Static properties of one-dimensional generalized Landau liquids

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The forward scattering f functions of the exact one-dimensional Landau theory are evaluated for the case of the Hubbard chain. The static charge and spin susceptibilities of the interacting system at arbitrary magnetic fields are derived as in a Fermi liquid. The connection to conformal field theory is studied in detail. All the critical exponents associated with the asymptotic behavior of the correlation functions are fully determined by the pseudoparticle interactions.

I. INTRODUCTION

The interest in strongly correlated electron models of phenomena like high-temperature superconductivity or the electronic properties of quasi-one-dimensional synthetic metals, has not subsided. As even the "simpler" one- and two-dimensional Hamiltonians are very difficult to solve, there has recently been an upsurge of interest in exact soluble models. The exact Bethe-ansatz solutions[1] of fermionic one-dimensional models, for example,[2] even if formally known for many years, have recently been shown to describe the interactions between pseudoparticles of a generalized Landau-liquid theory.[3] That allows the extension of the concept of a Landau liquid[4,5] to a wider class of many-body fermionic systems[3,6] and contributes to a deeper understanding of the physical content of these exact solutions.[7] The possibility of expressing the Bethe-ansatz solutions in terms of a Fermi-liquid-like formalism was considered in Ref. 8.

In this paper we present a detailed evaluation of the forward scattering f functions for the Hubbard chain, which were discussed in a recent paper.[3] Furthermore, we derive the compressibility and spin susceptibility of the model at arbitrary magnetic fields, and establish the connection to conformal field theory.[3,9,10]

Ten years ago Haldane[11] showed that a wide class of one-dimensional systems, which he called Luttinger liquids, share features characteristic of the exact soluble Luttinger-Tomonaga model.[12–14] All these exotic one-dimensional liquids are characterized by nonclassical exponents.[10,14,15] The key point in his approach is to recognize that, to a first approximation, the low-energy spectrum of these systems can be described in terms of noninteracting bosons, similarly to the Luttinger model (bosonization). He then adds correction terms in order to describe the nonlinear coupling between the bosonic collective modes. In the case of the Hubbard model his method requires the use of two effective Hamiltonians, which correspond to two decoupled harmonic (Gaussian) models, describing charge and spin-density fluctuations.[11] Haldane's description is, however, not valid for finite magnetic fields because when $H > 0$ the ground state is not spin-rotationally invariant, and there is a change in the nature of the gapless excitations.[6] Therefore, the zero-magnetic-field ground state is not a good starting point for the study of spin fluctuations in the presence of magnetic fields.

In the case of fermionic Luttinger liquids solvable by the Bethe ansatz,[1] our approach shows that in spite of the fully incoherent single particle spectral function characteristic of these systems, the collective modes, which describe these liquids, can be understood as pseudoparticles of a generalized Landau-liquid theory. Although this is only true at finite magnetic fields, where all gapless excitations can be described by particle-hole processes in the bands of the pseudoparticles,[6] the limit $H \rightarrow 0$ leads to the correct values for the zero-field static susceptibilities and critical exponents. Furthermore, the generalized theory provides the exact couplings between the collective modes that are derived in a simple manner from the Landau energy functional. Although, as in the case of the Fermi-liquid theory,[16–20] we restrict our study to the two-pseudoparticle forward scattering f functions, the exact theory contains all the higher-order interaction coefficients, which can be derived by taking functional derivatives of the Landau energy with respect to the pseudomomentum deviations. Based on the two-particle spectral properties of the new liquids, we show elsewhere that the Landau-liquid functional introduced in this work is valid over a finite domain of excitation energy where all states can be described by real Bethe-ansatz rapidities.[21]

The new generalized Landau liquids include the usual Fermi liquids[4,5,16–20] and the present one-dimensional Landau-Luttinger liquids. Since other Landau liquids are expected to exist in higher dimensions, our exact results may provide valuable information on the generalized theory in higher dimensions. The transitions between the phases considered here can be produced by changes of the electron-electron interaction. In the case of the (repul-

sive) Hubbard chain at fixed values of density $n = N/N_a$ and magnetic field H (and $H < H_c^0$, where H_c^0 is the critical field for the onset of ferromagnetism[6] at $U = 0$), in addition to the Fermi-liquid–Landau-Luttinger-liquid phase transition at $U = 0$ (Mott-Hubbard transition if $n = 1$), there is a Landau-Luttinger–Fermi-liquid (ferromagnetic) transition at $U = U_c$, where $U_c = U_c(n, H)$ is the value at which $H = H_c$.[6]

Our four general criteria for a Landau liquid are (i) the fact that in each phase the low-energy physics is fully controlled by the departure of the pseudo-momentum distribution(s) of pseudoparticles from its (their) value(s) in the interacting ground state—in addition, all the gapless elementary excitations can be expressed in terms of particle-hole processes in the bands of the latter; (ii) the forward-scattering interactions of the pseudoparticles have a predominant role in the low-energy physics, being regulated by the two-pseudoparticle f functions (second functional derivatives of the energy with respect to the deviations); (iii) in contrast to the energy, the total charge, magnetization, and momentum of the many-body system depend linearly on the deviations (i.e., only the first functional derivatives are nonzero); and (iv) the adiabatic continuity principle is valid within each phase, but not necessarily across the phase transitions—the change in symmetry induced by the particle-particle interactions at the phase transitions may produce singular changes in the structure of the eigenstate energy spectrum. Both the usual Fermi liquids[18,19] and the new one-dimensional Landau liquids meet these four criteria.

The paper is organized as follows: In Sec. II we derive the energy Landau functional for $n < 1$ and $0 < H < H_c$. The scale-invariant regime is studied in Sec. III. The evaluation of the charge and spin susceptibilities is presented in Sec. IV. In Sec. V we consider the connection to conformal field theory. The half-filled band case and the ferromagnetic phase are studied in Sec. VI. Finally, Sec. VII contains the concluding remarks.

II. THE LANDAU-ENERGY FUNCTIONAL

We consider the one-dimensional Hubbard model at arbitrary magnetic field

$$\hat{H} = -t\sum_{j,\sigma}(c_{j\sigma}^{\dagger}c_{j+1\sigma} + c_{j+1\sigma}^{\dagger}c_{j\sigma}) + U\sum_{j} c_{j\uparrow}^{\dagger}c_{j\uparrow}c_{j\downarrow}^{\dagger}c_{j\downarrow} + (\mu - U/2)\sum_{j,\sigma} c_{j\sigma}^{\dagger}c_{j\sigma} - \mu_0 H\sum_{j,\sigma}\sigma c_{j\sigma}^{\dagger}c_{j\sigma}\,, \qquad (1)$$

where $c_{j\sigma}^{\dagger}$ and $c_{j\sigma}$ are creation and annihilation operators for an electron with spin σ at site j, μ is the chemical potential, H is an external magnetic field, and μ_0 is the Bohr magneton. The shift in the chemical potential makes explicit the particle-hole symmetry of the model.[6] The Hamiltonian (1) describes N interacting electrons on a chain of N_a sites ($n = N/N_a$, $k_F = \pi n/2$). It consists of $N_{\downarrow}$ down-spin electrons and $N_{\uparrow}$ up-spin electrons ($k_{F\sigma} = \pi N_{\sigma}/N_a$). The spin density is $m = (k_{F\uparrow} - k_{F\downarrow})/2\pi$. We restrict our study to the case $\mu \geq 0$ and $H > 0$ ($n \leq 1$ and $N_{\uparrow} > N_{\downarrow}$). We note that, in the figures of the present paper, units such that $t = \mu_0 = 1$ are used.

In Ref. 6 we have shown that in the limit of a large system ($N_a \rightarrow \infty$, n fixed) the generalized Lieb and Wu equations[2] may be rewritten as

$$K(q) = q + \frac{1}{\pi}\int_{-k_{F\uparrow}}^{k_{F\uparrow}} dp' N_{\downarrow}(p')\tan^{-1}[\,S(p') - (1/u)\sin K(q)]\,, \qquad (2)$$

$$p = \frac{1}{\pi}\int_{-\pi}^{\pi} dq' M_c(q')\tan^{-1}[S(p) - (1/u)\sin K(q')] - \frac{1}{\pi}\int_{-k_{F\uparrow}}^{k_{F\uparrow}} dp' N_{\downarrow}(p')\tan^{-1}\{\tfrac{1}{2}[S(p) - S(p')]\}\,, \qquad (3)$$

where $u = U/4t$, the form of the pseudomomentum distributions $M_c(q)$ and $N_{\downarrow}(p)$ is determined by the choice of the sets of numbers $\{I_j\}$ and $\{J_{\alpha}\}$,[2] and for $U > 0$ the pseudomomenta q and p describe the quantum numbers of the many-body system [in the discrete case $q_j = (2\pi/N_a)I_j$, $p_{\alpha} = (2\pi/N_a)J_{\alpha}$]. We emphasize that the generalized equations (2) and (3) do not refer only to the ground state—they describe a large number of excited states that correspond to different choices of the functions $M_c(q)$, $N_{\downarrow}(p)$, which are the pseudomomentum distributions of the charge and spin pseudoparticles, respectively. Furthermore, the restrictions on the numbers I_j and J_{α} lead to a fermionic character for the charge and spin pseudoparticles, i.e., each pseudomomentum value in $M_c(q)$ and $N_{\downarrow}(p)$ cannot be occupied by more than one pseudoparticle. The relation of the functions $K(q)$ and $S(p)$ with the Lieb and Wu distribution functions $\rho(k)$ and $\sigma(\Lambda)$ is presented in Ref. 6.

Although our approach can be extended to excited states described by complex roots, we restrict the present study to excitations involving real rapidities because the former are not the lowest-lying excitations for $H > 0$.[6] For eigenstates involving only real rapidities, we always have $K(\pm\pi) = \pm\pi$, $S(\pm k_{F\uparrow}) = \pm\infty$. Then the limits of the pseudo-Brillouin-zones of the charge and spin pseudoparticles are $\{q = \pm\pi\}$ and $\{p = \pm k_{F\uparrow}\}$, respectively.

The energy of the many-body system can be written as[6]

$$E = \frac{N_a}{2\pi}\left[\int_{-\pi}^{\pi} dq M_c(q)[-2t\cos K(q)] - 2\mu_0 H\left(\frac{1}{2}\int_{-\pi}^{\pi} dq M_c(q) - \int_{-k_{F\uparrow}}^{k_{F\uparrow}} dp N_{\downarrow}(p)\right) + (\mu - U/2)\int_{-\pi}^{\pi} dq M_c(q)\right]. \qquad (4)$$

The ground state at arbitrary magnetic fields is described by distributions $M_c(q) = M_c^0(q)$ and $N_\downarrow(p) = N_\downarrow^0(p)$ given by

$$M_c^0(q) = \Theta(2k_F - |q|), \quad N_\downarrow^0(p) = \Theta(k_{F\downarrow} - |p|), \tag{5}$$

where $\{q = \pm 2k_F, p = \pm k_{F\downarrow}\}$ constitute the charge and spin pseudo-Fermi surfaces, respectively. As in a Fermi liquid,[4,18,19] the ground-state pseudomomentum distributions (5) do not depend on the (on-site) interaction. We are particularly interested in the low-energy states described by small deviations from the ground-state distributions

$$M_c(q) = M_c^0(q) + \delta_c(q), \quad N_\downarrow(p) = N_\downarrow^0(p) + \delta_\downarrow(p), \tag{6}$$

where $\delta_c(q)$ and $\delta_\downarrow(p)$ are the pseudomomentum deviations that measure the departure from $M_c^0(q)$ and $N_\downarrow^0(p)$.

The key point of the present Landau-liquid theory is to consider $K(q), S(p)$ and the energy (4) as functionals of the pseudoparticle deviations $\delta_c(q)$ and $\delta_\downarrow(p)$. Although the deviations are arbitrary in the sense that the expressions for the coefficients of the Landau-energy functional are independent of them, only suitable choices of these deviations describe true eigenstates of the many-body system.[6]

In agreement with our criteria for a Landau liquid presented in Sec. I, a crucial feature of the present one-dimensional Landau-liquid theory is that, unlike the energy and the functionals $K(q), S(p)$ defined by Eqs. (2) and (3), the total charge, magnetization, and momentum of the many-body system depend linearly on the deviations.[6] Thus, as in a Fermi liquid,[4,19,20] the backflow effect due to the pseudoparticle interaction renormalizes only the energies.

