Fluxon viscosity in high-T_c superconductors

Michael Golosovsky, Yehuda Naveh, and Dan Davidov
Racah Institute of Physics, Hebrew University of Jerusalem, Jerusalem 91904, Israel
(Received 12 August 1991)

We present a phenomenological model for the frequency-dependent fluxon complex resistivity in which the fluxon viscosity η is a central parameter. We use the high-frequency limit of this model to extract η from microwave transmission measurements through thin superconducting films in the presence of a magnetic field. The measured value of η allows a critical check of the model at low frequencies. The viscosity is anisotropic and consistent with anisotropic effective mass ratios of $\gamma=(m_c/m_a)^{1/2}=5-8$ for Y-Ba-Cu-O and $\gamma=12-15$ for Bi-Sr-Ca-Cu-O.

It is now accepted that the complex resistance of high-T_c superconductors is associated with fluxon dynamics and strongly depends on magnetic field, frequency, and temperature. The very early model of Gittleman and Rosenblum[1] assumed oscillations of damped fluxons in a harmonic pinning potential. According to this model the fluxon resistivity is dissipative at high frequencies but it is nondissipative at low frequencies. However, this model[1] was developed for zero temperature and cannot account for the low-frequency dissipative properties, particularly for the thermally activated dc resistivity which is very pronounced in high-T_c superconductors.[2] The dc resistivity of high-T_c superconductors in the presence of a magnetic field has been successfully explained by several models[3-5] which assume flux hopping between adjacent pinning sites. These models take into account pinning forces but do not depend explicitly on the fluxon viscosity. Clearly, there is a need for a more general model for the fluxon dissipation at different frequencies and temperatures. Initial attempts in this direction were made by Inui, Littlewood, and Coppersmith[6] and by Martinoli *et al.*,[7] who have used the analogy between a pinned fluxon and a Brownian particle in a potential well to calculate the low-frequency complex resistivity. The present paper extends the theoretical approach of Martinoli *et al.*[7] for wide frequency ranges and for different temperatures.

The phenomenological model assumes uncorrelated motion of fluxons in a harmonic pinning potential $U(x)=U_0[1-\cos(qx)]$. Here q^{-1} is the pinning length and U_0 is the amplitude of the pinning potential. Both of these parameters depend on temperature. Correlations in the fluxon motion may be accounted for by introducing magnetic-field-dependent U_0 and q^{-1}. A single pinning energy is assumed. The fluxons are regarded as "rigid rods" with length L and negligible mass.[8] The driving force for fluxon motion is the Lorentz force $F_{\text{ext}}=(J\Phi_0L/c)\sin\varphi$, where J is the current density, φ is the angle between the microwave current and the magnetic field, and Φ_0 is the flux quantum. The model also assumes the presence of a stochastic force F_{th} due to thermal fluctuations which satisfies $\langle F_{\text{th}}(t)F_{\text{th}}(0)\rangle=2\eta LkT\delta(t)$, where $\delta(t)$ is a δ function, η is the temperature-dependent viscosity coefficient per unit length, and k is the Boltzmann factor. The fluxon concentration is assumed to be uniform. Therefore, the fluxon motion is two dimensional and can be described by the Langevin equation of motion:[7]

$$\eta L\dot{x}+U_0q\sin(qx)=F_{\text{ext}}+F_{\text{th}}\ , \tag{1}$$

where x is the fluxon displacement. The moving fluxons produce an electric field $E=\dot{x}H/c$, where x can be calculated using Eq. (1). For an oscillating external current $J=J_0\exp(i\omega t)$, the rf resistivity associated with the fluxon motion is given by $Z_f(\omega)=E/J$. In the limit of small current ($J_0\ll U_0qc/\Phi_0L$) Eq. (1) can be solved approximately by a continued-fraction expansion.[9] One obtains, then, for the fluxon complex resistivity[7]

$$Z_f(\omega)=R_f\left[1+\frac{I_0^2(s)-1}{1-i(\omega/\omega_0)[I_0^2(s)-1]I_0(s)/I_1(s)}\right]^{-1}, \tag{2}$$

where $R_f=H|\sin\varphi|\Phi_0/\eta c^2$, $s=U_0/kT$, $\omega_0=U_0q^2/\eta L$, $I_0(s)$ and $I_1(s)$ are modified Bessel functions.[10] Here φ is the angle between the magnetic field H and the microwave current I_{mw} (see Fig. 1). The real part ρ_f and the imaginary part X_f of the fluxon complex resistivity, as calculated using Eq. (2), are given by the following expressions:

