Magnetotransport properties of two-dimensional electron gases under a periodic magnetic field

Deng Ping Xue* and Gang Xiao
Department of Physics, Brown University, Providence, Rhode Island 02912
(Received 18 October 1991)

We have calculated electronic and transport properties of two-dimensional electron gases (2DEG's) under a *periodic* magnetic field. In the perturbation framework, the magnetoresistances are similar to those of 2DEG's under a spatial potential modulation, although the classical picture of cyclotron motion differs from the $\mathbf{E}\times\mathbf{B}$ drift. A flat-band condition and an additional term in the Hall conductivity have been found in the current system. We have suggested experimental avenues for the study of magnetically modulated 2DEG's.

Transport properties of two-dimensional electron gases (2DEG's) in a perpendicular magnetic field and an in-plane grating potential were the topic of some recent publications.[1–8] The modulated 2DEG is realized in high-mobility $Al_xGa_{1-x}As$-GaAs heterojunctions by illuminating samples with interfering laser beams or by a periodic voltage-gate array on the top of heterojunction films. It has been observed that at magnetic fields (B) higher than 0.5 T, Shubnikov–de Haas–type oscillations (SdH) appear in the resistivity tensor elements ρ_{xx} and ρ_{yy}. At lower fields, a different oscillation dominates, which has been understood from both a classical and a quantum-mechanical point of view. But all of the works quoted above were confined to 2DEG's with electric modulation. Transport properties of 2DEG's with magnetic modulation, that is, in a *periodic magnetic field*, have received much less attention both theoretically and experimentally. Recently, Bending, von Klitzing, and Ploog[9] performed Hall and magnetoresistance measurements on a low-mobility 2DEG in a flux-lattice field of a type-II superconductor. They observed weak localization at extremely low B and suggested the application of the 2DEG as a detector of flux-lattice properties.[9]

In this paper we shall investigate theoretically a 2DEG subject to an oscillating magnetic field. Such a system would be achieved experimentally by depositing periodic lines of magnetic materials (e.g., iron) onto an $Al_xGa_{1-x}As$-GaAs heterojunction, using modern lithography. Another metallic overlayer (e.g., silver) that has a similar work function as iron can be deposited onto the magnetic lines. Electrically, both the magnetic and nonmagnetic metals serve as a uniform gate with no electric potential modulation. However, when a uniform magnetic field B_0 is applied, the magnetic lines become local micromagnets and produce a grating magnetic field. The proposed magnetic grating structure and the calculated oscillating field in the 2DEG are shown in Fig. 1. We will neglect the weak-localization correction to the conductivity, which is not very important in high-mobility electron gases.

We consider a 2DEG system subject to a 1D oscillating magnetic field centered around a uniform B_0 background:

$$\mathbf{B}=(B_0+B_1\cos Kx)\mathbf{z},\quad K=2\pi/a\ , \tag{1}$$

where a is the period of the field modulation. The Hamiltonian of the system is $H=1/2m(\mathbf{p}+e\mathbf{A})^2$, where the vector potential $\mathbf{A}=\mathbf{A}_0+\mathbf{A}_1$ is chosen as

$$\mathbf{A}_0=(0,B_0x,0),\quad \mathbf{A}_1=\left(0,\frac{B_1}{K}\sin Kx,0\right). \tag{2}$$

Omitting higher-order terms of B_1, one obtains $H=H_0+H_1$, where $H_0=1/2m(\mathbf{p}+e\mathbf{A}_0)^2$ is the Hamiltonian of the 2DEG in a uniform field, and $H_1=eB_1/mK(p_y+eB_0x)\sin Kx$ is the new perturbation term due to field modulation. The eigenvalue problem of