In a recent paper[6] we have studied the first-order term of the Landau-energy functional, which defines the bands for charge and spin pseudoparticles. In the following we evaluate in detail the Landau functional up to second order in the deviations.[3]

The solutions $K(q), S(p)$, which correspond to the general distributions introduced in Eq. (6), may be evaluated by inserting these distributions into the right-hand side (rhs) of Eqs. (2) and (3) and expanding in the small deviations $\delta_c(q)$ and $\delta_\downarrow(p)$. That leads to

$$K(q) = K_0(q) + K_1(q) + K_2(q) + \cdots, \tag{7}$$

$$S(p) = S_0(p) + S_1(p) + S_2(p) + \cdots, \tag{8}$$

where $K_j(q)$ and $S_j(p)$ are the jth-order terms. Equations (2) and (3) allow the systematic evaluation order by order of all terms of the expansions (7) and (8). By using a recursion procedure, we find that the solutions (7) and (8) may be simply written as

$$K(q) = K_0(\mathcal{Q}(q)), \quad S(p) = S_0(\mathcal{P}(p)), \tag{9}$$

where $K_0(q)$ and $S_0(p)$ are the ground-state solutions that correspond to the choice of distributions (5), and $\mathcal{Q}(q)$, $\mathcal{P}(p)$ are functionals of the form

$$\mathcal{Q}(q) = q + \mathcal{Q}_1(q) + \mathcal{Q}_2(q) + \cdots, \tag{10}$$

$$\mathcal{P}(p) = p + \mathcal{P}_1(p) + \mathcal{P}_2(p) + \cdots, \tag{11}$$

which can be obtained by solving Eqs. (2) and (3). The results (9) and (11) imply that the first- and second-order terms of the rhs of Eqs. (7) and (8) may be written as

$$K_1(q) = \frac{dK_0(q)}{dq}\mathcal{Q}_1(q), \tag{12}$$

$$K_2(q) = \frac{dK_0(q)}{dq}\mathcal{Q}_2(q) + \frac{1}{2}\frac{d^2K_0(q)}{dq^2}[\mathcal{Q}_1(q)]^2, \tag{13}$$

$$S_1(p) = \frac{dS_0(p)}{dp}\mathcal{P}_1(p), \tag{14}$$

$$S_2(p) = \frac{dS_0(p)}{dp}\mathcal{P}_2(p) + \frac{1}{2}\frac{d^2S_0(p)}{dp^2}[\mathcal{P}_1(p)]^2. \tag{15}$$

We note that the functions $dK_0(q)/dq$ and $dS_0(p)/dp$ obey the equations

$$\frac{dK_0(q)}{dq} = \frac{1}{2\pi\rho_c^0(K_0(q))}, \quad \frac{dS_0(p)}{dp} = \frac{1}{2\pi\rho_s^0(S_0(p))}, \tag{16}$$

where $2\pi\rho_c^0(k)$ and $2\pi\rho_s^0(v)$ $[\rho_s^0(v) = u\sigma(uv),\ v = \Lambda/u]$ are the usual ground-state distributions of Lieb and Wu.[2,6,7]

Solving Eqs. (2) and (3) to first order leads to

$$\mathcal{Q}_1(q) = \int_{-\pi}^{\pi} dq'\delta_c(q')\Phi_{cc}(q,q') + \int_{-k_{F\uparrow}}^{k_{F\uparrow}} dp'\delta_\downarrow(p')\Phi_{cs}(q,p'), \tag{17}$$

$$\mathcal{P}_1(p) = \int_{-\pi}^{\pi} dq'\delta_c(q')\Phi_{sc}(p,q') + \int_{-k_{F\uparrow}}^{k_{F\uparrow}} dp'\delta_\downarrow(p')\Phi_{ss}(p,p'), \tag{18}$$

where the functions $\Phi_{cc}(q,q')$, $\Phi_{cs}(q,p')$, $\Phi_{sc}(p,q')$, and $\Phi_{ss}(p,p')$ are the two-pseudoparticle phase shifts of forward scattering introduced in Refs. 3 and 6. They are defined by a set of four integral equations, which we consider below. These phase shifts define the S matrix for the gapless excitations at finite magnetic fields—at zero field the S matrix for elementary excitations is different because the spin-gapless excitations also acquire a different character.[6]

It is useful to define the functions $\tilde{\mathcal{Q}}_1(k)$ and $\tilde{\mathcal{P}}_1(v)$ such that

$$\tilde{\mathcal{Q}}_1(K_0(q)) = \mathcal{Q}_1(q), \quad \tilde{\mathcal{P}}_1(S_0(p)) = \mathcal{P}_1(p). \tag{19}$$

The analysis of the higher-order terms of the rhs of expansions (10) and (11), which are generated by using Eqs. (2) and (3), shows that all higher-order expressions

involve the two-pseudoparticle phase shifts. In the case of the second-order terms it is useful to introduce the functions $\mathcal{Q}_2^*(q)$ and $\mathcal{P}_2^*(p)$ given by

$$\mathcal{Q}_2^*(q) = \mathcal{Q}_2(q) - \frac{1}{2}\frac{d}{dq}\{[\mathcal{Q}_1(q)]^2\}\,, \tag{20}$$

$$\mathcal{P}_2^*(p) = \mathcal{P}_2(p) - \frac{1}{2}\frac{d}{dp}\{[\mathcal{P}_1(p)]^2\}\,, \tag{21}$$

as well as the related functions $\widetilde{\mathcal{Q}}_2^*(k)$ and $\widetilde{\mathcal{P}}_2^*(v)$ such that

$$\widetilde{\mathcal{Q}}_2^*(K_0(q)) = \mathcal{Q}_2^*(q),\quad \widetilde{\mathcal{P}}_2^*(S_0(p)) = \mathcal{P}_2^*(p)\,. \tag{22}$$

The two-pseudoparticle phase shifts $\Phi_{cc}(q,q')$, $\Phi_{cs}(q,p')$, $\Phi_{sc}(p,q')$, and $\Phi_{ss}(p,p')$ can be written as

$$\Phi_{cc}(q,q') = \widetilde{\Phi}_{cc}(K_0(q),K_0(q'))\,, \tag{23}$$

$$\Phi_{cs}(q,p') = \widetilde{\Phi}_{cs}(K_0(q),S_0(p'))\,, \tag{24}$$

$$\Phi_{sc}(p,q') = \widetilde{\Phi}_{sc}(S_0(p),K_0(q'))\,, \tag{25}$$

$$\Phi_{ss}(p,p') = \widetilde{\Phi}_{ss}(S_0(p),S_0(p'))\,, \tag{26}$$

where the auxiliary functions $\widetilde{\Phi}_{cc}(k,k')$, $\widetilde{\Phi}_{cs}(k,v')$, $\widetilde{\Phi}_{sc}(v,k')$, and $\widetilde{\Phi}_{ss}(v,v')$ were introduced in Refs. 3 and 6. It is useful to express them in terms of the universal functions $\bar{\Phi}_{cc}(x,x')$, $\bar{\Phi}_{cs}(x,y')$, $\bar{\Phi}_{sc}(y,x')$, and $\bar{\Phi}_{ss}(y,y')$, which are related to the auxiliary functions as

$$\widetilde{\Phi}_{cc}(k,k') = \bar{\Phi}_{cc}\left(\frac{\sin k}{u},\frac{\sin k'}{u}\right)\,, \tag{27}$$

$$\widetilde{\Phi}_{cs}(k,v') = \bar{\Phi}_{cs}\left(\frac{\sin k}{u},v'\right)\,, \tag{28}$$

$$\widetilde{\Phi}_{sc}(v,k') = \bar{\Phi}_{sc}\left(v,\frac{\sin k'}{u}\right)\,, \tag{29}$$

$$\widetilde{\Phi}_{ss}(v,v') = \bar{\Phi}_{ss}(v,v')\,. \tag{30}$$

Introducing the parameters

$$Q = K_0(2k_F)\,,\quad B/u = S_0(k_{F\downarrow})\,,$$
$$x_0 = \frac{\sin Q}{u}\,,\quad y_0 = B/u\,, \tag{31}$$

where Q and B are the usual cutoff parameters of the ground-state Lieb-Wu equations,[2,6,7] the universal functions, which are common to other one-dimensional Landau liquids, obey the following integral equations:

$$\bar{\Phi}_{cc}(x,x') = \frac{1}{\pi}\int_{-y_0}^{y_0} dy''\frac{\bar{\Phi}_{sc}(y'',x')}{1+(x-y'')^2}\,, \tag{32}$$

$$\bar{\Phi}_{cs}(x,y') = -\frac{1}{\pi}\tan^{-1}(x-y') + \frac{1}{\pi}\int_{-y_0}^{y_0} dy''\frac{\bar{\Phi}_{ss}(y'',y')}{1+(x-y'')^2}\,, \tag{33}$$

$$\bar{\Phi}_{sc}(y,x') = -\frac{1}{\pi}\tan^{-1}(y-x') + \int_{-y_0}^{y_0} dy'' G(y,y'')\bar{\Phi}_{sc}(y'',x')\,, \tag{34}$$

$$\bar{\Phi}_{ss}(y,y') = \frac{1}{\pi}\tan^{-1}\left(\frac{y-y'}{2}\right) - \frac{1}{\pi^2}\int_{-x_0}^{x_0} dx''\frac{\tan^{-1}(x''-y')}{1+(y-x'')^2} + \int_{-y_0}^{y_0} dy'' G(y,y'')\bar{\Phi}_{ss}(y'',y')\,, \tag{35}$$

where the kernel $G(y,y')$ reads

$$G(y,y') = -\frac{1}{2\pi}\left(\frac{1}{1+[(y-y')/2]^2}\right) \times\left[1-\frac{1}{2}\left(t(y)+t(y')+\frac{l(y)-l(y')}{y-y'}\right)\right] \tag{36}$$

and

$$t(y) = \frac{1}{\pi}\sum_{j=\pm 1}(j)\tan^{-1}(y+jx_0)\,, \tag{37}$$

$$l(y) = \frac{1}{\pi}\sum_{j=\pm 1}(j)\ln[1+(y+jx_0)^2]\,. \tag{38}$$

We emphasize that in this paper the inverse of the trigonometric and hyperbolic functions are denoted by using the superscript -1.

Introducing the distributions (6) in the energy functional (4) leads to the Landau energy expansion

$$E = E_0 + E_1 + E_2 + \cdots\,. \tag{39}$$

The use of Eqs. (7), (8), (12), (14), and (16) leads to

$$E_1 = \frac{N_a}{2\pi}\left\{\int_{-\pi}^{\pi} dq\delta_c(q)\left[\left(\mu-\frac{U}{2}\right)-\mu_0 H-2t\cos[K_0(q)]\right]+\int_{-k_{F\uparrow}}^{k_{F\uparrow}} dp\delta_\downarrow(p)2\mu_0 H+\int_{-Q}^{Q} dk(2t\sin k)\tilde{\mathcal{Q}}_1(k)\right\}. \tag{40}$$

With the help of Eqs. (17) and (19) this expression may be rewritten as

$$E_1 = \frac{N_a}{2\pi}\int_{-\pi}^{\pi} dq\delta_c(q)\epsilon_c(q)+\frac{N_a}{2\pi}\int_{-k_{F\downarrow}}^{k_{F\downarrow}} dp\,\delta_\downarrow(p)\epsilon_s(p)\,, \tag{41}$$

where

$$\epsilon_c(q) = \epsilon_c^0(q)+(\mu-U/2)-\mu_0 H,\quad \epsilon_s(p) = \epsilon_s^0(p)+2\mu_0 H\,, \tag{42}$$

and the bare spectra $\epsilon_c^0(q)$ and $\epsilon_s^0(p)$ are given by

$$\epsilon_c^0(q) = -2t\cos K_0(q)+2t\int_{-Q}^{Q} dk\tilde{\Phi}_{cc}(k,K_0(q))\sin k\,, \tag{43}$$

$$\epsilon_s^0(p) = 2t\int_{-Q}^{Q} dk\tilde{\Phi}_{cs}(k,S_0(p))\sin k\,. \tag{44}$$

The ground-state solutions $K_0(q)$ and $S_0(p)$ can be simply evaluated. The result is

$$\begin{aligned} K_0(q) &= q-\int_{-Q}^{Q} dk\tilde{\Phi}_{cc}(k,K_0(q))\,,\\ p &= \int_{-Q}^{Q} dk\tilde{\Phi}_{cs}(k,S_0(p))\,. \end{aligned} \tag{45}$$

We note that the pseudoparticle energy spectra $\epsilon_c(q)$ and $\epsilon_s(p)$ defined by Eq. (42) vanish at the pseudo-Fermi-points, i.e.,

$$\epsilon_c(\pm 2k_F) = \epsilon_s(\pm k_{F\downarrow}) = 0\,. \tag{46}$$

The combination of Eqs. (42) and (46) allows the derivation of the density and magnetization curves, which read

$$\begin{aligned} \mu(n) &= \frac{U}{2}-\epsilon_c^0(2k_F)\,|_x-\tfrac{1}{2}\epsilon_s^0(k_{F\downarrow})\,|_x\,,\\ H(m) &= -\frac{\epsilon_s^0(k_{F\downarrow})}{2\mu_0}\bigg|_y\,, \end{aligned} \tag{47}$$

where, depending on the constraints imposed to the system, either $x = H$ or $x = m$ and either $y = \mu$ or $y = n$, respectively. It is useful for the interpretation of the figures of Sec. IV, particularly in what concerns the behavior of the charge and spin susceptibilities across the ferromagnetic transition, to study the magnetization curve defined by the second equation of (47). It is represented in Fig. 1, in the case $y = n$, for $U = 10$, $0 < H < H_c$ and various densities. The envelope curve defines the critical field H_c at the different densities. Note the overlap of magnetization curves, which occurs for electronic densities close to $n = 1$.

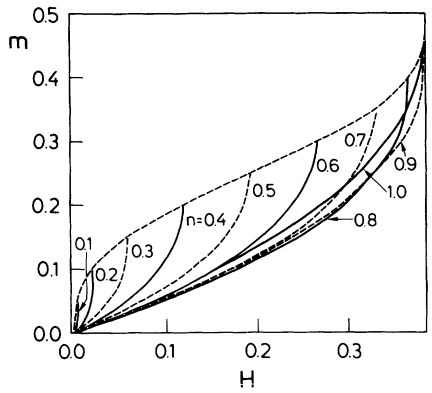

FIG. 1. The magnetization curve for $U = 10$ and various densities.

