

Optical interdonor hopping in multiple quantum wells

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(Received 16 July 1991)

It is shown that the hopping transitions from neutral center-of-well donors to ionized center-of-barrier donors in multiple-quantum-well GaAs-Ga_{1-x}Al_xAs heterostructures may result in an infrared-absorption maximum of the oscillator strength comparable to that for intradonor 1s-2p transitions. The weak magnetic-field dependence of the hopping transition energies makes such transitions hard to detect by variable-field magnetoabsorption techniques but they should show up in direct absorption measurements. Their detection would furnish direct information about the magnitude of electrostatic-potential fluctuations in multiple quantum wells.

I. INTRODUCTION

It is now well established that the energy spectrum of a shallow impurity in multiple-quantum-well (MQW) heterostructures depends strongly on the geometry of the structure and on the position of the impurity ion. Starting from the pioneering work of Bastard,¹ this idea has been gradually developed in both theory¹⁻⁶ and experiment⁷⁻¹¹ to the extent that nowadays, far-infrared absorption and magnetoabsorption spectra of shallow impurities in MQW's are used as sources of information on the doping profiles along the direction of growth of heterostructures.⁹ For obvious reasons, the case of shallow donors in GaAs-Ga_{1-x}Al_xAs MQW heterostructures is the most thoroughly studied.

When analyzing the position dependence of the energy spectrum of a shallow donor in MQW structure, one notes two types of extreme points: one in the middle of the wells; the other in the middle of the barriers. The first corresponds to the largest binding and transition energies; the second, to the smallest. These extreme points are responsible for the singularities in the electronic density of states (in the case of a continuous doping profile across the wells and barriers) which show up in the absorption spectra of intradonor transitions. Note that wherever the donors may be, in the well or in the barrier, the electronic wave functions are confined to the wells. The wave function of the ground state of an electron bound to a center-of-well donor may overlap with the wave function of the ground state of the nearby center-of-barrier donor. In the case of partial compensation, always existing in GaAs-Ga_{1-x}Al_xAs heterostructures, barrier donors are first candidates to lose their electrons in favor of acceptors. This opens a channel for interdonor infrared absorption—electrons from the occupied center-of-well donors may hop to ionized center-of-barrier ones. Such hopping transitions, representing the transition between the singularities of the density of states on the energy scale, might lead to a well-defined absorption peak of a measurable magnitude. The width of the peak should depend on the magnitude of the potential-energy fluctuations within the well. Our objec-

tive is to give quantitative support for these statements.

For simplicity, we consider in the following a MQW with planar doping in the central planes of all wells and all barriers. The reason for this is to focus attention on the most interesting donors. With a proper choice of surface concentrations of donors, the model predictions concerning the absorption due to interdonor hopping should apply for a broad class of doping profiles, from a strictly planar to a uniform distribution of donors.

In Sec. II we discuss the ground-state energies and wave functions for the two categories of donors *w* and *b* located at the middle planes of the wells and the barriers, respectively. In Sec. III we construct the states of an electron interacting simultaneously with two donors, *w* and *b*. In Sec. IV we calculate the absorption coefficient resulting from electronic transitions from occupied *w* donors to empty *b* donors. The quantitative results are presented and discussed in Sec. V.

II. DONOR GROUND STATE IN MQW STRUCTURE

We first list the models of a superlattice periodic potential used until now in the theoretical treatments of an impurity center in MQW. In the first attempt,¹ MQW was considered as a set of independent quantum wells and, as a consequence, the case of a single GaAs well sandwiched between infinitely high barriers was treated. In subsequent works,²⁻⁴ the single well with infinitely high bar-

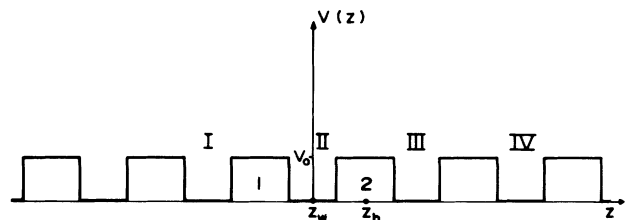


FIG. 1. Schematic representation of the model potential of the GaAs-Ga_{1-x}Al_xAs heterostructure.

riers was substituted by a single well of finite depth. Chaudhuri⁵ made an attempt to use more than one well; he performed variational calculations for the ground state of an impurity located at the center of a quantum well, allowing for the spreading of the impurity envelope wave functions to the first-neighbor wells. Extending this idea, Lane and Greene⁶ considered a model of a donor atom placed in any position of a periodic square-well potential. The variational wave functions of the ground state and low-lying excited states of a donor center were

$$V(z) = \begin{cases} V_0 & \text{for } \frac{1}{2}l < |z - z_w| < \frac{1}{2}l + d \text{ and } \frac{3}{2}l + d < |z - z_w| < \infty, \\ 0 & \text{for } 0 < |z - z_w| < \frac{1}{2}l \text{ and } \frac{1}{2}l + d < |z - z_w| < \frac{3}{2}l + d, \end{cases} \quad (1)$$

for the impurity placed in the center of the well. The parameters l and d denote the width of the well and the barrier, respectively.