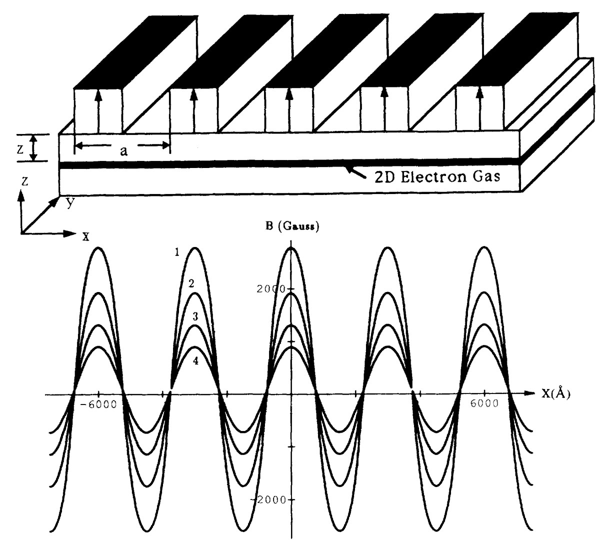

FIG. 1. Proposed 1D periodic magnetic grating structure to generate field modulation. The magnetic lines fabricated lithographically serve as local micromagnets. The lower graph is the calculated magnetic field at a 2DEG that is Z distance away from the bottom of the magnetic line (1500×1500 Å^2, a=3000 Å, material: iron). Note: for curve 1, Z=400 Å; 2, 600 Å; 3, 800 Å; 4, 1000 Å.

H_0 is well known, i.e, $E_n^0=\hbar\omega_c(n+\frac{1}{2})$, $\psi_{nk}=L^{-1/2}e^{iky}\phi_n(x-x_0)$, where $\omega_c=eB_0/m$ is the cyclotron frequency, L is the length of the system in the y direction, and $\phi_n(x-x_0)$ is the harmonic-oscillator function centered at $x_0=kl^2$, with $l=(\hbar/eB_0)^{1/2}$ being the magnetic length. It is justified to treat the term eB_0x in H_1 as a perturbation even in the limit where x goes to infinity. This is because the zeroth-order wave function is that of a harmonic oscillator that decays exponentially with $x-x_0$. Treating H_1 as a perturbation gives rise to the first-order wave functions

$$\psi_{nk}^{(1)}=\psi_{nk}+\sum_{m\neq n}\frac{\langle mk|H_1|nk\rangle}{E_n^0-E_m^0}\psi_{mk} \tag{3}$$

and eigenenergy

$$E_{nx_0}=E_n^0+V_n\cos Kx_0 , \tag{4}$$

$$V_n=\tfrac{1}{2}\hbar\omega_1 e^{-u/2}[L_n^1(u)+L_{n-1}^1(u)] , \tag{5}$$

where $\omega_1=eB_1/m$, $u=K^2l^2/2$, and $L_n^1(u)$ is the associated Laguerre polynomial. Therefore, under field modulation, Landau levels (LL's) broaden into minibands whose widths oscillate with B_0, a, and band index n. Although a similar feature is also seen in the 2DEG under electric modulation, there are substantial differences between magnetic and electric modulation. In particular, the different expression for V_n in (5) leads to a different flat-band condition,

$$L_n^1(u)+L_{n-1}^1(u)=0 , \tag{6}$$

which will manifest itself in transport properties. Using the asymptotic expression[10]

$$L_n^1(u)\simeq\pi^{-1/2}e^{u/2}u^{-3/4}n^{1/4}\cos\left[2(nu)^{1/2}-\frac{3\pi}{4}\right] \tag{7}$$

and $L_n^1(u)\simeq L_{n-1}^1(u)$ for large n, one obtains from Eqs. (6) and (7) that

$$\frac{2R_c}{a}=\lambda+\tfrac{1}{4} , \quad \lambda=1,2,3,\ldots , \tag{8}$$

where $R_c=(2n)^{1/2}l$ is the classical cyclotron radius at Fermi energy E_F. In the case of electric modulation, the flat-band condition for large n is $2R_c/a=\lambda-\frac{1}{4}$, different from Eq. (8) by a negative sign before $\frac{1}{4}$. From Eqs. (5) and (7), one observes that in the limit $E_F\gg\hbar\omega_c$ (i.e., large n), the electron bandwidth oscillates sinusoidally and is periodic in $1/B_0$ when n and a are fixed. For small n, the bandwidth still oscillates with $1/B_0$, but the flat-band condition of Eq. (8) no longer holds because neither Eq. (7) nor $L_n^1\simeq L_{n-1}^1$ is valid. The oscillation in the bandwidth and, hence, in the density of states (DOS) near E_F will have profound effects on transport properties.

