Quantum transport and pinning of a one-dimensional Wigner crystal

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We study electron transport along a strongly depleted one-dimensional channel and assume the limit of very weak disorder when electrons form the Wigner crystal. Pinning of this crystal by a weak potential of single impurity is considered in the quantum case. Zero-point oscillations of the crystal significantly diminish the pinning barrier. However, it remains finite at moderate densities of electrons ($na_B \lesssim 0.5$), in spite of the absence of the long-range order in one dimension. Charge transfer in the system occurs due to thermally assisted tunneling of the crystal through the pinning barrier, which results in a power-law temperature dependence of the conductance $\sigma(T)$. The nonlinear conductance $\sigma(U)$ at $T=0$ obeys a similar law.

I. INTRODUCTION

Gated semiconductor structures have a remarkable advantage in the study of electron-transport phenomena in mesoscopic systems: the electron density in the sample can be controlled by means of the gate voltage. At high densities, quasi-one-dimensional channels formed by the depletion of a two-dimensional (2D) electron gas in a heterostructure or in a Si metal-oxide-semiconductor field-effect transistor (MOSFET) reveal high ($\sigma > e^2/h$) conductance of a metallic type.[1,2] Moreover, for channels with a moderate length, $L < 1$ μm, the ballistic electron propagation can be observed. These facts prove that at sufficiently high 1D electron densities, $n > 1/a_B$, the interaction between electrons does not have drastic effects and a free-electron description is an adequate one (here, a_B is the Bohr radius for the semiconductor material). However, the Coulomb interaction becomes increasingly important for a stronger gate-induced depletion of the channel when $na_B \ll 1$. This interaction can control the crossover to the dielectric behavior of the channel conductance. The particular mechanism for such a crossover depends on the magnitude of the random fluctuations of electrostatic potential which forms the channel.

If the fluctuations are large, the channel breaks, forming a sequence of quantum dots,[3] and the transport is controlled by tunneling between them. Under certain conditions, the conventional Coulomb blockade determines the character of this tunneling.[4,5]

If fluctuations are small, a one-dimensional Wigner crystal (WC) can be formed. This crystal is pinned by weak inhomogeneities in the channel and the electron transport is determined by tunneling and activation of the crystal. Such a possibility for silicon MOSFET channels was first suggested by Scott-Thomas *et al.*[6] in order to explain their experimental data.

In this paper we study the pinning of a 1D Wigner crystal by a single impurity, creating a smooth potential for each electron (Fig. 1). The total potential felt by the crystal as a whole is a periodical one with the crystal period a. Therefore, the elementary charge transfer in the system occurs due to the shift of the WC by its period a. It can be provided either by thermal activation or by tunneling of the crystal through some effective barrier $V_{\rm pin}$ (pinning energy). Let us denote the height of this barrier in the classical limit, which corresponds to very low electron densities, as V_0. Here we evaluate the pinning energy $V_{\rm pin}$ for the quantum case. The difference between $V_{\rm pin}$ and its classical value V_0 is caused by zero-point oscillations of the WC that should wash out the periodic pinning potential felt by the crystal. It is natural to characterize the magnitude of this effect by the dimensionless parameter

$$\alpha = \frac{\pi\hbar}{msa} = \pi \left[\frac{na_B}{2\ln(8.0nD)} \right]^{1/2}, \tag{1}$$

which represents the relative magnitude of quantum fluctuations of the crystal period: $\delta a/a \sim \alpha$. Here, m is the electron mass, s is the sound velocity in the crystal calculated in the Appendix and given by

$$s^2 = \frac{2e^2 n}{\kappa m} \ln(8.0nD), \tag{2}$$

where κ is the dielectric constant, and D is the distance between the WC and a conducting plane (gate). Although we carry out all the derivations for the model of the WC with a closely situated gate ($D \gtrsim a$), the case in which the gate is absent is also briefly discussed in our paper.

The influence of quantum fluctuations on a 1D crystal can be very strong even at $\alpha \ll 1$. This manifests itself as a divergence of the correlator $\langle u(0)u(x)\rangle$, where $u(x)$ is the crystal displacement at the point x. At large x this correlator increases as $\alpha a^2 \ln(x/a)$ that leads to the loss of long-range order in a 1D crystal. It might be expected, then, that for a sufficiently long system the pinning potential will be completely smeared out at any finite α, i.e., $V_{\rm pin}=0$. We will show that this is not the case, because the barrier itself, however small it may be, suppresses the long-wave fluctuations responsible for the absence of long-range order. The remaining oscillatory modes of the WC renormalize the height of the barrier according to the formula

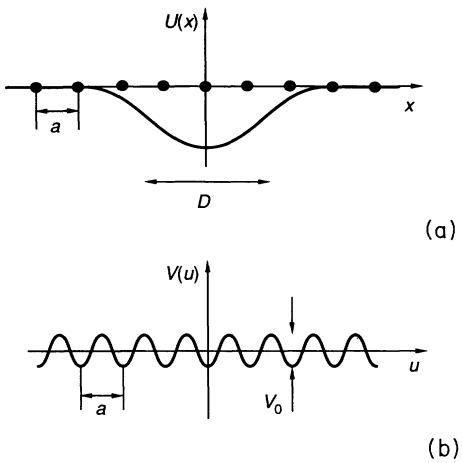

FIG. 1. Pinning of a one-dimensional Wigner crystal by a single impurity in the classical limit. (a) The crystal in a smooth impurity potential $U(x)$. Positions of electrons corresponding to the ground state are shown by circles. (b) The effective pinning potential $V(u)$ as a function of the shift of crystal u with respect to the impurity. The amplitude V_0 may be much less than $U(0)$.

$$V_{\rm pin}=V_0\left[\frac{\beta V_0}{ms^2}\right]^{\alpha/(1-\alpha)}, \tag{3}$$

where $V_0 \ll ms^2$, $\alpha<1$, and β is a numerical factor.

It is quite common to define the crystal, in contrast to the liquid, as an object which can be pinned by an arbitrarily weak potential. According to this definition, Eq. (3) justifies the use of the term "one-dimensional Wigner crystal" in spite of the absence of long-range order. We see from Eq. (3) that the pinning exists at any $\alpha<1$, that usually corresponds to $na_B \lesssim 0.5$ [see Eq. (1)]. One can contrast this behavior to the two-dimensional WC which undergoes quantum melting at very small 2D densities N: According to numerical simulations,[7] $Na_B^2 \approx 2\times 10^{-4}$. This fact results plausibly from an anomalously small value of shear modulus[8] and has no analogy in one-dimensional crystals where transverse degrees of freedom do not exist. That is why, in the sense of the definition given above, it can be much easier to obtain a one-dimensional Wigner crystal than a two-dimensional one.

