Resonant tunneling in a resistive wire

A. Knäbchen
Technical University Chemnitz, Institut of Theoretical Physics, P.O. Box 964, Chemnitz D-9010, Federal Republic of Germany
(Received 3 September 1991)

Using a recently developed superposition method, we consider resonant tunneling through a double-barrier structure in a resistive wire. The wire with finite mean free path permits phase-randomizing scattering; the barriers are described by elastic scatterers. We analyze in detail the evolving residual-resistivity dipole surrounding the barriers. We investigate the additional resistance due to the barriers and study the transition from completely coherent to completely incoherent transmission. The transport regime is governed by the ratio of the mean free path to the well width. We discuss the effect of phase-breaking scattering on the tunneling process leading to a decrease in the resonant transmission probability and a rise in the off-resonant one. A formula for the density of states in the well is also obtained.

I. INTRODUCTION

Resonant tunneling is attracting increasing attention in view of recent experiments in which an oscillatory behavior in the conductance of narrow channels with varying Fermi energy or, equivalently, electron density was observed. It is very likely that in the Si devices investigated the periodic conductance oscillations are related to the presence of two dominant scattering centers which define an isolated segment within the channel.[1,2] On the other hand, in high-mobility GaAs nanostructures one can adjust electrostatically created potential barriers in the one-dimensional electron gas.[3] Calculations performed very recently by de Aguiar and Wharam[4] based on the resonant tunneling concept have shown reasonable agreement with experimental data. However, there are effects beyond purely ballistic transport, particularly incoherent scattering events, leading to a broadening of the resonance and a decrease of the peak value. This corresponds to the measured conductance that is temperature independent at the lowest temperatures and that shows a continuously decreasing amplitude of the oscillations as the temperature is raised.[2] Therefore it is essential to have a theory of the influence of incoherent processes on resonant tunneling in order to make sensible contact with experiment. We develop such a theory in general terms below.

From a methodical point of view, different approaches have been proposed to describe coherent and incoherent propagation simultaneously.[5] In the present context we mention the Breit-Wigner formalism.[6,7] However, this method has only a limited range of applicability, namely, the crossover from coherent to sequential tunneling. Büttiker[8,7] derived an expression for the conductance which allows for both coherent and incoherent scattering processes. His approach models phase randomizing by connecting the well of the double-barrier structure via perfect conductors to a reservoir. The reservoir into which carriers are scattered with a tunable probability acts as an inelastic scatterer and permits phase breaking.

Our system under consideration, sketched in Fig. 1, is more natural than Büttiker's model. The two barriers are located in a wire at points $\pm a$. For simplicity we focus on a one-dimensional (one quantum channel) conductor. The conductor is not a perfect lead but a resistive wire, assumed to be filled with background scatterers of, on average, constant density and certain scattering amplitude each, giving rise to a finite mean free path (MFP), l. Thus, even an unperturbed lead of length L has the resistance[9]

$$\mathcal{R}_w = \frac{\pi\hbar L}{e^2 l}\ , \qquad (1)$$

where we have assumed—as in the following—$L \gg l$ to separate the studied section of the wire from the remaining parts of the whole circuit. l is introduced in our model as a mean-field quantity. It represents the net effect of the elastic interactions (cf. below) between the carrier and the background scatterers in the wire through which it moves after averaging over their positions. In the configurational average the wire behaves like a homo-

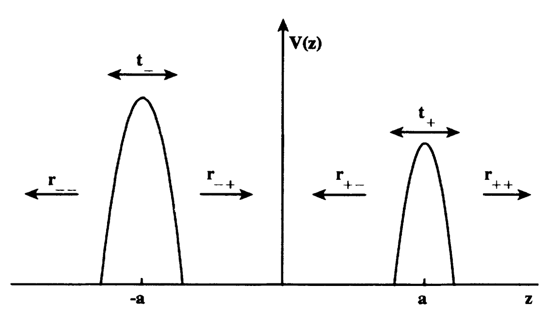

FIG. 1. Model of the double-barrier structure. The carrier propagation in an otherwise homogeneous wire (along the z axis) of the finite mean free path l is affected by the presence of two potential barriers [$V(z)$] located at points $\pm a$. The barriers are characterized by reflection amplitudes r and transmission amplitudes t and define a well of width $2a$.

geneous optical medium where l describes the loss of the coherent wave field.[10]

If we take now into account the double barrier, we have to distinguish between three regimes. First, when the distance between the barriers is small compared with the MFP, $2a \ll l$, we have coherent resonant tunneling. In this case the extra resistance arising from the obstacle is given by the Landauer formula[11,9]

$$\mathcal{R}_B = \frac{2\pi\hbar R}{e^2 T} \, , \tag{2}$$

where $T = 1 - R$ is the transmission probability for carriers to traverse both barriers. T is taken at the Fermi energy. Second, as the well width increases, $2a \simeq l$, incoherence gains significance. A carrier which traverses one of the barriers loses now, with non-negligible probability, its phase memory before escaping from the well, due to the interaction with the disordered background scatterers. Nevertheless, the carriers can execute many oscillations in the well before they scatter incoherently and, hence, the resistance comprises detailed information on the particular configuration of the perturbed wire. Accordingly, the additional resistance is determined by a complicated combination of both background and barrier parameters. The formula for that sequential tunneling will be given in Sec. V. The third case, $2a \gg l$, is again rather simple. During well traversal, each carrier undergoes phase memory loss and reaches the next barrier with a phase uncorrelated to that of the incident particle. If the phase randomization is complete, the extra resistance can be written as

$$\mathcal{R}_B = \frac{2\pi\hbar}{e^2} \left[\frac{R_-}{T_-} + \frac{R_+}{T_+} \right] = \mathcal{R}_- + \mathcal{R}_+ \, . \tag{3}$$

