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Frustration, scaling, and local gauge invariance

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A variety of two- and three-dimensional random frustrated systems with continuous and discrete symmetries are studied within the Migdal-Kadanoff renormalization-group scheme. The continuous-symmetry XY models are approximated by discretized clock models with a large number of clock states. In agreement with earlier studies of the random gauge XY model using a $T=0$ scaling approach, a nonzero transition temperature is observed in three-dimensional XY models with O(2) local gauge invariance. Our analysis points to the possible importance of local gauge invariance in determining the lower critical dimensionality of frustrated systems.

I. INTRODUCTION

Recently it has been conjectured[1] that the lower critical dimensionality (LCD) of spin glass models depends not only on the spatial dimension (D) and spin dimensionality (n) but also on whether the spin-spin interactions have local gauge invariance. The conjecture was based on detailed studies of two 3D models with $n=2$ (XY spins) described by the Hamiltonian

$$H=-\sum_{\langle ij\rangle}^{N} J_{ij}\cos(\phi_i-\phi_j-A_{ij}) , \quad (1)$$

where $\langle ij\rangle$ denotes a summation over neighboring grains and J_{ij} is the exchange coupling. In one of the models $J_{ij}=J$ and A_{ij} was chosen randomly to be 0 or π, so this model is identical to the bimodal XY spin glass. In the second model, J_{ij} is still chosen to be J but the A_{ij}'s are chosen randomly from a uniform interval between 0 and 2π. We shall henceforth refer to this latter model as the random gauge XY model. This model has O(2) local gauge invariance, which means that the ground-state energy of the system will not change if the spins are rotated by arbitrary amounts as long as the random gauge factors are also adjusted so that the probability distribution of the adjusted factors remains the same as the original distribution. Numerical studies of the zero temperature scaling behavior of the generalized stiffness[2-5] in the two models in three dimensions showed qualitatively different behaviors: the bimodal XY spin glass had no nonzero temperature spin glass phase (LCD > 3), whereas the random gauge model did (LCD < 3). These results and therefore the relevance of the local gauge invariance are still open to debate: Even though Monte Carlo simulations by Huse and Seung[6] were in accord with our results, an ε expansion[7] to order ε^3 and a zero temperature Migdal-Kadanoff recursion scheme[8] have suggested that the random-gauge model has an LCD greater than 3.

In this paper we study a variety of models some of which have local gauge invariance and others that do not. The models are studied within the framework of the Migdal-Kadanoff[9] renormalization group (MKRG) scheme. We use a discretization scheme for the XY spins; the scheme is self-consistent and allows us to extend our studies to arbitrary temperatures. Within the approximations we use, our results show very clearly the importance of local gauge invariance in determining the LCD of frustrated systems thus providing further support for our conjecture. We also use our numerical scheme to deduce values for the stiffness exponent y in $D=2$ and $D=3$ and provide estimates for the LCD of the random gauge XY model.

The random gauge XY model has been invoked[6] to explain the unusual magnetic properties of the mixed state of the oxide superconductors.[10] The basic idea is that defects, which are ubiquitous even in single crystal samples of the oxide superconductors, cause the magnetic flux lines in the mixed state to be frozen into a random array called the vortex glass.[11] While the phase ϕ of the complex order parameter changes discontinuously across structural defects, the Josephson effect promotes phase coherence between the domains in the absence of a magnetic field. In the presence of a magnetic field, the line integral of the vector potential **A** between the domains also contributes to the phase difference between them. This may be modeled by a random array of points representing domains in the sample with the phase factor A_{ij} determined by $2\pi/\Phi_0 \int_i^j \mathbf{A}\cdot dl$ (Φ_0 is the flux quantum for a Cooper pair). We expect that in the large field limit this model is in the same universality class as the random gauge model. It has been suggested[12] that the zero temperature scaling exponent y in the $D=2$ random gauge model may be estimated experimentally by measuring the resistivity of disordered thin film superconductors in an applied magnetic field. Specifically, the resistivity is predicted to be of the form $\exp[(-T_0/T)^p]$ with $p=1/(1+|y|)$ at low T with a crossover to a classical regime at higher T of the form $T^{1+1/|y|}$. The value of y was estimated to be -0.5 in Ref. 12.