$$\rho_f=R_f\frac{(\omega/\omega_1)^2+I_0^{-2}(s)}{1+(\omega/\omega_1)^2}\ , \tag{3a}$$

$$X_f=R_f[1-I_0^{-2}(s)]\frac{\omega/\omega_1}{1+(\omega/\omega_1)^2}\ , \tag{3b}$$

where

$$\omega_1=\omega_0\frac{I_0(s)I_1(s)}{I_0^2(s)-1}\ . \tag{4}$$

At low frequencies the pinning forces dominate (pinning regime) while at high frequencies the viscous forces dominate (flux-flow regime). The model predicts a crossover frequency [Eq. (4)] from the pinning regime to the flux-flow regime. Far below T_c the crossover frequency can be expressed as

$$\omega_1 \approx \omega_0 = U_0 q^2/\eta L \ . \tag{5}$$

Let us consider the fluxon dissipation in several limiting cases:

(1) Low frequencies $[\omega \ll \omega_1 I_0^{-2}(s)]$. In this limit the fluxon complex resistivity is dominated by ρ_f which is given by $\rho_f \approx R_f I_0^{-2}(s)$. This expression is similar to the result of Tinkham[4] $[\rho_f = \rho_n I_0^{-2}(s)]$ for the dc resistivity tail in a magnetic field. Far below T_c, $s = U_0/kT \gg 1$ and the modified Bessel functions may be approximated by exponentials as follows: $I_1(s) \sim I_0(s) \sim (2\pi s)^{-1/2} \exp(s)$. Then, Eq. (3) reduces to the thermally activated resistivity

$$\rho_f = \rho_0 \exp\left[-\frac{2U_0}{kT}\right]; \quad \rho_0 = \frac{2\pi U_0 H \Phi_0}{c^2 \eta k T} \ . \tag{6}$$

(2) Intermediate frequencies $[\omega_1 \gg \omega \gg \omega_1 I_0^{-2}(s))]$. In this region $X_f \gg \rho_f$ and the fluxon complex resistivity is inductive.

(3) High frequencies $(\omega \gg \omega_1)$. In this limit the fluxon complex resistivity is dominated by dissipation, $\rho_f \gg X_f$. Equation (3a) reduces to the result of Gittleman and Rosenblum[1] and the fluxon resistivity is given by

$$\rho_f = R_f = H|\sin\varphi|\Phi_0/\eta c^2 \ . \tag{7}$$

This flux-flow resistivity is significantly larger than that at low frequencies [Eq. (3a)] and is independent of pinning forces.

As clearly seen from Eqs. (5)-(7), the viscosity coefficient η is an important parameter even at low frequencies and certainly is the dominant parameter at high frequencies in the flux-flow region. Estimates[11,12] of the crossover frequency ω_0 indicate that at microwave frequencies and for materials with relatively low pinning forces the fluxons are in the flux-flow regime. Then, the viscosity coefficient can be determined by measuring the fluxon resistivity ρ_f using Eq. (7). As demonstrated previously,[12] an elegant method to measure the fluxon resistivity ρ_f in the microwave range is by microwave transmission through thin superconducting films in the presence of a magnetic field.

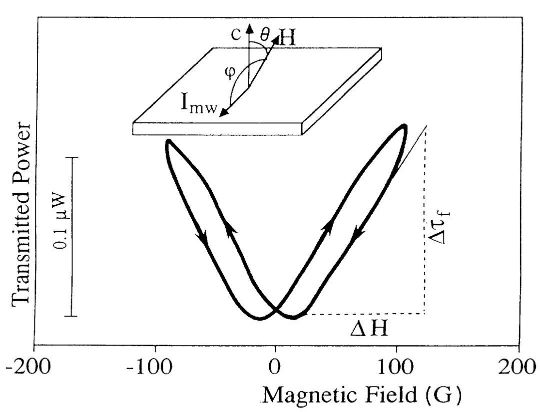

FIG. 1. Magnetic field dependence of the microwave transmission τ_f through an Y-Ba-Cu-O film for field orientation $\theta = 50°$ at $T = 74$ K; θ is defined in the figure. The slope is determined from the linear part of the curve as $S = \Delta\tau_f/\Delta H$. The inset demonstrates the orientation of the field H and microwave current I_{mw} with respect to the c axis of the film.

