Mechanisms of magnetoresistance in variable-range-hopping transport for two-dimensional electron systems

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The temperature and magnetic-field dependencies of hopping transport in dilutely doped δ layers have been measured under the conditions for which the variable-range mechanism applies. We trace the transition from negative magnetoresistance in low fields to positive magnetoresistance in high fields. In the range of intermediate fields, the resistance in the perpendicular orientation appears to be several times less than that in the parallel one. It is shown that this "inverted" relation cannot be accounted for in terms of the interference of alternative hopping paths alone. We note also a distinct nonmonotonic variation of the resistance with magnetic field in the perpendicular orientation. The results are explained by considering both the interference of alternative hopping paths and the influence of a magnetic field on the density of states at the Fermi level, which defines the magnitude of Mott's T_0 parameter in variable-range hopping.

I. INTRODUCTION

The effect of a magnetic field on hopping conduction has been studied in a large number of publications. The theory of magnetoresistance (MR) for both the nearest-neighbor and variable-range regimes (respectively, NNH and VRH) was originally developed by Shklovskii.[1,2] It is based on the concept of wave-function shrinkage for the localized states in a magnetic field. Shrinkage reduces the overlap between states and MR is inherently positive.

For two-dimensional (2D) electron systems the theoretical predictions were investigated by Timp and Fowler[3] for electrons at metal-oxide-semiconductora (MOS) interfaces in the presence of oxide charge. Their results were by and large in agreement with the theory. Since this time many other 2D systems have been examined. For In_2O_3 films,[4,5] GaAs metal-semiconductor transistor structures,[6] Ge bicrystals,[7] and most recently GaAs δ-doped layers,[8] a substantially different dependence of the hopping resistance on the magnetic field has been reported. In low fields, there was generally found significant negative MR.

One possible explanation of negative MR is the interference of amplitudes for the electron that has traversed different paths, corresponding to different sequences of scattering acts, between a given initial and final state.[9] The coherent superposition of partial tunneling amplitudes that stem from the different paths in VRH results in a high resistance. Phase factors acquired by the motion in a magnetic field will destroy the interference. This increases the probability of a hop and reduces the resistance. The arguments in Ref. 9 apply only in low fields for which the shrinkage of the wave functions can be neglected. They do not require a large number of scatterings. In Ref. 10 it is argued that the coherent mechanism applies also if there is interference between the amplitudes of direct tunneling and tunneling with one scattering only. The two paths span a triangular area.

The experiments on the δ-layer transport[8] were done at temperatures above 1.6 K. At such temperatures the concepts of VRH do not strictly apply in the system studied. The authors reported negative MR approaching 20% of the zero-field resistance. The effect was accounted for by the interference mechanism. References 9 and 10 predict an effect of such magnitude, but not exceeding 50%.

In the present work, we measure at lower temperatures in order to more closely approximate VRH conditions. Work is done with the same samples as in the previous publication. The results in Fig. 1 show that the negative MR becomes even more distinct in going to 0.17 K. It exceeds 50% of the zero-field resistance. When referred to the theoretically expected positive MR, the reduction is so large as to seriously question the interference mechanism as the dominant source of MR. Moreover, there is distinct structure in the MR of Fig. 1. The "hump" structure near 1.6T was seen weakly in Ref. 8 but becomes a dominant feature of the data at lower T. It is evident that the interference-based mechanism alone cannot provide a satisfactory explanation of the VRH-regime data.

An additional mechanism of negative MR was recently proposed in Ref. 11. It is based on the magnetic-field dependence of the energies of the localized states. Refer-

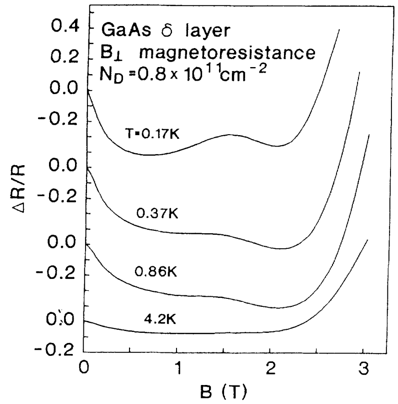

FIG. 1. Evolution of the negative magnetoresistance from the NNH regime (4.2 K in Ref. 8) to low temperatures where VRH applies. Note that $\Delta R(B)/R(0)$ exceeds 50% and is nonmonotonic.

ence 11 argues that wave-function shrinkage reduces the level repulsion effect and thus decreases the average activation energy. In other words, the spread of the energy levels making up the impurity band decreases. In turn, the density of states at the Fermi level rises with the well-known consequences for the T_0 parameter in VRH theory. In contrast to the interference mechanism, the discussion in Ref. 11 uses the conventional expression for the probability of a hop. We refer to it therefore as the "incoherent" mechanism and distinguish it from interference.

It will be shown that both the interference and incoherent mechanisms are operative in the experimental situation of Fig. 1. The resistance for VRH in a magnetic field will be the result of Shklovskii's shrinkage of the wave function, the interference for coherently added alternative paths, and the incoherent mechanism. Repeating the experiments of Ref. 8, in which both $B_\perp$ and $B_\parallel$ measurements are made on the δ layers, MR in the VRH regime will allow us to judge the different contributions. Although it is a smaller effect, the incoherent mechanism in a parallel field remains in force. The interference effect is absent.