The existence of pinning energy [Eq. (3)] manifests itself both in temperature dependence of linear conductance $\sigma(T)$ and in the I-V characteristic of the channel. At high temperatures, $\alpha V_{\rm pin} \lesssim k_B T \lesssim V_{\rm pin}$, the conductance has an activation behavior

$$\sigma(T)=\sigma_0 \exp(-V_{\rm pin}/k_B T) . \tag{4}$$

At lower temperatures, $k_B T \ll \alpha V_{\rm pin}$, the tunnel mechanism of transport of the WC dominates. The quasiclassical probability of tunneling of an elastic chain through a single potential barrier had been found by Larkin and Lee.[9] It was shown to vanish in the limit of an infinitely long chain. However, at finite temperatures, when the tunneling can be activation assisted, this probability, as shown in this paper, becomes finite and provides the conductivity

$$\sigma(T)=\sigma_0\left[\frac{k_B T}{\alpha V_{\rm pin}}\right]^{4/\alpha} . \tag{5}$$

Expressions (4) and (5) match at $k_B T \sim (\alpha/4)V_{\rm pin}$. The prefactor σ_0 in these formulas is of the order of e^2/h.

The main signature of a pinning state is the existence of threshold voltage $U_{\rm th}$ in the current-voltage dependence at which a steep increase of current occurs. By analogy with the classical limit, the threshold value $U_{\rm th}$ can be found as $U_{\rm th}=\pi V_{\rm pin}/e$. However, even at voltages far below the threshold, $U \ll U_{\rm th}$, and at $T=0$ the quantum tunneling gives rise to the finite nonlinear conductance

$$\sigma(U)=\sigma_0\left[\frac{U}{U_{\rm th}}\right]^{4/\alpha} . \tag{6}$$

The last result represents a zero-temperature analog of Eq. (5).

As one can see from Eq. (3), the pinning energy, and hence the threshold voltage, vanish at the point $\alpha=1$. This important prediction, as well as applicability of formula (3) to the case of large quantum fluctuations $\alpha \sim 1$, are planned to be discussed elsewhere.[10]

This paper is organized in the following way. In Sec. II a classical model of a 1D Wigner crystal interacting with a charged impurity is described. In Sec. III we give a simplified derivation of pinning energy for the quantum case. Formal calculation of this energy by the variational method is presented in Sec. IV. In Sec. V the quantum transport of a pinned WC is studied. The summary of results and their possible relation to the experiments[6,15] are given in Sec. VI.

II. DESCRIPTION OF THE MODEL

The properties of a Wigner crystal are determined, first of all, by the form of electron-electron interaction. Usually the channel, where electrons are situated, is accompanied by a metallic body (gate) spaced from the channel by some distance D larger than interelectron distance a (we shall assume that $D \gtrsim a$). The gate screens the Coulomb interaction between electrons and remakes it into a short-range repulsion. It manifests itself as a finite phase velocity s of the crystal oscillations with wavelengths larger than D. The value of s is calculated in the Appendix and is given by Eq. (2). Thus, as soon as one is interested in large-scale deformations, the WC can be represented by an elastic chain of particles connected by springs. The system is described by three parameters: the chain period a, the electron mass m, and the sound velocity s. This simplest model of the WC will be used in the present paper. The effects of the long-range Coulomb repulsion, which appear in the absence of the gate, will be discussed briefly in Sec. III and V.

Suppose now, that there is an impurity near the channel that produces, for an electron, a long-range potential $U(x)$ with the spatial scale $d \gtrsim a$ and with the small magnitude $U(0) \ll e^2/\kappa a$ (Fig. 1). When the chain is shifted as a whole by the distance u, its total energy $V(u)$

changes periodically with some amplitude V_0 which defines the pinning energy in the classical case. It is easy to relate V_0 to $U(0)$ in a general form in the approach which neglects the deformation of the WC caused by the impurity potential. In this case the energy of crystal-impurity interaction is given by the sum

$$V(u)=\sum_{i=-\infty}^{\infty} U(ai+u) . \tag{7}$$

The function $V(u)$ is periodic and can be expanded into a Fourier series [for the sake of simplicity we consider an even function $U(x)$]

$$V(u)=\frac{1}{a}\int_{-\infty}^{\infty} dx\ U(x)+\sum_{m=1}^{\infty} U(q_m)\cos(q_m u) , \tag{8}$$

where $q_m=2\pi m/a$ and

$$U(q)=\frac{2}{a}\int_{-\infty}^{\infty} dx\ U(x)\cos(qx) \tag{9}$$

is the Fourier transform of original potential $U(x)$. Because the first term in Eq. (8) does not depend on u, only the second term in Eq. (8) is important.

The Fourier transform $U(q)$ drops with q at a typical scale $q\sim 1/d$. This decrease at $q\gg 1/d$ is known to be at least exponentially fast provided the original potential $U(x)$ has no singularities

$$U(q)\propto \exp(-Aqd) ,$$

where coefficient A depends on the shape of the $U(x)$. In Eq. (8) all $q_m\gg 1/d$ so that $U(q_m)$ drops very fast with m. Hence, we may keep the only term in Eq. (8) with $q_1=2\pi/a$ and get

$$V(u)=V(0)+\frac{V_0}{2}\left[\cos\frac{2\pi u}{a}-1\right] , \tag{10}$$

where $V_0=2U(q_1)$ is given by Eq. (9) and is of the order of $U(0)\exp(-2\pi Ad/a)$.

For example, a charged impurity spaced off from the WC by a distance d such that $D\gg d\gtrsim a$ creates for each electron the Coulomb potential

$$U(x)=\frac{e^2}{\kappa}\frac{1}{(x^2+d^2)^{1/2}} . \tag{11}$$

This provides for the crystal, as follows from Eq. (9), the pinning energy

$$V_0=\frac{4e^2}{\kappa(da)^{1/2}}\exp\left[-\frac{2\pi d}{a}\right] . \tag{12}$$

Thus, if the charged center is shifted away from the axis of the WC as little as on $d=a$, the effective barrier height, compared with the original potential, will be suppressed by two orders of magnitude.

Note that the sinusoidal potential of form (10) had been used in the Hamiltonian,[11] which describes the interaction of a charge-density wave (CDW) with zero-radius impurity potential. This model is based on the assumption that the CDW itself is sinusoidal. We see that the same form of interaction appears automatically for a WC formed by the pointlike electrons, provided that, in contrast, the impurity potential is of long-range nature.