In the above model we allow the spreading of the impurity envelope wave function at most to the neighbor wells labeled I and III in Fig. 1. It is justified by the fact that the probability of finding an electron in the ground state within wells I and III is already much smaller than that of finding it in well II (less than 13% in the case of the heterostructures we are interested in); consequently in the next wells it must be substantially less. Similar arguments work for a donor impurity centered in the barrier. We propose the following model of the periodic potential for this case:

$$V(z) = \begin{cases} V_0 & \text{for } 0 < |z - z_b| < \frac{d}{2} \text{ and } \frac{d}{2} + l < |z - z_b| < \frac{3}{2}d + l \text{ and } \frac{3}{2}d + 2l < |z - z_b| < \infty, \\ 0 & \text{for } \frac{d}{2} < |z - z_b| < \frac{d}{2} + l \text{ and } \frac{3}{2}d + l < |z - z_b| < \frac{3}{2}d + 2l. \end{cases} \quad (2)$$

$V(z)$ given by (2) allows the spreading of the barrier impurity envelope wave function to the wells marked by I–IV in Fig. 1. The barrier height is given by $V_0 = 0.6\Delta E_g$,^{12–14} where the band-gap difference between the two bulk semiconductors is obtained from the expression¹⁵

$$\Delta E_g = (1.155x + 0.37x^2) \text{ eV}.$$

In the effective-mass approximation the dimensionless Hamiltonian for the impurity has the form

$$H_0 = -\nabla^2 - \frac{2}{r} + V(z), \quad (3)$$

where $V(z)$ is given by formula (1) for donors placed in the center of the well, and by formula (2) for donors placed in the center of the barrier. The energies are measured in effective rydbergs $R^* = m^*e^4/2\hbar^2\epsilon^2$ and the effective Bohr radius $a_B^* = \hbar^2\epsilon/m^*e^2$ is the unit of the distance. m^* and ϵ are the effective mass of the electron and the static dielectric constant, respectively. We neglect the difference between the effective masses in the well and the barrier materials and we use in both cases the effective mass equal to the bulk GaAs value $0.067m_e$ and the dielectric constant $\epsilon = 12.5$.

In the region of compositions x that we are interested in, the barrier height is many times larger than the effective rydberg; as a consequence the Coulomb binding energy is small compared to the subband energy and one can choose the variational wave function ψ for the ground state of the Hamiltonian H_0 as a product of the

expanded in terms of a fixed Gaussian set with 13 linear variational parameters.

In the present work we need to know the binding energies and wave functions of donors located in the center of the well and in the center of the barrier. Figure 1 shows a donor placed in position z_w coinciding with the center of a well and another one in the center of the neighbor barrier in position z_b . Following Chaudhuri,⁵ we propose the following model for the periodic potential along the z axis, which is normal to the interfaces of the superlattice,

ground-state eigenfunction of the square-well potential problem $f(z)$ and an envelope wave function $G(\rho, z)$

$$\psi(\rho, z) = Nf(z)G(\rho, z), \quad (4)$$

where N is the normalization constant and ρ, z denote cylindrical coordinates. The eigenenergy E of an electron in the potential given by (1) [or (2)] is obtained as a solution of the equation

$$-\frac{\gamma}{k}(1+a)p^3 + \left\{ 3 \left[\frac{\gamma}{k} \right]^2 + a \left[\left[\frac{\gamma}{k} \right]^2 - 2 \right] \right\} p^2 + \frac{\gamma}{k} \left\{ 3 - a \left[2 \left[\frac{\gamma}{k} \right]^2 - 1 \right] \right\} p - \left[\frac{\gamma}{k} \right]^2 (1+a) = 0, \quad (5)$$

for the donor placed in the center of the well, and as a solution of the equation

$$\frac{\gamma}{k} \left\{ \frac{\gamma}{k} + c \left[\left[\frac{\gamma}{k} \right]^2 a - 1 \right] \frac{1}{1+a} \right\} \left[\frac{\gamma}{k} bc + 1 \right] + \left\{ \frac{\gamma}{k} + c \left[\left[\frac{\gamma}{k} \right]^2 - a \right] \frac{1}{1+a} \right\} \left[\frac{\gamma}{k} b - c \right] = 0, \quad (6)$$

for the donor placed in the center of the barrier [$k = E^{1/2}$, $\gamma = (V_0 - E)^{1/2}$, and $a = \tanh(\gamma d)$, $b = \tanh(\gamma d/2)$, $c = \tan(kl)$, $p = \tan(kl/2)$]. Equations (5) and (6) result from matching of the wave functions at