To calculate transport coefficients, we follow the formulation of Refs. 5 and 11, which is derived from general Liouville equation and includes dissipation explicitly. Under the one-particle approximation, one has

$$\sigma_{\mu\mu}=\frac{\beta e^2}{\Omega}\sum_{\zeta}f_\zeta(1-f_\zeta)\tau(E_\zeta)\langle\zeta|v_\mu|\zeta\rangle^2+\frac{\beta e^2}{2\Omega}\sum_{\zeta,\zeta'}f_\zeta(1-f_{\zeta'})W_{\zeta\zeta'}(a_\mu^\zeta-a_\mu^{\zeta'})^2 , \quad \mu=x,y , \tag{9}$$

where $\beta=1/k_BT$, Ω is the volume of the system, $|\zeta\rangle$ the single electron state, $\tau(E_\zeta)\simeq\tau$ is the relaxation time, f is the Fermi-Dirac function, $W_{\zeta\zeta'}$ is the transition probability due to impurity scattering, and $a_\mu^\zeta=\langle\zeta|r_\mu|\zeta\rangle$. This formula has been successfully applied to electrically modulated 2DEG's (Ref. 5) and to other systems.[12] The first term in Eq. (9) describes the extended-state contribution that leads to Drude conductivity for free-electron gases. The second term results from the localized-state contribution that leads to SdH oscillations in a 2DEG under a magnetic field. In magnetically modulated 2DEG, wave functions are extended in the y direction, but are localized in the x direction. So the first term in Eq. (9) contributes to σ_{yy} and the second to σ_{xx}. Substituting $\psi_{nk}^{(1)}$ for $|\zeta\rangle$ and E_{nx_0} for E_ζ in Eq. (9), one obtains

$$\sigma_{yy}=\frac{2\pi^2\tau e^2l^2}{h\hbar a^2}\sum_n V_n^2\left[-\frac{\partial f}{\partial E}\right]_{E=E_{n0}} , \tag{10}$$

$$\sigma_{xx}=\frac{e^2\beta N_iU_0^2}{h\pi\Gamma al^2}\sum_n(2n+1)\int_0^a dx_0 f_{nx_0}(1-f_{nx_0}) , \tag{11}$$

where we have kept only the leading term of V_n^2, N_i is the impurity concentration, U_0 is the Fourier-transformed impurity potential, and Γ is the impurity broadening parameter. E_F is determined self-consistently from

$$N_s=2\sum_{n,k}f_{nk} , \tag{12}$$

where N_s is the total number of electrons. Taking a high-mobility $Al_xGa_{1-x}As$-GaAs heterojunction as an example ($m_{\text{eff}}\simeq0.07m_e$), numerical results for $\rho_{xx}\simeq\sigma_{yy}B_0^2/e^2n_s^2$ and $\rho_{yy}=\sigma_{xx}B_0^2/e^2n_s^2$ (n_s is area carrier density) are given in Fig. 2. The beating behavior reflects the resonance of three characteristic lengths, i.e., the Fermi wavelength $\lambda_F=2\pi/k_F$, the magnetic length l, and the modulation period a. In the following, we use the term SdH-type oscillation to refer to those resistivity oscillations stemming from oscillatory DOS's. The SdH-type oscillations overlap on a slowly oscillating envelope, and the former is a resonance between l and λ_F, whereas the latter is between l and a. At $B\leq0.3$ T, SdH-type oscillations are too fine to resolve, and ρ_{xx}, ρ_{yy} are out of phase, since V_n and the DOS are out of phase. But at $B\geq0.3$ T, SdH oscillations manifest themselves dramatically and ρ_{xx} and ρ_{yy} are in phase; this is better understood when we express Eqs. (10) and (11) in terms of DOS $D_n(E)$,

$$\sigma_{yy}\sim\sum_n\int dE\left[-\frac{\partial f}{\partial E}\right][V_n^2-(E-E_n^0)^2]D_n(E) , \tag{13}$$