III. QUANTUM SUPPRESSION OF THE PINNING BARRIER

Our goal now is to explore how the quantum oscillations of the chain affect the amplitude of the effective pinning potential. Suppose the pinning center is situated at $x=0$ in the middle of a chain with length L. The interaction between the chain and this center can be described by Eq. (10) with $u=u(0)$, where $u(x)$ is the local chain displacement at point x. This approach is justified since relevant oscillations, as we shall see later on, are the long-wavelength ones. The Hamiltonian of the Wigner crystal interacting with the pinning center has a form

$$H=\frac{1}{2m}\sum_k p_k^2+\frac{ms^2}{2}\sum_k k^2u_k^2-\frac{V_0}{2}\cos\left[\frac{2\pi}{a}u(0)\right] , \tag{13}$$

where p_k and u_k are canonically conjugated operators of momentum, and coordinate of the even oscillation mode with the wave number k ($k=2\pi m/L$; $m=0,1,\ldots,L/2a$), and u_k is defined by

$$u(x)=\left[\frac{2a}{L}\right]^{1/2}\left[\sum_{k>0} u_k\cos(kx)+\frac{u_0}{\sqrt{2}}\right] . \tag{14}$$

The terms related to the odd modes $\sin(kx)$ are omitted in Eqs. (13) and (14) because they do not interact with the center and produce a constant term in the Hamiltonian. The local displacement near the center $u(0)$ in Eq. (13) is given by the sum over even modes only,

$$u(0)=\left[\frac{2a}{L}\right]^{1/2}\left[\sum_{k>0} u_k+\frac{u_0}{\sqrt{2}}\right] . \tag{15}$$

First, we will calculate the shift ε in the ground-state energy due to the pinning potential given by the last term in Eq. (13), as a function of parameter α [Eq. (1)]. The problem of calculating ε is nontrivial, because the perturbation analysis does not work even at very small V_0 (we will further assume $V_0\ll ms^2$). It turns out that for increasingly long systems (i) the first-order correction to the energy vanishes at any α, and (ii) the second-order correction diverges (at $\alpha<\frac{1}{2}$).

Statement (i) can be easily checked by averaging the pinning term in Eq. (13) with the nonperturbed wave function of the ground state. The eigenfunction of the Hamiltonian (13) with $V_0=0$ is given by the product

$$\Psi_0\{u_k\}=B\prod_{k>0}\exp(-u_k^2/4\langle u_k^2\rangle) , \tag{16}$$

with

$$\langle u_k^2\rangle=\frac{\hbar}{2msk} , \tag{17}$$

and B being the normalization factor. After averaging the cosine in Eq. (13) by use of Eqs. (15) and (16), we get

$$\left\langle \Psi_0 \left| \cos\left[\frac{2\pi}{a}u(0)\right] \right| \Psi_0 \right\rangle \propto \exp\left[-\frac{4\pi^2}{aL}\sum_{k>0}\langle u_k^2\rangle\right], \quad (18)$$

which represents the well-known Debye-Waller factor.[12] After substituting Eq. (17) into Eq. (18) and replacing the sum over k by the integral, for the first-order correction $\varepsilon^{(1)}$ to the ground-state energy we obtain

$$\varepsilon^{(1)} = -\frac{V_0}{2}\exp\left[-\alpha\int_0^{\pi/a}\frac{dk}{k}\right]. \quad (19)$$

The integral in Eq. (19) diverges logarithmically at the lower limit and should be cut off at $k \sim 2\pi/L$. The final value of $\varepsilon^{(1)}$ vanishes as $L^{-\alpha}$ with increasing chain length L. It is a consequence of the lack of long-range order in a one-dimensional crystal. The second statement (ii) requires more cumbersome calculations and is planned to be proved elsewhere.[10]

As we shall demonstrate below, the actual shift of energy ε is finite and proportional to a noninteger power of V_0. The origin of this nonanalytical dependence is that even a very small pinning barrier V_0 produces, at $x=0$ roughly speaking, a node for each long-wavelength mode with a wave number less than some typical value $k_0 = k_0(V_0)$ (Fig. 2). The oscillations of these "soft" modes are suppressed near the pinning center. As a result, these modes do not participate in smearing out the pinning potential. In other words, the integral in Eq. (19) should be cut off at the lower limit by $k = k_0$, that yields

$$\varepsilon_1 = -\frac{V_0}{2}\exp\left[-\alpha\int_{k_0}^{\pi/a}\frac{dk}{k}\right] = -\frac{V_0}{2}\left[\frac{k_0 a}{\pi}\right]^{\alpha}. \quad (20)$$

Thus, the existence of nodes for long-wave oscillations is profitable in terms of energy because it leads to the restoration of a finite potential well with the depth $|\varepsilon_1|$. On the other hand, the nodes are also associated with the increase of kinetic energy of zero-point oscillations. Indeed, the existence of additional nodes at $x=0$ for the modes with $k < k_0$ causes their frequencies to shift upward to make them coincide with those of the nearest odd modes (Fig. 2), i.e., the frequencies are increased by $\Delta\omega_k = \pi s/L$. With the total number of shifted modes being $k_0 L/2\pi$, the kinetic energy increases by

$$\varepsilon_2 = \frac{\hbar}{2}\sum_{k<k_0}\Delta\omega_k = \frac{\hbar s k_0}{4}. \quad (21)$$

The wave number k_0 should be chosen in order to minimize the sum of kinetic and renormalized potential energy, Eqs. (21) and (20), respectively. At $\alpha < 1$ the net energy $\varepsilon = \varepsilon_1 + \varepsilon_2$ has its minimum at

$$k_0 = \frac{\pi}{a}\left[\frac{2V_0}{ms^2}\right]^{1/(1-\alpha)}. \quad (22)$$

The corresponding value of ε is given by

$$\varepsilon = -\frac{V_0}{2}(1-\alpha)\left[\frac{2V_0}{ms^2}\right]^{\alpha/(1-\alpha)}. \quad (23)$$

In Sec. IV this formula will be obtained again by means of a regular variational procedure.

At $\alpha > 1$ the sum $\varepsilon = \varepsilon_1 + \varepsilon_2$ has its minimum value $\varepsilon = 0$ at $k_0 = 0$. This result, which implies the absence of pinning at $\alpha > 1$, is discussed in Ref. 10, where the existence of a peculiarity at $\alpha = 1$ for the Hamiltonian (13) is confirmed by the perturbation analysis.