II. THE MIGDAL-KADANOFF RENORMALIZATION GROUP

A. The harmonic approximation

The MKRG scheme has been used with varying degrees of success in the study of frustrated systems such as spin glasses.[13–17] In spite of the uncontrolled nature of the approximation in dealing with real Euclidean systems, the scheme is nevertheless exact on a hierarchical lattice.[18] Indeed, the MKRG scheme has been successful in predicting correctly the LCD of the familiar random bond Ising,[16] XY and Heisenberg[15] spin glasses. The MKRG scheme provides a framework for determining the effective coupling on increasingly larger length scales. The hierarchical lattice for which the scheme is exact is shown in Fig. 1. For a scale factor b, $(b+1)$ spins in series are considered and a decimation carried out to eliminate all the internal degrees of freedom. To obtain the rescaled coupling b^{D-1} such decimated bonds are added in parallel.

The MKRG scheme for hierarchical lattices is exact when spin degrees of freedom are discrete. In this case the form of the Hamiltonian is preserved under the transformation. The study of systems with continuous symmetry, such as those given by Eq. (1), poses a problem since the renormalized Hamiltonian no longer has the simple cosine form. Traditionally, this problem has been circumvented in the study of scaling behavior of XY and Heisenberg spin glasses by making a $T=0$ approximation. This approximation,[15] which is uncontrolled, proceeds as follows: The Hamiltonian is expanded about the equilibrium angle between neighboring spins up to quadratic terms. The Gaussian form of the resulting partition function is taken to be valid over the entire range of angles. The virtue of such an approximation is that the quadratic form is preserved in a decimation process thereby facilitating the recursion scheme. We note again that this scheme may work only for $T=0$ and even then it is somewhat ad hoc.

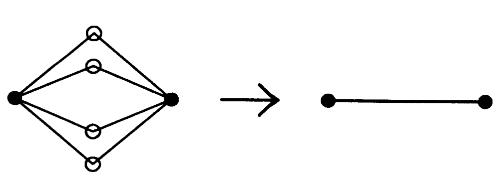

FIG. 1. Rescaling on a hierarchical lattice corresponding to $b=2$ and $D=3$.

Such an approximation has been recently applied to the random gauge XY model by Gingras.[8] He finds, in disagreement with our earlier scaling results, that the random gauge XY model does not order in $D=3$. Specifically, the Hamiltonian he studied was

$$H=-\sum_{\langle ij\rangle}^{N} K_{ij}\cos(\phi_1-\phi_j)-\sum_{\langle ij\rangle}^{N} D_{ij}\sin(\phi_i-\phi_j)\ , \quad (2)$$

which reduces in the special case of

$$K_{ij}=J\cos(A_{ij})\ , \quad (3a)$$

$$D_{ij}=J\sin(A_{ij})\ , \quad (3b)$$

to the isotropic random gauge model. Gingras also studied the random Dzyaloshinsky-Moriya model when K_{ij} and D_{ij} are chosen independently from Gaussian distributions. He reported similar scaling behavior for the effective couplings in the two cases.

We have rederived the recursion equations of Gingras and have reproduced his results. The recursion equations are based on a decimation procedure: when spin 2 is decimated from a row containing spins 1, 2, and 3, one obtains

$$K'_{13}=(K_{12}K_{23}-D_{12}D_{23})/L_{123}\ , \quad (4a)$$

$$D'_{13}=(K_{12}D_{23}-K_{23}D_{12})/L_{123}\ , \quad (4b)$$

where

$$L_{123}=(K_{12}^2+D_{12}^2)^{1/2}+(K_{23}^2+D_{23}^2)^{1/2}\ . \quad (4c)$$

We have also extended his calculations to the case where the rescaling factor $b=3$. The recursion relations now arise from the decimation of two sites and read (when spins 2 and 3 are decimated from a row containing spins 1, 2, 3, and 4)

$$K'_{14}=[K_{12}(K_{34}K_{23}-D_{34}D_{23}) -D_{12}(D_{34}K_{23}+D_{23}K_{34})]/M_{1234}\ , \quad (5a)$$

$$D'_{14}=[D_{12}(K_{23}K_{34}-D_{23}D_{34}) +K_{12}(D_{23}K_{34}+D_{34}K_{23})]/M_{1234}\ , \quad (5b)$$

where

$$M_{1234}=L_{12}L_{23}+L_{12}L_{34}+L_{23}L_{34}\ , \quad (5c)$$

with

$$L_{ij}=(K_{ij}^2+D_{ij}^2)^{1/2}\ . \quad (5d)$$

Our numerical studies of the two sets of recursion relations show, in agreement with Gingras, that the effective couplings decrease on increasing length scales suggesting that both the random gauge XY and the Gaussian Dzyaloshinsky-Moriya models have an LCD > 3. Indeed, there is even less of a tendency to order when $b=3$ compared to the $b=2$ case.