The pseudoparticle velocities are defined by the usual expression for the group velocity. They are given by

$$v_c(q) = \frac{d\epsilon_c(q)}{dq} = \frac{d\epsilon_c^0(q)}{dq}\,,\quad v_s(p) = \frac{d\epsilon_s(p)}{dp} = \frac{d\epsilon_s^0(p)}{dp}\,. \tag{48}$$

Two relevant parameters of the one-dimensional theory, which play an important role in the scale-invariant regime of Sec. III, are the pseudoparticle velocities at the pseudo-Fermi points,

$$v_c = v_c(2k_F)\,,\quad v_s = v_s(k_{F\downarrow})\,. \tag{49}$$

In Ref. 6 it is shown that the low-energy gapless excitations of the many-body system can be understood in terms of particle-hole processes in the pseudoparticle bands (42). Furthermore, that work presents a detailed study of the dependence of the bands (42)–(44) on the on-site repulsion, electronic density, and magnetic field. In that paper we did not study the f functions associated with the second-order term of the energy expansion (39) because these do not contribute to the spectra of the single-pair elementary excitations, which involve a single particle-hole process in the pseudoparticle bands. In the case of multipair excitations involving a number of particle-hole processes $N_{\text{p-h}}$ such that $N_{\text{p-h}}/N_a \to 0$ as $N_a \to \infty$, the excitation energy is additive and the second-order terms of the expansion (39) can also be neglected. These second-order contributions cannot be neglected in the case of multipair excitations involving a small but finite density of pseudoparticles. It follows that the f functions play a predominant role in the low-energy physics of the many-body system by regulating the forward-scattering interactions of the pseudoparticles.[3] As in a Fermi liquid,[4,18,19] they are the main ingredients for the derivation of the charge and spin static susceptibilities, which we present in Sec. IV.

In order to derive the second-order term of expansion (39), we use the results (7)–(22) in Eq. (4). After some simple algebra we obtain

$$E_2 = \frac{N_a}{4\pi^2}\left(\int_{-\pi}^{\pi} dq\delta_c(q)\frac{dK_0(q)}{dq}[4\pi t\sin K_0(q)]\mathcal{Q}_1(q) + \int_{-Q}^{Q} dk(4\pi t\sin k)\tilde{\mathcal{Q}}_2^*(k) + \frac{2\pi t\sin Q}{2\pi\rho_c(Q)}\sum_{j=\pm 1}[\mathcal{Q}_1(j2k_F)]^2\right). \quad (50)$$

The next step is the derivation of the function $\tilde{\mathcal{Q}}_2^*(k)$. This requires the introduction in Eqs. (2) and (3) of the second-order expansions (7) and (8), with first- and second-order terms of the form (12)–(15) and $\mathcal{Q}_1(q)$ and $\mathcal{P}_1(p)$ given by Eqs. (17) and (18). The explicit calculation is straightforward, but somewhat lengthy. Although the intermediate steps involve a large number of terms, the final result leads to a simple expression for the energy (50), which reads

$$E_2 = \frac{N_a}{4\pi^2}\Big(2\pi\int_{-\pi}^{\pi} dq\delta_c(q)v_c(q)\mathcal{Q}_1(q) + 2\pi\int_{-k_{F\downarrow}}^{k_{F\downarrow}} dp\delta_\downarrow(p)v_s(p)\mathcal{P}_1(p) + 2\pi v_c\frac{1}{2}\sum_{j=\pm 1}[\mathcal{Q}_1(j2k_F)]^2 + 2\pi v_s\frac{1}{2}\sum_{j=\pm 1}[\mathcal{P}_1(jk_{F\downarrow})]^2\Big). \quad (51)$$

The use of Eqs. (17) and (18) leads finally to the desired result,

$$E_2 = \frac{N_a}{4\pi^2}\int_{-\pi}^{\pi} dq\int_{-\pi}^{\pi} dq'\delta_c(q)\delta_c(q')\frac{1}{2}f_{cc}(q,q') + \frac{N_a}{4\pi^2}\int_{-k_{F\uparrow}}^{k_{F\uparrow}} dp\int_{-k_{F\uparrow}}^{k_{F\uparrow}} dp'\delta_\downarrow(p)\delta_\downarrow(p')\frac{1}{2}f_{ss}(p,p') + \frac{N_a}{4\pi^2}\int_{-\pi}^{\pi} dq\int_{-k_{F\uparrow}}^{k_{F\uparrow}} dp'\,\delta_c(q)\delta_\downarrow(p')f_{cs}(q,p'), \quad (52)$$

where the f functions $f_{cc}(q,q')$, $f_{ss}(p,p')$, and $f_{cs}(q,p')$, which are the second functional derivatives of the energy with respect to the deviations, are given by

$$f_{cc}(q,q') = 2\pi v_c(q)\Phi_{cc}(q,q') + 2\pi v_c(q')\Phi_{cc}(q',q) + 2\pi v_c\sum_{j=\pm 1}\Phi_{cc}(j2k_F,q)\Phi_{cc}(j2k_F,q') + 2\pi v_s\sum_{j=\pm 1}\Phi_{sc}(jk_{F\downarrow},q)\Phi_{sc}(jk_{F\downarrow},q'), \quad (53)$$

$$f_{ss}(p,p') = 2\pi v_s(p)\Phi_{ss}(p,p') + 2\pi v_s(p')\Phi_{ss}(p',p) + 2\pi v_s\sum_{j=\pm 1}\Phi_{ss}(jk_{F\downarrow},p)\Phi_{ss}(jk_{F\downarrow},p') + 2\pi v_c\sum_{j=\pm 1}\Phi_{cs}(j2k_F,p)\Phi_{cs}(j2k_F,p'), \quad (54)$$

$$f_{cs}(q,p') = 2\pi v_c(q)\Phi_{cs}(q,p') + 2\pi v_s(p')\Phi_{sc}(p',q) + 2\pi v_c\sum_{j=\pm 1}\Phi_{cc}(j2k_F,q)\Phi_{cs}(j2k_F,p') + 2\pi v_s\sum_{j=\pm 1}\Phi_{ss}(jk_{F\downarrow},p')\Phi_{sc}(jk_{F\downarrow},q). \quad (55)$$

We emphasize that although the starting energy expressions (4), (40), and (50) are not symmetric in the charge and spin contributions, the final expressions (41), and (51) and (52) for the first- and second-order terms of the energy functional, respectively, have symmetric charge and spin parts.

As in Fermi-liquid theory,[4,18,19] the higher-order terms of the energy lead to corrections in the pseudoparticle band expressions. Up to first order in the deviations the corrected bare bands read

$$\breve{\epsilon}_c^0(q) = \epsilon_c^0(q) + \frac{1}{2\pi}\int_{-\pi}^{\pi} dq'\delta_c(q')f_{cc}(q,q') + \frac{1}{2\pi}\int_{-k_{F\uparrow}}^{k_{F\uparrow}} dp'\delta_\downarrow(p')f_{cs}(q,p'), \quad (56)$$

$$\breve{\epsilon}_s^0(p) = \epsilon_s^0(p) + \frac{1}{2\pi}\int_{-\pi}^{\pi} dq'\delta_c(q')f_{cs}(q',p) + \frac{1}{2\pi}\int_{-k_{F\uparrow}}^{k_{F\uparrow}} dp'\delta_\downarrow(p')f_{ss}(p,p'). \quad (57)$$

The spectra (56) and (57) are useful for the evaluation of the charge and spin susceptibilities of Sec. IV.

The form of the rhs of Eqs. (43)–(45) and (53)–(55) shows that the first- and second-order Landau coefficients are fully controlled by the forward-scattering phase shifts. The same holds true for the remaining higher-order energy functional derivatives—all the effects of the on-site repulsion in the system are controlled exclusively by the two-pseudoparticle phase shifts.[6] At $T = 0$ and finite fields the many-body system (1) can be treated as a two-fluid Landau liquid with only forward scattering. The functions $\Phi_{cc}(q',q)$ and $\Phi_{sc}(p',q)$ [$\Phi_{ss}(p'',p)$ and $\Phi_{cs}(q'',p)$] are the phase shifts of the charge (spin) pseudoparticle of pseudomomentum q (p) due to a forward-scattering interaction with the charge (spin) and spin (charge) pseudoparticles of pseudomomenta q' (p'') and p' (q''), respectively.

In this section we have calculated the first- and second-order coefficients of the Landau-energy functional (39). As in Fermi-liquid theory,[4,18,19] the first-order coefficients (42) define the bands of the pseudoparticles and the second-order coefficients (53)–(55) are the f functions that regulate the interactions of the pseudoparticles.

Although the generalized Bethe-ansatz equations (2) and (3) allow, in principle, the exact evaluation of all

the coefficients, the calculations become very lengthy for orders larger than two. Nonetheless, as in Fermi-liquid theory, the second-order expansion controls the physics of the low-energy regime.

We emphasize the universal character of the Landau energy functional derived in this section, which provides the energy and momentum associated with all possible gapless excitations, provided that the corresponding deviations $\delta_c(q)$ and $\delta_\downarrow(p)$ are suitably chosen.

Finally, we note that the expressions for the pseudoparticle bands (42)–(45), as well as the equations for the phase shifts (32)–(38), may be written in an alternative form in the limit $H \rightarrow 0$. In Appendix A we introduce this alternative representation.

III. THE SCALE-INVARIANT REGIME

For $U > 0$, $n < 1$, and $0 < H < H_c$ the particle-hole processes in the charge and spin pseudoparticle bands (42) describe the low-lying gapless excitations of the interacting system.[6] The system becomes scale invariant in the limit of very small excitation energy ω and momentum k, when ω/k equals v_c and v_s for charge and spin excitations, respectively. It also becomes scale invariant for very small energies when the excitation momentum is very close to $\pm 4k_F$ ($\pm 2k_{F\downarrow}$) in the case of charge (spin) excitations. The scale-invariant regime is determined by the behavior of the f functions (53)–(55) at the pseudo-Fermi surfaces.

As in Fermi-liquid theory, it is useful to introduce the Landau parameters. In one dimension the pseudo-Fermi surfaces are comprised of two points, and thus one has only the symmetric and antisymmetric combinations of the f functions, which correspond to the first two terms in the usual Legendre expansion of the Fermi-liquid theory.[4,5,18,19] The present one-dimensional Landau liquid includes two types of fermionic pseudoparticles and then two pseudo-Fermi surfaces with two points each. As a result, the Landau parameters can be considered as the elements of the two matrices $\mathbf{F}^0$ and $\mathbf{F}^1$,

$$\mathbf{F}^i = \begin{bmatrix} F^i_{cc} & F^i_{cs} \\ F^i_{sc} & F^i_{ss} \end{bmatrix}, \quad i = 0,1, \tag{58}$$

with elements

$$F^i_{cc} = \frac{1}{2\pi} \sum_{j=\pm 1} (j)^i f_{cc}(2k_F, j2k_F), \quad i = 0,1, \tag{59}$$

$$F^i_{ss} = \frac{1}{2\pi} \sum_{j=\pm 1} (j)^i f_{ss}(k_{F\downarrow}, jk_{F\downarrow}), \quad i = 0,1, \tag{60}$$

$$F^i_{cs} = F^i_{sc} = \frac{1}{2\pi} \sum_{j=\pm 1} (j)^i f_{cs}(2k_F, jk_{F\downarrow}) = \frac{1}{2\pi} \sum_{j=\pm 1} (j)^i f_{cs}(j2k_F, k_{F\downarrow}), \quad i = 0,1. \tag{61}$$

In order to derive simple expressions for the Landau parameters (59)–(61), it is useful to introduce the matrices $\mathbf{R}^\pm$,

$$\mathbf{R}^\pm = \begin{bmatrix} \Phi_{cc}(2k_F, \pm 2k_F) & \Phi_{cs}(2k_F, \pm k_{F\downarrow}) \\ \Phi_{sc}(k_{F\downarrow}, \pm 2k_F) & \Phi_{ss}(k_{F\downarrow}, \pm k_{F\downarrow}) \end{bmatrix}, \tag{62}$$

with elements given by the forward-scattering phase shifts involving only pseudoparticles with momentum at the pseudo-Fermi surfaces. These define the S matrix for the elementary excitations of lowest energy. Based on Eqs. (53)–(55), it is straightforward to show that the Landau parameters (59)–(61) may be written as

$$F^i_{cc} = -v_c + v_c[\xi^i_{cc}]^2 + v_s[\xi^i_{sc}]^2, \quad i = 0,1, \tag{63}$$

$$F^i_{ss} = -v_s + v_s[\xi^i_{ss}]^2 + v_c[\xi^i_{cs}]^2, \quad i = 0,1, \tag{64}$$

$$F^i_{cs} = F^i_{sc} = v_c[\xi^i_{cc}\xi^i_{cs}] + v_s[\xi^i_{ss}\xi^i_{sc}], \quad i = 0,1, \tag{65}$$

where the ξ^i's are the elements of the matrices

$$\mathbf{Z}^i = \begin{bmatrix} \xi^i_{cc} & \xi^i_{cs} \\ \xi^i_{sc} & \xi^i_{ss} \end{bmatrix}, \quad i = 0,1, \tag{66}$$

defined as

$$\mathbf{Z}^i = \mathbb{1} + \mathbf{R}^+ + (-1)^i \mathbf{R}^-, \quad i = 0,1. \tag{67}$$

Here $\mathbb{1}$ is the 2×2 unit matrix. Except for the diagonal contributions arising from $\mathbb{1}$, the elements of the matrices $\mathbf{Z}^0$ and $\mathbf{Z}^1$ correspond to the symmetric and antisymmetric combinations of the forward-scattering phase shifts of pairs of pseudoparticles with momentum at the pseudo-Fermi surfaces, respectively.