It follows from Eq. (23) that $|\varepsilon|$ is strongly diminished compared with its classical value $|\varepsilon_{\text{class}}| = V_0/2$, if $\alpha \ln(ms^2/V_0)$ is large, although α itself may be small. We have shown that this decrease is due to the high-frequency modes with $k > k_0$. The same modes suppress the amplitude of the pinning potential that acts on the crystal. Hence, the renormalized amplitude of the pinning potential is $|\varepsilon_1|$. Thus, in our approach, which implies a sharp division on high ($k > k_0$) and low ($k < k_0$) modes, the effective pinning potential is given by

$$V_{\text{eff}}(\bar{u}) = -\frac{V_{\text{pin}}}{2}\cos(2\pi\bar{u}/a), \quad (24)$$

with $V_{\text{pin}} = 2|\varepsilon_1|$. Here, $\bar{u}(0)$ means the crystal displacement (15) averaged over the high-frequency modes. After substituting Eq. (22) into Eq. (20), for the pinning energy V_{pin} we obtain Eq. (3) with $\beta = 2$.

So far we have considered the case of a short-range interaction between electrons in the WC, which is provided by a closely situated gate. Our derivation was essentially based on the existence of a finite phase velocity $s = \omega_k/k$ for long-wave oscillations of the crystal. If the gate is absent, the phase velocity diverges in the long-wavelength limit, and the frequency spectrum ω_k is that of the 1D plasmon

$$\omega_k = s_0 k \ln^{1/2}\left[\frac{\gamma}{ka}\right], \quad (25)$$

where $\gamma \sim 1$ and $s_0 = 2e^2/\kappa m a$. As a result, the mean-squared amplitude of zero-point oscillations for mode k,

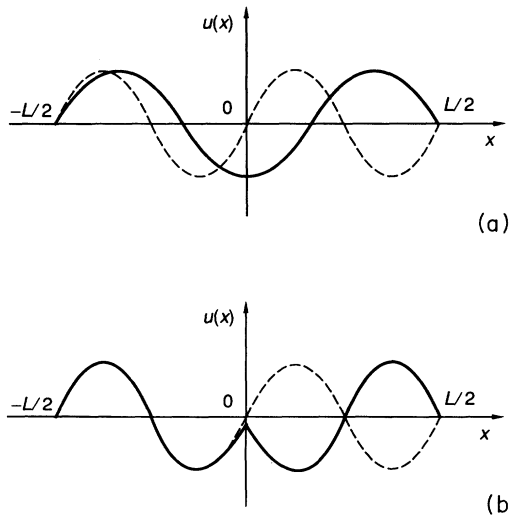

FIG. 2. The profile of an even oscillation mode (a) in the absence and (b) in the presence of a pinning center situated at $x=0$. Dashed curve shows the adjacent odd mode.

$$\langle u_k^2 \rangle = \frac{\hbar}{2m\omega_k} , \tag{26}$$

increases with decreasing k more slowly than given by Eq. (17). Consequently, the divergence of the sum over k in the exponent in Eq. (18) is slower, too, which should result in a weaker nonanalytic dependence of the pinning energy $V_{\rm pin}$ on the classical pinning potential V_0 than that given by Eq. (3). The expression for $V_{\rm pin}$ can be derived in the same way as it was done above for the short-range model, starting with Eqs. (25) and (26) instead of Eq. (17). The result can be represented in the form

$$V_{\rm pin} = V_0 \exp\left[-2\alpha^* \ln^{1/2}\left[\frac{ms_0^2}{V_0} \right] \right] , \tag{27}$$

where the quantum parameter α^* analogous to α [see Eq. (1)] is defined as

$$\alpha^* = \pi (n a_B/2)^{1/2} . \tag{28}$$

A comparison of Eqs. (27) and (3), where α is defined for a large but finite gate spacing D by Eq. (1), shows that the long-range nature of electron-electron interaction enhances the pinning by suppressing the quantum oscillations. In real structures, however, another factor may be more important, namely, the gate screening of the random potential produced by charged impurities that already diminishes the classical pinning energy V_0.

IV. VARIATIONAL CALCULATION

Here, we derive the ground-state energy of the Wigner crystal in the presence of a small pinning potential by means of a variational approach. It will be convenient to use dimensionless units in which $a=m=s=1$. The Hamiltonian (13) then takes the form

$$H = \tfrac{1}{2}\sum_k p_k^2 + \tfrac{1}{2}\sum_k k^2 u_k^2 - \frac{V_0}{2}\cos[2\pi u(0)] , \tag{29}$$

with the energy measured in units of ms^2, and $V_0 \ll 1$. Parameter $\alpha/\pi = \hbar/msa$ defined in Eq. (1) plays the role of dimensionless Plank constant: $[p_k, u_k] = -i\alpha/\pi$.

We shall construct the appropriate trial function, using results of Sec. III as a guide. Obviously, the trial function $\Psi\{u_k\}$ should depend on all normal-mode coordinates u_k. One's first inclination is to choose the function Ψ in the form of Eq. (16), considering $\langle u_k^2 \rangle$ as variational parameters. However, this would yield a senseless answer because the picture of the crystal oscillations, described by the wave function (16), does not provide decreasing oscillation amplitude near the pinning center at $x=0$: normal modes u_k are amplitudes of cosines in the coordinate space. However, as shown in the previous section, this decrease ("node") should exist in the crystal bound to a pinning center. This suggests that one should use a function Ψ which depends instead on linear combinations of u_k, e.g., as given by

$$\Psi_0\{u_k\} = B \prod_k \exp(-v_k^2/4\langle v_k^2 \rangle) , \tag{30a}$$

$$v_k = \sum_q \Gamma_{kq} u_q , \tag{30b}$$

where matrix Γ_{kq} should be chosen in order to produce an oscillation profile

$$f_k(x) = \sum_q \Gamma_{kq} \cos qx \tag{31}$$

which approaches the modulus of the sine for small k ("pinned" modes), and the cosine for large k ("non-pinned" modes).

A wave function satisfying this requirement is the eigenfunction of the ground state in the case of a small quantum parameter, $\alpha \ll \ln^{-1}(1/V_0)$, within the limits of which the Hamiltonian (29) can be exactly diagonalized. In this case, the magnitude of quantum fluctuations near the center is small compared with the lattice parameter, $\langle u^2(0) \rangle \ll 1$, so that the last term in Eq. (29) can be expanded in $u(0)$, yielding

$$H = \tfrac{1}{2}\sum_k (p_k^2 + k^2 u_k^2) - \frac{V_0}{2} + V_0 \pi^2 u^2(0) . \tag{32}$$

The harmonic Hamiltonian (32) is diagonalized by the transformation (30b) with the matrix

$$\Gamma_{kq} = \frac{c_k \Delta}{(k+\delta_k \Delta)^2 - q^2} , \tag{33a}$$

$$c_k = \frac{2}{\pi}\frac{k_0 k}{(k^2+k_0^2)^{1/2}} , \tag{33b}$$

$$\delta_k = \frac{1}{\pi}\arctan\left[\frac{k}{k_0} \right] , \tag{33c}$$

where $\Delta = 2\pi/L$ is the spacing of the frequency spectrum in the absence of pinning potential, and $k_0 = \pi^2 V_0$. As seen from Eqs. (33) at high wave numbers $k \gg k_0$, the quantity δ_k tends to zero, so that $\Gamma_{kq} \approx \delta_{kq}$. At small k, $\delta_k = \frac{1}{2}$, and the matrix Γ_{kq} gives the Fourier transform of $\sin(k|x|)$, that can be checked by substitution of Eq. (33a) into Eq. (31).