TABLE I. Calculated variational parameters λ , κ , and binding energies E_D for various well and barrier widths (l and d , respectively) and the two positions of donors, for $x=0.3$. l , d , and λ are in effective Bohr radius a_B^* .

l	d	Donor in the center of the well			Donor in the center of the barrier		
		λ	κ	E_D/R^*	λ	κ	E_D/R^*
1	0.5	0.80	1.00	1.71	2.10	1.54	0.95
1	1	0.82	1.05	1.89	2.50	1.59	0.78
1	2	0.98	1.24	2.04	3.17	1.66	0.60
2	2	1.17	1.32	1.71	3.46	1.64	0.50

the interfaces.

Until now two ground-state envelope wave functions were used—one of the spherical symmetry^{1,5} with a single variational parameter, and the other one²⁻⁴ more adequate from the point of view of the symmetry of the system and involving several linear variational parameters. We decided to choose the envelope wave function in the form

$$G(\rho, z) = \exp \left[-\frac{1}{\lambda} (\kappa^2 \rho^2 + z^2)^{1/2} \right], \quad (7)$$

where λ and κ are the variational parameters. This function takes into account the cylindrical symmetry of the system but, at the same time, remains quite simple for intuitive interpretations. The presence of the parameter κ makes the behavior of the electron in a plane parallel to the interfaces less dependent on its behavior in the z direction than in the case of a spherical function.

The variational ground-state binding energy $E_D(l, d)$ is given by

$$E_D = E - \min \langle \psi | H_0 | \psi \rangle. \quad (8)$$

The expectation value of the operator $-\nabla^2 + V(z)$ has the form

$$\begin{aligned} \langle \psi | [-\nabla^2 + V(z)] | \psi \rangle &= \left[k^2 + \frac{\kappa^2}{\lambda^2} \right] \\ &+ \frac{(1 - \kappa^2)}{\kappa^2} \frac{4\pi N^2}{\lambda^2} \\ &\times \int_0^{+\infty} dz z^2 f^2(z) E_1(2z/\lambda), \end{aligned} \quad (9)$$

where $E_1(x) = \int_x^\infty dt \exp(-t)/t$. For $\kappa=1$ (in the case of a spherically symmetric envelope wave function) expression (9) reduces to $(k^2 + 1/\lambda^2)$ and greatly simplifies the joint expression (9b) and (9d) in the Chaudhuri⁵ paper. In the case of $\kappa \neq 1$, two of the three integrations involved in the Coulomb energy $\langle \psi | (-2/r) | \psi \rangle$ must be evaluated numerically. The normalization constant N is expressed analytically and differs from the case of $\kappa=1$ only by a factor of $1/\kappa^2$.

For all quantitative illustrations we selected one typical concentration of Al ions: $x=0.3$ in the MQW heterostructures. The results of our calculations are summarized in Table I. The general conclusion for a donor at

the center of the well is that if the joint thickness of the well and barrier does not exceed two Bohr radii ($l + d \leq 2a_B^*$), the optimum value of the parameter κ is close to 1. As a consequence, the spherically symmetric envelope wave function proposed by Chaudhuri⁵ represents the proper choice in this case. For example, for $l=d=1a_B^*$ we obtained $\kappa=1.05$, and the binding energy $E_D=1.89R^*$. Lane and Greene⁶ with their many variational parameters obtained $E_D=1.93R^*$ in this case (the difference is only 2%). In the case of heterostructures for which $l+d > 2a_B^*$ the expected tendency to wider spreading of the envelope wave function in the z direction than in the plane parallel to the interfaces is observed. For example, for $l=d=2a_B^*$ we obtained $\kappa=1.32$; as a consequence, in the plane parallel to the interfaces the ‘‘Bohr radius’’ given by the ratio $\lambda/\kappa=0.89a_B^*$ is quite different from the one in the z direction ($\lambda=1.17a_B^*$). At the same time, the obtained binding energy $E_D=1.71R^*$ gives only 2.5% improvement compared with the case of $\kappa=1$.