The field-dependent microwave transmission through superconducting films τ_f was calculated recently[12,13] for $T_c > T > 0.8T_c$. For a thin film of thickness L on a thin substrate τ_f is given by

$$\tau_f = \frac{8\rho_b \rho_f}{L^2 Z_0^2} = \frac{8\rho_b \Phi_0 H|\sin\varphi|}{L^2 Z_0^2 c^2 \eta(\theta)} \ , \tag{8}$$

where ρ_b is the real part of the bulk complex resistivity (which does not depend on the magnetic field), Z_0 is the impedance of the waveguide, and θ is the angle between the magnetic field and the crystallographic axis (Fig. 1). We have demonstrated recently the validity of the Lorentz force in the high-frequency flux flow.[12] Therefore in this paper we emphasize the microwave transmission experiment with fixed value of $\varphi = 90°$ (i.e., $\sin\varphi = 1$) but with different orientations θ of the field with respect to the c axis (Fig. 1). Under these conditions the slope $S(S = d\tau_f/dH)$ is proportional to η^{-1} according to Eq. (8). As predicted theoretically,[14] η is anisotropic and the study of the slope S for different orientations θ yields $\eta(\theta)$.

The experimental setup for the microwave transmission experiment at 9.2 GHz was described elsewhere.[12,13] The film is mounted across the waveguide. The incident microwave power is ≈ 100 mW. A magnetic field was achieved by using two independent coils: a coil along the waveguide which produces a field perpendicular to the plane of the film and a Helmholtz pair which produces a magnetic field in the plane of the film. The orientation of the field with respect to the film was monitored by varying the ratio of currents in these two coils. The maximum field, $H = 500$ G, exceeds the lower critical field H_{c1} at the temperature of the experiment. At least five different Y-Ba-Cu-O films and four Bi-Sr-Ca-Cu-O films on MgO substrates were studied. We found a significant field-dependent microwave transmission (at $\omega/2\pi = 9.2$ GHz) in films prepared by metalorganic chemical vapor deposition (MOCVD) (dimensions of 1×1 cm^2 and thickness of ≈ 0.5 μ), as well as in films prepared by spray pyrolysis. These films exhibit critical temperatures of $T_c \approx 85$ K (Y-Ba-Cu-O) and $T_c \approx 80$ K (Bi-Sr-Ca-Cu-O), and critical currents of the order of 10^3 A/cm^2 at 77 K. However, the field-dependent transmission τ_f is very small in the Y-Ba-Cu-O films with high critical current such as those produced by the laser ablation method. We believe that the small τ_f in such films is due to the dominance of the pinning forces over the viscous forces, such that the microwave frequency is lower than the crossover frequency [Eq. (4)]. Figure 1 describes the field-dependent microwave transmission through a thin Y-Ba-Cu-O film at $T = 76$ K for $\theta = 50°$. As clearly seen, the microwave transmission τ_f is a linear function of the field H, as predicted by Eq. (8). A small hysteresis is also observed. However, the slope $S = d\tau_f/dH$ was calculated in a consistent way for the various orientations θ. Figure 2 exhibits the angular dependence of $S(\theta = 0°)/S(\theta)$ for well oriented thin films of Y-Ba-Cu-O (MOCVD) and

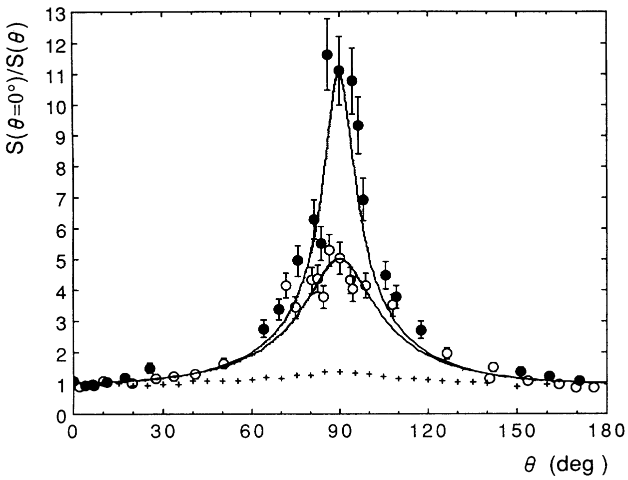

FIG. 2. Angular dependence of the $S^{-1}=(d\tau_f/dH)^{-1}$ [for well oriented films $S^{-1}\propto\eta$ according to Eq. (8)]. Open and closed symbols indicate well oriented films of Y-Ba-Cu-O at $T=77$ K and Bi-Sr-Ca-Cu-O at $T=64$ K, respectively; crosses indicate a poorly oriented Y-Ba-Cu-O film at $T=65$ K. The slope for each film is normalized to its value at $\theta=0°$. The solid lines are fit to Eq. (9) with $\gamma=5$ and $\gamma=11$.