In this paper we study the resistance for dilutely doped δ layers at temperatures T where VRH applies. We compare data in the two field configurations $B_\perp$ and $B_\parallel$. It is shown that both mechanisms of negative MR need to be considered to account for the experimental observations. Section II of the paper gives relevant experimental details. In Sec. III we discuss the dependence in zero field to demonstrate VRH transport. This is followed by the MR data. Section IV deals with the theoretical interpretation and comparison with experiments. Section V concludes the paper.

II. EXPERIMENTAL NOTES

The epitaxially grown GaAs samples used in the present work are described in Ref. 8. They contain 20 parallel Si-donor layers spaced 1000 Å apart. The sheet concentration is $N_D = 0.8\times10^{11}$ cm^{-2} with a possible error of 10%. Compensation is estimated to be of order 5%.

The low-T data in the range 0.1 to 1.0 K is obtained with the sample mounted inside the mixing chamber of a dilution refrigerator. Fields up to $7T$ in the $\perp$ and $\parallel$ configurations are used for the measurements. The data is restricted to be below a few times 10^7 Ω for the total sandwich of 18 active parallel layers. Top and bottom layers are sacrificial layers that are expected to be depleted.

III. EXPERIMENTAL RESULTS

A. T dependence in the absence of a field

The zero-field resistance data is shown in Fig. 2. Here the logarithm of the resistivity per layer is plotted against $T^{-1/3}$. The reason for choosing these axes is that in the VRH regime the temperature dependence of the resistance R follows Mott's law, which in 2D reads

$$R = R_0 \exp(T_0/T)^{1/3} . \tag{1}$$

Percolation theory predicts

$$T_0 = \frac{14}{g(E_F)a_B^{*2}} , \tag{2}$$

where $g(E_F)$ is the density of states at the Fermi level and a_B^* the Bohr radius of the donors. As seen in Fig. 2, below 0.5 K the data fall on a straight line. From its slope we find $T_0 = 840$ K. Because for GaAs $a_B^* = 100$ Å we thus obtain $g(E_F) = 2\times10^{14}$ (eV cm^2)$^{-1}$. A rough estimate of $g(E_F)$ can be obtained by assuming that $N_D = 0.8\times10^{11}$ donors per cm^2 are spread over an energy of the order of the activation energy in NNH. In Ref. 8 this energy is measured as $\varepsilon_3 = 1.1\times10^{-3}$ eV. It follows that $g(E_F) = 0.7\times10^{14}$ (eV cm^2)$^{-1}$, which compares reasonably well with the value above. The difference can be accounted for by the fact that for small compensation the impurity bandwidth is less than ε_3 (see, for example, Ref. 1).

The deviation from the straight-line behavior in Fig. 2

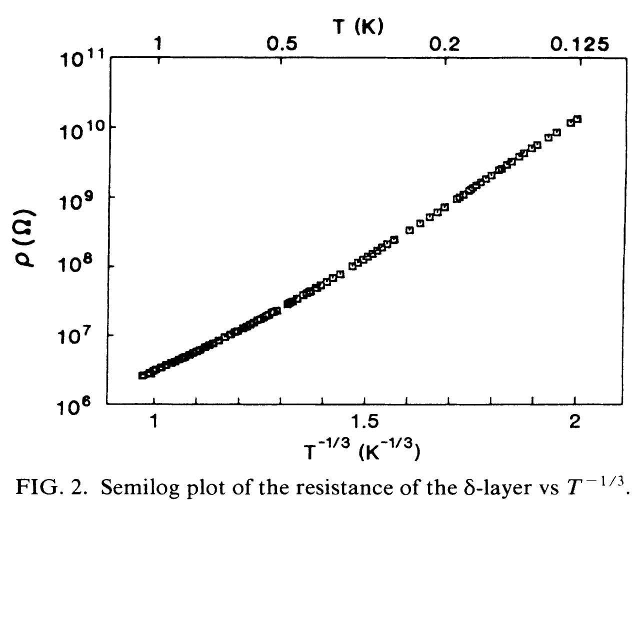

FIG. 2. Semilog plot of the resistance of the δ-layer vs $T^{-1/3}$.

is expected when either the maximum activation energy of a hop $\Delta E_{\max}$ becomes comparable to the impurity bandwidth or the maximum length of a hop $r_{\max}$ approaches the average distance between the donors. At 0.5 K we have

$$E_{\max} \approx T(T_0/T)^{1/3} = 0.5 \text{ meV} , \quad r_{\max} \approx (a_B^*/2)(T_0/T)^{1/3} = 600\text{Å} , \tag{3}$$

which is in fact comparable to $\varepsilon_3 = 1.1$ meV and $N_D^{-1/2} = 350$ Å. Therefore, above 0.5 K a gradual transition to the regime of NNH is taking place. Because of the steep temperature dependence of the resistivity, data could be obtained only above 0.1 K. This limits the VRH data to the temperature interval 0.5–0.1 K.