Here $\mathcal{R}_B$ is the sum of the resistances due to the individual scatterers, $\mathcal{R}_\pm$, where $\mathcal{R}_\pm$ are given by the Landauer formula and $T_\pm = 1 - R_\pm$ refer to the transmission through one barrier only.

The aim of this paper is to present a methodically new approach which describes the continuous transition from completely coherent tunneling through the double-barrier structure, formula (2), to the completely incoherent tunneling, formula (3). The transport regime is governed by the relation between MFP and well width. We can influence the ratio $l/2a$ either by a change of the distance, i.e., the sample design, or, alternatively, by a change of l. As is well known, the MFP depends on the carrier energy, which is controlled by the Fermi energy, i.e., the gate voltage in practice, and the temperature as well. In spite of some oversimplifications, our model, in comparison with previous attempts to incorporate phase-randomizing processes, thus has the advantage that the "incoherence measure," $l/2a$, can be related straightforwardly to properties of the real physical system.

We solve the problem defined above with the superposition method. This formalism rests on a simple and, going back to its origin, old idea:[12] the wave field and the corresponding density matrix, too, can be constructed by the superposition of waves emerging from all scatterers belonging to a given system. We have shown[9,13,14] that the superposition method, also in its lowest approximation, is well suited to treat the dc transport in problems with inhomogeneities or spatial restrictions. The main assumption of the superposition method as applied in the next section is $\lambda_F \ll l$, i.e., the MFP in the wire is large compared to the Fermi wavelength. We remark that Datta[15] derived recently a kinetic equation for steady-state quantum transport which is similar to the superposition formula. In generalization of our framework, however, he considers from the very beginning elastic and inelastic phase-breaking background scattering. Therefore we believe that our results remain valid if we use l as an effective parameter determined by both processes. Owing to this phenomenological interpretation, l is influenced by temperature effects, too.

Within the superposition method we handle only an elastic, time-independent scattering problem of noninteracting carriers. This aspect can be considered, up to a certain point, of course, as an advantage. It is worth noting that the influence of the electric field on the double-barrier structure[16] and the complicated effect due to the space-charge pileup (taking place because of the accumulation of electrons within the well) can be included in our model by an appropriate (self-consistent) calculation of the single-barrier reflection and transmission amplitudes, r and t (Fig. 1), respectively. Time-dependent feedback processes,[17] however, are beyond our scope.

The superposition method can be applied to the diffusion case with a given carrier-density gradient or to the mobility case with a driving force.[9] The diffusion case is mathematically much simpler than the force case and therefore we choose it to start with. This decision is only a formal matter because it is well known how to go from diffusivity to conductivity via the Einstein equivalence.

Our paper is organized in the following way. In Sec. II we outline briefly the basic formulas of the superposition method and give the Green functions. In Secs. III and IV we construct, in a two-step procedure, the solution in the diffusion picture. In Sec. V the result is transformed into the force picture, i.e., the density pileup arising from the barriers passes into a voltage drop and yields the extra resistance, and we discuss some limiting forms. The density of states in the well as a function of the Fermi energy is considered in Sec. VI, in which the role of incoherence is also indicated. Last, we give a short summary in Sec. VII.

II. SUPERPOSITION METHOD AND GREEN FUNCTIONS

The stationary diffusion current is driven by a carrier-density gradient that approaches its unperturbed value far from the double barrier in a homogeneous wire, $\text{grad}\rho = \text{const}$. Then, according to Lenk,[9] the current-induced carrier redistribution due to the obstacles, $\delta\rho$, obeys, in the same approximation as in the mentioned paper, an integral equation

$$\delta\rho(z)=\rho_{\rm ind}(z)+\frac{{\rm Im}k^2}{{\rm Im}G_w(0)}\int_{-\infty}^{\infty}dz'|G(z,z')|^2\delta\rho(z') . \quad (4)$$

G is the one-particle Green function

$$[\partial^2/\partial z^2+k^2-(2m/\hbar^2)V(z)]G(z,z')=-\delta(z-z') , \quad (5)$$

where $k=k'+ik''$ denotes the medium wave number. Its imaginary part $k''=(2l)^{-1}$ follows from the presence of background scatterers and is responsible for the attenuation of the coherent wave field.[10] In Eq. (5) the potential, $V(z)$ (Fig. 1), describes the barriers disturbing the propagation process. For convenience, $G=G_w+G_{\rm sc}$ is decomposed into an unperturbed wire term ($V\equiv 0$), $G_w(z,z')=(i/2k)\exp(ik|z-z'|)$, and a scattering part $G_{\rm sc}$.