As in Fermi-liquid theory,[4,18,19] while the symmetric Landau parameters given by expressions (59)–(61) with $i = 0$ regulate the compressibility and magnetic susceptibility, the antisymmetric Landau parameters given by expressions (59)–(61) with $i = 1$ control the charge and spin currents and the stiffnesses.[21] In this paper we limit our considerations to the static properties, the expressions for the compressibility and spin susceptibility in the presence of a magnetic field being derived in Sec. IV. In Figs. 2(a)–2(c) the symmetric functions $v_c + F^0_{cc}$, $v_s + F^0_{ss}$, and F^0_{cs}, respectively, are plotted vs the magnetic field H for $0 < H < H_c$, $U = 10$ and various densities. While $v_c + F^0_{cc}$ and $v_s + F^0_{ss}$ are positive, the charge-spin Landau parameter F^0_{cs} is negative. Notice the different behavior of the functions represented in Figs. 2(a) and 2(c) in the case of a more dilute system ($n = 0.3$) and of intermediate and large densities close to $n = 1$ ($n = 0.7$ and $n = 0.9$).

In Appendix B it is shown that the matrix $\mathbf{Z}^1$ introduced in Eq. (67) is nothing but the dressed charge matrix previously obtained in conformal field theory (we use here the same definition as Ref. 9, which is the transposition of that of Ref. 10). Within the present Landau-liquid theory it follows naturally from the phase shift (S matrix) dependence of the Landau parameters. These dominate the physics in the limit of vanishing excitation energy, where the theory becomes scale invariant. Further relations to conformal field theory results will be presented in Sec. V.

The structure of the coupled integral equations (32)–

(35) shows that the functions that determine the form of the phase shifts are not independent. This dependence assumes a simple and elegant form in the case of the phase shifts at the pseudo-Fermi-points (62). It may be expressed by the following relation:

$$\mathbf{Z}^0 = [[\mathbf{Z}^1]^{-1}]^T , \tag{68}$$

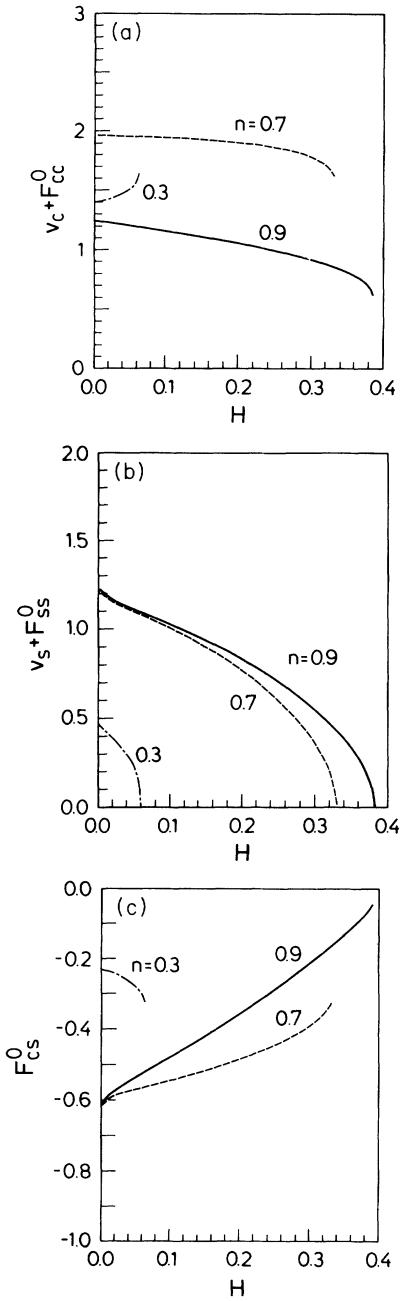

FIG. 2. The functions (63)–(65) (a) $v_c + F^0_{cc}$, (b) $v_s + F^0_{ss}$, and (c) F^0_{cs} vs the magnetic field H for $U = 10$ and various densities. When the system is diluted $v_c + F^0_{cc}$ (F^0_{cs}) increases (decreases) with increasing fields for $H < H_c$. For intermediate and large densities it becomes a decreasing (increasing) function of the magnetic field. $v_s + F^0_{ss}$ decreases with increasing fields for $H < H_c$. In (b) and (c) the close values of the $n = 0.7$ and $n = 0.9$ curves at $H = 0$ is purely accidental.

i.e., $\mathbf{Z}^0$ is the transposition of the inverse matrix of $\mathbf{Z}^1$.

The dependence of the elements of the dressed charge matrix on U, H, and n has been investigated previously in the context of conformal field theory.[9,10] Using the Wiener-Hopf method, Frahm and Korepin have studied the logarithmic singularities that occur as $H \to 0$ when $U \to \infty$.[10] We find that for $H \to 0$ the matrices (66) are singular at $U = 0$. While for $U \to 0$ they read

$$\mathbf{Z}^0 = \begin{bmatrix} 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & \sqrt{2} \end{bmatrix} , \quad \mathbf{Z}^1 = \begin{bmatrix} \sqrt{2} & 1/\sqrt{2} \\ 0 & 1/\sqrt{2} \end{bmatrix} , \tag{69}$$

at $U = 0$ the result is

$$\mathbf{Z}^0 = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} , \quad \mathbf{Z}^1 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} . \tag{70}$$

This singular behavior is absent at finite fields, where both as $U \to 0$ and at $U = 0$ the matrices (66) are given by (70). On the other hand, the above singular behavior does not show up in the static susceptibilities and critical exponents because the Landau parameters (59)–(61) [(63)–(65)], which are the macroscopic parameters that regulate these quantities, are always analytical. Their expression at $U = 0$ (and for $U \to 0$) is, for both $H \to 0$ and finite fields, given by

$$\mathbf{F}^0 = \begin{bmatrix} 0 & -v_c \\ -v_c & v_c \end{bmatrix} , \quad \mathbf{F}^1 = \begin{bmatrix} v_s & v_s \\ v_s & 0 \end{bmatrix} , \tag{71}$$

where for $U \to 0$ the velocities (49), which appear in these matrices, read

$$v_c = 2t \sin(k_{F\uparrow}) , \quad v_s = 2t \sin(k_{F\downarrow}) . \tag{72}$$

Since the Landau parameters (59)–(61) regulate all the low-energy physics, it is interesting to study their values in other limiting cases. For example, for $H \to 0$ the matrices (58) may be written as

$$\mathbf{F}^0 = \begin{bmatrix} \dfrac{v_s}{2} - v_c \left(1 - \dfrac{1}{\xi_0^2}\right) & -v_s \\ -v_s & v_s \end{bmatrix} ,$$

$$\mathbf{F}^1 = \begin{bmatrix} -v_c(1-\xi_0^2) & \frac{1}{2} v_c \xi_0^2 \\ \frac{1}{2} v_c \xi_0^2 & \frac{1}{4} v_c \xi_0^2 - \frac{1}{2} v_s \end{bmatrix} , \tag{73}$$

where ξ_0 is the function that regulates the thermodynamic quantities and critical exponents[10] in this limit. It is given by $\xi_0 = \xi_0(x_0)$, where the function $\xi_0(x)$ is defined by the following integral equation:

$$\xi_0(x) = 1 + \int_{-x_0}^{x_0} dx'' A(x - x'') \xi_0(x'') , \tag{74}$$

and the kernel $A(x)$ reads

$$A(x) = \frac{1}{\pi} \int_0^\infty d\omega \frac{\cos(\omega x)}{1 + e^{2\omega}} . \tag{75}$$

Equation (74) may be simply obtained from the equations for the phase shifts introduced in Appendix A. For

$0 < n < 1$, ξ_0 changes from $\xi_0 = \sqrt{2}$ at $U = 0$ to $\xi_0 = 1$ for $U \to \infty$. On the other hand, for $H \to 0$ the velocities (49) change from $v_c = v_s = 2t\sin(\pi n/2)$ when $U = 0$, to $v_c = 2t\sin(\pi n)$ and $v_s = 0$ for $U \to \infty$.

Another interesting limit is $H \to H_c$ (or $U \to U_c$). The matrices (58) in this limit are given by

$$\mathbf{F}^0 = \begin{bmatrix} 0 & -v_c\eta_0 \\ -v_c\eta_0 & v_c\eta_0^2 \end{bmatrix}, \quad \mathbf{F}^1 = \begin{bmatrix} v_s\eta_0^2 & v_s\eta_0 \\ v_s\eta_0 & 0 \end{bmatrix} = 0, \tag{76}$$

where η_0 and the velocities are simply given by

$$\eta_0 = \frac{2}{\pi}\tan^{-1}\left(\frac{\sin(\pi n)}{u}\right), \quad v_c = 2t\sin(\pi n), \quad v_s = 0. \tag{77}$$

Finally, in the limit $U \gg t$ and $H \ll H_c$ all the elements of $\mathbf{F}^0$ show logarithmic dependence due to the singular behavior as $H \to 0$. Nonetheless, in $\mathbf{F}^1$ only the Landau parameter F_{ss}^1 shows such dependence, which is introduced through the function β_0 defined below:

$$\mathbf{F}^0 = \begin{bmatrix} \frac{v_s}{2}\left(\frac{\alpha_0}{\beta_0}\right)^2 & -v_s\frac{\alpha_0}{\beta_0^2} \\ -v_s\frac{\alpha_0}{\beta_0^2} & v_s\left(\frac{2}{\beta_0^2} - 1\right) \end{bmatrix}, \tag{78}$$

$$\mathbf{F}^1 = \begin{bmatrix} 0 & \frac{1}{2}v_c\alpha_0 \\ \frac{1}{2}v_c\alpha_0 & \frac{1}{4}v_c(\alpha_0)^2 - v_s\left(1 - \frac{\beta_0^2}{2}\right) \end{bmatrix}. \tag{79}$$

Here the renormalized parameters read

$$\alpha_0 = 1 - \frac{4}{\pi^2}\frac{H}{H_c}, \quad \beta_0 = 1 + \frac{1}{4\ln(\sqrt{\pi^3/2e}H_c/H)}, \tag{80}$$

$$v_c = 2t\sin(\pi n)\left(1 - \frac{2\ln(2)}{u}\cos(\pi n)\right), \tag{81}$$

$$v_s = 2t\frac{1}{u}\frac{\pi k_F}{4k_{F\uparrow}}\left(1 - \frac{\sin(2\pi n)}{2\pi n}\right)\sin\left(\frac{\pi k_{F\downarrow}}{2k_{F\uparrow}}\right), \tag{82}$$

and the critical field H_c (Ref. 6) may be written as

$$H_c = \left(\frac{t}{\mu_0}\right)\frac{n}{u}\left(1 - \frac{\sin(2\pi n)}{2\pi n}\right). \tag{83}$$

The logarithmic terms (80), which appear in the expressions for the static susceptibilities derived in Sec. IV, cannot be obtained by the effective theory introduced by Haldane,[11] because the relevant spin fluctuations do not exist at $H = 0$.[11,6]

In this section we have introduced the Landau parameters which, as in Fermi-liquid theory, regulate the low-energy physics of the many-body system. They are obtained from the general expressions for the f functions derived in Sec. II.

The limiting expressions (71)–(83) are very useful in the study of the compressibility and spin susceptibility of Sec. IV.

IV. THE STATIC CHARGE AND SPIN SUSCEPTIBILITIES

In this section we derive exact expressions for the compressibility and spin susceptibility of the model (1) at arbitrary magnetic fields. Previous studies on this subject[22,23] were restricted to zero fields. We emphasize that, contrary to the $H = 0$ case, the two compressibilities

$$\chi_c\,|_H = -\frac{1}{n^2}\frac{1}{\partial\mu(n)/\partial n\,|_H}, \tag{84}$$

$$\chi_c\,|_m = -\frac{1}{n^2}\frac{1}{\partial\mu(n)/\partial n\,|_m}, \tag{85}$$

which refer to the system at fixed magnetic field and magnetization, respectively, are in general unequal for $0 < H < H_c$. The same holds true for the spin susceptibilities

$$\chi_s\,|_\mu = \frac{2\mu_0}{\partial H(m)/\partial m\,|_\mu}, \tag{86}$$

$$\chi_s\,|_n = \frac{2\mu_0}{\partial H(m)/\partial m\,|_n}, \tag{87}$$

which refer to the grand canonical and canonical ensembles, respectively. We find that the relation

$$\vartheta = \frac{\chi_c\,|_m}{\chi_c\,|_H} = \frac{\chi_s\,|_n}{\chi_s\,|_\mu}, \tag{88}$$

holds for the whole parameter space.

We consider the case $n < 1$ and $0 < H < H_c$. The cases $n = 1$ and (or) $H > H_c$ are studied in Sec. VI.