On the other hand, our results indicate that when the donor is placed in the center of the barrier the presence of the κ parameter in the variational envelope wave function is more important. For example, if $l=d=1a_B^*$, we obtained $\kappa=1.59$, $\lambda=2.50a_B^*$, and $E_D=0.78R^*$ ($\lambda/\kappa=1.57a_B^*$). In the case of $\kappa=1$ we have $E_D=0.74R^*$; this means that about 5% improvement occurs when we allow κ to act as a second variational parameter. Lane and Greene⁶ obtained 10% smaller binding energy, $E_D=0.71R^*$, for this case. This seems to indicate that the set of Gaussians selected by Lane and Greene⁶ was not extensive enough for the case of an electron kept at some distance from the Coulomb center. Summarizing, we can say that the simple trial functions (7), with two nonlinear variational parameters, reproduce quite well the previous results and in some cases give even better binding energy than the variational procedures involving 13 linear parameters.⁶

III. TWO-CENTER DONOR STATES

Let us consider now two donors: the neutral donor w situated in the middle of quantum well II at $\mathbf{R}_w(\rho_w=0, z_w)$ and the ionized donor b situated in the middle of the barrier closest to well II (barrier 1 or 2) at the position $\mathbf{R}_b(\rho_b, \phi_b=0, z_b)$. The electric field of the incident infrared radiation may thus force the electron to jump from the ground state of donor w to the ground

state of donor b . Note, however, that the absorption coefficient resulting from such interdonor jumps may not be calculated directly from the matrix element of the dipole moment between the ground states $|\psi_w\rangle$ and $|\psi_b\rangle$ of the donors w and b determined in Sec. II as eigenstates of the Hamiltonian (3) with the periodic potential $V(z)$ given by Eqs. (1) and (2), respectively. These states, corresponding to different electron energies, are not orthogonal since they are the ground states of two different Hamiltonians. Each of these states changes when there is another donor center in the vicinity. Therefore, before calculating transition probability we have to construct the eigenstates of the two-center Hamiltonian accounting for the simultaneous action of the two charged donor ions on the electron:

$$H = -\nabla^2 + V(z) - \frac{2}{r_w} - \frac{2}{r_b}, \quad (10)$$

where $r_w = |\mathbf{r} - \mathbf{R}_w|$ and $r_b = |\mathbf{r} - \mathbf{R}_b|$, and $V(z)$ is the periodic MQW potential shown in Fig. 1. [The fact that $V(z)$ here differs from the potentials $V(z)$ given by Eqs. (1) and (2) is not fundamental since the three potentials $V(z)$ differ from each other in the regions of space where the amplitudes of electronic wave functions ψ_w and ψ_b are practically vanishing.] In writing the Hamiltonian (10) we neglected the electrostatic interaction of the electron with other ionized donors and compensating acceptors which might be situated in the vicinity of our w - b donor pair. Unless the impurity concentration and the compensation are very high, the potential-energy difference between the centers w and b due to other charged centers remains small compared to the difference between binding energies E_{Dw} and E_{Db} . In this situation the effect of neighboring charged impurities may be ac-

counted for by introducing some dispersion of the energy of the electronic transitions from w to b .

When the average distance between the well and the barrier donors is much larger than the spatial extension of the ground-state wave functions ψ_w and ψ_b , the lowest eigenstates of the Hamiltonian (10) for typical donor pairs might well be approximated by the two orthogonal linear combinations of ψ_w and ψ_b :

$$\psi_1(\mathbf{r}) = c_w^1 \psi_w(\mathbf{r}_w) + c_b^1 \psi_b(\mathbf{r}_b), \quad (11)$$

$$\psi_2(\mathbf{r}) = c_w^2 \psi_w(\mathbf{r}_w) + c_b^2 \psi_b(\mathbf{r}_b), \quad (12)$$

where

$$c_b^{1,2} = \alpha_{1,2} c_w^{1,2}, \quad (13)$$

$$c_w^{1,2} = (1 + 2\alpha_{1,2} S + \alpha_{1,2}^2)^{-1/2},$$

and

$$\alpha_{1,2} = -\frac{H_{wb} - E_{1,2} S}{H_{bb} - E_{1,2}},$$

where $E_{1,2}$ denote the expectation values of the Hamiltonian (10) for the states (11) and (12) and the remaining abbreviations are defined as follows:

$$S = \langle \psi_w | \psi_b \rangle,$$

$$H_{wb} = \langle \psi_w | H | \psi_b \rangle, \quad (14)$$

$$H_{bb} = \langle \psi_b | H | \psi_b \rangle.$$

The lower state $|\psi_1\rangle$ is predominantly built from the state $|\psi_w\rangle$ while the state $|\psi_2\rangle$ is built from $|\psi_b\rangle$. The energy difference between the two states is given by the formula

