Theory of hot-electron magnetophonon resonance in quasi-two-dimensional quantum-well structures

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The linear and nonlinear (dc) electrical transport parallel to the walls of a quantum well, with a magnetic field $\mathbf{B}=B\hat{\mathbf{z}}$ applied normal to its barriers, is considered for an electron-phonon system, using the formalism of nonlinear response theory [Phys. Rev. B **40**, 5632 (1989)] developed previously. The structurally confined electron gas is assumed to interact with bulk phonons. Explicit expressions for hot-electron magnetophonon resonances are obtained for polar-LO-phonon scattering by computing the electric-field-dependent conductivity formula defined in the Ohm's-law form of a nonlinear electric current. Certain values of the electric field induce transitions of the carriers between neighboring Landau levels and the maxima of the ordinary magnetophonon resonance at weak electric field evolve to minima and vice versa. The conductivity (and hence the current) oscillates as a function of the magnetic field with electric-field-induced resonances occurring in the hot-electron regime when $P\omega_c=\omega_L^*$, where ω_c and ω_L^* are the cyclotron and effective phonon frequencies, respectively, and P is an integer. These peak positions are shifted to the higher B side from the ordinary magnetophonon resonance peaks at $P\omega_c=\omega_L$, where ω_L is the bare phonon frequency. The shift of the resonance peaks is proportional to F. Unlike the three-dimensional system, additional subsidiary resonance peaks are predicted even under very weak electric fields whenever the interelectric subband transitions are allowed to take place for a relevant energy separation between two subbands, leading to an additional oscillatory behavior. The possibility of these interelectric transitions is also discussed. The dependence of the conductivity (or current), energy relaxation rate, and Landau-level broadening on the electric and magnetic fields, the thickness of the well, and the temperature is shown explicitly. Some of the results obtained here are in accordance with those available in the literature.

I. INTRODUCTION

Since predicted by Gurevich and Firsov,[1] the magnetophonon resonance (MPR) effect is a powerful spectroscopic tool to investigate transport properties of semiconductors.[2–4] The magnetophonon effect arises from the resonant scattering of electrons quantized in Landau levels by phonons whenever the phonon energy is equal to an integral multiple of energy separations between two Landau levels. For weak fields, this gives the ordinary MPR condition as

$$\hbar\omega_L=P\hbar\omega_c=P\hbar eB/m^*, \quad P=1,2,3,\ldots, \tag{1.1}$$

where m^* is a suitably defined effective mass of an electron and ω_L is usually the longitudinal-optic (LO) phonon frequency. This inelastic scattering by optic phonons (the resonant phonon emission and/or absorption) acts strongly to relax carrier momentum and hence leads to changes in electron mobility, giving rise to a corresponding oscillatory dependence of the electric current (or conductivity) on applied magnetic field with period $1/B$. Therefore, the analysis of the oscillatory variations (e.g., the amplitude of the oscillations, the broadening of the resonance peaks due to the applied electric field as well as the scattering, and their dependence on physical parameters) gives very important information on the relaxive transport properties of semiconductors, such as carrier relaxation mechanism (i.e., the energy gain and loss processes), damping of the oscillations due to the electron-phonon interaction, and intracollisional field effects[5] (ICFE) as well as on the phonon frequencies and band structure (e.g., the effective mass m^*).

The ordinary and hot-electron MPR effects have been studied in considerable detail on three-dimensional (3D) systems[1–4,6–8] from both experimental and theoretical points of views. Unusual behavior of the MPR line shape (e.g., conversion of MPR maxima into minima or splitting of the MPR peaks) in the conductivity component σ_{xx} has been reported for n^+-n^--n^+ GaAs structures,[7] when the relevant currents or electric fields exceed certain values. Hot-electron MPR behaviors in 3D systems have been studied extensively and a new type of conduction (relaxation) mechanism[6,7] has been proposed. Recently there has, however, been a concerted attention given to the lower-dimensional systems. The linear and nonlinear transport properties of these systems have already been studied in a number of papers.[9–23] Warmenbol, Peeters, and Devreese[9] studied MPR effects in the 2D system (formed in a single heterojunction) theoretically in the framework of the momentum-balance equation. Mori *et al.*[10] also studied the same system using the Kubo formula and the Fang-Howard trial function. Concerning the hot-electron (nonlinear) MPR in quasi-2D quantum-well (superlattice) structures, to the best of our knowledge, we are not aware of theoretical work other than that of Ref. 16 and are still at an initial stage both experimentally and theoretically. It is therefore desired to develop a theory which could analyze MPR effects in quasi-2D quantum-well structures, ranging from very small (linear regime) to large electric fields (hot-electron regime).

The purpose of the present paper is to develop a theory of hot-electron MPR in quasi-2D semiconductor quantum-well (so-called the HEMT-type) structures, starting from the field-dependent conductivity formula[24] defined in the Ohm's-law form of the nonlinear current density, and to study the physical trends of the MPR effects in such structures. In order to obtain the analytical expression, we employ a simple model for a quantum-well structure, but it leads to correct physical trends of MPR effects of a quasi-2DEG (electron gas) interacting with phonons in a quantum-well structure, assuming the interaction with polar LO phonons is the dominant scattering mechanism. We evaluate the field-dependent transverse magnetoconductivity $\sigma_{xx}(F)$ for the quasi-2DEG confined in the quantum-well structure subjected to the crossed electric and magnetic fields, where a magnetic field **B** (in the z direction) is perpendicular to the barriers of the well and the electric field **F** (in the x direction) is along the lateral direction of their wall.

Our results show an oscillatory behavior of the relaxation rate and hence the conductivity as a function of the applied magnetic field with period $1/B$, showing MPR effects when $\omega_c\tau(F)\gg 1$ is satisfied. Here $\tau(F)$ is the field-dependent relaxation time. By increasing the electric field, those MPR maxima given by Eq. (1.1) for ordinary MPR in the conductivity (and hence electric current) can convert into the minima and vice versa, as in the case of 3D systems.[6,7] The electric-field-induced MPR (EFIMPR) in the hot-electron regime takes place when the condition $P\omega_c=\omega_L^*$ is satisfied contrary to the ordinary MPR condition (1.1). The EFIMPR peaks shifted to the higher B side from the ordinary MPR peaks are predicted. The "ordinary" and "hot electron" refer to the linear-response regime (very small applied electric field) and to the nonlinear, non-Ohmic regime (large applied electric field), respectively. Splitting of the MPR peaks (i.e., the appearance of subsidiary peaks) and the additional oscillations, which are attributed to *interelectric subband scattering,* are predicted in the conductivity component $\sigma_{xx}(F)$ (and hence the x component of the electric current) even in the linear (weak field) as well as the hot-electron (large electric field) regime when the relevant subband-energy separations are available for interelectric subband transitions. It is noted that the origin of these splittings in the MPR peaks in quasi-2D quantum-well structures is due to the interelectric subband scattering and is different from those in 3D systems,[6,7] where splitting occurs only in the hot-electron (high-field) regime and is due to the electric-field-induced inter-Landau-level scattering by LO phonons.[6] Finally, the (field-dependent) relaxation rate is inversely proportional to the thickness L_z of the well, and the $1/L_z^2$ dependence of $\sigma_{xx}(F)$ (or the electric current density) is predicted. The former result is in agreement with Refs. 13 and 17, whereas the latter result supports the finding of Vasilopoulos, Charbonneau, and Van Vliet[16] for the GaAs quantum well.

The paper is organized as follows. In Sec. II the model system in quantum-well structures is clarified and the field-dependent dc conductivity $\sigma_{xx}(F)$ formula defined in the Ohm's-law form of nonlinear electric current is presented. Field-induced relaxation processes and the intracollisional field effects are discussed in connection with the EFIMPR effects. In Sec. III the field-induced relaxation rate for bulk polar-LO-phonon scattering in the quasi-2D quantum-well structure is evaluated, including the effect of collision broadening of the Landau levels. The magnetophonon resonances under high electric fields (EFIMPR) as well as the ordinary MPR are discussed for such a system, where the special attention is given to the shift and splitting of the MPR peaks. Concluding remarks are given in Sec. IV. In the Appendix, the explicit expression for the MPR broadening parameter is derived for polar-LO-phonon scattering.