The scattering of carriers incident at a barrier gives rise to a current-induced coherent density change $\rho_{\rm ind}$ which reads

$$\rho_{\rm ind}=\rho_1+\rho_{2'}$$

$$\rho_1(z)=4k' \text{ grad } \rho\int_{-\infty}^{\infty}dz'{\rm Im}\left[G^*_{\rm sc}(z,z')\frac{\partial}{\partial z'}G_w(z,z')\right] , \quad (6a)$$

$$\rho_2(z)=i2k' \text{ grad } \rho\int_{-\infty}^{\infty}dz'G_{\rm sc}(z,z')\frac{\partial}{\partial z'}G^*_{\rm sc}(z,z') . \quad (6b)$$

Since $\rho_{\rm ind}$ is subject to a multiple scattering process in the wire which destroys phase relations, the coherent density is attenuated and localized within a few MFP around the barriers. Corresponding to Eq. (4), $\rho_{\rm ind}$ acts as an initial distribution of a transport process to follow. Asymptotically, this is classical diffusion. We emphasize, however, that only the pure-wire term $|G_w|^2$ in the integral on the right-hand side (rhs) of (4) is due to the diffusion pole in a q-space representation

$$\frac{{\rm Im}k^2}{{\rm Im}G_w(0)}\lim_{q\to 0}|G_w|^2(q)=1 , \quad (7)$$

responsible for the long-range behavior of the diffusive solution $\delta\rho(z)$. Nevertheless, the integrand comprises two further contributions with $G_{\rm sc}$ and $|G_{\rm sc}|^2$, respectively. Thus a defect acts twofold: it yields a coherent density perturbation and affects the following diffusion process.

Supposing that the width of the barriers is small compared to the MFP, we do not use the potential $V(z)$ in Eq. (5) directly, but characterize the barriers as usual by reflection and transmission amplitudes, r and t, defined in accordance with Fig. 1. These amplitudes are conveniently expressed in the form[7]

$$\begin{aligned} t_\pm&=T_\pm^{1/2}\exp(i\varphi_\pm) ,\\ r_{++}&=iR_+^{1/2}\exp[i(\varphi_++\tau_+)] ,\\ r_{-+}&=iR_-^{1/2}\exp[i(\varphi_-+\tau_-)] ,\\ r_{+-}&=iR_+^{1/2}\exp[i(\varphi_+-\tau_+)] ,\\ r_{--}&=iR_-^{1/2}\exp[i(\varphi_--\tau_-)] , \end{aligned} \quad (8)$$

where, for example, for the right barrier, $T_+=|t_+|^2$ and $R_+=|r_{++}|^2=|r_{+-}|^2$ are the transmission and reflection coefficient, respectively; φ_+ is the phase accumulated during barrier transversal; $\varphi_++\tau_+$ is the phase change associated with reflection for carriers incident from the right-hand side; and $\varphi_+-\tau_+$ is the phase change due to reflection of carriers from the left-hand side (lhs). Let us now apply Eqs. (8) to find $G_{\rm sc}$ outside the barriers. For brevity we do not present this calculation but only give the final result,

$$G_{\rm sc}(z,z')=\begin{cases}(r_{--}e^{-2ika}+t_-^2r_{+-}e^{2ika}Z^{-1})G_w(z+z'), & z'<-a\\ (t_-r_{+-}e^{2ika}Z^{-1})G_w(z+z')+(t_-Z^{-1}-1)G_w(z-z'), & -a<z'<a\\ (t_-t_+Z^{-1}-1)G_w(z-z'), & z'>a.\end{cases} \quad (9)$$

Equations (9) are valid in the domain $z<-a$. Obviously, the denominator $Z=1-r_{-+}r_{+-}e^{4ika}\equiv 1+(R_-R_+)^{1/2}e^{i\phi-2a/l}$ sums up the coherent contributions for one or more full revolutions in the well before losing phase memory. The phase increment associated with a revolution reads

$$\phi=4k'a+\varphi_+-\tau_++\varphi_-+\tau_- . \quad (10)$$

The phase accumulated by traversing the piece of wire between the barriers is

$$2ak'=2a(2mE)^{1/2}/\hbar , \quad (11)$$

where E is the energy of the carriers.

To get solutions for $z>a$ we have to interchange all indices and to transfer accordingly the domains of z' in (9). As far as we are interested in the density pileup it is sufficient to consider Eq. (4) on the lhs ($z<-a$) and rhs ($z>a$) of the double barrier (see below). Both intervals are described by the set of Green functions (9) and its counterpart. On the other hand, the Green function in the remaining region, $-a<z,z'<a$,

$$G_{\rm sc}(z,z')=\frac{i}{2kZ}[r_{-+}e^{2ika}(e^{ikz'}+r_{+-}e^{2ika}e^{-ikz'})e^{ikz}+r_{+-}e^{2ika}(e^{-ikz'}+r_{-+}e^{2ika}e^{ikz'})e^{-ikz}] , \quad (12)$$

yields the density of states in the well as discussed in Sec. VI.