B. The discretization scheme

We now turn to a discretization scheme that enables us to carry out the MKRG scheme at arbitrary temperatures and go beyond the harmonic approximation. The idea is simple and it is illustrated in Fig. 2: instead of allowing ϕ to be a continuous variable, we allow it to take one of many discrete values uniformly distributed between 0 and 2π. Due to limitation of computer storage, most of our studies were for $q=300$ for random systems and up to 900 for uniform models. The Hamiltonian is now defined for values of ϕ restricted to be $2\pi k/q$, where $k=0,1,2,\ldots,(q-1)$. We define

$$J_{ij}(q,k)=J_{ij}\cos(2\pi k/q - A_{ij}) \tag{6}$$

and find

$$J_{ij}(q,q+m)=J_{ij}(q,m)\ , \tag{7}$$

and

$$\sum_{k=0}^{q-1} J_{ij}(q,k)=0\ . \tag{8}$$

The recursion relation for the discretized clock model can be derived straightforwardly. For the one-dimensional decimation the recursion relation reads

$$J'_{13}(q,k)=k_BT\left[\ln F(q,k,t)-(1/q)\sum_{l=0}^{q-1}\ln F(q,l,T)\right]\ , \tag{9}$$

where

$$F(q,k,T)=\sum_{l=0}^{q-1}\exp[J_{12}(q,l)+J_{23}(q,\mathrm{mod}(q+k-l,q))]/k_BT\ . \tag{10}$$

Equation (9) is derived by noting that the renormalized Hamiltonian is characterized by renormalized exchange interactions and by a constant term. The latter was determined by imposing condition (8) on the rescaled couplings. Note that in the limit of T tending to 0 only the largest exponent contributes to the sum in Eq. (10). If the microscopic Hamiltonian involved anisotropic couplings (J^x different from J^y) then pair energies would depend not only on the difference in spin angles but also on their sum. In that case the discretization scheme would involve $q\times q$ and not just q couplings in the Hamiltonian.

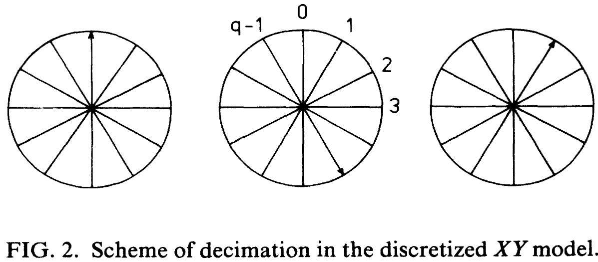

FIG. 2. Scheme of decimation in the discretized XY model.

The recursion scheme is completed by combining 2^{D-1} bonds decimated according to Eq. (9) into one rescaled bond which is a q-valued variable. For random systems, the numerical procedure is based on creating first a pool of N_P bonds, each decomposed into q components according to Eq. (6). One then picks N_P random batches of 2^D such bonds from the pool to generate a new pool of the coupling variables and the whole procedure is iterated. We typically consider N_P equal to 2000. Generally, the error bars due to statistics of the finite number of bonds in a pool are smaller than the size of the data points shown in the figures which display the results.

In the Ising case we have $q=2$ and then

$$\begin{aligned}J'_{13}(2,0)&=-J'_{13}(2,1)\\&=\tfrac{1}{2}k_BT[\ln\cosh(J_{12}+J_{23})/k_BT\\&\qquad-\ln\cosh(J_{12}-J_{23})/k_BT]\ ,\end{aligned} \tag{11}$$

which is a familiar result.