The function $\Psi\{u_k\}$ given by Eqs. (30) and (33) is an exact eigenfunction for $\alpha \ll \ln^{-1}(1/V_0)$ only. It is not the case for an arbitrary α when the coordinate $u(0)$ can reach a few neighboring minima of pinning potential, and Hamiltonian (29) is essentially nonharmonic. However, we can use the function $\Psi\{u_k\}$ for arbitrary α, as a trial function, treating k_0 in Eqs. (33a)–(33c) as a variational parameter (perfectly analogous to k_0 in Sec. III).

The form of Hamiltonian (29) in the representation v_k, $P_k = -i(\alpha/\pi)\partial/\partial v_k$, can be obtained by substitution of expressions

$$u_q = \sum_k \Gamma_{kq} v_k , \quad p_q = \sum_k \Gamma_{kq} P_k ,$$

as well as substituting Eq. (15), into Eq. (29), that yields

$$H = \tfrac{1}{2}\sum_k P_k^2 + \tfrac{1}{2}\sum_k (k+\delta_k\Delta)^2 v_k^2 - \frac{\pi\Delta}{4k_0}\left[\sum_k c_k v_k \right]^2 - \frac{V_0}{2}\cos\left[\frac{(\pi^3\Delta)^{1/2}}{k_0} \sum_k c_k v_k \right] , \tag{34}$$

where we have used Eq. (33a) for Γ_{kq} and the identity

$$\sum_{i=0}^{\infty}\frac{1}{z^2-i^2}=\frac{\pi}{2z}\cot(\pi z)\ .$$

After averaging the Hamiltonian (34) with the wave function (30), for the energy $E=\langle\Psi|H|\Psi\rangle$ we have

$$E=\sum_k\left[\frac{\alpha^2}{8\pi^2\langle v_k^2\rangle}+\frac{k^2}{2}\langle v_k^2\rangle\right]+\Delta\sum_k k\delta_k\langle v_k^2\rangle-\frac{k_0}{2\pi^2}\Omega-\frac{V_0}{2}e^{-\Omega}\ , \tag{35}$$

where

$$\Omega=\frac{\pi^3\Delta}{2k_0^2}\sum_k c_k^2\langle v_k^2\rangle\ . \tag{36}$$

In Eq. (35) we have omitted the terms of second order in Δ.

Parameters $\langle v_k^2\rangle$ in Eqs. (35) and (36), as well as parameter k_0, are variational ones. The last three terms in the right-hand side of Eq. (35) form a small correction to the zero-point energy due to the pinning. Hence, we can substitute into these terms the values $\langle v_k^2\rangle$ found from the minimization of the major (first) term in the energy (35): $\langle v_k^2\rangle=\alpha/2\pi k$. As a result, replacing the sums over k in Eqs. (35) and (36) by integrals over the interval $0<k<\pi$, we get

$$E-E_0=\frac{\alpha}{2\pi}\int_0^{\pi}dk\,\delta_k-\frac{k_0}{2\pi^2}\Omega-\frac{V_0}{2}e^{-\Omega}\ , \tag{37}$$

$$\Omega=\frac{\pi^2\alpha}{4k_0^2}\int_0^{\pi}dk\frac{c_k^2}{k}\ , \tag{38}$$

where $E_0=(\alpha/2\pi)\sum_k k$ is the zero-point energy in the absence of pinning. Substitution of Eq. (33), for δ_k,c_k, into Eqs. (37) and (38), and calculation of integrals over k yield

$$E-E_0=\frac{\alpha k_0}{2\pi^2}-\frac{V_0}{2}\left[\frac{k_0}{\pi}\right]^{\alpha}\ , \tag{39}$$

where we allowed k_0 to be much less than the Debye wave number $k=\pi$. At $\alpha<1$, the sum (39) has a minimum at

$$k_0=\pi(\pi V_0)^{1/(1-\alpha)}\ll\pi\ .$$

Finally, substituting the last value into (39) and returning to the dimensional notation, for the ground-state energy, we obtain

$$E-E_0=-\frac{V_0}{2}(1-\alpha)\left[\frac{\pi V_0}{ms^2}\right]^{\alpha/(1-\alpha)}\ . \tag{40}$$

Note that this result differs from the estimate (23) of Sec. III by the numerical factor $(\pi/2)^{\alpha/(1-\alpha)}$ only. An expansion of Eq. (40) at small values $\alpha\ll\ln^{-1}(ms^2/V_0)$ yields

$$E-E_0=-\frac{V_0}{2}\left[1-\alpha\ln\frac{ems^2}{\pi V_0}\right]\ .$$

This result coincides with the exact ground-state energy for the quadratic Hamiltonian (32) and might be obtained by a straightforward diagonalization of (32).

Expression (40) was obtained in the Debye approximation, i.e., for the frequency spectrum $\omega_k=sk$, $0<k<\pi/a$, which we used for the sake of simplicity. All the calculations can be repeated for another form of the spectrum given by

$$\omega_k=\frac{2s}{a}\sin\left[\frac{ka}{2}\right]\ , \tag{41}$$

which corresponds to a chain with the nearest-neighbor interaction and is a good approximation when the distance D between the gate and the WC is small: $D\lesssim a$. As a result, we arrive at Eq. (40) with a somewhat different numerical coefficient: $\pi^2/4$ instead of π. The functional dependence of $E-E_0$ versus V_0 remains unchanged because it is determined by the long-wave part of the phonon spectrum only.

V. TRANSPORT OF THE WIGNER CRYSTAL

The form of effective pinning potential $V_{\rm eff}(u)$ obtained in Sec. III, Eqs. (24) and (3), permits one to determine the threshold voltage $U_{\rm th}$ at which the depinning of the WC occurs. In an analogy with a classical problem, $U_{\rm th}$ is given by the formula

$$U_{\rm th}=\frac{a}{e}\left[\frac{dV_{\rm ef}}{d\bar{u}}\right]_{\max}=\pi\frac{V_{\rm pin}}{e}\ . \tag{42}$$

For $\alpha\ll1$, the voltage $U=U_{\rm th}$ corresponds to the sharp increase in the WC conductance. The finite current, however, can pass through the system even far below the threshold ($U\ll U_{\rm th}$). In this section we shall calculate both linear and nonlinear conductance of the WC in this region, and shall estimate the voltage of crossover between these two regimes.

