this configuration.

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