III. INDUCED DENSITY

To evaluate explicitly the induced density (6) we neglect the regions of the barriers in the integrals over z'. Remember that their widths should be small on the l scale. This approximation allows us to use only the Green functions (9) defined outside the scatterers. From Eqs. (6) we obtain for $|z|>a$

$$\rho_{\rm ind}(z)=\mathrm{sgn}(z)l\,\mathrm{grad}\rho R_{l,r}\exp(-|z\pm a|/l) \quad \text{for} \begin{cases} z<-a \\ z>a \end{cases}, \tag{13}$$

where

$$\begin{aligned} R_{l,r} &= 1-T_{\mp}[1-R_{\pm}\exp(-2a/l)]/|Z|^2 \\ &= 1-T_{\mp}(1-e^{-2a/l})/|Z|^2-T_-T_+e^{-2a/l}/|Z|^2 \end{aligned} \tag{14}$$

is the coherent reflection coefficient for carriers incident at the double barrier from the lhs (rhs). It is convincing that $1-R_{l,r}$ is determined by two processes, namely, transmission through one obstacle followed by incoherent scattering before reaching the next one [second term in the last line of (14)], and coherent transmission through the double barrier,

$$T_c=T_-T_+\exp(-2a/l)/|Z|^2 \; . \tag{15}$$

In the limiting cases $a \gg l$, $R_{r,l}\to 1-T_{\pm}=R_{\pm}$, and $a \ll l, R_{r,l}\to 1-T_-T_+/|Z|^2$, the induced density (13) reduces to the simpler result for one obstacle,[9] where in the latter case the scatterer consists of the two consecutive barriers.

According to (13) there is, related to the diffusion current $\sim -\mathrm{grad}\rho$, a density pileup in front of the double barrier and a deficiency behind it. This agrees with the concept one has in mind if a current is hindered by an obstacle. We mention, however, that $\rho_{\rm ind}(z)$ is in general not an antisymmetric function of z and, therefore, does not form a coherent density dipole, cf. the behavior of the total density change $\delta\rho$ (Sec. IV). Finally, we point to the fact that besides the smooth term given in (13) there are oscillatory interference terms. Their very existence is a consequence of our nonclassical procedure. As discussed in Refs. 9 and 14, they are irrelevant for the solution of the diffusion process and, hence, can be omitted here.

IV. TOTAL DENSITY

The integral equation (4) can be easily satisfied by keeping in mind that the corresponding one-dimensional problem of a single obstacle (located at point $z=0$) can be solved by a simple dipole distribution $\delta\rho(z)=\alpha\,\mathrm{sgn}(z)$, $\alpha=$const.[9] Now the ansatz $\delta\rho(z)=\alpha\,\mathrm{sgn}(z)$ for $|z|>a$ and $\delta\rho=\beta$ within the well ($|z|<a$), $\alpha,\beta=$const, proves to be correct. The unknown parameters, α and β, can be obtained from two independent equations which may be chosen to be Eq. (4) in the domains $z<-a$ and $z>a$ for simplicity. Substitution of our ansatz and the induced density (13) into (4) yields after a straightforward calculation

$$\pm l\,\mathrm{grad}\rho R_{r,l}+\tfrac{1}{2}\beta S_{r,l}\mp\tfrac{1}{2}\alpha[1+\exp(-2a/l)](1-R_{r,l}) = 0 \quad \begin{cases} z>a \\ z<-a \end{cases}, \tag{16}$$

where we have suppressed as in (13) unimportant oscillatory terms. Note that the undamped ansatz term $\pm\alpha$ on the lhs of (4) is canceled by an equivalent one originating from the pure-wire contribution $|G_w|^2$ in the integrand on the rhs. From Eqs. (16) we derive the total density

$$\delta\rho(z)=\mathrm{sgn}(z)l\,\mathrm{grad}\rho[T_{\rm tot}^{-1}-2/(1+e^{-2a/l})], \quad |z|>a \; , \tag{17a}$$

$$\delta\rho=2l\,\mathrm{grad}\rho(R_l-R_r)/[S_l(1-R_r)+S_r(1-R_l)] \; , \quad |z|<a \; , \tag{17b}$$

where coefficients are defined by

$$S_{l,r}=[1-\exp(-2a/l)]T_{\mp}[1+R_{\pm}\exp(-2a/l)]/|Z|^2 \tag{18}$$

and

$$T_{\rm tot}=\frac{1+\exp(-2a/l)}{2}[S_l(1-R_r)+S_r(1-R_l)] \times(S_l+S_r)^{-1} \; . \tag{19}$$