II. ELECTRIC-FIELD-DEPENDENT MAGNETOCONDUCTIVITY FOR QUANTUM-WELL STRUCTURES

A. Preliminaries: Model and basic formulas

We consider the high-field transport of electron gas in a quasi-two-dimensional quantum-well structure, where a static magnetic field $\mathbf{B}(\|\hat{z})$ and a dc electric field $\mathbf{F}(\|\hat{x})$ are, respectively, applied perpendicularly to the barriers of the potential well (such as realized in the hetero interface) and along the lateral direction of their walls. For the sake of simplicity, the quantum well is modeled by a rectangular potential well of infinite depth and width L_z. Applying the effective-mass approximation for conduction electrons confined in the quantum well and taking the z abscissa origin at one interface, the one-particle Hamiltonian (h_F) for such electrons subject to the crossed electric (**F**) and magnetic (**B**) fields, its normalized eigenfunctions ($\langle \mathbf{r}|\lambda\rangle$) and eigenvalues ($E_\lambda$), in the Landau gauge of vector potential **A**, are respectively, given by[16]

$$h_F=(\mathbf{p}+e\,\mathbf{A})^2/2m^*+eFx,\quad \mathbf{A}=(0,Bx,0)\ , \tag{2.1}$$

$$\langle \mathbf{r}|\lambda\rangle=\langle \mathbf{r}|N,n,k_y\rangle =(2/L_yL_z)^{1/2}\phi_N(x-x_\lambda)\times\exp(ik_yy)\sin(k_zz)\ , \tag{2.2}$$

$$k_z:=n\pi/L_z,\quad n=1,2,3,\ldots;\quad N=0,1,2,\ldots\ , \tag{2.2a}$$

$$\phi_N(x)=(1/2^N\pi^{1/2}l^BN!)^{1/2}\exp(-x^2/2l_B^2)H_N(x/l_B)\ , \tag{2.2b}$$

$$E_\lambda=E_{Nn}(k_y) =(N+\tfrac{1}{2})\hbar\omega_c+\epsilon_n(k_z)-\hbar V_dk_y-m^*V_d^2/2\ , \tag{2.3}$$

$$\epsilon_n(k_z):=\hbar^2k_z^2/2m^*=:\epsilon_0n^2,\quad \epsilon_0:=\hbar^2\pi^2/2m^*L_z^2\ , \tag{2.3'}$$

where $\omega_c:=eB/m^*$ is the cyclotron frequency, m^* is the effective mass (assumed spherical) of a conduction electron with the electric charge $-e(e>0)$, and $\phi_N(x-x_\lambda)$ represents harmonic-oscillator wave functions, centered at $x=x_\lambda:=-l_B^2(k_y+eF/\hbar\omega_c)$. k_y is the wave vector in the y direction. It is noted that the wave function in the z direction is assumed to vanish at $z=0$ and L_z. Here $l_B:=(\hbar/m^*\omega_c)^{1/2}$ is the radius of the ground-state electron orbit in the (x,y) plane. N and n denote the Landau- and subband-level indices, respectively. It should be noted that the electron energy spectrum in the quasi-2D quantum well is level quantized in the z direction and is given as a function of L_z. $V_d(:=F/B)$ is the center drift velocity of the electron. We shall occasionally designate a set of quantum numbers (N,n,k_y) by a greek letter λ. $\lambda\pm1$ will then indicate the state $(N\pm1,n,k_y)$. x_λ is the center of cyclotron orbit corresponding to the particular state (N,n,k_y). The dimensions of the sample are assumed to be $\Omega:=L_xL_yL_z=:A_0L_z$, where A_0 is the interface area. In the following treatment, we assume the vibrational spectrum in the quasi-two-dimensional quantum-well structure is identical with that in a bulk material, i.e., that the phonons, to a first approximation, are not affected by the presence of the quantum well. Deviations from this bulk behavior, such as interface modes or slab modes, are neglected. The electron-phonon interaction Hamiltonian is then generally expressed by[25]

$$h_{e\text{-ph}}=\sum_{\mathbf{q}}[C(\mathbf{q})b_{\mathbf{q}}e^{i\mathbf{q}\cdot\mathbf{r}}+C^*(\mathbf{q})b_{\mathbf{q}}^\dagger e^{-i\mathbf{q}\cdot\mathbf{r}}]\ , \tag{2.4}$$

where $b_{\mathbf{q}}^\dagger$ and $b_{\mathbf{q}}$ are, respectively, the creation and annihilation operators for phonons with wave vector $\mathbf{q}$. $C(\mathbf{q})$ denotes the Fourier component of the electron-phonon coupling potential, the form of which depends on the type of interaction.

For the calculations of the field-dependent conductivity component $\sigma_{xx}(F)$ [cf. Ref. 24, Eq. (3.18)] for the quasi-2DEG system subjected to the crossed electric and magnetic fields, we need the following matrix elements in the representation (2.2):

$$|\langle\lambda|j_x|\lambda'\rangle|^2=(el_B\omega_c/\sqrt{2})^2[(N+1)\delta_{\lambda',\lambda+1}+N\delta_{\lambda',\lambda-1}]\ , \tag{2.5}$$

$$\delta_{\lambda',\lambda\pm1}:=\delta_{N',N\pm1}\delta_{n',n}\delta_{k_y',k_y}\ , \tag{2.5$'$}$$

$$|\langle\lambda|\exp(\pm i\mathbf{q}\cdot\mathbf{r})|\lambda'\rangle|^2=|J_{NN'}(x_\lambda,\pm q_x,x_{\lambda'})|^2 \times|F_{nn'}(\pm q_z)|^2\delta_{k_y,k_y'\pm q_y}\ . \tag{2.6}$$

In Eq. (2.5), j_x is the x component of a single-electron current operator and the Kronecker symbol (2.5′) expresses the selection rule, which arises during the integration of the matrix element with respect to y and z. The overlap integrals $J_{NN'}$ and $F_{nn'}$ in Eq.(2.6) are, respectively, defined as

$$J_{NN'}(x_\lambda,\pm q_x,x_{\lambda'}) :=\int_{-\infty}^{\infty}\phi_N^*(x-x_\lambda)e^{\pm iq_xx}\phi_{N'}(x-x_{\lambda'})dx\ , \tag{2.6a}$$

$$F_{nn'}(\pm q_z):=(2/L_z)\int_0^{L_z}e^{\pm iq_zz}\sin(n\pi z/L_z) \times\sin(n'\pi z/L_z)dz\ . \tag{2.6b}$$

$|J_{NN'}(x_\lambda,\pm q_x,x_{\lambda'})|^2$ and $|F_{nn'}(\pm q_z)|^2$ are then, respectively, given by

$$|J_{NN'}(x_\lambda,\pm q_x,x_{\lambda'})|^2 =|J_{NN'}(u)|^2=\frac{N_n!}{N_m!}e^{-u}u^{N_m-N_n}[L_{N_n}^{N_m-N_n}(u)]^2\ , \tag{2.7}$$

with $N_n:=\min\{N,N'\}$ and $N_m:=\max\{N,N'\}$, where

$$u:=l_B^2q_\perp^2/2,\quad q_\perp:=(q_x^2+q_y^2)^{1/2}\ ,$$

and $L_N^M(u)$ are Laguerre polynomials,[26] and by

$$|F_{nn'}(\pm q_z)|^2=|F_{nn'}(t)|^2=\begin{Bmatrix}\sin(t)\\ \cos(t)\end{Bmatrix}^2\frac{(\pi^2nn')^2t^2}{\{t^2-[(\pi/2)(n-n')]^2\}^2\{t^2-[(\pi/2)(n+n')]^2\}^2}\ , \tag{2.8}$$

where $t:=L_zq_z/2$. The derivation of the above expressions proceeds as in the case of the usual Landau wave functions when $\sin(k_zz)$ in Eq. (2.2) is replaced by $\exp(ik_zz)$. It should be noted that the upper sin( ) is for n and n' both even or both odd, the lower cos( ) is for one of them even and the other odd; hence for interelectric subband scattering $(n'\neq n)$, the term with cos( ) must be taken in Eq. (2.8).