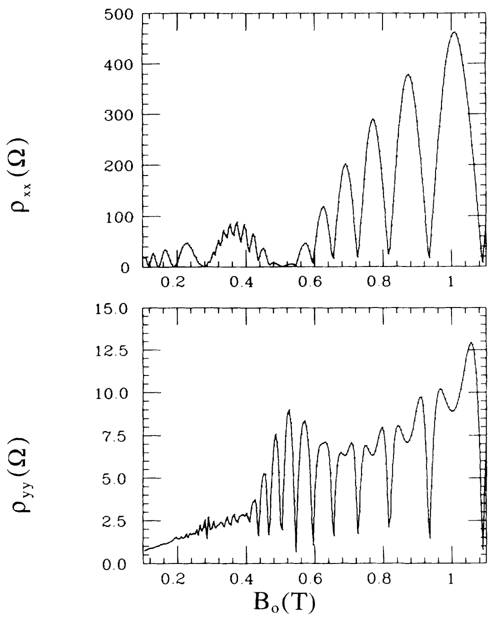

FIG. 2. Calculated ρ_{xx}, ρ_{yy} as a functions of magnetic field B_0 at $T=1$ K, $a=3000$ Å, and $B_1=0.07$ T for a 2DEG sample with mobility $\mu=1.3\times10^6$ cm^2/V s, area carrier density $n_s=3.16\times10^{11}$ cm^{-2}, impurity concentration $N_i=1\times10^8$ cm^{-2}, and $\Gamma=0.01$ meV.

$$\sigma_{xx}\sim\sum_n(2n+1)\int dE\left[-\frac{\partial f}{\partial E}\right]D_n(E)\ . \tag{14}$$

At low T, $(-\partial f/\partial E)\approx\delta(E-E_f)$. If E_F lies in the gap of adjacent LL's, both σ_{xx} and σ_{yy} are zero. This guarantees that σ_{xx} and σ_{yy} have the same minimum positions and thus are in phase as far as SdH-type oscillations are concerned.

In Fig. 2, the oscillations in ρ_{xx} and ρ_{yy} result from changing the electron population in different LL's by varying the LL spacing (i.e., varying B_0). The population in different levels may also be varied by changing n_s through a gate voltage while fixing the magnetic field. This approach may have certain advantages over changing B_0. In particular, if the field modulation is provided by a flux-lattice of a type-II superconductor overlayer, the variation in B_0 also changes the periodicity of the field modulation. This complicates data analysis, even though the problem is interesting in its own right. Figure 3 displays a typical result of ρ_{xx} as a function of n_s. The beating behavior occurs again: the oscillations with a short period are caused by sweeping E_F through subsequent LL's, in other words, by resonance between λ_F and l; the slowly varying envelope reflects the variation of bandwidth, i.e., the resonance between λ_F and a.

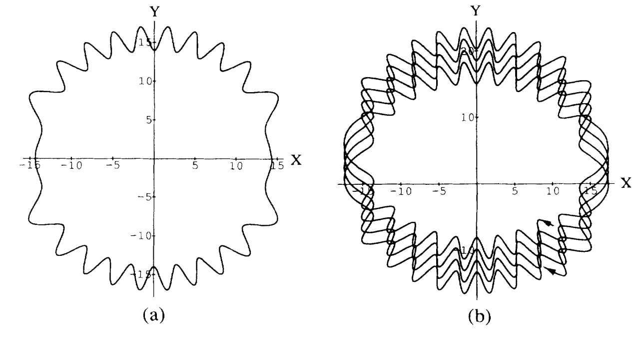

FIG. 4. Classical trajectories of an electron in a periodic magnetic field with $a=3000$ Å, $B_1/B_0=0.1$ (x,y units: 100 Å). (a) Off resonance: $2R_c/a=10+\frac{1}{4}$ ($B_0=$ 0.60 T); the average drift velocity is zero, and the electron forms a closed orbit. (b) On resonance: $2R_c/a=10+\frac{3}{4}$ ($B_0=$ 0.57 T); there is a drift of orbit center in the y direction.