Let us first consider these relations and then return to (17). $S_{l,r}$ can be viewed as probability for a carrier to escape from the well through the left (right) barrier. Owing to the prefactor $[1-\exp(-2a/l)]$ which determines the amount of particles suffering phase breaking in the piece of wire between both barriers, $S_{l,r}$ denotes an incoherent transmission coefficient. Consequently, only the fraction $S_{l,r}/(S_l+S_r)$ of carriers moving initially in the well leaves the double barrier to the left (right). Since the prefactor mentioned does not appear in the quotient this probability gives the total escape rate. In the light of that interpretation it seems to be reasonable to call $T_{\rm tot}$ from Eq. (19) the total transmission probability of both barriers, including coherent and incoherent transversing processes as well. Indeed, transmission through the sample from the right to the left, for instance, should be associated with the term $(1-R_r)$ multiplied by the factor we have just discussed. This picture becomes more convincing when we transform $T_{\rm tot}$ into

$$T_{\rm tot}=T_c+T_{ic} \; , \tag{20}$$

where T_c, from Eq. (15), is the probability for a carrier to transverse the double barrier coherently and

$$T_{ic}=S_lS_r/(S_l+S_r) \tag{21}$$

is the transmission probability for carriers which have suffered phase-breaking events. It is self-evident that the coherent coefficient, T_c from Eq. (15), cannot be evaluated as if no incoherent scattering takes place within the well, since it is also influenced by the presence of these processes. Equations in a similar sense as (20) and especially formula (21) were originally proposed by Büttiker.[8,7] Interestingly, Büttiker starts from the Landauer approach[18,11] and models the phase-randomizing agent by a reservoir.

The density change (17) appears in response to the double barrier and occurs in two steps at $\pm a$. It is superimposed on the wire density gradient and, therefore, associated with the extra resistance (cf. Sec. V) due to the barriers. The piled-up density inside the well (17b) reaches a value between those of the lhs and rhs (17a); in particular $\delta\rho=0$ holds for equal scatterers, $R_-=R_+$. Guided by our analysis we see the long-range change $\delta\rho$ as a result of diffusive propagation and recombination of excess and deficit carriers generating initially the localized coherent density. This picture is not new. Really, it was introduced by Landauer more than 30 years ago.[18] In his well-known 1957 paper, he pointed out that the transport field arising from a scatterer is a dipole field called residual resistivity dipole. And, in fact, the total density $\delta\rho(z)$ outside the well (17a) is an antisymmetric function of z, i.e., a dipole distribution, in difference to the local quantity $\rho_{\rm ind}(z)$ (13).

V. EXTRA RESISTANCE

At the beginning of this section, we transcribe our result to the more familiar situation where the current is driven by a constant electric field. In a charge-compensating background of (on the average) constant density the long-range diffusion dipole (17) is reflected by a voltage drop established within a microscopic screening length. In particular, if we are interested in the overall drop following from the scatterers, the density difference between both sides of the double barrier [Eq. (17a)] counts. Solution (17) gives the density change on each energy shell and, generally, voltage drop, resistance, etc. are related to the sum of all these contributions weighted by the derivation of the Fermi function and the density of states. This procedure is outlined in Ref. 9. Here we assume a degenerate electron gas where the spread kT is small compared to the width of the resonances. With regard to the experimental situation we note, however, that this is only for the sake of simplicity and the general case is not excluded. Guided by Ref. 9 we deduce for the extra resistance due to the barriers (see the Appendix)

$$\mathcal{R}_B=\frac{2\pi\hbar}{e^2}[T_{\rm tot}^{-1}-2/(1+e^{-2a/l})]\ , \tag{22}$$

where $T_{\rm tot}$ is taken at the Fermi energy. Equation (22) as well as Eqs. (1)–(3) are valid for a single electron, i.e., the factor for spin degeneracy is omitted everywhere or, for example, the Coulomb blockage[19] prevents the double occupation of a state.

In Fig. 2 the additional resistance due to the double barrier, $\mathcal{R}_B$, from Eq. (22), is shown as function of l (in units of the width $2a$). The upper curve shows the maximum resistance (off-resonant transmission) and the lower curve shows the minimum resistance (peak transmission). The condition for resonance of the transmission probability $T_{\rm tot}$ from Eq. (19) is $|Z|^2\rightarrow\min$ [see Eq. (9)] and thus

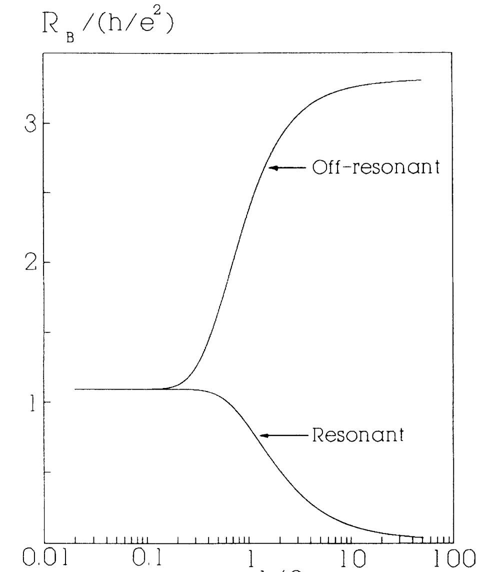

FIG. 2. Resistance of a double-barrier structure for off-resonant and resonant transmission as a function of the ratio of mean free path to well width, $l/2a$. The two barriers forming the well have transmission probability $T_-=0.7$ and $T_+=0.6$.

$\phi=(2m+1)\pi$ [Eq. (10)], where m is an integer. This expression determines the phase [Eq. (11)] accumulated at resonance and the corresponding resonant energy E_r. On the other hand, the transmission is minimal or off-resonant for $\phi=2m\pi$. Generally, the upper and the lower curve give the amplitude of the resistance oscillations with varying energy for each ratio $l/2a$.