Bi-Sr-Ca-Cu-O (spray pyrolysis), as well as for a poorly oriented thick Y-Ba-Cu-O film (spray pyrolysis). As clearly seen, $S(\theta)$ exhibits its lowest value for the magnetic field parallel to the film ($\theta=90°$) and its highest value for the field perpendicular to the film ($\theta=0°$). Both $S(\theta=0°)$ and $S(\theta=90°)$ vary with temperature (not shown here). However, their ratio $S(\theta=0°)/S(\theta=90°)$ is almost temperature -independent for $T_c>T>0.8T_c$.

The mosaic spread in all our films was studied by high-resolution x-ray technique. Generally speaking, for well oriented films (mosaic spread of 0.4°–2°) the anisotropy of the slope $S(\theta=0°)/S(\theta=90°)$ was found to be $\approx 4-6$ for Y-Ba-Cu-O films and $\approx 9-11$ for Bi-Sr-Ca-Cu-O films. For poorly oriented Y-Ba-Cu-O films (large width of the x-ray rocking curve) the anisotropy is ≈ 1.5, i.e., remarkably smaller (Fig. 2). These results suggest that the anisotropy of $S(\theta)$ is due to the anisotropic viscosity $\eta(\theta)$ and not to some shape factor.[13] The magnitude of $\eta(\theta)$ may be estimated using Eq. (8). We obtain $\eta(0°)=(2-4)\times10^{-8}$ cgs units for Y-Ba-Cu-O at 70 K and $\eta(0°)=(0.4-1)\times10^{-8}$ cgs units for Bi-Sr-Ca-Cu-O at 64 K (using $\rho_n=10^3$ $\mu\Omega$ cm). These estimates are consistent with the magnitude of the viscosity of Abrikosov fluxons.[12]

The coefficients $\eta(\theta=0°)$ and $\eta(\theta=90°)$ represent viscosities for the fluxons along the c axis moving in the a-b plane, and for the fluxons in the a-b plane moving in the direction of the c axis, respectively. In the notation of Hao and Clem[14] these parameters are defined as $\eta_a^{(c)}$ and $\eta_c^{(a)}$ (here $\eta_j^{(i)}$ represent viscosity of fluxon in the i direction moving along the j axis). These authors have demonstrated[14] that $\eta_c^{(a)}/\eta_a^{(c)}\approx0.75\gamma$, where γ is the anisotropic mass ratio, $\gamma=(m_c/m_a)^{1/2}$. Using the experimental data (Fig. 2), we find temperature-independent ratios $\eta_c^{(a)}/\eta_a^{(c)}\approx4-6$ for Y-Ba-Cu-O and $\approx9-11$ for Bi-Sr-Ca-Cu-O. This yields according to Hao and Clem[14] a temperature-independent effective mass anisotropy $\gamma\approx5-8$ for the former and $\gamma\approx12-15$ for the latter. These estimates for γ agree with the values of the effective mass obtained from the magnetic anisotropy.[15] They are slightly larger than those obtained from anisotropic critical fields.[11,16] The temperature independence of the $\eta_c^{(a)}/\eta_a^{(c)}$ can be explained in the framework of the anisotropic Landau-Ginzburg theory. The angular dependence of the viscosity can be obtained also using the following interpolation formula:[13]

$$\eta(\theta)=\eta(0°)(\gamma^{-2}\sin^2\theta+\cos^2\theta)^{-1/2}\ . \tag{9}$$

This expression fits well the experimental values of $S^{-1}(\theta)$ (solid lines in Fig. 2) and yields $\gamma=5$ for Y-Ba-Cu-O and $\gamma=11$ for Bi-Sr-Ca-Cu-O.