$$E_2 - E_1 = \frac{[(H_{bb} + H_{ww} - 2H_{wb}S)^2 - 4(H_{bb}H_{ww} - H_{wb}^2)(1 - S^2)]^{1/2}}{(1 - S^2)}, \quad (15)$$

where $H_{ww} = \langle \psi_w | H | \psi_w \rangle$. Figure 2 shows $\Delta E = E_2 - E_1$ as a function of variable ρ for three heterostructures with the same well thickness, $l = 1a_B^*$, and different barrier thicknesses: $d = 0.5a_B^*$, $d = 1a_B^*$, and $d = 2a_B^*$. One can see that the energy ΔE required for the hopping transition of the electron from the donor w to the donor b is not identical for all w - b pairs, but it is a decreasing function of the hopping distance ρ finally tending to $E_{ob} - E_{ow}$ for large ρ [E_{ob} and E_{ow} denote the expectation values of the Hamiltonian (3) with $V(z)$ given by formulas (2) and (1), respectively]. This leads to asymmetric broadening of the expected absorption peak. Note that the closely spaced pairs, for which the approximations (11) and (12) might be poor, can be excluded from further considerations by imposing the upper limit of the photon energies.

IV. ABSORPTION COEFFICIENT

We now derive the formula for the absorption coefficient using the pair model. Let N_w and N_b be the surface concentrations of the neutral center-of-well

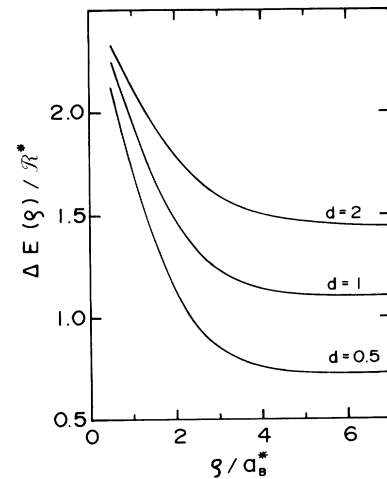


FIG. 2. The energy difference between the two lowest states of the two-center Hamiltonian (10) as a function of the variable ρ for three GaAs-Ga_{0.7}Al_{0.3}As heterostructures with the same well thickness $l = 1a_B^*$ and different barrier thicknesses: $d = 0.5a_B^*$, $d = 1a_B^*$, and $d = 2a_B^*$.

donors w and the ionized center-of-barrier donors b , respectively. Ionized donors w or neutral donors b are ineffective for hopping in the energy range of interest. We choose the origin of the coordinate system at the position of a neutral donor w . The expected number of ionized donors b with the radial coordinates in the range $(\rho, \rho+d\rho)$ is equal to $2N_b 2\pi\rho d\rho$. (The factor of 2 reflects the existence of two barriers neighboring with a well.) The results of Sec. III indicate that, except for the smallest radial coordinates ρ , the electronic wave functions for both considered states of a w - b donor pair are given by the formulas similar to those in nondegenerate perturbation theory. This means that the existence of a few donors b in the vicinity of a given donor w does not invalidate in general the pair model. If the donors b are not too close to one another, the admixtures of their wave functions to the ground-state wave function of the donor w are approximately additive. As a result the transition rate W_{wb} from the donor w to all nearby donors b may be calculated independently. The rate is given by the formula

$$W_{wb} = \frac{4\pi^2 e^2}{c\hbar^2 \epsilon^{1/2}} \langle \psi_1 | \mathbf{r} \cdot \mathbf{n} | \psi_2 \rangle^2 I(\omega), \quad (16)$$

where \mathbf{n} denotes the unit vector parallel to the electric field of radiation, which we assume to be parallel to the interface planes. $I(\omega)$ is the intensity of the radiation per unit interval of the angular frequency ω . The dipole matrix element has the form

$$D = \langle \psi_1 | \mathbf{r} \cdot \mathbf{n} | \psi_2 \rangle = (c_w^1 c_b^2 + c_b^1 c_w^2) \langle \psi_w | \mathbf{r} \cdot \mathbf{n} | \psi_b \rangle + \mathbf{R}_b \cdot \mathbf{n} c_b^1 c_b^2. \quad (17)$$

The two terms in the above expression have opposite signs and partially compensate. If we overlooked the nonorthogonality of the wave functions ψ_w and ψ_b we would get only the first term with the bracket replaced by 1 overestimating the transition rate.