The specific feature of the structure studied is the small degree of compensation (see Sec. II and Ref. 8). For intermediate compensations the dependence $\ln R \propto T^{-1/2}$ instead of Mott's law might be expected. Such a dependence is caused by the Coulomb gap at the Fermi level in the single-electron density of states.[1] The temperature below which the $T^{-1/2}$ dependence applies is determined from the condition that $E_{\max}$ [see Eq. (3)] is of the order of the width of the Coulomb gap. For intermediate compensations the width of the Coulomb gap is comparable to the width of the impurity band. However, in the case of low compensations the gap is narrow[1] and the $T^{-1/2}$ dependence might be expected only at very low temperatures.

B. Magnetoresistance measurements

Figure 3 shows the dependencies of the resistance on the magnetic field oriented parallel to the plane of the δ layer and perpendicular to it. The cases of parallel and perpendicular orientations of magnetic field are label $B_{\parallel}$ and $B_{\perp}$, respectively. The $B_{\parallel}$ data is a longitudinal resistance measured with the current along the field ($B_{\parallel}^{\parallel}$ in Ref. 8). In order to cover a wide range of resistance the data are plotted on a log scale.

Both temperatures $T = 0.39$ and 0.6 K at which the measurements were performed correspond to the VRH regime of conduction. The results of MR measurements

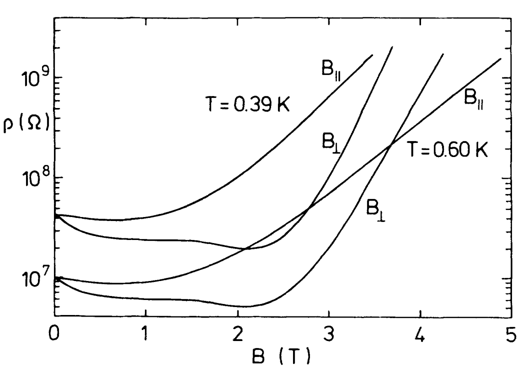

FIG. 3. Magnetic-field dependence of the resistance plotted logarithmically for $T = 0.6$ and 0.39 K for parallel and perpendicular field.

for higher temperatures ($2\text{K} < T < 4.5$ K) corresponding to the NNH regime are reported in Ref. 8.

As in the earlier work, the $B_{\perp}$ data shows a sizable negative MR which at $T = 0.39$ K exceeds 50% of the resistance at $B = 0$. There is a weak negative effect for $B_{\parallel}$. In the range of magnetic fields $1.5T < B < 2.5T$ the $B_{\parallel}$ data rises with B while the MR for $B_{\perp}$ orientation is negative. In sufficiently strong fields $B > 2.5T$ both resistances increase, whereas the ⊥-data remain well below the corresponding ∥ case for most of the field range. This contradicts the expected relation between the resistances in $B_{\perp}$ and $B_{\parallel}$ orientations and demands explanation.

Indeed, the rapid increase of the resistance with B results from the orbital shrinkage effect.[1] This effect, however, is much stronger in $B_{\perp}$ orientation than in $B_{\parallel}$. In what follows the relation $R_{\parallel}(B) > R_{\perp}(B)$ is referred to as the "inverted" relation between the resistances. In Ref. 8 it was argued that this relation is the result of interference. The magnitude of the effect now makes it clear that the interference mechanism alone is not enough to give a consistent picture. The relative effect in comparing the resistances is just too large. Moreover, there is also a nonmonotonic variation of the ⊥-resistance. The "hump" structure gives a MR which is decreasing while the $B_{\parallel}$ data are strongly rising. The nonmonotonic variation of the $B_{\perp}$ data is also seen for current work on InP samples with comparable doping density. It is a general feature of such data.

The present experiments and data such as in Fig. 3 make it apparent that large negative MR in δ-layer VRH transport is a feature to be explained. The observations are not limited to the NNH regime and high temperatures of the earlier work.[8]

IV. THEORY

The challenge for the theory is to explain the striking "inverted" relation between the resistance in $B_{\perp}$ and $B_{\parallel}$ orientations, as well as to account for the nonmonotonic variation of the resistance. Up to now two different mechanisms for the diminishing of the resistance in magnetic field have been proposed. They are the interference mechanism[9,10,12] and the incoherent mechanism.[11] We shall investigate the possibility of applying each of them to the treatment of the experimental data.

A. Interference mechanism

The estimate in Eq. (3) for the maximal hopping length shows that even for the lowest temperature of the measurements, $r_{\max}$ does not exceed $3N_D^{-1/2}$. Therefore the number of donors which can scatter the electron, tunneling from the initial to the final state in the process of the hop, is small. It allows us to use the model of single scattering[10] to study the effect of interference.