The general expression for the electric-field-dependent dc conductivity $\sigma_{i,j}(F)\,(i,j=x,y,z)$, which is defined in the Ohm's-law form of nonlinear electric current density, for an electron-phonon system has been derived in Ref. 24, assuming that phonons remain at thermal equilibrium (a situation that can often be arranged).[25] This nonlinear version of the conductivity formula (based on one-particle resolvent superoperator theory) is reduced to the Kubo formula[27] for dc conductivity when $\lim_{F\to0}\sigma_{i,j}(F)$ and is the basis for the present theory. When a uniform electric field is applied in the x direction, the quasi-2D version of the field-dependent transverse magnetoconductivity $\sigma_{xx}(F)$ can be evaluated from Eq. (3.18) of Ref. 24. After performing the sum over the λ_2 state with the use of Eq. (2.5), $\sigma_{xx}(F)$ can be readily expressed in the representation (2.2) as

$$\sigma_{xx}(F)=\frac{e^2l_B^2\omega_c}{\Omega}\sum_\lambda(N+1)[f(E_\lambda^0)-f(E_{\lambda+1}^0)] \times A_{\lambda,\lambda+1}(F)\ , \tag{2.9}$$

where the spectral density $A_{\lambda,\lambda+1}$ is given by

$$A_{\lambda,\lambda+1}(F)=\frac{\Gamma_{\lambda,\lambda+1}(F)}{[E_{\lambda+1}-E_\lambda+\Delta_{\lambda,\lambda+1}(F)]^2+[\Gamma_{\lambda,\lambda+1}(F)]^2} . \tag{2.9$'$}$$

It is noted that the quantities Γ and Δ play a role of the width and the shift in the spectral line shape, respectively, and depend on the strength of the applied electric and magnetic fields. Equation (2.9) with (2.9′) is valid irrespective of the strength of the electric field F as well as the magnetic field B since the entire effect of the fields is included in the eigenstate energies E_λ of the Hamiltonian h_F. As seen from Eq. (2.9′), if there is no collision (i.e., $\Gamma,\Delta\rightarrow 0$), the spectral density and hence the conductivity component (2.9) diverges as expected when the magnetic field is absent. In the presence of collisions, assuming $\Gamma_{\lambda,\lambda+1},\Delta_{\lambda,\lambda+1}\ll\hbar\omega_c(=E_{\lambda+1}-E_\lambda)$, which is usually satisfied and which is in fact the condition to observe the oscillatory behavior of electric-field-induced magnetophonon resonances as will be discussed in Sec. III, the field-dependent transverse magnetoconductivity (2.9) for the relaxive transport can then be approximated as

$$\sigma_{xx}^{\rm osc}(F)\approx\frac{e^2 l_B^2}{\hbar^2\omega_c\Omega}\sum_\lambda (N+1)[f(E_\lambda^0)-f(E_{\lambda+1}^0)] \times\Gamma_{\lambda,\lambda+1}(F) . \tag{2.10}$$

If $\hbar\omega_c\leq\Gamma_{\lambda,\lambda+1}$, the oscillatory structure of the spectral density and hence of $\sigma_{xx}(F)$ due to the EFIMPR dissolves. A spatially uniform distribution $f(E)$ is strictly a Fermi-Dirac distribution function, which can be replaced by the Boltzmann distribution for nondegenerate semiconductors, viz.,

$$f(E_\lambda^0)=:f_{N,n}(k_z)\approx\exp[\beta_e(\zeta-E_{Nn}^0)] , \tag{2.11}$$

where $E_{Nn}^0=(N+\frac{1}{2})\hbar\omega_c+\epsilon_n(k_z)$, $\beta_e:=1/k_B T_e$ with k_B being Boltzmann's constant, T_e the "hot-electron" temperature, and ζ the Fermi energy. It should be noted that Eq. (2.11) is independent of an applied electric field (instead, which depends on the effective electron temperatures T_e in the hot-electron regime) and is spatially uniform (k_y independent) since we are considering a uniform system.[24] A field-dependent Γ is a measure of field-induced electronic relaxation processes and the (field-dependent) energy relaxation (or collision) time $\tau(F)$ can be estimated from $\tau\approx\hbar/\Gamma$. Hereafter, $\Gamma_{\lambda_1\lambda_2}$ is referred to as the relaxation rate associated with the states λ_1 and λ_2. It should be noted that the field-dependent Γ is responsible for nonlinearities with respect to the electric field. This field-induced electronic relaxation process is known as the intracollisional field effect.[5] The effect of the applied electric field on the relaxation (or collision) processes can therefore be studied theoretically by examining $\Gamma(F)$. Except for the Ohmic conduction it is not permissible for hot-electron transport to expand it in powers of F. The detailed derivation of this quantity and its general expression in the lowest-order approximation for the scattering processes can be seen in Ref. 24. Since $|\lambda\rangle=|N,n,k_y\rangle$, the summation over λ means $\sum_\lambda=\sum_{N,n,k_y}$, where the one summation over k_y with periodic boundary conditions will be transformed into the integral as

$$\sum_{k_y}(\cdots)=\frac{L_y}{2\pi}\int_{-L_x/2l_B^2-eF/\hbar\omega_c}^{L_x/2l_B^2-eF/\hbar\omega_c} dk_y(\cdots) . \tag{2.12}$$

The upper and lower limits are obtained from the facts that electrons should be within the crystal dimensions in the x direction, i.e., $-L_x/2\leq x\leq L_x/2$ and that the functions $\phi_N(x-x_\lambda)$ are centered at $x_\lambda=-l_B^2(k_y+eF/\hbar\omega_c)$. Here L_x is assumed to be much larger than l_B. It is noted that in the high-field regime, the energy gained between collisions (intercollisional field effect) and that gained during each collision (ICFE) are both important; the former effect is entered through the energy difference terms whereas the latter through $\Gamma(F)$ via the eigenstate energies E_λ of the Hamiltonian h_F in the energy-conserving δ functions in Γ as will be shown explicitly in Sec. II B. Although these effects due to the applied electric field are seen implicitly in Eq. (2.9) with Eq. (2.9′), we defer the discussion of the field-induced relaxation (or collision) processes for the specific interactions until Sec. III. Equation (2.9) with Γ evaluated from Eq. (2.13) in Sec. II B is strictly valid when the scatterers (phonons) remain at equilibrium.[24] As an approximation, however, it could be used for the case when phonons are not at equilibrium, i.e., for not too strong electric field. Finally, we note that the field-dependent (transverse magneto) conductivity $\sigma_{xx}(F)$ is, as seen from Eqs. (2.9) or (2.10), related to the relaxation rate $\Gamma_{\lambda,\lambda+1}(F)$ and hence the collisional (relaxive) conduction process is associated with the electronic transition between the states λ and $\lambda+1$ in the scattering (collision) processes. In other words, electric currents are induced by those electrons which hop successively between magnetically and quantum mechanically localized states by absorbing the field energy during collision (phonon emission and/or absorption) processes as a direct consequence of their interaction with phonons, leading to changes in the electron mobility. Accordingly the electronic transport properties (e.g., electronic relaxation processes, ICFE, ordinary and hot-electron magnetophonon resonances, etc.) in the quasi-2D quantum-well structures can be studied by examining the behavior of Γ as a function of relevant physical parameters introduced in the theory.