When discussing the linear conductance of a channel connected to "perfect" leads with metallic conductivity, we neglect the electron-electron interaction inside the leads. This allows us to relate the conductance to the transmission coefficient of the channel by means of the Landauer formula

$$\sigma=\frac{e^2}{\hbar}\int dE\,D(E)\left|\frac{df}{dE}\right|\ , \tag{43}$$

where $f(E)$ is the Fermi distribution, and $D(E)$ is the transmission coefficient for electrons with the energy E counted from the Fermi level. Although electrons are assumed to be noninteracting outside the channel, the value of $D(E)$ is determined by the electron-electron interactions *inside* the channel.[13] Transport through the channel at low temperatures T occurs due to tunnel hops of the crystal on a period a through the effective pinning barrier. The quasiclassical probability of this tunneling at $T=0$ for a similar system was calculated by Larkin and Lee,[9] who studied the tunneling of the charge-density wave. We shall review briefly the concepts of this derivation in terms of Wigner crystal parameters, and

generalize this result for the case of finite temperatures.

Following Ref. 9, we shall treat the tunneling process as a quasiclassical one that occurs through the barrier with the effective height $V_{\rm pin}$ [Eq. (3)]. This approach is also suitable in the case in which the pinning barrier is strongly suppressed by quantum oscillations, and will be justified later on. The tunneling probability is given by $D\sim\exp(-2A/\hbar)$, where A is the minimal value of the tunnel action, for the shift of the crystal by a from an undeformed ground state to the equivalent one. Obviously, a long chain cannot tunnel with all its particles moving simultaneously because action A will contain a huge mass. Hence the tunneling should consist of two stages. First, a finite segment of WC tunnels through the barrier on the distance a, creating a deformed area with the length l_0 around the center. Then this deformation, which corresponds to a classically forbidden state of WC, relaxes spreading along the remaining part of the chain.

At the first stage, action A_1 is given by

$$A_1=a\left[\frac{ml_0}{a}\left[V_{\rm pin}+\frac{ms^2}{l_0}a\right]\right]^{1/2}, \tag{44}$$

where ml_0/a is the tunneling mass, and the total tunnel barrier consists of two parts: the pinning barrier $V_{\rm pin}$ and the deformation energy $\sim ms^2a/l_0$ of the tunneling segment. The action at the relaxation stage (that follows the "local" tunneling through the pinning barrier) does not depend on $V_{\rm pin}$ and can be represented in the following form:

$$\begin{aligned}A_2&=\int_0^{t_L}[V_{\rm def}(t)+|T_{\rm def}(t)|]dt\\&=2\int_0^{t_L}V_{\rm def}(t)dt\ ,\end{aligned} \tag{45}$$

where t_L is the time necessary for spreading of the deformation onto the whole crystal length; $T_{\rm def}(t)<0$ and $V_{\rm def}(t)$ are kinetic and potential energies of the deformation, respectively, and we have assumed that the total energy E is that of the ground state: $E=V_{\rm def}+T_{\rm def}=0$.

The energy $V_{\rm def}(t)$ can be estimated in terms of characteristic size $l(t)$ of the deformed segment at instant t as

$$V_{\rm def}(t)=\frac{ms^2a}{l(t)}\ . \tag{46}$$

The deformed region, starting from the size l_0, expands in time with the sound velocity s, so that we have

$$l(t)=l_0+st\ . \tag{47}$$

Substituting Eqs. (47) and (46) into Eq. (45) and changing the integration variable t to l, yields

$$A_2=\frac{2}{\pi}msa\int_{l_0}^{L}\frac{dl}{l}=\frac{2}{\pi}msa\ln\frac{L}{l_0}\ . \tag{48}$$

The numerical coefficient $2/\pi$ in Eq. (48) does not follow, of course, from our simplified derivation. It can be obtained, for example, by a solution of the harmonic equation of motion that permits one to calculate the function $V_{\rm def}(t)$ for the crystal with negative kinetic energy, as was done in Ref. 9.

The length l_0 should be found from the minimization of total action

$$A(l_0)=A_1(l_0)+A_2(l_0)\ ,$$

Eqs. (44) and (48). This sum has a minimum at

$$l_0=ms^2a/V_{\rm pin}\ .$$

Finally, for the tunneling probability we get

$$D=\exp\left[-\frac{2}{\hbar}(A_1+A_2)\right]=\exp\left[-\frac{4}{\alpha}\ln\left[\frac{V_{\rm pin}L}{ms^2a}\right]\right], \tag{49}$$

where α and $V_{\rm pin}$ are given by Eqs. (1) and (3), respectively.

Note that the inverse length l_0^{-1} coincides with the wave number k_0 (22) that separates the pinned (or classical) and nonpinned (or "quantum") oscillation modes. The main contribution to the action $A=A_1+A_2$ comes from A_2 [we have $A_2/A_1=\ln(L/l_0)$] which mainly involves, as seen from Eq. (47), the tunneling of long-wavelength modes with $l\gg l_0$. At the same time, the renormalization of the pinning barrier down to $V_{\rm pin}$, Eq. (3), is due to the modes with $l\lesssim l_0$. Thus our suggestion regarding a quasiclassical nature of tunneling in the quantum case is justified. However, the effective potential barrier is diminished due to the high-frequency oscillations.

The tunneling rate (49) of a WC interacting with an impurity and, hence, the linear conductance at zero temperature, vanish for increasingly long sample. This is because the deformation energy in Eq. (45) approaches too slowly the zero limit that corresponds to the ground state: $V_{\rm def}(t)\propto 1/t$. At finite temperatures, however, the crystal can tunnel, being excited above the ground state by a finite energy $E>0$, so that the deformation energy, after the tunneling event, is also finite: $V_{\rm def}\sim E$. According to Eq. (46), the energy corresponds to the size $l_E=ms^2a/E$ of the deformed region. In order to obtain the action $A_2(E)$ for the finite energy E, we should replace L with l_E at the upper limit of the integral (48). The final formula for the tunneling probability $D(E)$ has a form

$$D(E)=\exp\left[-\frac{4}{\alpha}\ln\frac{V_{\rm pin}}{E}\right], \tag{50}$$

with E being in the interval

$$ms^2a/L\ll E\ll V_{\rm pin}\ .$$

Equations (49) and (50) are written apart from numerical coefficients in the argument of logarithm. Physically, expression (50) represents the probability of tunneling for a long-wave soliton with a length $\sim l_E=ms^2a/E$.