In order to derive the thermodynamic quantities (84)–(87), we use the expressions (47) with the bare bands replaced by the spectra $\tilde{\epsilon}_c^0(q)$ and $\tilde{\epsilon}_s^0(p)$ introduced in Eqs. (56) and (57). As in Fermi-liquid theory,[4,18,19] this allows taking into account the changes induced by the momentum deviations in the bare pseudoparticle spectra. The evaluation of the susceptibilities involves small changes $\delta 2k_F$ and $\delta k_{F\downarrow}$ in the pseudo-Fermi-momenta $2k_F$ and $k_{F\downarrow}$, respectively. The appropriate pseudoparticle distributions are of the form

$$\begin{aligned} M_c(q) &= \Theta\left(2k_F + \delta 2k_F - |q|\right), \\ N_\downarrow(p) &= \Theta\left(k_{F\downarrow} + \delta k_{F\downarrow} - |p|\right). \end{aligned} \tag{89}$$

Expanding these distributions around the ground-state functions (5) leads to distributions of the form (6) with the deviations given by

$$\delta_c(q) = \delta(2k_F - |q|)\delta 2k_F, \quad \delta_\downarrow(p) = \delta(k_{F\downarrow} - |p|)\delta k_{F\downarrow}, \tag{90}$$

where $\delta(x)$ is the usual delta function. Inserting the deviations (90) in the rhs of Eqs. (56) and (57) for the bands $\tilde{\epsilon}_c^0(q)$ and $\tilde{\epsilon}_s^0(p)$, and replacing the obtained expressions in Eqs. (47), which define the density and magnetization curves, leads to

$$\left.\frac{\partial\mu(n)}{\partial n}\right|_x = -\left[\left(v_c + F^0_{cc} + \frac{1}{2}F^0_{cs}\right)\frac{\delta 2k_F}{\delta n} + \left(F^0_{cs} + \frac{1}{2}(v_s + F^0_{ss})\right)\frac{\delta k_{F\downarrow}}{\delta n}\right]\Bigg|_x , \quad x = H, m , \tag{91}$$

$$\left.\frac{\partial H(m)}{\partial m}\right|_y = -\frac{1}{2\mu_0}\left(F^0_{cs}\frac{\delta 2k_F}{\delta m} + (v_s + F^0_{ss})\frac{\delta k_{F\downarrow}}{\delta m}\right)\Bigg|_y , \quad y = \mu, n . \tag{92}$$

As expected, the expressions for the derivatives (91) and (92) are regulated by the Landau parameters (59)–(61). On the other hand, the changes $\delta 2k_F$ and $\delta k_{F\downarrow}$ in the pseudo-Fermi momenta are related with the changes δn in the density and δm in the spin density as follows:

$$\delta 2k_F = \pi\delta n , \quad \delta 2k_F - 2\delta k_{F\downarrow} = 2\pi\delta m . \tag{93}$$

Equations (93) follow from the fact that both the number of spin-up electrons $N_\uparrow$ and of spin-down electrons $N_\downarrow$ are good quantum numbers of the system.[2]

In order to calculate the compressibility at constant magnetic field, Eq. (84), we make use both of the condition $\delta H = 0$ and of Eq. (93) to derive the following expressions for the derivatives of the rhs of Eq. (91)

$$\left.\frac{\delta 2k_F}{\delta n}\right|_H = \pi , \quad \left.\frac{\delta k_{F\downarrow}}{\delta n}\right|_H = -\pi\left(\frac{F^0_{cs}}{v_s + F^0_{ss}}\right) . \tag{94}$$

Inserting these expressions in the rhs of Eq. (91) allows the derivation of the following expressions for the compressibility (84)

$$\chi_c|_H = \frac{1}{\pi n^2}\frac{L_0}{(L_1)^2} , \tag{95}$$

where the functions L_0 and L_1 are given by

$$L_0 = \frac{1}{4}(v_s + F^0_{ss}) ,$$
$$L_1 = \frac{1}{2}\sqrt{(v_c + F^0_{cc})(v_s + F^0_{ss}) - (F^0_{cs})^2} . \tag{96}$$

Equation (95) provides the exact form for the charge susceptibility at constant magnetic field, which, as in Fermi-liquid theory,[18,19] is determined by the symmetric Landau parameters introduced in Sec. III.

Combining the condition $\delta m = 0$ with Eq. (93), leads to

$$\left.\frac{\delta 2k_F}{\delta n}\right|_m = \pi , \quad \left.\frac{\delta k_{F\downarrow}}{\delta n}\right|_m = \frac{\pi}{2} . \tag{97}$$

The compressibility at constant magnetization (85) is readily obtained by inserting the derivatives (97) in the rhs of Eq. (91). The result is

$$\chi_c|_m = \frac{1}{\pi n^2}\frac{1}{L_2} . \tag{98}$$

Here L_2 is given by

$$L_2 = v_c + F^0_{cc} + (v_s + F^0_{ss})/4 + F^0_{cs} . \tag{99}$$

On the other hand, the condition $\delta\mu = 0$ allows the evaluation of the derivatives of the rhs of Eq. (92) in the case $y = \mu$. We obtain

$$\left.\frac{\delta 2k_F}{\delta m}\right|_\mu = 2\pi\left(\frac{(v_s + F^0_{ss})/2 + F^0_{cs}}{2(v_c + F^0_{cc}) + (v_s + F^0_{ss})/2 + 2(F^0_{cs})}\right) , \tag{100}$$

$$\left.\frac{\delta k_{F\downarrow}}{\delta m}\right|_\mu = -2\pi\left(\frac{v_c + F^0_{cc} + (F^0_{cs})/2}{2(v_c + F^0_{cc}) + (v_s + F^0_{ss})/2 + 2(F^0_{cs})}\right) . \tag{101}$$

Combining Eqs. (92), (100), and (101) leads to the following exact expression for the spin magnetic susceptibility at constant chemical potential:

$$\chi_s|_\mu = \frac{\mu_0^2}{\pi}\frac{L_2}{(L_1)^2} . \tag{102}$$

Finally, in the case of the canonical ensemble the condition $\delta n = 0$ leads to

$$\left.\frac{\delta 2k_F}{\delta m}\right|_n = 0 , \quad \left.\frac{\delta k_{F\downarrow}}{\delta m}\right|_n = -\pi , \tag{103}$$

and the spin susceptibility at constant electronic density, Eq. (87), may be evaluated. The result is

$$\chi_s|_n = \frac{\mu_0^2}{\pi}\frac{1}{L_0} . \tag{104}$$

Equations (95), (98), (102), and (104) allow us to derive the function (88), which reads

$$\vartheta = \frac{(L_1)^2}{L_0 L_2} . \tag{105}$$

For all densities $0 < n < 1$ and finite values of $U < U_c$, ϑ changes from $\vartheta = 1$ at $H = 0$ to $\vartheta = 0$ at $H = H_c^{(-)}$. While, for $H \to 0$, $\chi_c|_H = \chi_c|_m$ and $\chi_s|_\mu = \chi_s|_n$, when $0 < H < H_c^{(-)}$, $\chi_c|_H \neq \chi_c|_m$ and $\chi_s|_\mu \neq \chi_s|_n$. At $H = H_c^{(-)}$, $\chi_c|_H = \chi_s|_\mu = \infty$ and $\chi_c|_m$, $\chi_s|_n$ remain finite. The study of the dynamical properties of the model (1) shows that $\chi_c|_H$ and $\chi_s|_\mu$ are the most interesting susceptibilities because they correspond to the limit $k \to 0, \omega \to 0$ of the Fourier transforms of the charge and spin retarded correlation functions.[21] Since these charge and spin susceptibilities have qualitatively the same kind of dependence on H, U, and n as the functions (98) and (104), respectively, we concentrate our study mainly on the functions (95) and (102). The relation of the latter with the former is simply given by Eqs. (88) and (105).

The compressibility at constant magnetic field (95) and the spin magnetic susceptibility at constant chemical potential (102) can be rewritten as the sum of two terms proportional to $1/v_c$ and $1/v_s$. The use of Eqs. (63)–(65) and (68) leads to

$$\chi_c\,|_H = \frac{1}{\pi n^2}\left(\frac{(\xi^1_{cc})^2}{v_c}+\frac{(\xi^1_{sc})^2}{v_s}\right), \tag{106}$$

$$\chi_s\,|_\mu = \frac{\mu_0^2}{\pi}\left(\frac{(\xi^1_{cc}-2\xi^1_{cs})^2}{v_c}+\frac{(\xi^1_{sc}-2\xi^1_{ss})^2}{v_s}\right). \tag{107}$$

On the other hand, the functions (98) and (104) can be rewritten as

$$\chi_c\,|_m = \frac{1}{\pi n^2}\left(\frac{1}{v_c(\xi^0_{cc}+\xi^0_{cs}/2)^2+v_s(\xi^0_{sc}+\xi^0_{ss}/2)^2}\right), \tag{108}$$

$$\chi_s\,|_n = \frac{\mu_0^2}{\pi}\left(\frac{1}{v_c(\xi^0_{cs}/2)^2+v_s(\xi^0_{ss}/2)^2}\right). \tag{109}$$

In the limit $H \to 0$ the expressions (95), (98), (102), and (104) reproduce the results of Refs. 22 and 23. Since these papers contain a detailed study on the zero-field compressibility and spin susceptibility, respectively, in the figures below we concentrate our attention mainly on the $H > 0$ case. We emphasize that Eqs. (95), (98), (102), and (104) are not valid in the ferromagnetic phase. Following the results of Sec. VI, in the figures below the compressibilities (84) and (85) and spin susceptibilities (86) and (87) are in the non-half-filling ferromagnetic phase simply given by $\chi_c\,|_H = \chi_c\,|_m = 1/[2\pi t n^2 \sin(\pi n)]$ and $\chi_s\,|_\mu = \chi_s\,|_n = 0$, respectively.

In Figs. 3(a) and 3(b) the inverse of the functions $\chi_c\,|_H$ and $\chi_s\,|_\mu$, respectively, is plotted vs U for $n = 0.7$ and various values of the magnetic field. These functions diverge at the value $U = U_c$ at which the system becomes ferromagnetic ($U_c \to \infty$ as $H \to 0$). For small densities and $U < U_c$ the compressibility is always an increasing function of the on-site repulsion. On the other hand, for intermediate and large densities close to $n = 1$, as in Fig. 3(a), the compressibility decreases for small values of the on-site interaction, going through a minimum at an intermediate value $U = U^*$, such that $0 < U^* < U_c$. For $U^* < U < U_c$ it becomes an increasing function of U, diverging as $U \to U_c$. The spin susceptibility of Fig. 3(b) also diverges as $U \to U_c$, increasing with increasing U in all the domain $0 < U < U_c$. In Fig. 3(c) we plot the spin density vs the on-site repulsion for the same values of the magnetic field and electronic density as in Figs. 3(a) and 3(b).

At constant values of the magnetic field H and of the on-site repulsion U, the Hubbard chain is ferromagnetic for densities between $n = 0$ and the critical density $n = n_c$ at which $H = H_c$ (Ref. 6) ($n_c \to 0$ as $H \to 0$). In Figs. 4(a) and 4(b) the inverse of the compressibility $\chi_c\,|_H$ and of the spin magnetic susceptibility $\chi_s\,|_\mu$, respectively, are plotted vs the electronic density for $U = 10$ and three values of the magnetic field. Figure 4(c) presents the spin density as a function of n for the same values of U and H as Figs. 4(a) and 4(b). As a result of the ferromagnetic transition, both the charge and spin susceptibilities diverge at $n = n_c^{(+)}$, being decreasing functions of n for electronic densities $n > n_c$ close to n_c and both going through minima located between $n = n_c$ and $n = 1$. Due to the metal-insulator transition the compressibility diverges as $n \to 1$, while the spin susceptibility has in general a finite value at $n = 1$. On the other hand, we find in Sec. VI that the compressibility vanishes at $n = 1$, i.e., it is singular across the

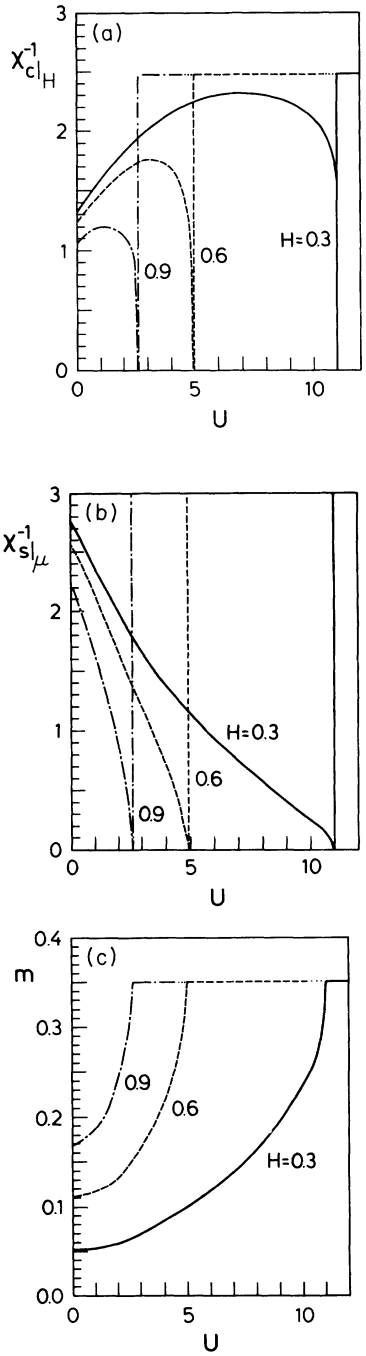

FIG. 3. (a) The inverse of the compressibility $\chi_c\,|_H$, (b) the inverse of the spin susceptibility $\chi_s\,|_\mu$, and (c) the spin density as a function of U at the electronic density $n = 0.7$ and values of the magnetic field $H = 0.3$ (solid line), $H = 0.6$ (dashed line), and $H = 0.9$ (dashed-dotted line). The discontinuities occur at $U = U_c$, where the system becomes a ferromagnetic Fermi liquid. Contrary to the compressibility curves, $\chi_s\,|_\mu$ vanishes in the Fermi-liquid ferromagnetic phase.

metal-insulator transition.