Consider the electron hop from occupied donor 1 to empty donor 2 (Fig. 4). Suppose that the tunneling in the process of the hop is accompanied by the scattering of the electron by donor 3. Then the total amplitude of tunneling reads

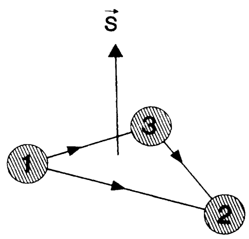

FIG. 4. Schematic drawing of interfering, neighboring hopping paths that generate a triangle of area **S**. Donor 3 scatters the electron hopping from donor 1 to donor 2.

$$A = A_{1\to 2} + A_{1\to 3\to 2} \ , \tag{4}$$

where the first term is the amplitude of the direct tunneling and the second one is the amplitude of tunneling with scattering. If we denote the resistance of the hop as $R_0 \exp \xi_{12}$ then the contribution to ξ_{12} caused by the interference of the partial amplitudes in (4) takes the form

$$\xi_{12}^{\text{int}} = -\ln(1+\tau)^2 \ , \tag{5}$$

$$\tau = \frac{A_{1\to 3\to 2}}{A_{1\to 2}} \ . \tag{6}$$

In a magnetic field the overlap integral between any pair of donors i and j acquires the phase factor $\exp(i\varphi_{ij})$, where

$$\varphi_{ij} = \frac{\pi \mathbf{B}}{2\Phi_0} \cdot (\mathbf{r}_i \times \mathbf{r}_j) \tag{7}$$

and Φ_0 is the flux quantum. The interference parameter τ acquires the phase factor $\exp(i\varphi)$ with

$$\varphi = \varphi_{13} + \varphi_{32} - \varphi_{12} = 2\pi \frac{\mathbf{B}\cdot\mathbf{S}}{\Phi_0} \ , \tag{8}$$

where $\mathbf{S}$ is the vector area of the triangle $1\to 2\to 3$. The phase factor differs from unity only in $B_\perp$ orientation for which Eq. (5) takes the form

$$\xi_{12}^{\text{int}} = -\ln \left[1 + \tau^2 + 2\tau \cos \left[2\pi \frac{\mathbf{B}\cdot\mathbf{S}}{\Phi_0} \right] \right] \ . \tag{9}$$

According to the perturbation method in percolation theory,[1] to calculate the interference correction to the total resistance, one averages (9) over the position of donor 3 in energy and in space. Since the position of donor 3 determines the values of parameters τ and S, this is equivalent to averaging over τ and S with some distribution function $F(\tau, S)$, i.e.,

$$\xi^{\text{int}}(B) = \int_0^\infty dS \int_{-\infty}^\infty d\tau \, F(\tau, S) \xi_{12}^{\text{int}} \ . \tag{10}$$

For $F(\tau,S)$ we choose the model Lorentzian distribution of τ

$$F(\tau,S) = \frac{f(S)\tau_S}{\pi(\tau^2 + \tau_S^2)} \ , \qquad \int_0^\infty dS \, f(S) = 1 \ , \tag{11}$$

where the width of the distribution τ_S decreases with increasing S. The averaging over τ can be performed analytically and gives

$$\xi^{\text{int}}(B) = -\int_0^\infty dS \, f(S) \left[\ln \left[\frac{1+\tau_S^2}{1+\tau_S} \right] + \ln \left[1 + \frac{2\tau \left| \sin \frac{2\pi BS}{\Phi_0} \right|}{1+\tau_S^2} \right] \right] \ . \tag{12}$$

We see that it is the second term in the square brackets that is responsible for the magnetoresistance caused by the phase factors (9). For $B=0$ this term vanishes. For small B it describes the negative magnetoresistance in the form

$$\xi^{\text{int}}(B) \propto -B \ , \tag{13}$$

in agreement with the prediction.[9] The interference mechanism[9] provides a possible explanation of the observed $R_\perp(B)$ in the low-field limit.

It is significant to note that for arbitrary B, S, and τ_S the second term in Eq. (12) does not exceed ln2. In other words, if one calculated the hopping resistance in $B_\perp$ orientation without taking into account the phase factors in overlap integrals, then the interference mechanism would decrease the result by not more than two times. From the data in Fig. 3 it is seen that at $T=0.6$ K and $B=2.2T$ we have $R_\perp(2.2T)=0.23R_\parallel(2.2T)$. If the resistance in field B was governed by the effect of shrinkage only, one would have $R_\perp(B) > R_\parallel(B)$. Thus, the reduction in $R_\perp$ caused by the interference cannot explain the "inverted" relation between $R_\perp$ and $R_\parallel$ observed experimentally. Also the origin of the additional structure in $R_\perp(B)$ dependence near $B=1.5T$ remains unclear. To explain the experimental data, another mechanism of strong suppression of positive MR is required.

B. Incoherent mechanism

We explain the essence of the incoherent mechanism of negative MR with the help of Fig. 5. The density of states in the 2D impurity band is plotted schematically. The degree of compensation in the samples studied is estimated to be small. It means that almost all the donors in the δ layer are occupied by the electrons and the Fermi level is placed in the tail of the density of states. The magnetic field diminishes the spread of the energy levels by decreasing the overlap of the neighboring donors. As a result the impurity band narrows with magnetic field and the tail becomes steeper. From the condition of conservation of the total number of the electrons it follows that the Fermi-level shifts with magnetic field toward the center of the impurity band. If the decay of the density of states in the tail is rapid enough, such a shift results in an increase of the density of states at the Fermi level $g(E_F)$. According to Mott's law [see Eqs. (1) and (2)] $\ln R \propto g(E_F)^{1/3}$. The increase of $g(E_F)$ with B causes

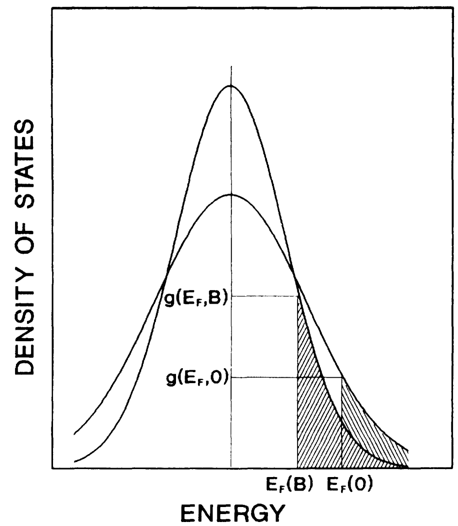

FIG. 5. Schematic form of the density of states in the impurity band in zero and infinite magnetic field.

negative MR. It is obvious that this mechanism competes with the conventional mechanism of positive MR since both are caused by the shrinkage of the impurity wave functions.