B. Field-induced relaxation processes and intracollisional field effect

In order to evaluate the field-dependent transverse magnetoconductivity (2.9) or (2.10), it is necessary to calculate the field-dependent relaxation rate $\Gamma(F)\,[\approx\hbar/\tau(F)]$, which is a measure of the electronic relaxation due to the collisions between an electron and phonons including the effect of electric fields during each collision (ICFE).[5] In this section, the general expression for the quasi-two-dimensional version of the relaxation rate $\Gamma_{\lambda_1,\lambda_2}(F)$ is explicitly given for any electron-phonon coupling within the first-order Born approximation of the scattering processes. When an electron gains the energy from the field during each collision process (ICFE), the relaxation rate $\Gamma_{\lambda_1,\lambda_2}$ associated with the electronic transitions between the states $|\lambda_1\rangle$ and $|\lambda_2\rangle$, accompanied by the absorption or emission of a phonon, is generally given by taking the imaginary part of the field-dependent irreducible electron self-energy [cf. Ref. 24, Eq. (3.17a)]. Within the first-order Born approximation in the scattering processes, the general form of the field-dependent Γ for an electron-phonon system is given by Eq. (3.19a) of Ref. 24. Using the representation given by Eq. (2.2), the quasi-2D version of this quantity associated with the electronic transition between the states $|\lambda_1\rangle$ and $|\lambda_2\rangle$ can be evaluated as

$$\Gamma_{\lambda_1,\lambda_2}(F)=\frac{A_0}{2l_B^2}\sum_{\mathbf{q}}\sum_{N_3,n_3}|C(\mathbf{q})|^2|J_{N_2N_3}(u)|^2|F_{n_2n_3}(q_z)|^2\{(N_\mathbf{q}+1)\delta[E_{N_1n_1}(k_{1y})-E_{N_3n_3}(k_{2y}-q_y)-\hbar\omega_\mathbf{q}]$$
$$+N_\mathbf{q}\delta[E_{N_1n_1}(k_{1y})-E_{N_3n_3}(k_{2y}+q_y)+\hbar\omega_\mathbf{q}]\}$$
$$+\frac{A_0}{2l_B^2}\sum_{\mathbf{q}}\sum_{N_3,n_3}|C(\mathbf{q})|^2|J_{N_3N_1}(u)|^2|F_{n_3n_1}(q_z)|^2\{(N_\mathbf{q}+1)\delta[E_{N_3n_e}(k_{1y}-q_y)-E_{N_2n_2}(k_{2y})+\hbar\omega_\mathbf{q}]$$
$$+N_\mathbf{q}\delta[E_{N_3n_3}(k_{1y}+q_y)-E_{N_2n_2}(k_{2y})-\hbar\omega_\mathbf{q}]\}\ , \tag{2.13}$$

where the summation over the intermediate states k_{3y} has already been carried out by using the relation (2.12). It should be noted that the relaxation rate $\Gamma_{\lambda_1,\lambda_2}(F)$ is associated with the electronic transitions between the states $|\lambda_1\rangle$ and $|\lambda_2\rangle$ due to the electric-field-induced inelastic scattering in the collision processes. Here $N_\mathbf{q}$ is the equilibrium distribution function for a phonon with momentum $\hbar\mathbf{q}$ and energy $\hbar\omega_\mathbf{q}$:

$$N_\mathbf{q}=[\exp(\beta\hbar\omega_\mathbf{q})-1]^{-1}\ , \tag{2.14}$$

where $\beta:=1/k_BT$ with T being the (lattice) temperature. It should be noted that the δ functions in Eq. (2.13) express the law of energy conservation in one-phonon collision (absorption and emission) processes, where the effect of the electric field (ICFE) is entered exactly through the exact eigenstate energy E_λ of an electron [cf. Eq. (2.3)]. The strict energy-conserving δ functions in Eq. (2.13) imply that when the electron undergoes a collision by absorbing the energy from the field, its energy can only change by an amount equal to the energy of a phonon involved in the transitions. This in fact leads to electric-field-induced magnetophonon resonances, whereby $\hbar\omega_c \gg \Gamma(F)$ [or $\omega_c\tau(F)\gg 1$] is satisfied. In other words, the EFIMPR in the quasi-2D quantum-well structure is due essentially to the *electric-field-induced inter-Landau-level (inelastic resonant phonon) scattering* analogous to the bulk situations studied by Mori *et al.*[6] If we neglect the field dependence ($F\to 0$) in Eq. (2.13), this reduces to the usual case where collisions are instantaneous and the result exhibits the usual phonon emission and absorption processes seen in the imaginary part of the lowest-order irreducible electron self-energy obtained from the Green's function approach,[28] giving rise to the ordinary MPR for the Ohmic (weak-field) case,[15] where the ICFE is not effective.

III. MAGNETOPHONON RESONANCES UNDER HIGH ELECTRIC FIELDS

The general expression (2.13) for the field-dependent relaxation rate in the quasi-two-dimensional version will be used to evaluate $\Gamma_{\lambda,\lambda+1}(F)$ for a specific electron-phonon interaction, explicitly. In this paper, we consider the scattering of electrons by polar LO phonons. For the scattering by randomly distributed impurities, an expression for the impurity collision time $\tau(F)\,(\approx\hbar/\Gamma)$ is simply obtained from Eq. (2.13) by discarding the factors $N_\mathbf{q}$ and $N_\mathbf{q}+1$, discarding the phonon energy $\hbar\omega_\mathbf{q}$ in the argument of the δ functions, and also replacing $|C(\mathbf{q})|^2$ by $N_i|V(\mathbf{q})|^2$, where $V(\mathbf{q})$ is the Fourier component of the scattering potential and N_i is the number of impurities. We shall consider the impurity effect to the field-induced relaxation processes and the effect of screening in a quantum well elsewhere.

A. Polar optical-mode phonon scattering

The Fourier component of the interaction potential $C(\mathbf{q})$ for polar-LO-phonon scattering may be given by the Fröhlich interaction potential[28]

$$|C(\mathbf{q})|^2=\frac{r\pi\alpha\hbar(\hbar\omega_L)^{3/2}}{\Omega(2m^*)^{1/2}q^2}=:\frac{D}{\Omega q^2}=\frac{D}{\Omega(q_\perp^2+q_z^2)}\ . \tag{3.1}$$

Here D is the constant of the polar interaction; α is the dimensionless (polaron) coupling constant given by

$$\alpha\equiv\frac{e^2}{4\pi\hbar}\left[\frac{m^*}{2\hbar\omega_L}\right]^{1/2}\left[\frac{1}{\kappa_\infty}-\frac{1}{\kappa_0}\right]\ , \tag{3.1'}$$

where κ_0 and κ_∞ are the static and high-frequency dielectric constants of the material, respectively. We assume that the phonons are those in a bulk (i.e., three-dimensional) and those that are dispersionless (i.e., $\hbar\omega_\mathbf{q}\approx\hbar\omega_L\approx$const, where ω_L is the polar-LO-phonon frequency).