In order to study the effect of correlations on the density dependence of the susceptibilities $\chi_c|_H$ and $\chi_s|_\mu$, we compare in Figs. 5(a) and 5(b) the values at $U = 0$ and $U = 10$ of the inverse of these functions vs n for magnetic field $H = 0.1$. In Fig. 5(c) we plot the spin density vs the electronic density for the same values of magnetic field and U as Figs. 5(a) and 5(b). While the dependence of the compressibility on n is qualitatively different in the cases of the noninteracting and interacting systems, the

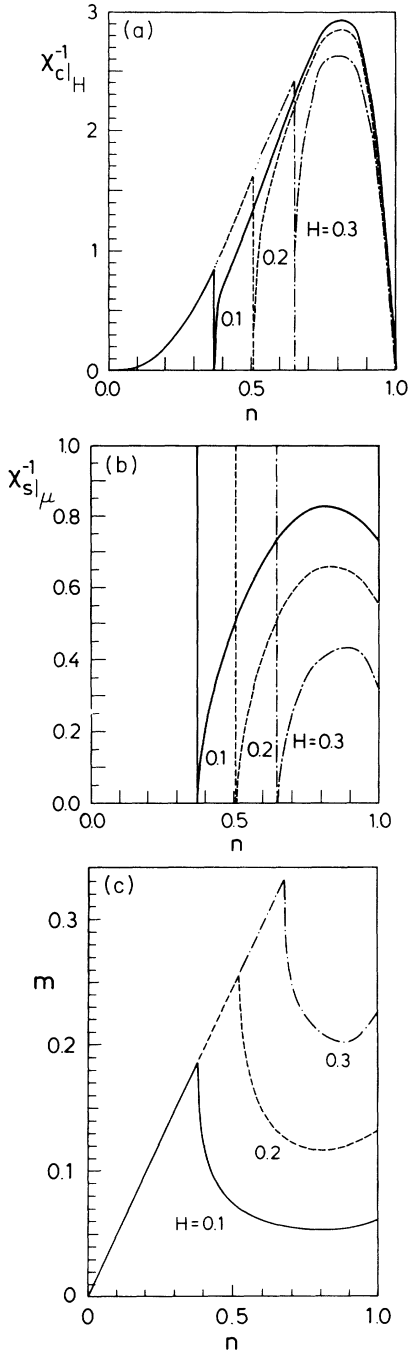

FIG. 4. (a) The inverse of the compressibility $\chi_c|_H$, (b) the inverse of the spin susceptibility $\chi_s|_\mu$, and (c) the spin density as a function of the electronic density n for $U = 10$ and values of the magnetic field $H = 0.1$ (solid line), $H = 0.2$ (dashed line), and $H = 0.3$ (dashed-dotted line). The discontinuities, which occur for $n = n_c$, correspond to the transition to the Fermi-liquid ferromagnetic phase. Due to the metal-insulator transition, the compressibility is also singular at $n = 1$, where $\chi_c|_H \rightarrow \infty$ for $n \rightarrow 1$ yet $\chi_c|_H = 0$ at $n = 1$.

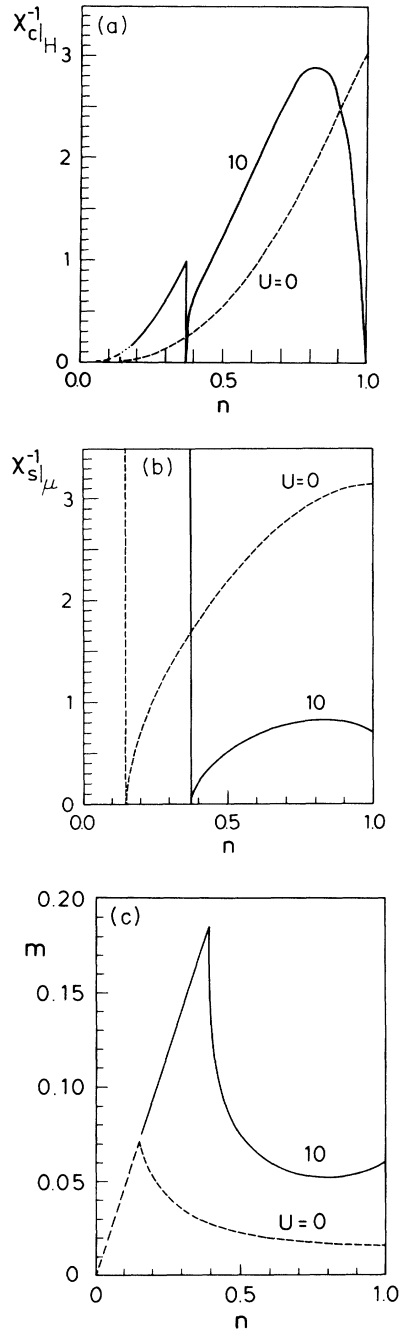

FIG. 5. (a) The inverse of the compressibility $\chi_c|_H$, (b) the inverse of the spin susceptibility $\chi_s|_\mu$, and (c) the spin density as a function of the electronic density n for $H = 0.1$ and values of the on-site repulsion $U = 0$ (dashed line) and $U = 10$ (solid line). Note the different behavior at $U = 0$ and $U = 10$ of $\chi_c|_H$ around $n = 1$ due to the Mott-Hubbard transition. Contrary to the spin susceptibility (b), in the presence of the magnetic field the on-site repulsion changes qualitatively the dependence of $\chi_c|_H$ on the electronic density.

on-site repulsion produces only a quantitative change in the susceptibility curves of Fig. 5(b). We emphasize the effect of the Mott-Hubbard transition in the type of the dependence of the compressibility for densities close to $n = 1$. The form of the compressibty at $U = 0$ and $U = 10$ is also different in the neighborhood of n_c.

In Figs. 6(a)–6(c) the inverse of the functions $\chi_c|_H$ and $\chi_s|_\mu$, and the spin density, respectively, are plotted vs the magnetic field for $U = 10$ and various densities. Both the charge and spin susceptibilities are for $H < H_c$ increasing functions of the magnetic field.

The dependence of the susceptibilities $\chi_c|_m$ and $\chi_s|_n$ on the different parameters is qualitatively similar to the dependence of the compressibility and spin susceptibilities represented in Figs. 3(a), 3(b), 4(a), 4(b), 5(a), 5(b), 6(a), and 6(b). The main difference is that while the latter diverge at the ferromagnetic transition on the side of the Landau-Luttinger phase, the former remain in general finite as $U \to U_c$, $n \to n_c$ $(n < n_c)$, or $H \to H_c$. Thus, it is interesting to study the susceptibilities $\chi_c|_m$ and $\chi_s|_n$ at $H = H_c^{(-)}$. In Figs. 7 and 8 the inverse of these two functions, respectively, is plotted as a function of the electronic density for various values of U and $H = H_c$. We note that as the value of H_c depends on n and U, the fixed quantity in Figs. 7 and 8 is not the magnetic field H, but the spin density m, which is such that $m \to n/2$. As in the case of the noninteracting system, when $m \to n/2$ both $\chi_c|_m$ and $\chi_s|_n$ diverge at $n = 0$ and $n = 1$. The effect of the electronic correlations is merely to increase the spin susceptibility and supress the compressibility.

Finally, in order to illustrate that the two kinds of compressibilities (84) and (85) and spin susceptibilities (86) and (87) have the same type of dependence on, for example, the magnetic field, we consider fixed values of U and n and plot the compressibilities and spin susceptibilities vs H. Figs. 9, 10, and 11 contain information about the field dependence of the four susceptibilities (95), (98), (102), and (104). While Figs. 9 and 10 represent $\chi_c|_m$ and $\chi_s|_n$, respectively, for $n = 0.7$ and various values of U, in Fig. 11 we plot the function (105) vs the magnetic field for the same values of n and U as in the former figures. This provides information about the functions $\chi_c|_H$ and $\chi_s|_\mu$, which have the same kind of dependence on H as the susceptibilities of Figs. 9 and 10, respectively, except that the former diverge as $H \to H_c$.

In order to better understand the behavior of the susceptibilities plotted in the figures discussed above, it is convenient to study their expressions in the limits considered in Sec. III.

In the case $U = 0$, we obtain

$$\chi_c|_H = \frac{1}{\pi n^2}\left(\frac{1}{v_c}+\frac{1}{v_s}\right), \quad \chi_c|_m = \frac{4}{\pi n^2}\frac{1}{v_c+v_s}, \tag{110}$$

$$\chi_s|_\mu = \frac{\mu_0^2}{\pi}\left(\frac{1}{v_c}+\frac{1}{v_s}\right), \quad \chi_s|_n = \frac{4\mu_0^2}{\pi}\frac{1}{v_c+v_s}. \tag{111}$$

These expressions reproduce the noninteracting values correctly.

The $H = 0$ compressibility expression has previously been obtained[22] and the zero-field magnetic susceptibility was studied numerically by Shiba.[23] Our general ex-

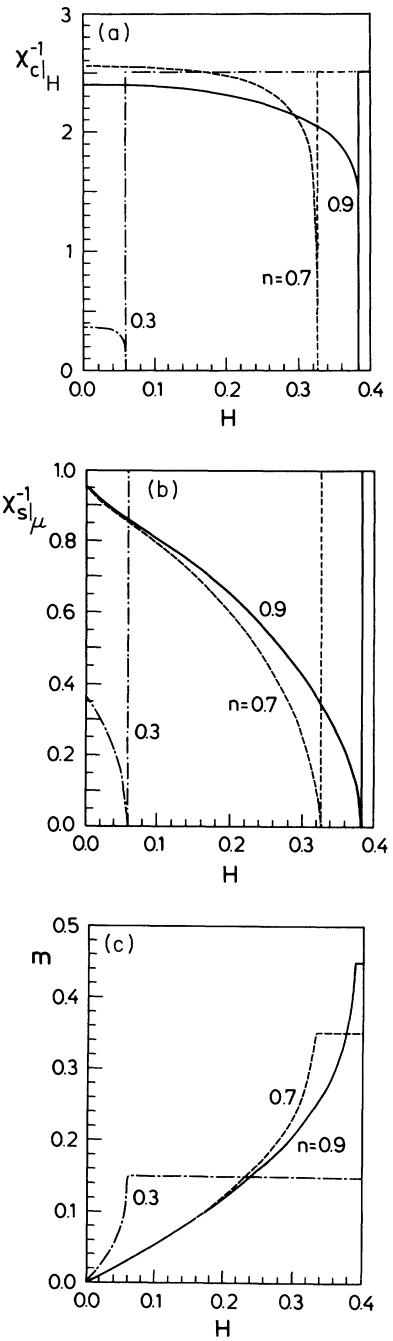

FIG. 6. (a) The inverse of the compressibility $\chi_c|_H$, (b) the inverse of the spin susceptibility $\chi_s|_\mu$, and (c) the spin density as a function of H for $U = 10$ and values of the electronic density $n = 0.3$ (dashed-dotted line), $n = 0.7$ (dashed line), and $n = 0.9$ (solid line). The discontinuities occur at $H = H_c$, where the system becomes a ferromagnetic Fermi liquid. For $H < H_c$ the charge and spin susceptibilities are increasing functions of the magnetic field.

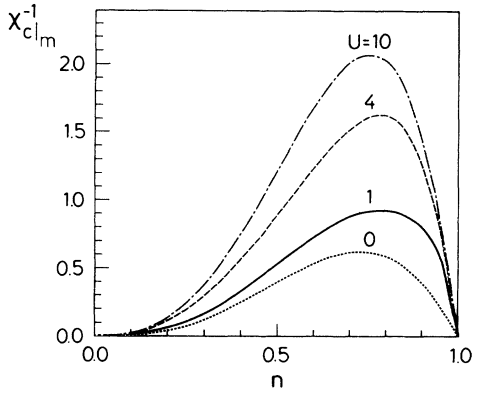

FIG. 7. The inverse of the compressibility $\chi_c|_m$ as a function of the electronic density n for $H = H_c^{(-)}$ $(m \to n/2)$ and various values of U.

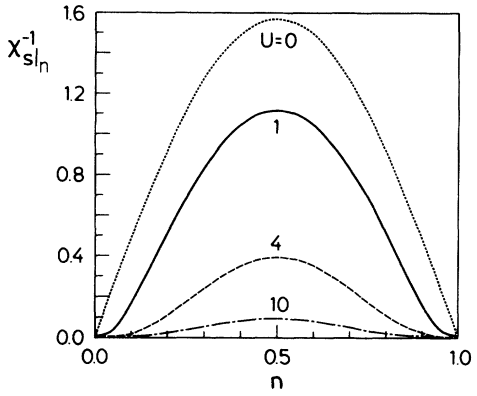

FIG. 8. The inverse of the spin susceptibility $\chi_s|_n$ as a function of n for $H = H_c^{(-)}$ $(m \to n/2)$ and the same values of U as Fig. 7.

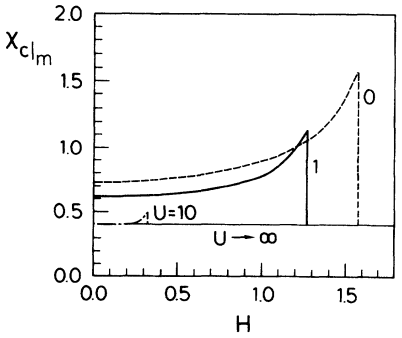

FIG. 9. The compressibility $\chi_c|_m$ as a function of H for $n = 0.7$ and various values of U. The $U = 10$ line is at $H = 0$ slightly under the $H > H_c$ line.

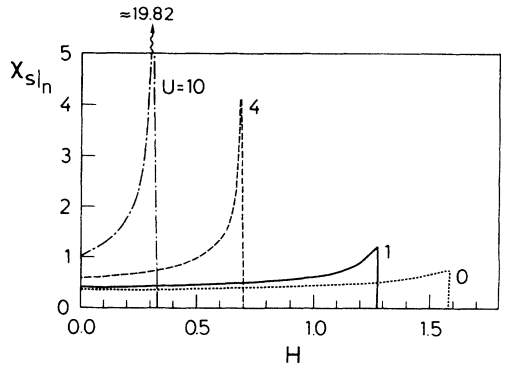

FIG. 10. The spin susceptibility $\chi_s|_n$ as a function of H for $n = 0.7$ and various values of U.

pressions fully agree with these results in the limit of zero magnetic field, where the susceptibilities read

$$\chi_c|_H = \chi_c|_m = \frac{\xi_0^2}{\pi n^2 v_c}, \quad \chi_s|_\mu = \chi_s|_n = \frac{2\mu_0^2}{\pi v_s}. \tag{112}$$

All the functions (84)–(87) are singular at the ferromagnetic transition. For $H = H_c^{(-)}$ we obtain

$$\chi_c|_H = \infty, \quad \chi_c|_m = \frac{1}{\pi n^2 v_c (1-\eta_0/2)^2}, \tag{113}$$

$$\chi_s|_\mu = \infty, \quad \chi_s|_n = \frac{4\mu_0^2}{\pi v_c \eta_0^2}.$$

The results for $H > H_c$ are presented in Sec. VI.