To examine the role of incoherent mechanism in magnetoresistance we first derive the expression for the conventional positive MR in 2D case. The overlap integral between the wave functions of two donors i and j can be written in the form

$$V(r_{ij})=V_0\exp[i\varphi_{ij}-s(r_{ij})]\ , \tag{14}$$

where the phase φ_{ij} is defined by Eq. (7) and V_0 is a prefactor. The analytical expression for the exponent $s(r_{ij})$ can be obtained in the limit of low and high magnetic field:[1]

$$s(r_{ij})=\begin{cases}\dfrac{r_{ij}}{a_B^*}+\dfrac{r_{ij}^3 a_B^*\sin^2\Theta_{ij}}{24\lambda^4} & \text{if } r_{ij}\ll\dfrac{\lambda^2}{a_B^*}\\[2ex] \dfrac{r_{ij}|\cos\Theta_{ij}|}{a_B^*}+\dfrac{r_{ij}^2\sin^2\Theta_{ij}}{4\lambda^2} & \text{if } r_{ij}\gg\dfrac{\lambda^2}{a_B^*}\ ,\end{cases} \tag{15}$$

where

$$\sin^2\Theta_{ij}=\frac{(\mathbf{B}\times\mathbf{r}_{ij})^2}{B^2 r_{ij}^2} \tag{16}$$

and

$$\lambda=\left[\frac{\hbar c}{eB}\right]^{1/2} \tag{17}$$

is the magnetic length. The resistance corresponding to the pair of donors i and j is equal to $R_0\exp\xi_{ij}$ with

$$\xi_{ij}=2s(r_{ij})+\frac{|\varepsilon_i|+|\varepsilon_j|+|\varepsilon_i-\varepsilon_j|}{2T}\ , \tag{18}$$

where ε_i and ε_j are the energies of the donors i and j measured from the Fermi level. According to the percolation theory, the total resistance of the system is equal to $R_0\exp\xi$ with ξ defined as the minimal value for which the resistance with $\xi_{ij}<\xi$ constitutes the infinite cluster. The typical hopping length in zero field is of the order of $a_B^*(T_0/T)^{1/3}$. Comparing this value to λ^2/a_B^*, we find that the magnetic field separating the low- and high-field limits is

$$B_c=\frac{\hbar c}{e a_B^{*2}}\left[\frac{T}{T_0}\right]^{1/3} \quad\text{or}\quad \lambda_c^2=a_B^{*2}\left[\frac{T_0}{T}\right]^{1/3}\ . \tag{19}$$

It can be shown that $\xi(B)$ is the universal function of the ratio B/B_c:

$$\xi(B)=\left[\frac{T_0}{T}\right]^{1/3}K\left[\frac{B}{B_c}\right]\ , \tag{20}$$

where the single-scale function K differs for the $B_\perp$ and $B_\parallel$ orientations. For both limits of B the asymptotical form of K can be found with the use of the method of invariants in percolation theory.[1] One finds

$$K_\perp(x)=\begin{cases}1+\dfrac{x^2}{360}, & x\ll 1\\[2ex] \dfrac{x^{1/2}}{4}+\dfrac{3}{2x}\ln x, & x\gg 1\end{cases} \tag{21}$$

and

$$K_\perp(x)=\begin{cases}1+\dfrac{x^2}{720}, & x\ll 1\\[2ex] \dfrac{1}{8}\left[\dfrac{35\pi}{2}\right]^{2/5}x^{1/5}, & x\gg 1\end{cases} \tag{22}$$

for the two orientations of the field. The numerical coefficient $\frac{1}{360}$ in Eq. (21) coincides with that obtained in Ref. 3. A similar result for $\xi_\perp(B)$ was obtained in Ref. 2 when the Coulomb gap in the density of states at energies close to the Fermi level was taken into account. If we rewrite Eq. (21) for $x\ll 1$ as

$$\xi(B)-\xi(0)=\frac{1}{360}\left[\frac{a_B^*}{\lambda}\right]^4\xi^3(0)\ , \tag{23}$$

the result in Ref. 2 has the form of Eq. (23) with the coefficient $\frac{1}{500}$ instead of $\frac{1}{360}$. The asymptotics in Eqs. (21) and (22) are shown in Fig. 6. Unfortunately, for the intermediate values of the magnetic field the functions $K_\parallel$ and $K_\perp$ cannot be found analytically. Nevertheless, one can assume from Fig. 6 that the low-field asymptotics are valid approximately up to $B/B_c=10$. For $T=0.6$ K we have $B_c=0.54T$. Therefore the interval of validity of low-field asymptotics is estimated as $B\le 5T$.