III. UNIFORM FERROMAGNETIC XY MODEL

We have tested our discretization scheme on the ferromagnetic XY model with q varying between 2 and 900. In this case the A_{ij} in Eq. (6) are 0 and the microscopic and renormalized $J(q,k)$ have a positive maximum at $k=0$ and a negative minimum at $k=q/2$ (clock angle ϕ equal to 0 and π, respectively). Note that in this model we have the symmetry $J(q,k)=J(q,q-k)$ which is lacking in the random gauge model. The $T=0$ scaling of the maximum is shown in Fig. 3. The scaling exponent y for the Ising case is equal to $D-1$ whereas for the continuous symmetry model it should be equal to $D-2$. We find that y is equal to $D-2$ for $q>2$ on length scales shorter than some critical value $L_c(q)$ which diverges as q tends to infinity. In Fig. 3, the first six iterations show essen-

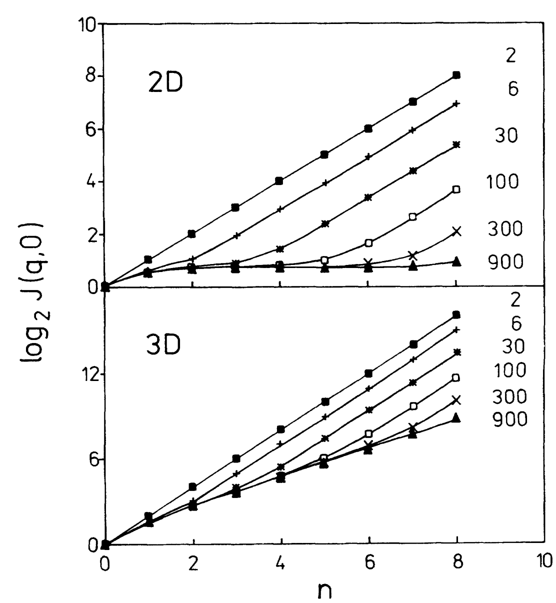

FIG. 3. Scaling of $J(q,0)$ for $D=2$ and $D=3$ for various values of q for uniform ferromagnetic models shown on the right-hand side of the figure.

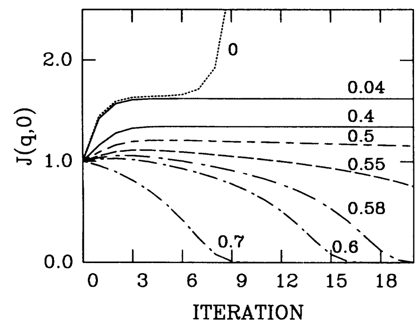

FIG. 4. Iterations of $J(q,0)$ for the uniform ferromagnetic model with $q=900$ for temperatures indicated in the plot (in units of J/k_B). The $T=0$ line was obtained from the recursion relations which were explicitly considered for this limiting T.

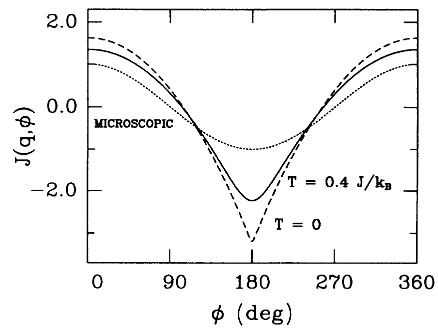

FIG. 6. Angular dependence of $J(q,k)$ in the ferromagnetic model with $q=900$. The dotted line corresponds to the microscopic cosine interactions. The remaining lines correspond to the fixed point situations at $T=0$ (dashed line) and $T=0.4J/k_B$ (solid line).

tially no crossover to the Ising-like behavior for $q=300$.

We found that $L_c(q)$ becomes very large when one calculates finite temperature couplings, even when T is infinitesimal. This is shown in Fig. 4 for the 2D XY model with $q=900$. No trace of a crossover is seen up to 20 iterations at $T=0.04J/k_B$ in this case: the effective coupling has reached a fixed point. For temperatures less than $0.44J/k_B$, $J(q,0)$ settles on a T-dependent fixed point value which is suggestive of a line of fixed points. This line seems to terminate at a Kosterlitz-Thouless[19] transition temperature of $0.44J/k_B$. At higher T's the coupling rescales to zero, indicating paramagnetic behavior. Figure 5 shows the behavior of the normalized helcity modulus defined as the fixed point value of $J(q,0)$ normalized to its $T=0$ value. The plot clearly indicates a sharp transition to the paramagnetic phase.[20]

We now focus on the angular (or k) dependence of $J(q,k)$. This is shown in Fig. 6. The microscopic coupling is given by the cosine function. Below the Kosterlitz-Thouless transition the fixed point "potential" develops a pronounced minimum at $\phi=\pi$ and a maximum at $\phi=0$. The values at the extremal points are no longer symmetric. The difference between the maximum and minimum shrinks with T, becoming equal to 0 at the critical point.