Let us proceed with the evaluation of $\Gamma_{\lambda,\lambda+1}(F)$ for the polar-LO-phonon scattering. Using the interaction potential given by Eq. (3.1) in Eq. (2.13), the quasi-2D version of the relaxation rate $\Gamma_{\lambda,\lambda+1}$ for polar-LO-phonon scattering can be written as

$$\Gamma_{\lambda,\lambda+1}(F)=\frac{A_0D}{2(2\pi)^3l_B^2}\sum_{N'n'}\int_{-\infty}^{\infty}dq_x\int_{-\infty}^{\infty}dq_y I_{nn'}(q_\perp)|J_{NN'}(u)|^2\{(N_0+1)\delta[(N-N'-1)\hbar\omega_c+\hbar\omega_{nn'}-\hbar V_d q_y-\hbar\omega_L]$$
$$+N_0\delta[(N-N'-1)\hbar\omega_c+\hbar\omega_{nn'}+\hbar V_d q_y+\hbar\omega_L]\}$$
$$+\frac{A_0D}{2(2\pi)^3l_B^2}\sum_{N'n'}\int_{-\infty}^{\infty}dq_x\int_{-\infty}^{\infty}dq_y I_{nn'}(q_\perp)|J_{N'N}(u)|^2\{(N_0+1)\delta[(N'-N-1)\hbar\omega_c-\hbar\omega_{nn'}+\hbar V_d q_y+\hbar\omega_L]$$
$$+N_0\delta[(N'-N-1)\hbar\omega_c-\hbar\omega_{nn'}-\hbar V_d q_y-\hbar\omega_L]\}\ , \tag{3.2}$$

where $\omega_{nn'}:=(n^2-n'^2)\epsilon_0/\hbar$, N_0 is the polar-LO-phonon distribution function given by Eq. (2.14) with $\omega_{\mathbf{q}}=\omega_L$, and $I_{nn'}(q_\perp)$ is given by

$$I_{nn'}(q_\perp):=\int_{-\infty}^{\infty}\frac{|F_{nn'}(q_z)|^2}{q_z^2+q_\perp^2}dq_z$$
$$=\pi L_z\left[\frac{1+\delta_{nn'}}{[(n-n')^2\pi^2+q_\perp^2L_z^2]}+\frac{1}{[(n+n')^2\pi^2+q_\perp^2L_z^2]}\right](1-\epsilon_{nn'}) \tag{3.3}$$

and

$$\epsilon_{nn'}:=\frac{q_\perp L_z[1\mp\exp(-q_\perp L_z)]}{[(n-n')^2\pi^2+q_\perp^2L_z^2][(n+n')^2\pi^2+q_\perp^2L_z^2]}\,\frac{32\pi^2n^2n'^2}{\{[(n-n')^2\pi^2+q_\perp^2L_z^2]+(1+\delta_{nn'})[(n+n')^2\pi^2+q_\perp^2L_z^2]\}}\ . \tag{3.3'}$$

In the above expression, the upper sign has to be taken if n and n' are both even or both odd, and the lower sign if n is odd and n' is even or vice versa. To derive Eq. (3.2), we have transformed the sum over $\mathbf{q}$ into the integral form in a usual way. It should be noted that N' and n' indicate intermediate Landau and subband states, respectively. Equation (3.2) is a basis for further calculations of the field-induced relaxation rate $\Gamma_{\lambda,\lambda+1}(F)$ (for polar-LO-phonon scattering) in the quasi-2D quantum-well structure.

The evaluation of Γ in Eq. (3.2) involves the further integrations either with respect to $q_\perp$ (or u) and θ in the cylindrical coordinates, or to q_x and q_y in the Cartesian coordinates. The $q_\perp$ dependence of the matrix element means that we have to take full account of directional dependence implicit in the energy conserving δ functions in Eq. (3.2). The integral over q_y can be done immediately, but the resulting integral over q_x must be done separately for each N and N', and is very difficult to evaluate analytically. To simplify the calculations, we replace $\hbar V_d q_y$ (as is often done[16]) in the argument of the δ functions by the potential-energy difference $eF\Delta\bar{x}$ across the spatial extent $\Delta\bar{x}$ of a Landau state, where $\Delta\bar{x}$ is a constant of the order of the magnetic length l_B. This approximation is equivalent to assuming an effective phonon momentum as $\hbar q_y\approx eB\Delta\bar{x}$. It is then convenient to evaluate Γ in the cylindrical coordinates: the integral over the $\mathbf{q}$ space in Eq. (3.2) can be reduced to the integrals with respect to θ and $q_\perp$ (or u), where the θ integration gives 2π. Therefore, Eq. (3.2) takes the form

$$\Gamma_{\lambda,\lambda+1}(F)\approx\frac{\Lambda}{\hbar}\sum_{N'n'}\int_0^{\infty}du\ I_{nn'}(q_\perp)|J_{NN'}(u)|^2\{(N_0+1)\delta[(N'-N+1)\omega_c-\omega_{nn'}+\omega_L^*]+N_0\delta[(N'-N+1)\omega_c-\omega_{nn'}-\omega_L^*]\}$$
$$+\frac{\Lambda}{\hbar}\sum_{N'n'}\int_0^{\infty}du\ I_{nn'}(q_\perp)|J_{NN'}(u)|^2\{(N_0+1)\delta[(N'-N-1)\omega_c-\omega_{nn'}+\omega_L^*]$$
$$+N_0\delta[(N'-N-1)\omega_c-\omega_{nn'}-\omega_L^*]\}\ , \tag{3.4}$$

where $\Lambda:=A_0D/8\pi^2l_B^4$, $\omega_L^*(:=\omega_L+eF\Delta\bar{x}/\hbar)$ is an effective phonon frequency, and the exact overlap integral $I_{nn'}(q_\perp)$ is given by Eq. (3.3). Now that the directional dependence implicit in the δ functions in Eq. (3.2) is suppressed by introducing the effective phonon frequency as seen in Eq. (3.4), the integral over $q_\perp$ (and hence u) can be easily evaluated analytically for the electronic transport in the (x,y) plane. If N' is very large, we may approximate $N'\pm1\approx N'$. Setting $N'-N=-P$ in the emission term and $N'-N=P$ in the absorption term, Eq. (3.4) can be written in a simple form:

$$\Gamma_{\lambda,\lambda+1}(F)\approx\frac{2\Lambda}{\hbar}\sum_{n'P}\int_0^{\infty}du\ I_{nn'}(q_\perp)[|J_{N,N-P}(u)|^2(N_0+1)\delta(-P\omega_c-\omega_{nn'}+\omega_L^*)$$
$$+|J_{N,N+P}(u)|^2N_0\delta(+P\omega_c-\omega_{nn'}-\omega_L^*)]\ . \tag{3.5}$$

In this paper, we shall consider the transport in the (x,y) plane, i.e., only the case for large $q_\perp$ since the electric field is in the x direction, we expect the largest contribution to the current comes from the processes involving large momentum transfer in the x direction, i.e., those processes with larger q_x and consequently large $q_\perp$ and small q_z. In this case, the exact expression $I_{nn'}(q_\perp)$ given by Eq. (3.3) reduces to the result of Vasilopoulos, Charbonneau, and Van Vliet:[16]

$$I_{nn'}(q_\perp)\approx\frac{\pi}{q_\perp^2 L_z}(2+\delta_{n'n}) \tag{3.6}$$

for $q_\perp \gg q_z$. Substituting Eq. (3.6) into Eq. (3.5) and noticing that[16,26]

$$\int_0^\infty |J_{N,N\pm P}(u)|^2 u^{-1}du=1/P,\quad P=1,2,3,\ldots \tag{3.7}$$

we obtain Eq. (3.5) associated with the transport in the (x,y) plane as

$$\Gamma_{\lambda,\lambda+1}(F)\approx 3\Lambda'\sum_P (2N_0+1)\delta(P-\omega_L^*/\omega_c)/P$$
$$+2\Lambda'\sum_{n'(\neq n)}\sum_P\{[(N_0+1)\delta(P-(\omega_L^*-\omega_{nn'})/\omega_c)+N_0\delta(P-(\omega_L^*+\omega_{nn'})/\omega_c)]/P\}\ , \tag{3.8}$$

where $\Lambda' := (\pi l_B^2/L_z\hbar\omega_c)\Lambda$, which has a dimension of energy. Applying Poisson's summation formula[29] for the sum $\sum_P$ in Eq. (3.8), we obtain

$$\Gamma_{\lambda,\lambda+1}(F)\approx 3\Lambda'\left[\frac{(2N_0+1)}{x}\left[1+2\sum_{s=1}^{\infty}\cos(2\pi sx)\right]\right]$$
$$+2\Lambda'\sum_{n'(\neq n)}\left[\frac{N_0+1}{x(1-y)}\left[1+2\sum_{s=1}^{\infty}\cos[2\pi sx(1-y)]\right]+\frac{N_0}{x(1+y)}\left[1+2\sum_{s=1}^{\infty}\cos[2\pi sx(1+y)]\right]\right]\ , \tag{3.9}$$