Finally, for $U \gg t$ and $H \ll H_c$ there arises logarithmic magnetic-field dependence in the susceptibilities, which follows from the singular nature of the $H = 0$ point—the change of symmetry induced by the magnetic field alters the "spinon" nature of the zero-field spin excitations.[6] In this regime we find the following results:

$$\chi_c|_H = \frac{1}{\pi n^2 v_c}, \quad \chi_c|_m = \frac{1}{\pi n^2} \frac{2\beta_0^2}{v_c 2\beta_0^2 + v_s(1-\alpha_0)^2}, \tag{114}$$

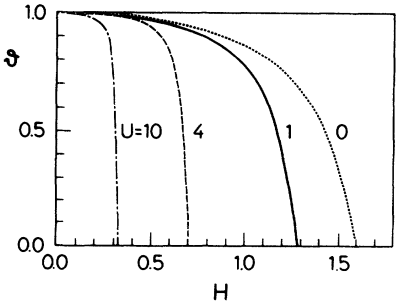

FIG. 11. The function ϑ as a function of H for $n = 0.7$ and the same values of U as Fig. 10.

$$\chi_s|_\mu = \frac{\mu_0^2}{\pi}\left(\frac{(1-\alpha_0)^2}{v_c}+\frac{2\beta_0^2}{v_s}\right), \quad \chi_s|_n = \frac{2\mu_0^2\beta_0^2}{\pi v_s}. \tag{115}$$

The parameters and functions occurring in Eqs. (110)–(115) were defined in Sec. III.

In this section we have shown that the compressibilities and magnetic susceptibilities of the Hubbard chain can be evaluated as in a Fermi liquid, their expressions being determined by the symmetric Landau parameters. In the presence of a magnetic field the charge compressibilities at constant magnetic field and at constant magnetization are unequal. The same holds true for the spin magnetic susceptibilities at constant chemical potential or constant number of electrons.

The main effect of the on-site correlations for magnetic fields $0 < H < H_c$ is to increase the spin magnetic susceptibilities and to supress the compressibilities. We stress that our general exact expressions agree with all previously known limiting values.

V. CONNECTION TO CONFORMAL FIELD THEORY

The Landau energy functional defined by Eqs. (39), (41), and (52) can describe all the low-energy physics of model (1), provided that in each particular situation adequate deviations $\delta_c(q)$ and $\delta_\downarrow(p)$ are chosen. It provides the universal and most general framework for the description of the low-energy physics of the present one-dimensional Landau liquid. For example, the choice of suitable deviations allows the evaluation of the energy spectra for the elementary single-pair charge and spin excitations.[6] These can be described in terms of a single particle-hole process in the charge or spin-pseudoparticle band (42). Also the low-temperature thermodynamics can be described in the framework of this functional. The corresponding deviations involve Fermi-Dirac distributions for the charge and spin pseudoparticles.[6] On the other hand, we have shown in the preceding section that the particular choice of deviations introduced in Eqs. (90) allows the evaluation of the compressibility and spin magnetic susceptibility at arbitrary magnetic fields. We found that the pseudoparticle forward-scattering interactions fully regulate these thermodynamic quantities.

In this section we show that the excitation energy spectrum of conformal field theory,[9,10] which determines the critical exponents of the model, may also be obtained by inserting suitable deviations in the universal energy functional of the Landau-liquid theory. Moreover, we show that these exponents are also fully regulated by the pseudoparticle forward scattering interactions. The present results are a generalization of those introduced in Ref. 3, which did not consider changes δN and $\delta N_\downarrow$ in the number of electrons and down spins, respectively.

With starting point in the ground-state distributions (5), let us consider small changes δN and $\delta N_\downarrow$ in the number of electrons and down-spin electrons, respectively, associated with small changes in the pseudo-Fermi momenta as follows:

$$\delta N = \frac{N_a}{\pi}\delta 2k_F, \quad \delta N_\downarrow = \frac{N_a}{\pi}\delta k_{F\downarrow}. \tag{116}$$

In addition, let us also consider particle-hole processes in the pseudoparticle charge and spin bands of the following two kinds:

(i) From pseudomomenta close to $-2k_F(-k_{F\downarrow})$ to pseudomomenta in the neighborhood of $2k_F(k_{F\downarrow})$, or vice versa. This originates small changes q_c and q_s in the pseudo-Fermi momenta $2k_F$ and $k_{F\downarrow}$, respectively. The numbers of charge and spin pseudoparticles transferred, D_c and D_s, respectively, are given by

$$D_c = \frac{N_a}{2\pi}q_c, \quad D_s = \frac{N_a}{2\pi}q_s. \tag{117}$$

(ii) The second kind of particle-hole process occurs around the same pseudo-Fermi point $\pm 2k_F$ or $\pm k_{F\downarrow}$. Let $q_p^\pm$ and $q_h^\pm$ denote the pseudomomenta of the "particles" and "holes" around $\pm 2k_F$, respectively, and $p_p^\pm$ and $p_h^\pm$ denote the pseudomomenta of the particles and holes around $\pm k_{F\downarrow}$, respectively. The number of particle-hole processes around $\pm 2k_F$ and $\pm k_{F\downarrow}$, $N_c^\pm$ and $N_s^\pm$, respectively, are given by

$$N_c^\pm = \pm\frac{N_a}{2\pi}\left(\sum_p q_p^\pm - \sum_h q_h^\pm\right),$$
$$N_s^\pm = \pm\frac{N_a}{2\pi}\left(\sum_p p_p^\pm - \sum_h p_h^\pm\right), \tag{118}$$

where the indices p and h refer to particle and hole summations. In addition to referring exclusively to pseudoparticles of momenta in the close neighborhood of the pseudo-Fermi points, all the processes considered above are assumed to involve a small finite density of pseudoparticles. This implies that $|\delta 2k_F|$, $|q_c|$, $|2k_F \mp q_p^\pm|$, $|2k_F \mp q_h^\pm| \ll 2k_F$ and $|\delta k_{F\downarrow}|\,|p_s|$, $|k_{F\downarrow} \mp p_p^\pm|$, $|k_{F\downarrow} \mp p_h^\pm| \ll k_{F\downarrow}$. The appropriate distributions to describe these processes are given by

$$M_c(q) = \Theta(2k_F + \delta 2k_F + (\mathrm{sgn}\ q)q_c - |q|) + \frac{2\pi}{N_a}\left(\sum_p [\delta(q-q_p^+)+\delta(q-q_p^-)] - \sum_h [\delta(q-q_h^+)+\delta(q-q_h^-)]\right), \tag{119}$$

$$N_{\downarrow}(p) = \Theta(k_{F\downarrow} + \delta k_{F\downarrow} + (\mathrm{sgn}\, p)p_s - |p|) + \frac{2\pi}{N_a}\left(\sum_p [\delta(p - p_p^+) + \delta(p - p_p^-)] - \sum_h [\delta(p - p_h^+) + \delta(p - p_h^-)]\right) . \quad (120)$$

Expanding around the ground-state distributions (5), leads to the following deviations:

$$\delta_c(q) = \delta(2k_F - |q|)[\delta 2k_F + (\mathrm{sgn}\, q)q_c] + \frac{2\pi}{N_a}\left(\sum_p [\delta(q - q_p^+) + \delta(q - q_p^-)] - \sum_h [\delta(q - q_h^+) + \delta(q - q_h^-)]\right) , \quad (121)$$

$$\delta_{\downarrow}(p) = \delta(k_{F\downarrow} - |p|)[\delta k_{F\downarrow} + (\mathrm{sgn}\, p)p_s] + \frac{2\pi}{N_a}\left(\sum_p [\delta(p - p_p^+) + \delta(p - p_p^-)] - \sum_h [\delta(p - p_h^+) + \delta(p - p_h^-)]\right) . \quad (122)$$

It is a simple exercise to show that inserting the deviations (121) and (122) in the energy Landau functional reproduces the excitation energy spectrum obtained in Refs. 9 and 10. To evaluate the integrals of the first-order term (41), we expand $\epsilon_c(q)$ and $\epsilon_s(p)$ around the pseudo-Fermi points. To the two leading orders in the density of pseudoparticles, only the terms E_1 and E_2 of the energy functional (39) contribute. This is why our second-order truncated Landau expansion leads to the exact low-energy physics in the scale-invariant regime. After some straightforward algebra, we obtain the excitation spectrum $E(P)$ associated with the deviations (121) and (122). It is given by

$$E = E_0 + \frac{2\pi}{N_a}[(v_c + F_{cc}^1)(D_c)^2 + (v_s + F_{ss}^1)(D_s)^2 + 2F_{cs}^1 D_c D_s] + \frac{2\pi}{N_a}\left[(v_c + F_{cc}^0)\left(\frac{\delta N}{2}\right)^2 + (v_s + F_{ss}^0)\left(\frac{\delta N_{\downarrow}}{2}\right)^2 + 2F_{cs}^0\left(\frac{\delta N}{2}\right)\left(\frac{\delta N_{\downarrow}}{2}\right)\right] + \frac{2\pi}{N_a}[v_c(N_c^+ + N_c^-) + v_s(N_s^+ + N_s^-)] , \quad (123)$$

which, following from the relations (63)–(65), may be rewritten as

$$E = E_0 + \frac{2\pi}{N_a}[v_c(\Delta_c^+ + \Delta_c^-) + v_s(\Delta_s^+ + \Delta_s^-)] , \quad (124)$$

where

$$\Delta_c^{\pm} = \frac{1}{2}\left\{ \xi_{cc}^1 D_c + \xi_{cs}^1 D_s \pm \left[\xi_{cc}^0\left(\frac{\delta N}{2}\right) + \xi_{cs}^0\left(\frac{\delta N_{\downarrow}}{2}\right)\right]\right\}^2 + N_c^{\pm} , \quad (125)$$

$$\Delta_s^{\pm} = \frac{1}{2}\left\{ \xi_{sc}^1 D_c + \xi_{ss}^1 D_s \pm \left[\xi_{sc}^0\left(\frac{\delta N}{2}\right) + \xi_{ss}^0\left(\frac{\delta N_{\downarrow}}{2}\right)\right]\right\}^2 + N_s^{\pm} . \quad (126)$$

On the other hand, the excitation momentum reads

$$P = \frac{2\pi}{N_a}(\Delta_c^+ - \Delta_c^- + \Delta_s^+ - \Delta_s^-) + 2D_c 2k_F + 2D_s k_{F\downarrow} . \quad (127)$$

Finally, the use of the relation (68) allows us to express the matrix elements $\xi_0' s$ appearing in the rhs of Eqs. (125) and (126) in terms of the elements $\xi_1' s$ of the dressed charge matrix. The obtained expressions are precisely the finite-size energy correction, conformal field dimensions $\Delta_c^{\pm}$, $\Delta_s^{\pm}$, and momentum obtained in Refs. 9 and 10. The leading term in the asymptotic expansion of the correlation functions decays with critical exponents obtained from (125) and (126) by minimizing with respect to D_c, D_s [i.e., by minimizing with respect to the deviations $\delta_c(q)$ (121) and $\delta_{\downarrow}(p)$ (122)].

It follows from the results obtained in this section that all the critical exponents of the model are fully determined by the forward scattering interactions of the Landau-liquid pseudoparticles. These critical exponents are studied in detail in Refs. 10.

VI. THE HALF-FILLED BAND CASE AND THE FERROMAGNETIC PHASE

Both at half filling and in the ferromagnetic phase the low-lying excitations are restricted to sub-Hilbert spaces where the Landau theory becomes a one-fluid theory with only one kind of pseudoparticle. On the one hand, at half-filling, where the chemical potential μ is such that $\mu < \Delta_c$, and Δ_c is the Hubbard gap,[2,6,24] the only allowed deviations are $\delta_{\downarrow}(p)$. On the other hand, in the ferromagnetic phase, where $H > H_c$, the charge deviations $\delta_c(q)$ are the only allowed deviations.