The linear conductance of a channel connected to metallic leads is given by formula (43). This implies that the transport through the channel occurs due to solitons which are excited at the ends of the channel by electrons arriving from the leads. Because the channel transmission coefficient $D(E)$ is mainly limited by the small tun-

neling probability of solitons passing through the impurity potential, we will estimate $D(E)$ at $E>0$, as given by Eq. (50). At energies $E \gg k_B T$ the Fermi function $f(E)$ can be replaced by the exponential $\exp(-E/k_B T)$, so that for conductance σ we get

$$\sigma \approx \frac{e^2}{\hbar} \int \frac{dE}{k_B T} \exp\left[-\frac{E}{k_B T} - \frac{4}{\alpha} \ln \frac{V_{\text{pin}}}{E} \right] . \tag{51}$$

At small $\alpha \ll 1$, the integrand in Eq. (51) has a sharp maximum at

$$E = 4 k_B T/\alpha . \tag{52}$$

Evaluating the integral in (51) near this maximum, we arrive, apart from a numerical coefficient, at Eq. (5) for σ with $\sigma_0 \sim e^2/\hbar$.

Thus we have obtained the temperature dependence of the linear conductance provided at low temperatures by activation-assisted tunneling. Formula (5) for the conductance is valid until the activation energy E given by Eq. (52) is less than the barrier height V_{pin}, i.e., at temperatures

$$T \lesssim T_c = \alpha V_{\text{pin}}/4 k_B .$$

In the temperature interval $T_c \lesssim T \lesssim V_{\text{pin}}/k_B$, the major contribution to the current comes from the activation over the barrier V_{pin}, and Eq. (4) takes place.

The finite energy E that is necessary for the crystal to complete the tunneling in a finite time can be provided not only by thermal activation but also by the external voltage applied. At low temperatures the conductance $\sigma(U)$ will then be nonlinear and proportional to $D(eU)$, where the tunneling probability $D(E)$ is given by Eq. (49). As a result, for $\sigma(U)$ we obtain the power-law dependence (6). At a given temperature T and voltage U the conductance is determined either by Eq. (6), or by the linear formula (5), whichever gives larger results. The crossover to the nonlinear regime occurs at $U \sim \alpha k_B T/e$. It is noteworthy that at $\alpha \ll 1$, the nonlinear behavior of the conductance takes place at voltages much smaller than the temperature (expressed in eV).

To conclude this section, we discuss briefly how the results of the conductance are changed in the case of the long-range Coulomb interaction between electrons in a WC (i.e., for the channel without a screening gate). At the stage of the tunnel relaxation that dominates the tunneling probability, the potential energy V_{def} of the deformed crystal segment scales with its length l as

$$V_{\text{def}} \propto \frac{1}{l} \ln\left[\frac{l}{a} \right] , \tag{53}$$

which differs from Eq. (46) by a logarithmic factor. The scaling relation between the length l and the time t has a form

$$t \propto \frac{l}{\ln^{1/2}(l/a)} \tag{54}$$

[cf. Eq. (25)]. As a result, the integral $\int dl/l$ in the tunnel action (48) will be replaced by

$$\int_{l_0}^{L} \frac{dl}{l} \to \int_{l_0}^{L} \frac{dl}{l} \ln^{1/2}\left[\frac{l}{a} \right] = \tfrac{2}{3} \ln^{3/2}\left[\frac{L}{l_0} \right] . \tag{55}$$

Hence, for the action A_2 we have

$$A_2 = \frac{4}{3\pi} m s_0 a \ln^{3/2}\left[\frac{L}{l_0} \right] , \tag{56}$$

where s_0 is introduced in Eq. (25). The further derivation of the conductance σ proceeds in the same way as given above, and we get

$$\sigma \sim \frac{e^2}{\hbar} \exp\left[-\frac{8}{3\alpha^*} \ln^{3/2}\left[\frac{\alpha^* V_{\text{pin}}}{T} \right] \right] , \tag{57}$$

with α^* defined by Eq. (28). We see that the temperature dependence $\sigma(T)$ is stronger than that given by the short-range formula (5). The field dependence $\sigma(U)$ [Eq. (6)] transforms, of course, in the same way.

VI. CONCLUSION

We considered in this paper the pinning of a one-dimensional electron solid by a single-impurity potential. Although the long-range order is absent in 1D, this does not eliminate the pinning even by an arbitrarily weak potential. This was demonstrated above for the case in which the impurity potential is not only weak but also smooth (this already makes the pinning barrier very small in a classical limit). Obviously, pinning becomes stronger for sharper potentials. It follows from the variational analysis (cf. Secs. III and IV) that our results are valid even if the impurity potential $U(x)$ is not smooth. Particularly, for

$$\alpha \gg \ln^{-1}(m s^2/V_0)$$

the renormalized pinning barrier is determined by the first Fourier harmonic of the impurity potential V_0 and quantum parameter α as given by Eq. (3); here V_0 and α are defined by Eqs. (9), (10), and (1). Higher harmonics of pinning potential are strongly smeared out by quantum oscillations: The magnitude of rth harmonic $U(q_r)$ is diminished by the factor $(V_0/ms^2)^{r^2\alpha}$. Thus, our result (3) for the effective pinning potential is valid as well for a broader class of weak pinning centers.

Formulas (5) and (6), which predict a power-law behavior for both temperature and voltage dependence of the conductance, were also derived for a weak smooth impurity potential. However, this power law is universal with respect to a model of potential. Particularly, it remains valid in the case of an arbitrarily high tunnel barrier: It follows from the discussion in Sec. V that the exponent $4/\alpha$ in Eqs. (5) and (6) is determined by tunneling at large imaginary times and does not depend on the barrier shape.

The studied case of a single impurity may serve as a starting point for an investigation of pinning by more complex configurations. In particular, pinning of a 1D Wigner crystal by two impurities, which leads to phenomena similar to the well-known Coulomb blockade, will be studied separately.[14] Here we list briefly the main

predictions for this model of pinning and its possible relation to the experiments reported in Refs. 6 and 15.