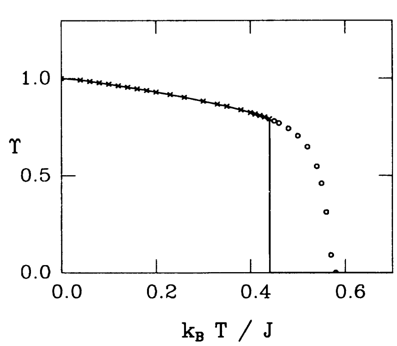

FIG. 5. Normalized helicity modulus for the ferromagnetic model with $q=900$ as a function of temperature. The open circles indicate $J(q,0)$ after 20 iterations for those temperatures at which an evidence for a gradual decrease in $J(q,0)$ is seen. Further iterations would result in the helicity modulus being zero. The data points denoted by the crosses are for temperatures at which no decrease in $J(q,0)$ is seen within 20 iterations.

MKRG schemes have not been unequivocally successful when applied to the 2D XY model. José *et al.* (JKKN)[21] showed that their MKRG scheme, based on expansion in a Fourier series and Migdal-Kadanoff recursion relations for the Fourier coefficients, gave numerical results apparently consistent with the Kosterlitz-Thouless line of fixed points.[20] However, a more careful study involving a mapping to the Villain model[22] indicated that this was not quite correct.[21] Indeed, the flows are toward a paramagnetic fixed point at any nonzero T, thereby barely missing the subtle Kosterlitz-Thouless transition.

Does our discretized clock model have a genuine line of fixed points in the manner of Kosterlitz and Thouless or is the behavior we observe in Figs. 4–6 akin to that of the Villain model in the JKKN analysis? The form of the interaction at the apparent fixed point shown in Fig. 6 is visually very similar to that presented by JKKN (except for a scale factor of 2) and is very close to the Villain form. Thus we expect that our approach is probably similar to that of JKKN in that the line of fixed points is almost but not quite present. Future studies to clarify this point would be desirable.

IV. FRUSTRATED MODELS

We now turn to the analysis of several 2D and 3D frustrated models. We have studied various models corresponding to special choices of A_{ij}, q, and sets of couplings in the Hamiltonians described by Eqs. (1) and (2) in which the ϕ_i variable takes on discrete values between 0 and $(q-1)2\pi/q$. The quenched random angle A_{ij} may also be now chosen from q' discrete values. The Ising bimodal spin glass is a special case when $q=q'=2$. We will see that the random gauge model is obtained for larger values of $q=q'$ ($q=300$ or more).

Other classes of models correspond to Hamiltonian (2), where J_{ij} and D_{ij} are chosen independently from a Gaussian distribution. The Ising Gaussian spin glass is a special case of $q=2$ with $D_{ij}=0$. For $q \gg 1$ and $D_{ij} \neq 0$ we should mimic the random Dzyaloshinsky-Moriya model. If we set $D_{ij}=0$ and take larger q we obtain the bimodal XY spin glass and the Gaussian XY spin glass for the corresponding distributions of the couplings J_{ij}. Of all these models the random gauge system has an O(2) local gauge invariance, which we hope will be recovered in our discretization scheme when $q=q' \gg 1$. On the other hand, the bimodal and Gaussian Ising spin glasses have the Z_2 gauge invariance symmetry. The random Dzyaloshinsky-Moriya model provides a good example for tests of the relevance of the local gauge invariance: for Gaussian J_{ij} and D_{ij} couplings the model has O(2) local gauge invariance and should behave like the random gauge model, but the local gauge invariance is destroyed if either J_{ij} or D_{ij} are non-Gaussian. $J(q,\phi)$ denote $J(q,k)$ for k corresponding to the discrete angle ϕ. A typical plot of $J(q,\phi)$ is shown in Fig. 7—for the $q=300$ random gauge model. The dotted line depicts one selected microscopic coupling. It is of the shifted cosine form. Two typical couplings picked from the pool of couplings obtained after eight iterations are shown by the solid lines. The structure is more complicated and a clear departure from the cosine form is seen. In general, each set of $J(q,\phi)$ attributed to a single bond coupling can be characterized by its maximum and minimum values. We are interested in how these extremal values scale for the models studied.