For $l\gg a$, when the transport through the barriers is coherent, our result (22) reduces to the Landauer formula (2) given in the Introduction. We find in this limit $T_{ic}=0$ (21) and $T=T_{\rm tot}=T_c$ [Eqs. (20) and (15)]. The validity of the Landauer formula for an obstacle in a wire was confirmed within the framework of the superposition method in Ref. 9. With decreasing MFP a small number of carriers is scattered incoherently during well traversal and thus we are in the crossover region from coherent to sequential tunneling. Figure 2 shows that an increasing number of sequential processes (decreasing l) leads to an approach of the maximum resistance and the minimum resistance caused both by a rise in the minimum transmission probability and by a decrease in the maximum transmission probability. Incoherent processes lead therefore to lower peak values in the transmission, but raise the off-resonant transmission. We emphasize, however, that this behavior of $\mathcal{R}_B$ is only of importance if $T_\pm\ll 1$, i.e., for example, particles are incident upon two consecutive barriers of height much greater than their energy E. Otherwise the resistance of the wire [see Eq. (1)] would contribute significantly to the total resistance. To study the crossover from coherent resonant tunneling to sequential tunneling in the case $T_\pm\ll 1$ it is possible to apply the Breit-Wigner scattering formalism. Detailed discussions were given by Stone and Lee[6] and

Büttiker.[7] Therefore, the representation of the analogous results in our model can be omitted.

For $2a \gg l$, i.e., $\exp(-2a/l)$ close to 0, only small corrections remain from the completely incoherent transmission. These coherent corrections are due to a tiny fraction of carriers that can execute one or more full revolutions in the well before losing phase memory. The correction term to the incoherent result (3) depends therefore on the phase (10) accumulated during well traversal and is sensitive to the geometrical arrangement of the barriers. Equation (22) yields in the case where the carriers can undergo at most one complete revolution before leaving the well [by expanding Eq. (22) to first order in $\exp(-2a/l)$]

$$\mathcal{R}_B = \frac{2\pi\hbar}{e^2}\left[\frac{R_-}{T_-}+\frac{R_+}{T_+}+2\sqrt{R_-R_+}\,\frac{T_-+T_+}{T_-T_+}\times\exp\left[-\frac{2a}{l}\right]\cos\phi\right] . \quad (23)$$

The last formula is valid independent of the magnitude of the transmission probabilities $T_\pm$ and confirms clearly the resonance condition given above. If the Fermi energy is such that $\phi=(2m+1)\pi$, we have maximum transmission, and for $\phi=2m\pi$ we have minimal transmission. For $2a \gg l$, Eq. (23) gives exactly the resistance (3) for completely incoherent transmission. In this limit every carrier traversing the well is scattered incoherently, the coherent transmission probability T_c [Eq. (15)] vanishes and $S_{l,r}=T_\mp$. Hence, the total transmission probability (20) reads $(T_-^{-1}+T_+^{-1})^{-1}$. In the case of strong incoherent scattering the resistance contains no detailed information on the separation of the barriers but is the sum of the individual resistances. In fact, the density pileup within the well (17b) reduces then to $\delta\rho = l\,\mathrm{grad}\,\rho(R_-/T_- - R_+/T_+)$ so that the density difference between both sides of each obstacle obeys the Landauer formula separately as indicated by the rhs of (3).

VI. DENSITY OF STATES

Besides the transport process the density of states in the well, n, depends on the degree of sequential tunneling and its discussion illustrates the effect of incoherent scattering, too. The density of states (DOS) is given by the imaginary part of the corresponding Green function, namely,

$$n(z,E)=(2m/\pi\hbar^2)\mathrm{Im}G(z,z) . \quad (24)$$

According to our decomposition of G [see Eq. (5)], the DOS comprises an unperturbed term due to the wire,

$$n_w(E)=(2m/\pi\hbar^2)\mathrm{Im}G_w(z,z) = m/\pi\hbar^2k' = (\pi\hbar)^{-1}\sqrt{m/2E} , \quad (25)$$

where we have suppressed the small imaginary part of k. If we are not interested in density variations on the scale of a Fermi wavelength it is sufficient to use a DOS which has been averaged over a small z volume several times larger than the scale set by the Fermi wavelength. Supposing that the well is wide compared to the Fermi wavelength to render the averaging procedure valid, Eq. (24) yields

$$n(E)/n_w(E)=(1-s^2)/(1+2s\cos\phi+s^2) , \quad (26)$$

with $s=\sqrt{R_-R_+}\exp(-2a/l)$. Nevertheless there are oscillatory density terms. These terms are even dominant for small barriers, $R_\pm \ll 1$, and $2a \gg l$. The reduced DOS (26) in the resonant well is shown as function of $E-E_r$ for three ratios $l/2a$ in Fig. 3. Equations (10) and (11) are used to obtain the energy dependence. Here the energy is measured in units of an attempt-to-escape frequency,

$$f=\hbar k_r/4am=(2a)^{-1}\sqrt{E_r/2m} , \quad (27)$$

where k_r is the wave number of a carrier in the well at the resonant energy. Obviously, as the number of incoherent processes increases ($l/2a$ decreases), the DOS in the well becomes less sharply peaked at the resonance energy and broadens. Furthermore the minimum resistance