Now we average the transition rate over all possible directions of the unit electric-field vector \mathbf{n} with respect to vector \mathbf{R}_b and we sum up the contributions from all donor pairs. This leads to the following formula for the absorption coefficient for a structure with j two-layer periods:

$$\alpha = j 4\pi N_w N_b \hbar \omega \int_0^\infty d\rho \rho \frac{4\pi^2 e^2}{c\hbar^2 \epsilon^{1/2}} \langle |D|^2 \rangle \delta(\omega - \Delta E / \hbar), \quad (18)$$

where $\langle |D|^2 \rangle$ denotes the directional average of the modulus squared of the dipole matrix element (17), and $I(\omega)$ was replaced by $\delta(\omega - \Delta E / \hbar)$. In Fig. 3 we plotted the function $\rho \langle |D|^2 \rangle$ as a function of ρ . One can see that the dominant contribution to the total oscillator strength for interdonor transitions comes from pairs having the radial coordinate ρ of the order of 2–4 Bohr radii a_B^* .

In deriving Eq. (18) we neglected the electrostatic interaction of the electron in the states localized at donors w and b with other ionized donors and compensating acceptors present in the sample. In principle this interaction introduces an additional term in the two-center

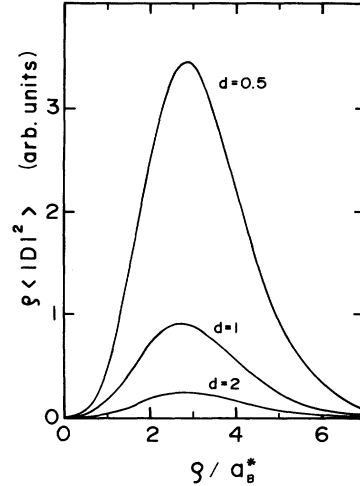


FIG. 3. The function $\rho \langle |D|^2 \rangle$, where $\langle |D|^2 \rangle$ denotes the directional average of the modulus squared of the dipole matrix element (17), as a function of the variable ρ for the same three GaAs-Ga_{0.7}Al_{0.3}As heterostructures as in Fig. 2.

Hamiltonian (10) which modifies both the energies and the wave functions of the two-center states (11), (12), and (15). However, as long as the electrostatic potential-energy difference ΔV_{wb} between ions w and b remains much smaller than $\Delta E(\rho)$ given by Eq. (15), we can account for the electrostatic interaction with the impurities by first-order perturbation theory. This amounts to adding the additional term ΔV_{wb} to $\Delta E(\rho)$ in the argument of the δ function in Eq. (18). If we approximate the probability distribution of ΔV_{wb} by a Gaussian we get, for the absorption coefficient, the formula

$$\alpha = j 4\pi N_w N_b \hbar \omega \frac{4\pi^2 e^2}{c\hbar^2 \epsilon^{1/2}} \times \int_0^\infty d\rho \rho \langle |D|^2 \rangle \times \frac{1}{(2\pi)^{1/2} \sigma} \exp \left\{ - \frac{[\hbar \omega - \Delta E(\rho)]^2}{2\sigma^2} \right\}. \quad (19)$$

Strictly speaking, the dispersion σ of the probability distribution should depend on the distance between w and b . We simplify the problem by using, for all pairs, the common value of σ proper for pairs giving the dominant contribution to the oscillator strength.

V. RESULTS AND DISCUSSION

One can show (following Morgan¹⁶) that for a completely random spatial distribution of point charges, and in the absence of any screening, the difference between the potential energies at two arbitrary points \mathbf{r}_1 and \mathbf{r}_2 is a random variable with the Gaussian distribution of zero mean value and the dispersion

$$\sigma = 4[\pi |\mathbf{r}_1 - \mathbf{r}_2| (a_B^*)^2 N_{\text{total}}]^{1/2} R^*.$$

Here N_{total} denotes the average volume concentration of charged centers. In MQW with $N_b \approx 10^{10} \text{ cm}^{-2}$, $l=d$

$=1a_B^*$, and $|\mathbf{r}_1 - \mathbf{r}_2| = 3a_B^*$, we get $\sigma \approx 1R^*$. This seems to be a serious overestimate of the magnitude of the root-mean-square potential-energy difference in real MQW with the above characteristics. The relatively low temperature of the growth, together with the important lateral mobility of atoms during the process of molecular-beam epitaxy, seem to favor strong spatial correlations between the negatively and positively charged impurities. As a result, the spatial distribution of charged impurities in MBE-grown systems should resemble the random distribution of electric dipoles composed of closely spaced ionized-donor-ionized-acceptor pairs rather than the random distribution of uncorrelated ionized donors and acceptors. The calculation, or even the estimation, of the dispersion of the distribution of the potential-energy differences in such correlated system with planar doping of donors and volume doping of acceptors would require many *ad hoc* hypotheses. We note, however, that the potential-energy differences in a system of randomly distributed electric dipoles are at least an order of magnitude lower than in the system of random uncorrelated charges of the same volume concentration. In our opinion the information on the dispersion σ of the potential-energy distribution in real MQW should come from experiment. In order to visualize the sensitivity of the absorption spectrum on the dispersion σ of the distribution of the transition energies for interdonor hopping we performed our numerical calculations for three different values of $\sigma = 0.05R^*$, $0.1R^*$, and $0.2R^*$.