In the above consideration it was assumed that the density of states at the Fermi level $g(E_F)$ does not depend on the magnetic field. On the other hand, in the beginning of this subsection it was shown that $g(E_F)$ in-

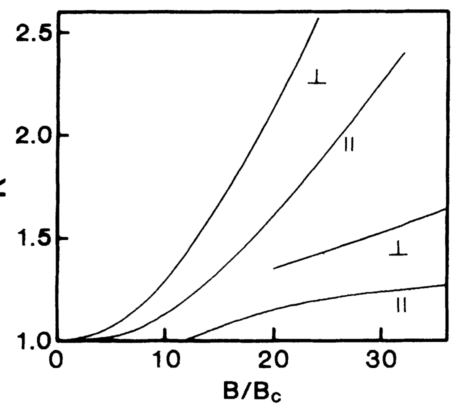

FIG. 6. Asymptotics of the normalized log resistance as a function of the normalized magnetic field for $B_\perp$ and $B_\parallel$ orientations.

creases with B. To take this dependence into account, let us introduce the dimensionless functions

$$G_{\perp,\parallel}(B)=\frac{g_{\perp,\parallel}(E_F,B)}{g(E_F,0)} . \tag{24}$$

With this definition the expressions for the log resistance in both orientations can be rewritten in the form

$$\xi_{\perp,\parallel}=\left[\frac{T_0}{T}\right]^{1/3} G_{\perp,\parallel}^{-1/3}(B)K_{\perp,\parallel}\left[\frac{B}{B_c G_{\perp,\parallel}^{1/3}(B)}\right] . \tag{25}$$

It is seen that if the increase of $G^{1/3}(B)$ with B predominates over the increase of the function K which is responsible for positive MR, then Eq. (25) describes negative MR. At high enough fields $G(B)$ gains saturation at some certain value, the resistance passes through a minimum, and the transition from negative MR to positive MR takes place.

Let us return to the discussion of the experimental data in Figs. 1 and 3. The most striking fact to be explained is the "inverted" relation between the resistances in $B_\perp$ and $B_\parallel$ orientations for $B<3.5T$. An explanation is provided by the expressions in Eq. (25). The ratio

$$\frac{\xi_\perp(B)}{\xi_\parallel(B)}=\frac{K_\perp\left[\dfrac{B}{B_c}G_\perp^{1/3}\right]}{K_\parallel\left[\dfrac{B}{B_c}G_\parallel^{1/3}\right]}\left[\frac{G_\parallel(B)}{G_\perp(B)}\right]^{1/3} \tag{26}$$

represents the product of two factors. The first is greater than unity while the second is less than unity. As a result $\xi_\perp$ may become less than $\xi_\parallel$ within some interval of B. The relation $G_\parallel(B)<G_\perp(B)$ is the consequence of the fact that the increase of the density of states at the Fermi level with B is caused by the shrinkage of the impurity wave functions. The effect is stronger in $B_\perp$ orientation than in $B_\parallel$.

The calculation of magnetoresistance caused by the incoherent mechanism reduces to the evaluation of normalized density of states at the Fermi level $G(B)$. The latter evaluation can be carried out in terms of the following simple model. We assume that the spread of the energies of donors within the impurity band originates exclusively from the quantum-mechanical overlapping of their wave functions (i.e., the Lifshitz model[13]). Then the typical width of the impurity band is determined by the splitting of the energy levels of two donors at distance $N_D^{-1/2}$. We are interested in the situation when the Fermi level is in the tail of the impurity band. The states in the tail are the result of the splitting of impurity levels within close pairs of donors at distances much less than $N_D^{-1/2}$. The magnitude of splitting of two levels i and j is equal to $|V(r_{ij})|$. For the density of states in the tail we have

$$g(E)=\frac{N_D^2}{2}\int d^2r\,\delta(|V(r)|-E) . \tag{27}$$

The position of the Fermi level is determined by the condition

$$\not{k}N_D=\int_{E_F}^{\infty}dE\,g(E)=\frac{N_D^2}{2}\int d^2r\,\Theta(|V(r)|-E_F) , \tag{28}$$

where $\Theta(x)$ is the unit-step function and $\not{k}$ is the degree of compensation. To find the function $G(B)$, one substitutes the expression for $V(r)$ in Eq. (28), finds the dependence $E_F(B)$, and substitutes it into Eq. (27). For low magnetic field the result has the form

$$G_\perp(B)=\exp\left[\frac{q^{3/2}a_B^{*4}}{12\lambda^4}\right],\quad G_\parallel(B)=\exp\left[\frac{q^{3/2}a_B^{*4}}{24\lambda^4}\right] . \tag{29}$$

The dimensionless parameter q is

$$q=\frac{2}{\pi\not{k}N_D a_B^{*2}} . \tag{30}$$

We see that both $G_\perp(B)$ and $G_\parallel(B)$ increase with B confirming the result of qualitative considerations. It is also seen from Eq. (29) that $G_\perp(B)>G_\parallel(B)$.