At half filling the many-body system (1) is a Landau liquid only with spin pseudoparticles. The one-fluid theory for half filling includes only the phase shift $\Phi_{ss}(p, p')$. The corresponding universal phase shift (30) obeys the integral equation

$$\bar{\Phi}_{ss}(y,y') = \frac{1}{\pi}\tan^{-1}\left(\frac{y-y'}{2}\right) + \int_{-y_0}^{y_0} dy'' G(y,y'')\bar{\Phi}_{ss}(y'',y') , \quad (128)$$

where the kernel $G(y,y'')$ reads

$$G(y,y') = -\frac{1}{2\pi}\left(\frac{1}{1+[(y-y')/2]^2}\right) . \quad (129)$$

Equations (128) and (129) are the half-filling version of Eqs. (35) and (36), which become much simpler in the limit $n \to 1$. The f function, which regulates the forward scattering of the spin pseudoparticles, reads

$$f_{ss}(p,p') = 2\pi v_s(p)\Phi_{ss}(p,p') + 2\pi v_s(p')\Phi_{ss}(p',p) + 2\pi v_s \sum_{j=\pm 1}\Phi_{ss}(jk_{F\downarrow},p)\Phi_{ss}(jk_{F\downarrow},p') . \quad (130)$$

The half-filling Landau-liquid theory has only two Landau parameters, which correspond to the symmetric and antisymmetric combinations of the f function (130) at the pseudo-Fermi points. They can be written as

$$F^i_{ss} = \frac{1}{2\pi}\sum_{j=\pm 1}(j)^i f_{ss}(k_{F\downarrow},jk_{F\downarrow}) = -v_s + v_s[\xi^i_{ss}]^2 , \quad i = 0,1 , \quad (131)$$

where

$$\xi^i_{ss} = 1+\Phi_{ss}(k_{F\downarrow},k_{F\downarrow})+(-1)^i\Phi_{ss}(k_{F\downarrow},-k_{F\downarrow}) , \quad i = 0,1 . \quad (132)$$

As in the case $n < 1$ for the dressed charge matrix [see Eq. (67) with $i = 1$], at half filling ξ^1_{ss} is nothing but the function also obtained in conformal field theory, which controls the critical exponents.[10]

In the limit $H \to 0$ all the equations can be solved in closed form. Following the results of Appendix A, as $n \to 1$ the universal phase shift (128) and the function $S_0(p)$ are defined by the following equations:

$$\bar{\Phi}_{ss}(y,y') = B(S_0(p) - S_0(p')) , \quad p = \int_0^\infty d\omega \frac{\sin[\omega S_0(p)]}{\omega\cosh\omega} J_0(u^{-1}\omega) , \quad (133)$$

where

$$B(x) = \frac{1}{\pi}\int_0^\infty d\omega \frac{\sin(\omega x)}{\omega(1+e^{2\omega})} , \quad (134)$$

and in the second equation of (133), which defines the inverse function of $S_0(p)$, J_0 is the Bessel function of zero order.

Since charge deviations are not allowed, at half filling the compressibilities vanish. The use of the method applied in Sec. IV for $n < 1$, leads for $H < H_c$ to the following expression for the spin susceptibilities:

$$\chi_s|_{n=1} = \chi_s|_{\mu} = \frac{\mu_0^2}{\pi L_0} , \quad L_0 = \frac{1}{4}(v_s + F^0_{ss}) . \quad (135)$$

On the other hand, at $n = 1$ and $H > H_c$ both charge and spin deviations are not allowed and all the susceptibilities vanish.

In the ferromagnetic phase the double-occupancy restrictions imposed by the on-site repulsion do not play any role because the ground state belongs to a sub-Hilbert space where double occupancy is excluded. Contrary to the Landau-Luttinger phase where the single-particle spectral function is fully incoherent, the ferromagnetic phase of the model (1) is again a Fermi liquid, the spectral function of the spin-up electrons showing a δ peak. At fixed values of n and $H < [2t\sin^2(\pi n/2)]/\mu_0 = H^0_c$, the transition to the ferromagnetic phase occurs at a value $U = U_c = U_c(n,H)$ at which $H = H_c$ (the expression for the critical field H_c is given in Ref. 6). U_c changes from $U_c = 0$ as $H \to H^0_c$, to $U_c = \infty$ as $H \to 0$. Since in the ferromagnetic phase only charge deviations are allowed, the relevant phase shift is $\Phi_{cc}(q,q')$. This vanishes both at $H^{(-)}_c$ and $H^{(+)}_c$ (or $U^{(-)}_c$ and $U^{(+)}_c$) because the cutoff $y_0 = B/u$ of the rhs of Eq. (32) vanishes smoothly when $k_{F\downarrow} \to 0$. Then, contrary to the singular Fermi-liquid–Landau-Luttinger-liquid transition, which occurs at $U = 0$, the transition to the Fermi-liquid phase at $U = U_c$ is smooth.

The ferromagnetic phase is an one-fluid Fermi liquid of noninteracting charge pseudoparticles (up-spin electrons), such that

$$\epsilon_c(q) = -2t[\cos(q) - \cos(2k_F)] , \quad f_{cc}(q,q') = 0 . \quad (136)$$

Since spin deviations are not allowed, the spin susceptibilities vanish. When $n < 1$ the charge susceptibilities are by given

$$\chi_c|_H = \chi_c|_m = \frac{1}{\pi n^2 v_c} , \quad v_c = 2t\sin(\pi n) , \quad (137)$$

which, as expected, equals the result for the ferromagnetic phase of the noninteracting system.

The Figs. 3(a), 3(b), 4(a), 4(b), 5(a), 5(b), 6(a), 6(b), 9, and 10 show that for U finite all the susceptibilities are singular across the ferromagnetic transition.

VII. CONCLUDING REMARKS

In this paper we have derived the forward-scattering f functions for the one-dimensional generalized Landau-liquid theory in the case of the Hubbard chain. As in a Fermi liquid,[4,5,16–20] the truncated second-order Landau-energy functional regulates the low-energy physics. The study of the different excitations requires the use of adequate charge and spin deviations in the energy functional. The truncated expansion leads to the exact physics in the scale-invariant regime.

We have introduced the two types of deviations that allow the exact evaluation of the static susceptibilities at arbitrary magnetic fields and of the conformal field dimensions, which regulate the critical exponents associated with the asymptotics of correlation functions, respectively. Our results show that the equilibrium prop-

erties of a one-dimensional Landau liquid can be studied as in a Fermi liquid, the thermodynamic quantities being determined by the Landau parameters and pseudoparticle velocities. They also confirm the Landau-liquid nature of the Hubbard chain, and thus clarify the deep nature of the one-dimensional fermionic Luttinger liquids at finite magnetic fields. Furthermore, in the case of the one-dimensional Hubbard model the generalized Landau theory also clarifies the notion of charge and spin decoupling, which is an inherent property of this many-body system.

In Ref. 21 we show that the pseudoparticles are the transport carriers of the many-body system. Therefore, their forward-scattering interactions control all the low-energy physics. As in the case of the one-dimensional Landau liquids and of Fermi-liquid theory, in the generalized Landau liquids, which we expect to exist at $D=2$ forward scattering plays a relevant role. There is evidence that forward scattering plays such a dominant role in the two-dimensional Hubbard model.[25] However, which Landau-liquid theory is most appropriate to describe the physics of this model still remains an open question.[26]

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APPENDIX A: THE ZERO-MAGNETIC-FIELD CASE

As $H \rightarrow 0$ the cutoff B introduced in Eq. (31) tends to $B \rightarrow \infty$. That allows the introduction of an alternative representation, which is obtained from the general representation of Sec. II by Fourier transform.

For example, Eqs. (43)–(45), which define the pseudoparticle bare bands, are for $H \rightarrow 0$ equivalent to the equations

$$\epsilon_c^0(q) = -2t\cos K_0(q) - 4t\int_0^\infty d\omega \frac{\cos\left(\omega\frac{\sin K_0(q)}{u}\right)}{\omega(1+e^{2\omega})}\Upsilon_1(\omega)\,, \tag{A1}$$

$$\epsilon_s^0(p) = -2t\int_0^\infty d\omega \frac{\cos[\omega S_0(p)]}{\omega\cosh\omega}\Upsilon_1(\omega)\,, \tag{A2}$$

$$q = K_0(q) + 2\int_0^\infty d\omega \frac{\sin\left(\omega\frac{\sin K_0(q)}{u}\right)}{\omega(1+e^{2\omega})}\Upsilon_0(\omega)\,, \tag{A3}$$

$$p = \int_0^\infty d\omega \frac{\sin[\omega S_0(p)]}{\omega\cosh\omega}\Upsilon_0(\omega)\,, \tag{A4}$$

where the functions $\Upsilon_0(\omega)$ and $\Upsilon_1(\omega)$ are solutions of the following integral equations

$$\Upsilon_0(\omega) = \frac{1}{2\pi}\int_{-Q}^{Q} dk\cos\left(\omega\frac{\sin k}{u}\right) + \int_{-\infty}^{\infty} d\omega'\Gamma(\omega,\omega')\Upsilon_0(\omega')\,, \tag{A5}$$

$$\Upsilon_1(\omega) = \frac{1}{2\pi}\int_{-Q}^{Q} dk(\sin k)\sin\left(\omega\frac{\sin k}{u}\right) + \int_{-\infty}^{\infty} d\omega'\Gamma(\omega,\omega')\Upsilon_1(\omega')\,, \tag{A6}$$

respectively, and the kernel $\Gamma(\omega,\omega')$ is given by

$$\Gamma(\omega,\omega') = \frac{\sin\left[\left(\frac{\sin Q}{u}\right)(\omega-\omega')\right]}{\pi(\omega-\omega')}\frac{1}{1+e^{2|\omega'|}}\,. \tag{A7}$$

This kernel was already obtained in Ref. 7. Equations (A5) and (A6) can be solved in close form for $n=1$. The result is

$$\Upsilon_0(\omega) = J_0(\omega/u)\,,\quad \Upsilon_1(\omega) = J_1(\omega/u)\,, \tag{A8}$$

where J_0 and J_1 are Bessel functions. This is consistent with the half-filling expressions obtained by Lieb and Wu.[2]

On the other hand, Eqs. (32)–(35), which define the phase shifts, are equivalent to

$$\bar{\Phi}_{cc}(x,x') = -B(x-x') + \int_{-x_0}^{x_0} dx'' A(x-x'')\bar{\Phi}_{cc}(x'',x')\,, \tag{A9}$$

$$\bar{\Phi}_{cs}(x,y') = -\frac{1}{2\pi}tan^{-1}\left[\sinh\left(\frac{\pi}{2}(x-y')\right)\right] + \int_{-x_0}^{x_0} dx'' A(x-x'')\bar{\Phi}_{cs}(x'',y')\,, \tag{A10}$$

$$\bar{\Phi}_{sc}(y,x') = -\frac{1}{2\pi}\tan^{-1}\left[\sinh\left(\frac{\pi}{2}(y-x')\right)\right] + \frac{1}{4}\int_{-x_0}^{x_0} dx'' \frac{\bar{\Phi}_{cc}(x'',x')}{\cosh\left[\frac{\pi}{2}(y-x'')\right]}\,, \tag{A11}$$

$$\bar{\Phi}_{ss}(y,y') = B(y-y') + \frac{1}{4}\int_{-x_0}^{x_0} dx'' \frac{\bar{\Phi}_{cs}(x'',y')}{\cosh[\frac{\pi}{2}(y-x'')]}\,, \tag{A12}$$

where the functions $A(x)$ and $B(x)$ are given by Eqs. (75) and (134), respectively.

APPENDIX B: THE DRESSED CHARGE MATRIX

In order to show that the matrix $\mathbf{Z}^1$ (66) is the dressed charge matrix of conformal field theory, it is useful to

express it as

$$\mathbf{Z}^1 = \begin{bmatrix} \xi^1_{cc}(x_0) & \xi^1_{cs}(x_0) \\ \xi^1_{sc}(y_0) & \xi^1_{ss}(y_0) \end{bmatrix} , \tag{B1}$$

where, following Eqs. (67), (62), and (23)–(30), the functions $\xi^1_{cc}(x)$, $\xi^1_{cs}(x)$, $\xi^1_{sc}(y)$, and $\xi^1_{ss}(y)$ are given by

$$\xi^1_{cc}(x) = 1 + \bar{\Phi}_{cc}(x, x_0) - \bar{\Phi}_{cc}(x, -x_0) , \tag{B2}$$

$$\xi^1_{cs}(x) = \bar{\Phi}_{cs}(x, y_0) - \bar{\Phi}_{cs}(x, -y_0) , \tag{B3}$$

$$\xi^1_{sc}(y) = \bar{\Phi}_{sc}(y, x_0) - \bar{\Phi}_{sc}(y, -x_0) , \tag{B4}$$

$$\xi^1_{ss}(y) = 1 + \bar{\Phi}_{ss}(y, y_0) - \bar{\Phi}_{ss}(y, -y_0) . \tag{B5}$$

On the other hand, based on Eqs. (32)–(35), it is straightforward to show that the functions (B2)–(B5) obey the following integral equations:

$$\xi^1_{cc}(x) = 1 + \frac{1}{\pi}\int_{-y_0}^{y_0} dy'' \frac{\xi^1_{sc}(y'')}{1 + (x - y'')^2} , \tag{B6}$$

$$\xi^1_{cs}(x) = \frac{1}{\pi}\int_{-y_0}^{y_0} dy'' \frac{\xi^1_{ss}(y'')}{1 + (x - y'')^2} , \tag{B7}$$

$$\xi^1_{sc}(y) = t(y) + \frac{1}{\pi}\int_{-y_0}^{y_0} dy'' G(y, y'')\xi^1_{sc}(y'') , \tag{B8}$$

$$\xi^1_{ss}(y) = 1 + \frac{1}{\pi}\int_{-y_0}^{y_0} dy'' G(y, y'')\xi^1_{ss}(y'') , \tag{B9}$$

where in the rhs of Eq. (B8) $t(y)$ is given by Eq. (37). Finally, we emphasize that the kernel (36) may be rewritten in integral representation as

$$G(y, y') = -\frac{1}{2\pi}\left(\frac{1}{1 + [(y - y')/2]^2} - \frac{2}{\pi}\int_{-x_0}^{x_0} dx'' \frac{1}{[1 + (y - x'')^2][1 + (y' - x'')^2]}\right) . \tag{B10}$$

It follows from Eqs. (B6)–(B10) that $\mathbf{Z}^1$ is the dressed charge matrix of Refs. 9 and 10. We remember that we use here the same definition as Ref. 9, which is the transposition of that of Ref. 10.

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