where $x:=\omega_L^*/\omega_c$ and $y:=\omega_{nn'}/\omega_L^*$. As seen from Eq. (3.9), the oscillatory inverse relaxation time $1/\tau(:=\Gamma/\hbar)$ shows the singular behavior, which is traced back to the δ-function singularities as seen in Eq. (3.8). Therefore, the electric-field-dependent transverse magnetoconductivity (2.10) shows the *resonant* behaviors: electric-field-induced magnetophonon resonances at $P\omega_c=\omega_L^*$ and at $P\omega_c=\omega_L^*\pm\omega_{nn'}$ (P is an integer). Those resonances involving the terms $\omega_{nn'}$, which arise from the interelectric virtual subband transitions between n and n', reflect the subband structure. The relaxation rate for polar-LO-phonon scattering diverges whenever the above conditions are satisfied. These divergences are physically attributed to the hybrid quantization of the electron energy spectrum: the appearance of Landau levels due to the presence of a magnetic field and of electronic subbands associated with quantum-well structures. The above conditions for the EFIMPR give the resonance magnetic fields (i.e., the EFIMPR peak positions at) B_P, B_P^+, and B_P^-:

$$B_P=[B_0+(m^*\Delta\bar{x}/\hbar)F]/P\ , \tag{3.10a}$$

$$B_P^{\pm}=B^P\pm m^*\omega_{nn'}/(eP),\quad (n'\neq n) \tag{3.10b}$$

where $B_0(:=m^*\omega_L/e)$ is the fundamental field for the ordinary MPR. It is very interesting to point out that additional EFIMPR peaks (subsidiary peaks) would appear at $B_P^{\pm}$ on both sides of the EFIMPR peaks at B_P whenever the interelectric (nonresonant) virtual subband transitions $(n\rightarrow n')$ can take place for the relevant energy separation between subbands. These subsidiary peaks are characteristics to the quasi-2DEG system in the quantum-well structure. It should be noted that the origin of the appearance of the subsidiary peaks in the 3DEG system[6,9] and the quasi-2DEG system studied here is *not* the same. The subsidiary peaks in the 3DEG system[6,9] appear only in the high-field regime whereas those in the quasi-2DEG system in the quantum-well structure would appear in both the low- and high-field regimes and are due to the interelectric subband scattering (i.e., *interelectric nonresonant virtual subband transitions*). Note that within the approximation made for phonon momentum, the EFIMPR peak positions (B_P and $B_P^{\pm}$) are shifted from the ordinary MPR peaks at $B_P^0(:=B_0/P)$ for a low-field (Ohmic) case by $m^*\Delta\bar{x}F/\hbar\approx m^*F/(eB_0 l_{B_0})$, where l_{B_0} is the cyclotron radius at the magnetic field B_0. The effect of electric fields in the scattering processes (ICFE) in the quasi-2D quantum-well structure is, therefore, to shift the ordinary MPR peak positions to higher magnetic fields. It should be noted that the amplitudes of these peaks depend strongly on (lattice) temperature via the phonon occupation number N_0. It is evident from Eqs. (3.10a) and (3.10b) that the shift of these resonance peaks from the ordinary MPR peaks is proportional to the electric field F.

The unpleasant divergences in Γ [Eq. (3.9)] due to the δ-function singularities (associated with the complete quantization of the electron energy spectrum [cf. Eq. (2.3)] in the presence of a magnetic field) indicate the deficiency of the present theory. These divergences may be removed by including higher-order electron-phonon scattering terms. The simplest way to avoid the divergences is that each δ function in Eq. (3.8) is approximated by Lorentzians of field-induced width and shift zero by introducing a width parameter γ_i. Employing this collision-broadening model,[10,13–15] Eq. (3.8) is then expressed by

$$\Gamma_{\lambda,\lambda+1}(F)\approx 3\Lambda'(2N_0+1)\frac{x}{x^2+(\gamma_1/\hbar\omega_c)^2}\left[1+2\sum_{s=1}^{\infty}e^{-2\pi s(\gamma_1/\hbar\omega_c)}\cos(2\pi s x)\right]$$
$$+2\Lambda'(N_0+1)\sum_{n'(\neq n)}\left[\frac{x(1-y)}{x^2(1-y)^2+(\gamma_2/\hbar\omega_c)^2}\left[1+2\sum_{s=1}^{\infty}e^{-2\pi s(\gamma_2/\hbar\omega_c)}\cos[2\pi s x(1-y)]\right]\right]$$
$$+2\Lambda' N_0\sum_{n'(\neq n)}\left[\frac{x(1+y)}{x^2(1+y)^2+(\gamma_3/\hbar\omega_c)^2}\left[1+2\sum_{s=1}^{\infty}e^{-2\pi s(\gamma_3/\hbar\omega_c)}\cos[2\pi s x(1+y)]\right]\right], \quad (3.11)$$

where the quantities in the large parentheses can be further calculated by applying

$$\Psi(a,b):=1+2\sum_{s=1}^{\infty}e^{-2\pi s a}\cos(2\pi s b) = \frac{\sinh(2\pi a)}{\cosh(2\pi a)-\cos(2\pi b)}, \quad a>0 . \quad (3.12)$$

The exponential factors play a role of the effect of field-induced-collision damping due to the combined effect of scatterings (or collisions) and electric fields (ICFE). If broadening (i.e., the width parameter γ_i) is not included, the damping factors $e^{-2\pi s(\gamma_i/\hbar\omega_c)}$ ($i=1$, 2, and 3), do not appear in Eq. (3.11) [cf. Eq. (3.9)], and the Γ diverges in the EFIMPR as we have seen in Eq. (3.9). It should be noted that as seen in Eq. (3.11), the additional oscillatory structure [the second and third terms in Eq. (3.11)] due to interelectric subband transitions could be seen in the quasi-2D quantum-well structure. It is interesting to note that the relaxation rate (3.11) depends explicitly on the number of the occupied subband n, but *not* on the Landau level index N.

The oscillatory (nonlinear) electric current density $\langle J_x\rangle/\Omega$ can be evaluated from $\langle J_x\rangle/\Omega=\sigma_{xx}^{\rm osc}(F)F$ by making use of Eqs. (2.10) and (3.11). Here $\mathbf{J}(=\sum_k \mathbf{j}^{(k)})$ is the total current operator. Noting that Eq. (3.11) and the Fermi distribution functions in Eq. (2.10) do not explicitly depend on k_y, $\sigma_{xx}^{\rm osc}(F)$ in Eq. (2.10) can be expressed as

$$\sigma_{xx}^{\rm osc}(F)=\frac{e^2}{2\pi\hbar^2\omega_c L_z}\times\sum_{N,n}(N+1)[f_{N,n}(k_z)-f_{N+1,n}(k_z)]\Gamma(F) , \quad (3.13)$$

where we have carried out the one summation with respect to k_y in $\sum_\lambda$ by making use of Eq. (2.12). Provided that $\Delta\bar{x}$ is independent of N and that the f's in Eq. (3.13) are replaced by the Boltzmann distribution (2.11) for nondegenerate semiconductors, we can further perform the sum over N (if N is large) by writing $\sum N e^{-\alpha N}=-(\partial/\partial\alpha)\sum e^{-\alpha N}$ and summing the geometric series. We obtain Eq. (3.13) for nondegenerate semiconductors as

$$\sigma_{xx}^{\rm osc}(F)\approx\frac{e^2\exp(\alpha)}{4\pi\hbar^2\omega_c L_z\sinh(\alpha/2)}\times\sum_n\exp[\beta_e(\zeta-n^2\epsilon_0)]\Gamma(F) , \quad (3.13')$$

where $\alpha:=\beta_e\hbar\omega_c$ and $\Gamma(F)$ is given by Eq. (3.11). As seen from Eq. (3.13'), the amplitude of $\sigma_{xx}^{\rm osc}(F)$ has a significant dependence on electron concentration as well as the electron and lattice temperature. We conclude that Eq. (3.13) [or Eq. (3.13') for nondegenerate semiconductors] with Eq. (3.11) gives a general description of the high-field magnetophonon oscillations in the quasi-2D quantum-well structure but no simple general formula exists for the damping factors γ_i ($i=1,2,3$).