Unfortunately Eq. (29) is not applicable to the experimental situation under study for the following reason. As shown in Ref. 14, Eq. (14) is not valid if the distance between the donors is less than 3.7 a_B^*. In our case the average distance $N_D^{-1/2}\approx 3a_B^*$. The other argument against the applicability of the simple model is that it does not take into account the classical Coulomb shifts of the energy levels caused by the charged donors and acceptors. To find the exact dependencies $G_\perp(B)$ and $G_\parallel(B)$ a computer simulation of the impurity band in a magnetic field is required.

In the absence of such a calculation one can choose some reasonable form for the dependencies $G_\perp(B)$ and $G_\parallel(B)$ and study the possibility of reproducing the experimental magnetoresistance using Eq. (25). Both $G_\perp(B)$ and $G_\parallel(B)$ in the limit of large B saturate at some value G_∞. This value can be estimated from the experimental curves in Fig. 3. If we assume that within some interval of B, in which $G_\perp(B)$ is saturated, the low-field asymptotics for $K_\perp$ is still valid, then the dependence $\xi_\perp(B)$ would be quadratic. Thus fitting the experimental curve $\ln R_\perp(B)$ to a B^2 dependence and extrapolating it to $B=0$ yields the intercept $R_\perp(0)$ which is related to $R(0)$ as

$$\ln\left[\frac{R(0)}{R_\perp(0)}\right]=\left[\frac{T_0}{T}\right]^{1/3}(1-G_\infty^{-1/3})\ . \tag{31}$$

Equation (31) permits one to estimate the value G_∞. For temperatures $T=0.39$ and 0.6 K the ratio $R(0)/R_\perp(0)$ is 137 and 52, respectively, leading to values of $G_\infty=4.0$ and 3.7. Note that although the ratio varies with temperature by a factor of 2.5, G_∞ remains constant to within 10%.

For the numerical calculations the following model dependencies were chosen:

$$G_{\perp,\parallel}=\frac{3\exp(B^2/b_{\perp,\parallel}^2)}{2+\exp(B^2/b_{\perp,\parallel}^2)}\ , \tag{32}$$

where $b_\perp$ and $b_\parallel$ are some adjusting parameters. It is seen from Eq. (32) that $G_\infty=3$. At low fields these expressions are consistent with Eq. (29). Substituting into Eq. (25) and using the low-field asymptotics for the functions $K_{\perp,\parallel}$ we get

$$\xi_\perp(B)=\left[\frac{T_0}{T}\right]^{1/3}\left[G_\perp^{-1/3}+\frac{B^2}{360B_c^2}G_\perp^{-1}\right]\ . \tag{33}$$

For $\xi_\parallel(B)$ the number 360 is to be replaced by 720.

Figure 7 shows the normalized log resistance in $\xi_\perp(B)/\xi(0)$ as a function of normalized magnetic field $B/b_\perp$ calculated with the use of Eq. (33) for several values of the parameter $\nu=b_\perp/B_c$. It is seen that while the parameter changes within the narrow interval $8.9<\nu<11.2$ the form of the curves undergoes a drastic transformation. For the smallest $\nu=8.9$ curve 1 exhibits the negative MR behavior beginning from zero field. The relative depth of the minimum is 6%. For $T=0.6$ K such 6% corresponds to a 50% decrease of the resistance. For $\nu<8.9$ the magnitude of negative MR is even greater than 50%. With the increase of ν the positive MR behavior of the curves changes. Nevertheless each curve exhibits in its beginning something like a plateau. This slowing of the increase of the resistance is caused by the incoherent mechanism. Comparing the curves in Fig. 7 and the $B_\perp$ curves in Fig. 3 one can conclude that the incoherent mechanism permits one to reproduce roughly the shape of the experimental dependencies. The typical value $b_\perp\approx 10B_c\approx 5T$ also appears reasonable.

To study the relation between the resistance in $B_\perp$ and $B_\parallel$ orientations in Fig. 8 curve 1 from Fig. 7 is shown together with the $\xi_\parallel(B)$ for two values of the ratio $b_\parallel/b_\perp$. We see that the "inverted" relation between $R_\perp$ and $R_\parallel$ is reproduced by the incoherent mechanism, if the appropriate value of the parameter $b_\parallel/b_\perp$ is chosen. For $b_\parallel/b_\perp=2.2$ (curve 2 in Fig. 8) the maximal ratio of the resistances is 5.2 for $T=0.6$ K. The value $b_\parallel/b_\perp\approx 2$ is in reasonable agreement with the simple model considered earlier. Returning to the experimental dependencies $R_\perp(B)$ in the region of negative MR (Fig. 3) we realize that there are two characteristic regimes of magnetic field which determine the behavior of these curves. The first $0<B<0.5T$ corresponds to the sharp decrease of the resistance starting from $B=0$. This decrease is accounted for by the interference mechanism of negative MR. The characteristic value of B in which the interference correction saturates was estimated in Ref. 9 to be of the order of $B_c(T_0/T)^{1/6}$. With $B_c\sim 0.5T$ we expect saturation near $0.2T$, a value that agrees reasonably with the experimental data. The slope of the curves in Fig. 3 increases with the decrease of T as expected.