Suppose that two weak pinning centers are situated in the channel at sufficiently short distance L such that one can neglect the crystal deformation between the impurities (the weak-pinning case). The effective pinning potential will be given by the sum of two terms of form (10) with the phase difference $2\pi L/a$ between two cosines. The amplitude of the total potential and, hence, the height of the pinning barrier $V_{\rm pin}$, will change periodically with the electron concentration $n=1/a$ resulting in sharp conductance oscillations in the gate voltage V_g shown in logarithmic scale in Fig. 3(a). A different mechanism of conductance oscillations versus V_g is well known for an electron gas separated in some region from the rest of the channel by two high tunnel barriers. Here the oscillations result from the existence of Coulomb levels due to discreteness of an electron charge,[4,5] the electron-electron interaction being described in the mean-field approach. This model does not account for the short-range correlations of electrons. Both models predict the same period of oscillations ΔV_g. However, the shape of oscillations $\ln\sigma(V_g)$ is different in these two cases. In the mean-field theory, the function $\ln\sigma(V_g)$ at low temperatures should consist of straight segments with a constant slope ("saw-teeth") analogous to that shown in Fig. 3(d). This is because the activation energy is defined by the Coulomb level closest to the Fermi energy. In the case of the strongly correlated system (WC) the situation is more complicated. The form of oscillations varies depending on the pinning strength and on the regime of transport (activation or activated tunneling). In one particular case it may look sawlike [Fig. 3(d)]. Generally, however, it differs qualitatively from the saw-teeth dependence [Figs. 3(a), 3(b), and 3(c)]. Apart from this, the ratio of the activation energy ε_a of the linear conductance to the threshold voltage $U_{\rm th}$ should be different in the case of weak (or intermediate) pinning and in the mean-field model (i.e., Coulomb blockade): $\varepsilon_a/eU_{\rm th}$ equals $1/\pi$ [cf. Eq. (42)] and 1, respectively. In Refs. 6 and 15 the conductance oscillations $\sigma(V_g)$ were observed for two different quasi-one-dimensional structures based on silicon and GaAs, respectively. In both cases, as one can see from data reported for a minimum of $\sigma(V_g)$, the ratio $\varepsilon_a/eU_{\rm th}$ was approximately one third. The pinning model could also explain a general increase in the ratio of the conductance values $\sigma_{\rm max}/\sigma_{\rm min}$ in adjacent maxima and minima with electron concentration n decreasing, which was observed in Refs. 6 and 15. The mean-field theory predicts this ratio,

$$\sigma_{\rm max}/\sigma_{\rm min}\sim\exp[-e^2/2C(L)k_BT]\ ,$$

where $C(L)$ is the segment capacitance, to be independent on n. In the weak-pinning model the ratio

$$\sigma_{\rm max}/\sigma_{\rm min}\sim\exp(-2V_0/k_BT)$$

is sensitive to the electron concentration n since the pinning energy V_0, as discussed in Sec. II, tends to increase with decreasing n. However, a clearer argument in favor of one or another model would be, in our opinion, the study of the oscillation curve in the logarithmic scale $\ln\sigma(V_g)$, where the difference between two models, as demonstrated in Fig. 3, should be observable. Since these data are absent in the experimental papers discussed, we refrain from drawing a final conclusion on this subject.

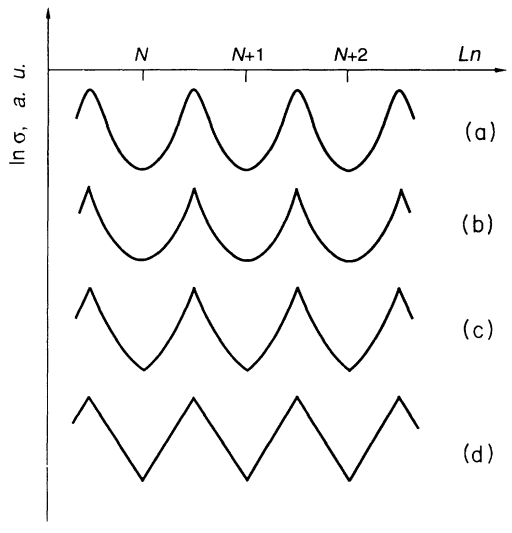

FIG. 3. Oscillatory dependence of the conductance σ vs the electron concentration n in a Wigner crystal pinned by two impurities located at distance L from each other. A few particular cases which differ by pinning strength and temperature regime are shown schematically in (a)–(d). The entire channel length is assumed to be sufficiently short. Graphs are shifted and normalized to the same oscillation magnitude. (a) Very weak pinning. (b) and (c) The crossover from the weak to the strong pinning. (d) Very strong pinning, intermediate temperatures. This case is analogous to the mean-field prediction (Coulomb blockade).

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APPENDIX

We calculate here the electrostatic energy and the sound velocity of a chain formed by electrons with a constant period $a=n^{-1}$ and placed at distance D along a conducting plane (gate). Parameters D, a, and the chain length L are supposed to meet the condition $L\gg D\gtrsim a$. The sound velocity s is defined by the general formula

$$s^2=\frac{n}{m}\frac{d^2\tilde{E}}{dn^2}\ , \tag{A1}$$

where m is the mass of electron, and $\tilde{E}(n)=E/L$ is the chain energy per unit length. Interaction energy of N electrons, including potentials of their electrostatic images, is given by

$$E=\frac{N}{2}\left[\sum_{i\neq 0}\frac{e^2}{\kappa a|i|}-\sum_{i=-\infty}^{\infty}\frac{e^2}{\kappa[4D^2+(ai)^2]^{1/2}}\right]. \quad \text{(A2)}$$

The second sum in Eq. (A2) can be replaced by an integral: For $D>a/2$, this replacement leads, as one can see, to an error of less than 2% in the final calculation for s. As a result, we have

$$E=\frac{Ne^2}{2\kappa a}\left[\sum_{i\neq 0}\frac{1}{|i|}-\int_{-\infty}^{\infty}dx\frac{1}{(4D^2+x^2)^{1/2}}\right]. \quad \text{(A3)}$$

Both the integral and the sum in Eq. (A3) diverge logarithmically; however, two divergences cancel each other. We cut the sum on large $|i|=M$ and the integral, correspondingly, on $|x|=Ma$; hence,

$$E=\frac{Ne^2}{2\kappa a}\left[2(\ln M+\gamma)-2\ln\frac{Ma}{D}\right]$$

$$=\frac{e^2}{\kappa}Ln^2[\ln(nD)+\gamma], \quad \text{(A4)}$$

where $\gamma=0.577$ is the Euler constant. For the second derivative of $\tilde{E}=E/L$ we obtain

$$\frac{d^2\tilde{E}}{dn^2}=\frac{2e^2}{\kappa}\left[\ln(nD)+\tfrac{3}{2}+\gamma\right]. \quad \text{(A5)}$$

Note, that the quantity (A5) is proportional to a directly measurable value, namely, to the inverse capacitance c per unit length between the channel and the gate: $c^{-1}=(\kappa/e^2)\tilde{E}''(n)$. Substituting Eq. (A5) into Eq. (A1), we arrive at formula (2) of the main text.

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