It is seen from Eq. (3.11) that the relaxation rate Γ for polar-LO-phonon scattering is proportional to Λ', which is inversely proportional to the crystal dimension in the z direction, i.e., the thickness of the quantum well L_z. Therefore the oscillatory relaxation rate Γ is inversely proportional to the thickness of the well, while the field-dependent transverse magnetoconductivity (3.13) and the electric current density vary as $1/L_z^2$. These results agree with the theoretical findings obtained from the different approach.[13,16] It should be noted that Eq. (3.11) is tied to the approximation $\hbar V_d q_y\approx eF\Delta\bar{x}$ ($\Delta\bar{x}\approx l_B$), and is strictly valid for the case $q_\perp\gg q_z$. The major contribution to $\Gamma_{\lambda,\lambda+1}(F)$ and hence $\sigma_{xx}^{\rm osc}(F)$ comes from the process involving small momentum transfer along the magnetic field. It should be noted that the oscillation in Eq. (3.11) [and hence in Eq. (3.13) or Eq. (3.13')] is damped at strong electric fields since the γ's are generally dependent on the field strength F (see the Appendix).

1. *Narrow wells*

Let us consider the case where the well width L_z is so small that the energy separation between adjacent subband levels is very large and hence no interelectric subband transition between the levels ϵ_n and $\epsilon_{n'}$ can take place by varying the magnetic or electric field. We assume the case where only intraelectric subband transitions ($n\to n'=n$) are allowed to take place. In this case, $\omega_{nn'}=0$ (i.e., $y=0$) and Eq. (3.11) is simplified considerably. Noting that $\gamma_i=\gamma$ ($i=1$, 2, and 3) for $n'=n$, Eq. (3.11) can be written as

$$\Gamma_{\lambda,\lambda+1}(F)\approx 3\Lambda'(2N_0+1)\frac{x}{x^2+(\gamma/\hbar\omega_c)^2}\Psi\left[\frac{\gamma}{\hbar\omega_c},x\right] , \quad (3.14)$$

where $\Psi(\gamma/\hbar\omega_c, x)$ is given by Eq. (3.12). Here the resonance width γ may be given by Eq. (A2) for small broadening of the Landau levels. (See the Appendix for the simple derivation of an explicit expression for γ.) Therefore, those subsidiary EFIMPR peaks (due to the interelectric subband transitions) do not appear in this case. Equation (3.14) shows that the period of the oscillations is given under the condition of $\omega_L^*/\omega_c = P$ (i.e., $x = P$). Evidently, if the broadening is not included [i.e., $\gamma \to 0$ in Eq. (3.14)], Γ and hence $1/\tau$ diverge at the resonance [i.e., the EFIMPR at B_P given by Eq. (3.10a)]. For vanishingly small electric field (i.e., $F \to 0$), Eq. (3.14) leads to the ordinary MPR at $\omega_L = P\omega_c$ for $\tau\omega_c \gg 1$, and is identical with the result of Vasilopoulos,[15] who applied the same model for the linear (weak field) case. For $\omega_L = P\omega_c$ the cosine factor in Eq. (3.14) [cf. Eq. (3.12)] becomes $\cos(2\pi seF\Delta\bar{x}/\hbar\omega_c)$. Hence, by varying the electric field at the same magnetic field, the ordinary MPR maxima (at $\omega_L = P\omega_c$) in the conductivity can evolve to minima and vice versa. This behavior as well as the shifting of the resonant peaks with increasing electric field is in agreement with the result of Vasilopoulos, Charbonneau, and Van Vliet[16] for GaAs quantum wells and has been observed in polar materials.[7]

2. Wide wells

When the thickness L_z of the well increases, the energy separation between subbands ϵ_n and $\epsilon_{n'}$ becomes closer. Therefore, various (virtual) transitions from n to any n' could take place by varying the electric field. In this case, we have to take into account all possible interelectric subband transitions ($n \to n'$). This means that we have to sum over the possible intermediate states n' in Eq. (3.11). The second and third terms in Eq. (3.11) are difficult to evaluate analytically for all $n'(\neq n)$. For an estimate of the contribution of these terms arising from the intersubband transitions, we shall however take into account only those transitions ($n \to n' = n \pm 1$) to the neighboring subbands $\epsilon_{n\pm 1}$ in addition to the intrasubband transitions ($n \to n' = n$) within the subband ϵ_n. In this case, we can evaluate the result (3.14) plus two additional terms corresponding to an upward ($n' = n + 1$) and a downward ($n' = n - 1$) transition. These additional terms arising from the interelectric subband transitions ($n \to n' = n \pm 1$) are not the same because the subband energy spectrum ϵ_n is proportional to n^2 and is not equidistant. Corresponding to Eq. (3.14) for a narrow well, we obtain from Eq. (3.11) the expression of the field-dependent oscillatory relaxation rate for a wide well as

$$\Gamma_{\lambda,\lambda+1}(F) \approx 3\Lambda'(2N_0+1)\frac{\omega_L^*/\omega_c}{(\omega_L^*/\omega_c)^2+(\gamma/\hbar\omega_c)^2}\Psi\left[\frac{\gamma}{\hbar\omega_c}, \frac{\omega_L^*}{\omega_c}\right] + 2\Lambda'(N_0+1)\sum_{\pm}\frac{\omega_1^\pm/\omega_c}{(\omega_1^\pm/\omega_c)^2+(\gamma/\hbar\omega_c)^2}\Psi\left[\frac{\gamma}{\hbar\omega_c}, \frac{\omega_1^\pm}{\omega_c}\right]$$
$$+ 2\Lambda' N_0 \sum_{\pm}\frac{\omega_2^\pm/\omega_c}{(\omega_2^\pm/\omega_c)^2+(\gamma/\hbar\omega_c)^2}\Psi\left[\frac{\gamma}{\hbar\omega_c}, \frac{\omega_2^\pm}{\omega_c}\right], \tag{3.15}$$

where the functional form of Ψ is given by Eq. (3.12), $\omega_1^\pm := \omega_L^* + (1 \pm 2n)\epsilon_0/\hbar$, $\omega_2^\pm := \omega_L^* - (1 \pm 2n)\epsilon_0/\hbar$, and γ may be given by Eq. (A2). It is interesting to point out that unlike the case for a narrow well, Eq. (3.15) exhibits additional complexity of oscillations due to the interelectric subband transitions characterized by the energy separation between the bottom of each pair of subbands. For simplicity, we have again assumed the same γ for the field-induced collisional damping in the additional associated terms. The interelectric subband transitions (scattering) between the subband ϵ_n and $\epsilon_{n\pm 1}$ could be seen by studying the behavior of the EFIMPR extrema by varying the electric field while keeping magnetic field constant. The possibility of these transitions between the well levels ϵ_n is likely to be realized in wide wells ($L_z \leq 1\,\mu$m) (Ref. 16) at very strong magnetic fields such that $\omega_{nn'} \ll \omega_c$. The subsidiary (EFIMPR) peaks could appear at $P\omega_c = \omega_i^\pm$ ($i = 1$ and 2). It should be noted that these subsidiary peaks and additional oscillations in Eq. (3.15) are attributed to the interelectric subband scattering inherent in a quasi-2DEG system formed in the quantum-well structure. At $P\omega_c = \omega_L$, the factors cos( ) in Eq. (3.15) [cf. Eq. (3.12)] become $\cos(2\pi seF\Delta\bar{x}/\hbar\omega_c)$ for the first term and $\cos\{2\pi s[eF\Delta\bar{x} + (1\pm 2n)\epsilon_0]/\hbar\omega_c\}$ and $\cos\{2\pi s[eF\Delta\bar{x} - (1\pm 2n)\epsilon_0]/\hbar\omega_c\}$ for the second and the third terms, respectively. Therefore, starting from an ordinary magnetophonon extremum corresponding to $F = 0$, and increasing the electric field F while keeping the magnetic field constant, an additional oscillatory behavior of the extremum amplitudes due to the interelectric subband transitions or the conversion of the MPR maxima into minima and vice versa could be seen at very strong magnetic fields such that $\omega_{n,n\pm 1} \ll \omega_c$. Although the present theory supports the results of Ref. 16, to check the validity of the present results, it would be desirable to perform experiments similar to those of Eaves *et al.*[7] for thickness less than 1 μm. Finally, we should notice that the oscillation in the relaxation rate [(3.14) or (3.15)] is damped at strong electric fields since γ, as given by Eq. (A2), is roughly proportional to the field strength F and the oscillatory conductivity is further damped by T_e via the electron distribution function.