Our results for these models are shown in Figs. 8 and 9. The former is for the maximal coupling and the latter for the minimal one. The scaling of the average maximal and minimal couplings is seen to be virtually identical so we focus on Fig. 8. Strikingly, the models with the local gauge invariance built in are characterized by a LCD < 3, whereas those without the invariance do not have a nonzero transition temperature in $D=3$. As mentioned before, the random Dzyaloshinsky-Moriya model with Gaussian exchange couplings is locally gauge invariant and indeed in $D=3$ it has a phase transition similar to that in Ising spin glasses. On the other hand, the Dzyaloshinsky-Moriya model with bimodal exchange couplings is not locally gauge invariant and this model does not undergo a finite-T phase transition. The data

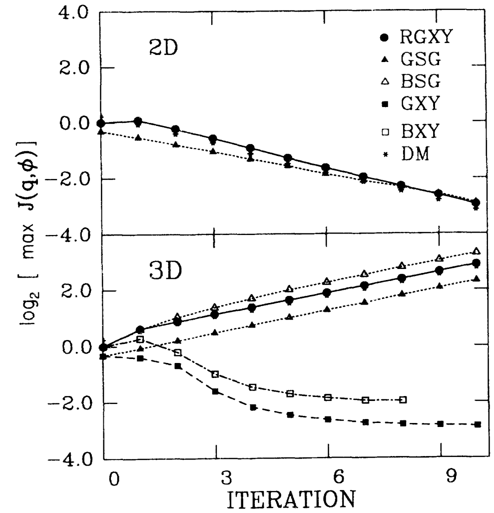

FIG. 8. Scaling of max $J(q,\phi)$ for various frustrated random systems. For GSG (Gaussian Ising spin glass) and BSG (bimodal Ising spin glass) $q=2$. For *RGXY* (random gauge *XY* model), *GXY* (Gaussian *XY* spin glass), and DM (random Dzyaloshinski-Moriya with Gaussian exchange couplings) $q=300$. For *BXY* (bimodal *XY* spin glass) $q=900$. In each case the pool consisted of 2000 bonds.

points corresponding to the latter model are virtually indistinguishable from those for the bimodal XY spin glass and are not shown in Fig. 8. Our results, and especially the different behaviors of the random Dzyaloshinsky-Moriya model with Gaussian and non-Gaussian couplings, are a confirmation of our previous conjecture on the role played by local gauge invariance in determining the LCD of frustrated systems.

The $T=0$ scaling exponent y in $D=3$ is found in our discretized MKRG scheme to be the same for the Gaussian and bimodal Ising spin glasses, the random gauge model, and the random Dzyaloshinsky-Moriya model

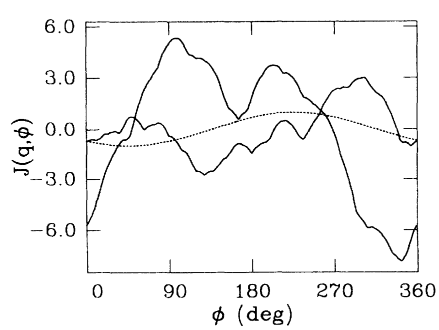

FIG. 7. $J(q,\phi)$ vs ϕ for the 3D random gauge XY model with $q=300$. The dotted line is a typical microscopic coupling. the solid lines show two typical couplings after eight iterations.

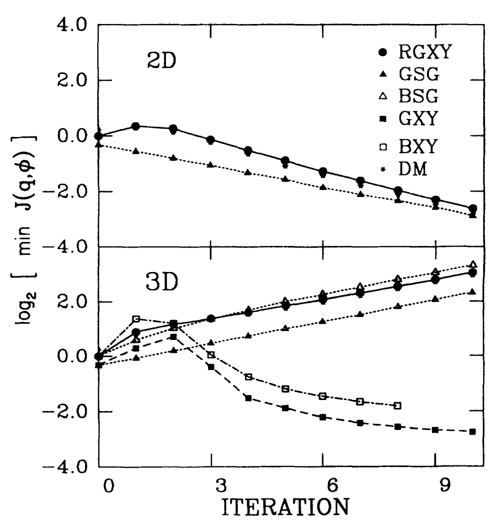

FIG. 9. Similar to Fig. 8 but for the minimal value of $J(q,\phi)$.