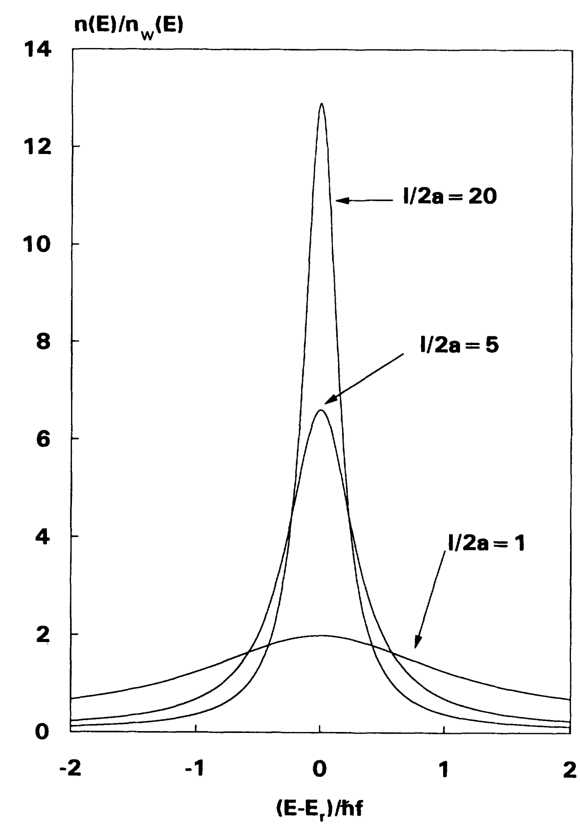

FIG. 3. Reduced density of states in the resonant well for three different ratios of the mean free path l to the well width $2a$, i.e., for different degrees of incoherent scattering, as a function of energy. E_r is the resonant energy and f is the well frequency; $n_w(E)$ is the density of states in the homogeneous wire. The reflection coefficients are $R_-=R_+=0.9$.

at $\phi=(2m+1)\pi$ coincides with an enhanced DOS and the minimal transmission at $\phi=2m\pi$ is accompanied by a reduced DOS.

In the crossover region from coherent to sequential tunneling it is convenient to represent the phase randomization by a small imaginary part of the energy, $\Gamma/2$. Thus, if we allow for complex ϕ, the denominator in (26) vanishes at the energy $E=E_r-i\Gamma/2$. The total decay width $\Gamma=\Gamma_c+\Gamma_{ic}$ comprises a coherent width,

$$\Gamma_c=-\hbar f \ln(R_-R_+) , \tag{28a}$$

and an incoherent contribution,

$$\Gamma_{ic}=\hbar f 4a/l . \tag{28b}$$

Expanding Eq. (26) near resonance yields for the number of states in the well per unit energy

$$\frac{dN}{dE}=2an(E)=\frac{1}{\pi}\frac{\Gamma/2}{(E-E_r)^2+(\Gamma/2)^2} . \tag{29}$$

We emphasize that this Lorentzian-shaped DOS is determined by the total width, Γ. An expression similar to (29) was derived in Ref. 7.

VII. SUMMARY

We have dealt with resonant tunneling through a double-barrier structure. The barriers are embedded in a resistive wire in which phase randomizing takes place. The resistance of the wire and its phase-breaking property are expressed in our model by a single phenomenological parameter, the mean free path. We have discussed the limiting regimes of completely coherent tunneling (2) and completely incoherent tunneling (3) and have shown that our result (22) describes the continuous transition between the two as a function of well width and MFP. In contrast to a customary approach[7,8] we considered the double barrier and the wire in a more convincing model abandoning perfect leads and ideal reservoirs; suppositions on the specific nature of the contact between the leads and the reservoirs or barriers, respectively, are unnecessary. That we are able to study the double barrier in the three different transport regimes without extensive assumptions is in itself an important result of our method.

Figure 2 shows that, for off-resonant transmission, incoherent scattering increases the transport through the barriers but destroys the resonant transmission. Thus, not only coherent resonant tunneling but also incoherent scattering can give rise to structure in the conductance. With decreasing MFP, i.e., normally with rising temperature, the amplitude of the conductance oscillations (or resistance oscillations according to Fig. 2, respectively) decreases. This behavior agrees qualitatively with the experimental data on narrow channels.[2] Nevertheless, more work has to be done to attain a realistic description of the experiments.

In concluding this paper, we remark that the superposition method applied proves to be powerful to treat problems with localized perturbations. Particularly, in the extension of well-known investigations on double-barrier structures, we have given a detailed analysis of the evolving residual-resistivity dipole surrounding the obstacles [see Eqs. (17)]. Our calculations confirm Landauer's idea[18] that the formation of the residual-resistivity dipole is the microscopic mechanism which yields the voltage drop across a scatterer or barrier.

ACKNOWLEDGMENT

I wish to thank Professor Dr. R. Lenk for many stimulating and productive discussions.