IV. CONCLUDING REMARKS

In this paper we have presented a theory of hot-electron MPR and investigated the ordinary (linear) and hot-electron (nonlinear) MPR effects in a HEMT-type structure, where a quasi-2DEG formed in an infinitely deep quantum well is subjected to a magnetic field ($\mathbf{B}\|\hat{\mathbf{z}}$) applied perpendicularly to the barriers. The electric-field-dependent conductivity formula[24] $\sigma_{xx}(F)$, which is defined in the Ohm's-law form of electric current, is the basis for the present theory. In the limit of high magnetic fields, where $\tau\omega_c \gg 1$ is satisfied for small broadening of the Landau levels, the field-dependent transverse magnetoconductivity $\sigma_{xx}(F)$ is directly proportional to the energy relaxation rate $\Gamma_{\lambda,\lambda+1}(F)$ associated with the electronic transitions between the states λ and $\lambda+1$. Thus the behavior of $\sigma_{xx}(F)$ is essentially determined by the behavior of $\Gamma_{\lambda,\lambda+1}(F)$. This relaxation rate of the quasi-2DEG formed in the quantum-well structure was evaluated within the lowest-order approximation of the collision processes for polar-LO-phonon scattering and its behavior (relaxive transport process) was discussed in connection with hot-electron MPR effects.

We found that the $\Gamma_{\lambda,\lambda+1}(F)$ is inversely proportional to the thickness of the layers L_z and hence that $\sigma_{xx}(F)$ has a $1/L_z^2$ dependence. These results agree with the theoretical results[13,16] obtained from the different approach to the same model system although the theoretical result of Ref. 14 shows a $1/L_z$ dependence on $\sigma_{xx}(0)$. The relaxation rate $\Gamma_{\lambda,\lambda+1}(F)$ for polar-LO-phonon scattering shows the quasi-2D EFIMPR at $P\omega_c=\omega_L^*,\omega_L^*\pm\omega_{nn'}$ in the hot-electron (nonlinear) regime and the quasi-2D ordinary MPR at $P\omega_c=\omega_L,\omega_L\pm\omega_{nn'}$ in a weak electric field limit. Here P is an integer. It should be noted that subsidiary peaks could appear if interelectric subband transitions ($n\rightarrow n'$) take place for a relevant energy separation ($\omega_{nn'}\neq 0$) between two subbands ϵ_n and $\epsilon_{n'}$. These divergences are traced back to the δ-function singularity of the density of states. In order to obtain the finite results for $\Gamma_{\lambda,\lambda+1}(F)$ and hence $\sigma_{xx}(F)$, we have introduced collision broadening of the Landau levels in the manner of replacing the δ functions in $\Gamma_{\lambda,\lambda+1}(F)$ by Lorentzians with broadening (damping) parameters γ. The amplitudes of such resonances are therefore expressed in terms of the width parameter γ of the Landau levels, and $\sigma_{xx}(F)$ [$\propto\Gamma_{\lambda,\lambda+1}(F)$] shows oscillatory behaviors. Using the analytical expression for $\sigma_{xx}(F)$ [Eq. (3.13) with Eq. (3.11)] describing the EFIMPR, quasi-2D EFIMPR are calculated, whereby a straightforward application of the collision-broadening parameter γ of Landau levels, the amplitude of such resonances are calculated in terms of the width of the Landau levels. We have obtained the explicit expression of the resonance width γ [cf. Eq. (A2)]. To check the theoretical results, the oscillations in the variation of the relaxation rate and hence of the field-dependent transverse magnetoconductivity as a function of magnetic field should be manifested in the resistivity measurements[25] in the presence of magnetic field for a quasi-2DEG formed in a quantum-well structure. It is noted that our result for $\Gamma_{\lambda,\lambda+1}(F)$ and hence $\sigma_{xx}(F)$ in the nonlinear case is tied to the approximation $\hbar V_d q_y \approx eF\Delta\bar{x}$ ($\Delta\bar{x}\approx l_B$). This led to the additional structure in the EFIMPR, which depends on the strength of the applied electric field.

In conclusion, we have developed a theory of hot-electron MPR and derived the analytical expression describing the MPR effects of quasi-2DEG formed in quantum-well structures, ranging from very small (linear regime) to large electric fields (nonlinear regime). It is hoped that the predictions made by the present theory will need more experimental verifications in the future.

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APPENDIX

An expression for the resonance width γ introduced in Eqs. (3.14) and (3.15) is given explicitly. For the EFIMPR at $\omega_L^*=P\omega_c$, $\Psi(\gamma/\hbar\omega_c,x)$ in Eq. (3.14) is equal to $\coth(\pi\gamma/\hbar\omega_c)$ and Eq. (3.14) takes the form

$$\frac{\pi\gamma}{\hbar\omega_c}=3\left[\frac{\pi^2\Lambda'}{\hbar\omega_c}\right](2N_0+1)\times\frac{\pi x}{(\pi x)^2+(\pi\gamma/\hbar\omega_c)^2}\coth\left[\frac{\pi\gamma}{\hbar\omega_c}\right]. \tag{A1}$$

To obtain the self-consistent equation (A1) for γ, we have approximated $\Gamma_{\lambda,\lambda+1}$ on the left-hand side of Eq. (3.14) as γ, assuming the width parameters γ_i to be the same for all associate states. In this way the value of $\Gamma_{\lambda,\lambda+1}$ on the left-hand side of Eq. (3.14) was made self-consistent with $\Gamma_{\lambda,\lambda+1}\approx\gamma$. It should be noted that Eqs. (3.14) and (A1) are independent of N provided $\Delta\bar{x}$ is independent of N, and are strictly valid for the case $q_\perp \gg q_z$. The self-consistent solution of Eq. (A1) determines the resonance width γ [$\approx\Gamma(F)$], from which we can theoretically estimate the field-induced resonant relaxation time $\tau(F)$ due to the polar-LO-phonon scattering and the ICFE in a quasi-2D quantum-well structure. Equation (A1) determines the field-dependent resonance width $\gamma(F)$ associated with the transitions between N and $N+1$ through Eq. (3.14). For $\pi\gamma \ll \hbar\omega_c$, we may approximate $\coth X\approx 1/X$ and Eq. (A1) gives the approximate result for the field-induced resonance width (i.e., the EFIMPR width):

$$\gamma(F)\approx\left[\frac{\delta[(\delta^2+4K)^{1/2}-\delta]}{2\pi^2}\right]^{1/2}\hbar\omega_c\ , \tag{A2}$$

where $K:=(3\pi/\hbar\omega_L^*)\Lambda'(2N_0+1)$ and $\delta:=\pi\omega_L^*/\omega_c$. It is noted that this broadening associated with Landau level N and $N+1$ is due essentially to the inter-Landau-level scattering between N and $N+1$, where the electric-field-induced inelastic phonon scattering is taking place. As seen from Eq. (A2), the resonance width γ is influenced by the strength of the applied electric field F as well as the lattice temperature (via N_0). Finally, we note that this result for $\gamma(F)$ [$\approx\hbar/\tau(F)$] could be used to estimate the high-field mobility of the quasi-2DEG confined in quantum-well structures through $\mu(F)=e\tau(F)/m^*$.

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