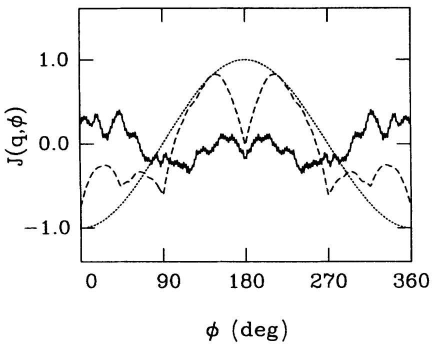

FIG. 10. The form of individual couplings for the 3D bimodal XY model after 0 (dotted line), 4 (broken line), and 8 (solid line) iterations.

with the Gaussian exchange couplings. Its value is equal to

$$y_{\rm MK} = 0.26 \quad (D=3) . \tag{12}$$

This should be compared to the exponent y of 0.19 obtained by the transfer matrix method for the 3D Ising spin glasses[3,4,17] and y of order 0.3 for the random gauge model obtained by the method of $T=0$ quenches.[1] It has been suggested[6,11] that the y exponent for the random gauge model might be the same as that for the 3D Ising spin glasses. Our MKRG results agree with this expectation. However, defect-energy calculations by Gingras[23] and Reger *et al.*,[24] based on large-sample statistics, yield y of order 0.05.

The corresponding calculations in $D=2$ show that whereas the random gauge model continues to be similar to the Gaussian Dzyaloshinsky-Moriya one, they are different from the Gaussian spin glass: $y=-0.36$ for the random gauge and Gaussian Dzyaloshinsky-Moriya models and it is -0.24 for the Gaussian Ising spin glass, indicating a $T=0$ phase transition in each case. For the random gauge XY model the MKRG scheme of Gingras[8] yields $y=-0.74$ and the $T=0$ scaling calculation of Fisher *et al.*[11] gave $y=-0.5$. It is seen that the discretized MKRG scheme is consistently closer to the numerical results on the stiffness.

We also find evidence that the LCD's for the random gauge and the Gaussian Ising spin glass models are slightly different and are 2.59±0.05 and 2.55±0.05, respectively. The procedure to find the LCD is similar to the one outlined in Ref. 16 for the case of Ising spin glasses. The idea is to combine in effect 2^{D-1} decimated bonds into a rescaled coupling where D is no longer restricted to be an integer. For instance, for the Gaussian Ising spin glass model at its lcd the required number of decimated bonds is about 2.92. Thus one has to combine two decimated bonds with a weight of 1 and to take the third decimated bond with a weight of 0.92. The behavior of the 2D bimodal Ising spin glass model is totally different from that of the 2D Gaussian Ising spin glass

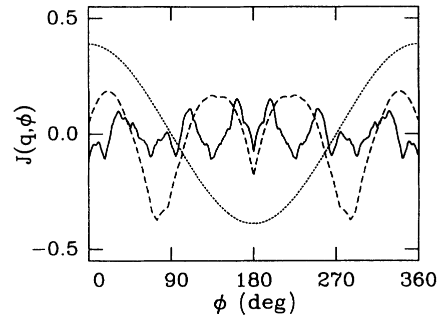

FIG. 11. Same as in Fig. 10 but for the Gaussian XY model.

and this point is discussed further in Ref. 17.

Consider now the Gaussian and bimodal XY spin glasses for which the LCD appears to be larger than 3. Figure 8 shows that the average maximal value of $J(q,\phi)$ does not behave as L^y, but that an effective y changes continuously from a value of order -1 and asymptotically reaches a value of -0.1. Numerical studies of the stiffness for the bimodal XY spin glass[1] suggest $y=-1$. On the other hand similar studies of the $D=3$ Gaussian XY spin glass[15] gave y of order -0.45. The bimodal and Gaussian XY models need not be in the same universality class in 3D (i.e., below the LCD), but our MKRG analysis suggests that they might be. The reason for the apparent dependence of y on the length scale can be grasped from Figs. 10 and 11 which show forms of typical couplings. In the initial iterations these forms have pronounced structures which disappear only after 6–7 iterations leading to the fixed point behavior.