We now use the values of η to estimate the preexponential factor ρ_0 in the fluxon thermally activated dc resistivity [Eq. (6) of our phenomenological model]. For Bi-Sr-Ca-Cu-O single crystals we find $\rho_0=10^4$ $\mu\Omega$ cm using $\eta(0°)=0.4\times10^{-8}$ cgs units at $T=64$ K (this work) and $2U_0=500$ K at the same temperature and at $H=1$ T (Ref. 2). This estimate is in rough agreement with the experimental value of $\rho_0=10^5$ $\mu\Omega$ cm found in Ref. 2. The very small anisotropy of ρ_0 (Ref. 2) can be explained using the relation $\rho_0\propto U_0/\eta$ (Eq. 6) such that the anisotropy of U_0 (Ref. 2) is partially compensated by the anisotropy of η. Note that Eq. (6) for the thermally activated dc resistivity is very similar to the prediction of the Anderson flux-creep model:[17]

$$\rho_f=\frac{2\nu_0\Phi_0^2L}{c^2kT}\exp\left[-\frac{2U_0}{kT}\right]\ . \tag{10}$$

Here the attempt frequency ν_0 was previously attributed to some phonon frequency.[5,18] However, a comparison of Eqs. (6) and (10) yields $\nu_0=\pi U_0H/\Phi_0\eta L$. Interestingly, this expression for ν_0 is very close to that of ω_0 [Eq. (5)] assuming[4] $q\approx(H/\Phi_0)^{1/2}$. Indeed, ω_0 was estimated previously[11] to be $\omega_0\approx10^{10}-10^{11}$ Hz for Y-Ba-Cu-O single crystals, which is close to the estimates of ν_0 found from the dc resistivity measurements[2] ($\nu_0\approx10^{12}$ Hz) and from the ac susceptibility[18] ($\nu_0\approx10^9-10^{12}$ Hz). This provides further evidence for the validity of the proposed model at different frequencies.

The authors gratefully acknowledge Professor G. Koren from Technion, Haifa, and Mr. M. Levinsky and Professor M. Schieber from Xsirius Superconductivity, Israel, for providing some of the films. We are grateful to Professor Y. Yeshurun, Professor J. E. Fischer, and Dr. L. Burlachkov for valuable discussions.

[1]J. I. Gittleman and B. Rosenblum, Phys. Rev. Lett. **16**, 734 (1966).
[2]T. T. M. Palstra, B. Batlogg, R. B. van Dover, L. F. Schneemeyer, and J. V. Waszczak, Phys. Rev. B **41**, 6621 (1990).
[3]Y. Yeshurun and A. P. Malozemoff, Phys. Rev. Lett. **60**, 2202 (1988).
[4]M. Tinkham, Phys. Rev. Lett. **61**, 1658 (1988).
[5]P. H. Kes, J. Aarts, J. van den Berg, C. J. van der Beek, and J. Mydosh, Supercond. Sci. Technol. **1**, 242 (1989).
[6]M. Inui, P. B. Littlewood, and S. N. Coppersmith, Phys. Rev. Lett. **63**, 2421 (1989).
[7]P. Martinoli, Ph. Fluckiger, V. Marsico, P. K. Srivastava, Ch. Leemann, and J. L. Gavilano, Physica B **165&166**, 1163 (1990). Similar results were recently obtained by M. W. Coffey and T. R. Clem, Phys. Rev. Lett. **67**, 386 (1991) and by E. H. Brandt, Phys. Rev. Lett. **67**, 2219 (1991).
[8]A. M. Portis, K. W. Blazey, K. A. Müller, and J. G. Bednorz, Europhys. Lett. **5**, 467 (1988).
[9]W. Dietrich, I. Peschel, and W. R. Schneider, Z. Phys. B **27**, 177 (1977).
[10]P. M. Morse and H. Feshbach, *Methods in Theoretical Physics* (McGraw-Hill, New York, 1953), p. 884.
[11]D. H. Wu and S. Sridhar, Phys. Rev. Lett. **65**, 2074 (1990).
[12]M. Golosovsky, D. Davidov, E. Farber, M. Schieber, and T. Tsach, Phys. Rev. B **43**, 10 390 (1991).
[13]M. Golosovsky, Y. Naveh, and D. Davidov, Physica **C180**, 164 (1991).
[14]Z. Hao and J. R. Clem, IEEE Trans. Magn. **27**, 1086 (1991).
[15]M. Tuominen, A. M. Goldman, Y. Z. Chang, and P. Z. Jiang, Phys. Rev. B **42**, 412 (1990).
[16]L. Krusin-Elbaum, A. P. Malozemoff, Y. Yeshurun, D. C. Cronemeyer, and F. Holtzberg, Phys. Rev. B **39**, 2936 (1989).
[17]P. W. Anderson, Phys. Rev. Lett. **9**, 309 (1962).
[18]A. P. Malozemoff, T. K. Worthington, Y. Yeshurun, F. Holtzberg, and P. H. Kes, Phys. Rev. B **38**, 7203 (1988).