APPENDIX: TRANSITION FROM THE DIFFUSION TO THE FORCE CASE

In the ensemble of all noninteracting carriers, moving with different energies, the density of the diffusion problem reads in a homogeneous wire

$$\begin{aligned} \rho(z)&=\frac{2m}{\pi\hbar^2}\mathrm{Im}G_w(z,z)f(E-\mu(z)) , \\ \mathrm{grad}\,\rho&=n_w(E)\frac{df}{dE}\left[-\frac{d\mu}{dz}\right] , \end{aligned} \tag{A1}$$

where f is the one-particle distribution function and $\mu(z)$ the varying chemical potential. According to the Einstein equivalence, $-d\mu/dz$ is proportional to a driving force F, i.e., the overall density gradient can be replaced by a constant field of force. The current has the value

$$I=F\int dE\left[-\frac{df}{dE}\right]n_w(E)D(E) \tag{A2}$$

with $D(E)=\hbar k' l(E)/m$ as the $1d$ diffusivity. Now we use the fact that the Einstein equivalence holds not only for the current itself but for any current-proportional quantity.[20] Especially in the force case, a definite asymptotic density difference between both sides of the double barrier corresponds to the distribution (17a) we have found in the diffusion picture. With the transcription rule (A1) we get

$$\begin{aligned} \Delta\rho&\equiv\delta\rho(z>a)-\delta\rho(z<a) \\ &=2F\int dE\left[-\frac{df}{dE}\right]n_w(E)l(E)[T_{\mathrm{tot}}^{-1}-2/(1+e^{-2a/l})] \end{aligned} \tag{A3}$$

for an ensemble of noninteracting particles. For interacting carriers, $\Delta\rho$ must be compensated by a screening process. Indeed, the electrostatic potential difference ΔU created by the final, screened dipole distribution yields such a shift in the band bottoms on both sides that the resulting carrier densities are equalized. These quantities fulfill therefore the relation

$$\Delta\rho = e\,\Delta U \int dE \left[-\frac{df}{dE}\right] n_w(E) \, . \tag{A4}$$

Now (A2)–(A4) determine the resistance,

$$\mathcal{R}_B = \frac{\Delta U}{eI} = \frac{2\int dE \left[-\frac{df}{dE}\right] n_w(E) l(E) [T_{\rm tot}^{-1} - 2/(1+e^{-2a/l})]}{e^2 \int dE \left[-\frac{df}{dE}\right] n_w(E) D(E) \int dE \left[-\frac{df}{dE}\right] n_w(E)} , \tag{A5}$$

where the spin factor has been omitted everywhere. For a degenerate gas with a sharp Fermi energy (A5) reduces to the simpler result (22).

[1]J. H. F. Scott- Thomas, S. B. Field, M. A. Kastner, H. I. Smith, and D. A. Antoniadis, Phys. Rev. Lett. **62**, 583 (1989).

[2]S. B. Field, M. A. Kastner, U. Meirav, J. H. F. Scott-Thomas, D. A. Antoniadis, H. I. Smith, and S. J. Wind, Phys. Rev. B **42**, 3523 (1990).

[3]U. Meirav, M. A. Kastner, and S. J. Wind, Phys. Rev. Lett. **65**, 771 (1990).

[4]F. M. de Aguiar and D. A. Wharam, Phys. Rev. B **43**, 9984 (1991).

[5]G. Czycholl and B. Kramer, Solid State Commun. **32**, 945 (1979); Y. Gefen and G. Schön, Phys. Rev. B **30**, 7323 (1984); A. D. Stone, M. Ya. Azbel, and P. A. Lee, *ibid.* **31**, 1707 (1985).

[6]A. D. Stone and P. A. Lee, Phys. Rev. Lett. **54**, 1196 (1985).

[7]M. Büttiker, IBM J. Res. Develop. **32**, 63 (1988).

[8]M. Büttiker, Phys. Rev. B **33**, 3020 (1986).

[9]R. Lenk, Phys. Status Solidi B **161**, 797 (1990).

[10]R. Lenk, Phys. Status Solidi B **154**, 733 (1989).

[11]R. Landauer, Philos. Mag. **21**, 863 (1970).

[12]L. L. Foldy, Phys. Rev. **67**, 107 (1945).

[13]R. Lenk, Phys. Status Solidi B **162**, 227 (1990).

[14]A. Knäbchen, J. Phys. Condens. Matter **3**, 6989 (1991).

[15]S. Datta, Phys. Rev. B **40**, 5830 (1989); J. Phys. Condens. Matter **2**, 8023 (1990).

[16]B. Ricco and M. Ya. Azbel, Phys. Rev. B **29**, 1970 (1984).

[17]C. Presilla, G. Jona-Lasinio, and F. Capasso, Phys. Rev. B **43**, 5200 (1991).

[18]R. Landauer, IBM J. Res. Dev. **1**, 223 (1957); **32**, 306 (1988).

[19]H. van Houten and C. W. J. Beenakker, Phys. Rev. Lett. **63**, 1893 (1989).

[20]R. Lenk, Phys. Status Solidi B **158**, 653 (1990).