The stability of an ordered phase at low temperatures depends on the scaling behavior of low-energy excitations. It would be interesting to investigate how the local gauge invariance, or lack of it, affects low energy excitations in spin systems. The presence of the local gauge invariance, within the MKRG scheme, yields the zero-temperature exponent y for the 3D gauge glass model to be close to that for the Ising spin glass. Equality of these exponents would not mean, however, that the two models belong to the same universality class. For instance, the ferromagnetic Ising and Potts models have a LCD of 1, but they are in different universality classes. Gingras[25] has shown recently, using a mean-field theory, that the gauge glass is in a different universality class than the Ising spin glass.

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[1]M. Cieplak, J. R. Banavar, and A. Khurana, J. Phys. A **24**, L145 (1991); please note that the captions to Fig. 1 in this reference are incorrect: the labels for the random-grain and gauge glass systems are interchanged.
[2]J. R. Banavar and M. Cieplak, Phys. Rev. Lett. **48**, 832 (1982); J. Phys. C **16**, L755 (1983).
[3]A. J. Bray and M. A. Moore, in *Heidelberg Colloquium on Glassy Dynamics,* edited by L. Van Hemmen and I. Morgenstern (Springer, Berlin, 1987), p. 121 and references therein; A. J. Bray, Comments Cond. Mater. Phys. **14**, 21 (1988).
[4]W. L. McMillan, Phys. Rev. B **30**, 476 (1984).
[5]K. Binder and A. P. Young, Rev. Mod. Phys. **58**, 801 (1986).
[6]D. A. Huse and H. S. Seung, Phys. Rev. B **42**, 1059 (1990).
[7]M. A. Moore and S. Murphy, Phys. Rev. B **42**, 2587 (1990).
[8]M. J. P. Gingras, Phys. Rev. B **43**, 13 747 (1991).
[9]A. A. Migdal, Zh. Eksp. Teor. Fiz. **69**, 1457 (1975) [Sov. Phys. JETP **42**, 743 (1975)]; L. P. Kadanoff, Ann. Phys. (NY) **100**, 359 (1975).
[10]M. Tinkham and C. J. Lobb, in Solid State Physics **42**, edited by H. Ehrenreich and D. Turnbull (Academic, Boston, 1989), p. 91.
[11]M. P. A. Fisher, Phys. Rev. Lett. **62**, 1415 (1989); D. S. Fisher, M. P. A. Fisher, and D. A. Huse, Phys. Rev. B **43**, 130 (1991).
[12]M. P. A. Fisher, T. A. Tokuyasu, and A. P. Young, Phys. Rev. Lett. **66**, 2931 (1991).
[13]B. W. Southern and A. P. Young, J. Phys. C **10**, 2179 (1977); S. Kirkpatrick, Phys. Rev. B **15**, 1533 (1977).
[14]P. W. Anderson and C. M. Pond, Phys. Rev. Lett. **40**, 903 (1978).
[15]B. W. Morris, S. G. Colborne, M. A. Moore, A. J. Bray, and J. Canisius, J. Phys. C **19**, 1157 (1986); J. R. Banavar and A.J. Bray, Phys. Rev. B **38**, 2564 (1988).
[16]R. G. Caflisch and J. R. Banavar, Phys. Rev. B **32**, 7617 (1985).
[17]M. Cieplak and J. R. Banavar, J. Phys. A **23**, 4385 (1990).
[18]See, e.g., R. G. Caflisch and A. N. Berker, Phys. Rev. B **29**, 1279 (1984).
[19]J. M. Kosterlitz and D. J. Thouless, J. Phys. C **6**, 1181 (1973).
[20]M. E. Fisher, M. N. Barber, and D. Jasnow, Phys. Rev. A **8**, 1111 (1973); T. Ohta and D. Jasnow, Phys. Rev. B **20**, 139 (1979).
[21]J. V. Jose, L. P. Kadanoff, S. Kirkpatrick, and D. Nelson, Phys. Rev. B **16**, 1217 (1977).
[22]J. Villain, J. Phys. (Paris) **36**, 581 (1975).
[23]M. J. P. Gingras (unpublished).
[24]J. D. Reger, T. A. Tokuyasu, A. P. Young, and M. P. A. Fisher, Phys. Rev. B **44**, 7147 (1991).
[25]M. J. P. Gingras, Phys. Rev. B **44**, 7139 (1991).