The regime $0.5T<B<4T$ relates to the position of the second minimum. This range can be accounted for by incoherent mechanism. Note, that in low fields the behavior of the dependencies Eq. (33) is rather complicated. For instance, curve 3 in Fig. 7, which corresponds to $\nu=10.4$, first increases with B, then passes through a maximum, decreases, and passes through a minimum. The negative correction to the resistance, caused by the interference mechanism, shifts this part of the curve to below $R_\perp(0)$. Therefore, the additional structure in the dependencies $R_\perp(B)$ can be accounted for by the combined action of the interference and the incoherent mechanisms.

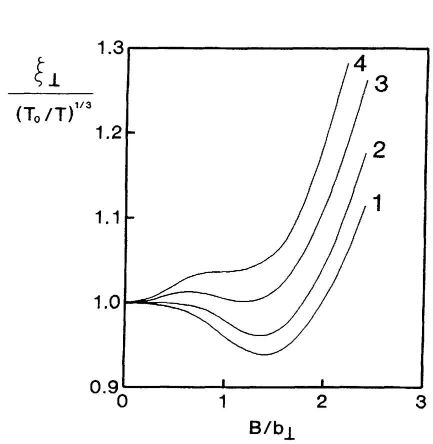

FIG. 7. The dependence of the normalized log resistance on the normalized magnetic field for $B_\perp$ orientation for various values of parameter ν: (1) $\nu=9$, (2) $\nu=9.49$, (3) $\nu=10.39$, (4) $\nu=11.22$.

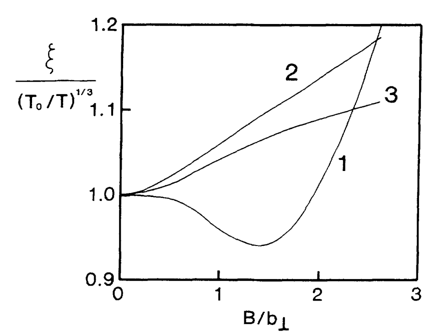

FIG. 8. The dependence of the normalized log resistance on the normalized magnetic field for $B_\parallel$ orientation for two values of parameter $b_\parallel/b_\perp$. (2) $b_\parallel/b_\perp=2.2$, (3) $b_\parallel/b_\perp=2$. Curve 1 is the same as in Fig. 7.

V. CONCLUSIONS

The experimental data presented in this paper indicate that alongside the interference mechanism of negative MR at low magnetic fields some additional suppression of positive MR plays an important role in the region of intermediate fields. As an explanation we propose the incoherent mechanism, which allows us to explain qualitatively the features of the experimental data.

We have restricted the consideration to the temperatures corresponding to the VRH regime of conduction. In this regime the action of both mechanisms is most pronounced. In the NNH regime the interference mechanism is not operative while the incoherent mechanism still applies.

Let us discuss the action of incoherent mechanism in the NNH regime. In this regime the resistance is proportional to $\exp(\varepsilon_3/T)$, where ε_3 is the activation energy. According to the conventional theory[1] in a lightly doped semiconductor ε_3 is equal to the difference between the energy positions of the Fermi level and the isolated donor level. If the overlapping between the donors at average distance is much less than the typical Coulomb shifts of the donor levels produced by the charged impurities in the system (classical impurity band[1]), then ε_3 is independent of B. If the overlap is strong enough, it causes the magnetic-field-dependent corrections to ε_3. There are two reasons for such corrections. The first is that the reduction of the overlap with B moves the Fermi level towards the center of impurity band (see Sec. IV B and Fig. 5). Consequently, the activation energy decreases with B. Probably such a decrease explains the negative MR observed in Ref. 8 for 2 K$<T<$4.5 K and $B<2T$ in the same samples that were studied in the present work. The other overlap-induced correction to ε_3 results from the splitting of the energy levels of donors between which the hops take place. The suppression of this splitting in magnetic field creates the tendency for $\varepsilon_3(B)$ to increase.

The dependence of both corrections on B is caused by the orbital shrinkage effect. Therefore they are quadratic in B in the low-field limit. The competition of opposite tendencies may cause nonmonotonic dependence ε_3 on B. Such a behavior (the decrease for $B<2T$ and the increase for $B>2T$) was observed in Ref. 8.

It should be emphasized in conclusion that the concentration of donors in the samples studied was about 40% less than the critical concentration $1.3\times10^{11}/\mathrm{cm}^{-2}$ corresponding to the Mott transition. For this reason, we have neglected the magnetic-field dependence of the resistance caused by the proximity of the Fermi level to the mobility threshold as it is discussed in Ref. 15.

As a final remark, we address the question of inhomogeneity in the layers. It has been argued in the literature that sample inhomogeneities will lead to structure in the magnetoresistance. Several samples prepared from the same wafer have reliably reproduced the magnetic-field dependences discussed here. The overall trends have also been observed in other wafers with somewhat different densities. From our work with δ-layer transport in the past seven years (materials GaAs and InP) intralayer inhomogeneities have never been an issue. This is a credit to the mature MBE technology with rotating wafers and precise control of the doping sources. Real concern attaches to the interlayer variations in the electron density of the 20 successive layers that contribute in parallel to conduction. We must expect the top and the bottom layers to be depleted by charge trapping at the surface or the substrate interface. This will raise their resistance and lead to a maximal increase of the measured value by 10%. In no way will it account for the large variations found in the resistance. We argue that the effects discussed in the paper are basic physics and are not an artifact of sample preparation.

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