In Fig. 4 we give the absorption coefficient as a function of photon energy for GaAs-Ga_{0.7}Al_{0.3}As heterostructure with 100 periods with $l = 1a_B^*$, $d = 0.5a_B^*$. Simi-

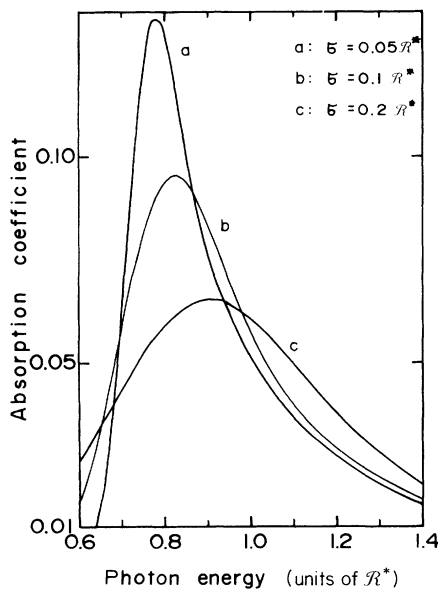


FIG. 4. The absorption coefficient vs photon energy for GaAs-Ga_{0.7}Al_{0.3}As heterostructure with 100 periods with well thickness $l = 1a_B^*$ and barrier thickness $d = 0.5a_B^*$ ($N_w = N_b = 2 \times 10^{10} \text{ cm}^{-2}$), for three values of dispersion σ : $0.05R^*$, $0.1R^*$, and $0.2R^*$.

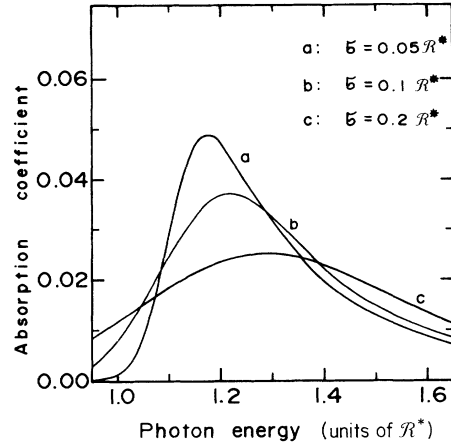


FIG. 5. The absorption coefficient vs photon energy for GaAs-Ga_{0.7}Al_{0.3}As heterostructure with 100 periods with well thickness $l = 1a_B^*$ and barrier thickness $d = 1a_B^*$ ($N_w = N_b = 2 \times 10^{10} \text{ cm}^{-2}$), for three values of dispersion σ : $0.05R^*$, $0.1R^*$, and $0.2R^*$.

lar results of the absorption coefficient for the heterostructures with $l = 1a_B^*$, $d = 1a_B^*$ and $l = 1a_B^*$, $d = 2a_B^*$ are given in Figs. 5 and 6, respectively. The maximal value of absorption coefficient in our ranges of parameters characterizing the GaAs-Ga_{0.7}Al_{0.3}As heterostructures under consideration is 0.13 for $\hbar\omega = 4.55 \text{ meV}$, for the MQW with $l = 1a_B^*$, $d = 0.5a_B^*$, and $\sigma = 0.05R^*$ ($N_w = N_b = 2 \times 10^{10} \text{ cm}^{-2}$).

The position of the maximum of each absorption curve is determined by the following factors: the weak dependence of $\Delta E(\rho)$ on ρ for $\rho > 4a_B^*$, the rapid increase of $\Delta E(\rho)$ for ρ lower than $4a_B^*$, where the element $\rho \langle |D|^2 \rangle$ has its maximal value (see Figs. 2 and 3), and by the value of σ which, via the function $\exp\{-[\hbar\omega - \Delta E(\rho)]^2 / 2\sigma^2\}$,

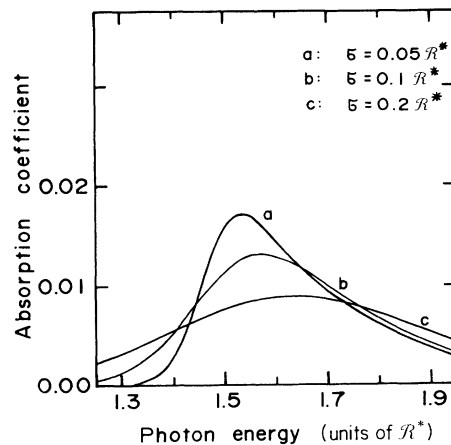


FIG. 6. The absorption coefficient vs photon energy for GaAs-Ga_{0.7}Al_{0.3}As heterostructure with 100 periods with well thickness $l = 1a_B^*$ and barrier thickness $d = 2a_B^*$ ($N_w = N_b = 2 \times 10^{10} \text{ cm}^{-2}$), for three values of dispersion σ : $0.05R^*$, $0.1R^*$, and $0.2R^*$.

decides the range of ρ contributing mostly to the absorption coefficient for fixed photon energy. The first factor tends to locate the maximum of the absorption curve quite close to the energy $\Delta E(\rho > 5a_B^*)$, while the second factor produces the shift of the maximum towards higher energies; finally, the dispersion σ decides which of these two tendencies dominate.

One can see in Figs. 4–6 that the shift of the maximum of the absorption coefficient with increasing dispersion σ is quite pronounced. The shift, as well as the width of the maximum, can provide information about the magnitude of potential-energy fluctuations in MQW heterostructures. Unfortunately, contrary to the case of intradonor transitions, the interdonor transition energies only weakly depend on the magnetic field. This makes the variable-field magnetoabsorption techniques inappropriate for the detection of the interdonor transitions and the direct, i.e., interferometric absorption measurements

have to be used for their detection. We believe, however, that the experimental effort is worth the investment, since the observation of the interdonor absorption could bring quite unique and relatively direct information on the magnitude of potential-energy fluctuations and on the correlation between the positions of the oppositely charged impurities in MBE-grown structures.

As has already been pointed out, our model predictions concerning the absorption due to interdonor hopping should apply not only to the case of planar doping in the central planes of wells and barriers, but to a broad class of doping profiles. In the case of uniform distribution of donors all barrier (well) donors which have similar binding energies of their ground states contribute in a similar way to the absorption under consideration. It means that by properly increasing surface concentrations of donors we should obtain absorption curves similar to those for the case of the strictly planar doping.

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¹G. Bastard, *Phys. Rev. B* **24**, 4714 (1981).

²C. Mailhot, Yia-Chung Chang, and T. C. McGill, *Phys. Rev. B* **26**, 4449 (1982).

³Ronald L. Greene and K. K. Bajaj, *Solid State Commun.* **45**, 825 (1983).

⁴Ronald L. Greene and K. K. Bajaj, *Phys. Rev. B* **31**, 913 (1985).

⁵S. Chaudhuri, *Phys. Rev. B* **28**, 4480 (1983).

⁶Pat Lane and Ronald L. Greene, *Phys. Rev. B* **33**, 5871 (1986).

⁷N. C. Jarosik, B. D. McCombe, B. V. Shanabrook, J. C. Comas, J. Ralston, and G. Wicks, *Phys. Rev. Lett.* **54**, 1283 (1983).

⁸E. Glaser, B. V. Shanabrook, R. J. Wagner, R. L. Hawkins, W. J. Moore, and D. Musser, *J. Phys. (Paris)* **48**, 239 (1987).

⁹S. Huant, M. Grynberg, G. Martinez, B. Etienne, B. Lambert, and A. Regreny, *Solid State Commun.* **65**, 1467 (1988).

¹⁰J.-M. Mercy, B. D. McCombe, W. Beard, J. Ralston, and G. Wicks, *Surf. Sci.* **196**, 334 (1988).

¹¹S. Huant, R. Stepniewski, G. Martinez, V. Thierry-Mieg, and B. Etienne, *Superlatt. Microstruct.* **5**, 331 (1989).

¹²R. C. Miller, A. C. Gossard, D. A. Kleinman, and O. Munteanu, *Phys. Rev. B* **29**, 3740 (1984).

¹³R. C. Miller, D. A. Kleinman, and A. C. Gossard, *Phys. Rev. B* **29**, 7085 (1984).

¹⁴H. Okimura, S. Misawa, S. Yoshida, and S. Gonda, *Appl. Phys. Lett.* **46**, 377 (1985).

¹⁵H. J. Lee, L. Y. Juravel, J. C. Woolley, and A. J. Springthorpe, *Phys. Rev. B* **21**, 659 (1980).

¹⁶T. N. Morgan, *Phys. Rev.* **139**, A343 (1965).