The magnetoresistance oscillation is at low B_0 in Fig. 2 has a semiclassical explanation. In the case of an electrically modulated 2DEG, Beenakker[13] pointed out that a similar oscillation in ρ_{xx} results from a resonance between the periodic cyclotron motion and the oscillating $\mathbf{E}\times\mathbf{B}$ drift of the orbit center. There, the electric field $\mathbf{E}$ is due to the potential modulation. In a magnetically modulated 2DEG, the drift of the orbit center is not caused by $\mathbf{E}\times\mathbf{B}$, since $\mathbf{E}=0$. Instead, it is due to the spatial variation of the cyclotron frequency in a nonuniform field. Following Beenakker's guiding-center-drift approach, we show here the semiclassical picture of ρ_{xx} oscillation. Under a modulating magnetic field of Eq. (1),

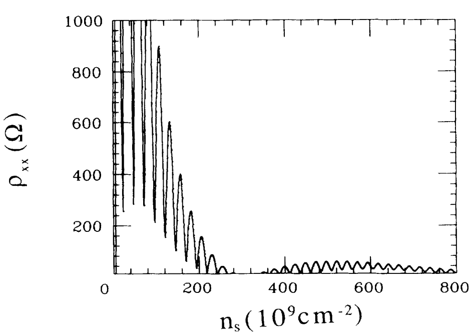

FIG. 3. Calculated ρ_{xx} as a function of carrier concentration n_s controlled by a gate voltage ($B_0=0.5$ T). Parameters are the same as those in Fig. 2.

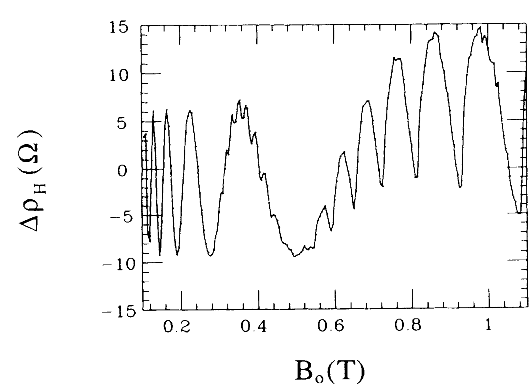

FIG. 5. Deviation of Hall resistivity of a field-modulated 2DEG from that of an unmodulated 2DEG, $\Delta\rho_H=\rho_H(B_1)-\rho_H(B_1=0)$, as a function of magnetic field with the same parameters in Fig. 2.

the angular velocity is $\dot{\phi}=\omega_c+\omega_1\cos Kx$. The guiding center (X,Y) of an electron with velocity (v_x,v_y) and position (x,y) is $X=x-y_y/\dot{\phi}$, $Y=y+v_x/\dot{\phi}$. The velocity of the guiding center is $\dot{X}=\ddot{\phi}v_y/(\dot{\phi})^2$, $\dot{Y}=-\ddot{\phi}v_x/(\dot{\phi})^2$. The time-averaged drift velocity of the center ($v_d^{(x)}$ and $v_d^{(y)}$) is obtained by integrating $\dot{X}$ and $\dot{Y}$ along the orbit, $v_d^{(x)}=0$ and $v_d^{(y)}=\omega_1 R_c\sin(Kx)J_1(KR_c)$ (J_1 is the Bessel function of first order). In the calculation, we have kept only the lowest order in ω_1 and have made use of the relations $v_k=-v_F\sin\phi$, $v_y=v_F\cos\phi$. The drift motion is along the y direction perpendicular to the direction of the field modulation. The square average of $v_d^{(y)}$ over X is

$$\langle (v_d^{(y)})^2\rangle=\frac{\omega_1^2R_c}{\pi K}\cos^2\left[KR_c-\frac{3\pi}{4}\right], \qquad (15)$$

where the asymptotic expansion of J_1 is used. Equation (15) contributes $\delta D=\tau/2\langle (v_d^{(y)})^2\rangle$ to the element D_{yy} of the two-dimensional diffusion matrix. Using Einstein's relation $\rho=h^2/4\pi me^2D^{-1}$, we obtain

$$\rho_{xx}=\rho_0\frac{\omega_1^2\tau^2}{\pi KR_c}\cos^2\left[\frac{2\pi R_c}{a}-\frac{3\pi}{4}\right], \qquad (16)$$

with $\rho_0=h/k_Fv_Fe^2\tau$. Equation (16) accounts for the low-field ρ_{xx} oscillation in Fig. 2 and predicts the same position of the ρ_{xx} minima (i.e., the flat-band condition) as obtained from the quantum-mechanical calculation [see Eq. (8)]. The calculated classical trajectories are shown in Fig. 4 for a resonating case ($2R_c/a=10.75$) where the drift is large and for an off-resonating case ($2R_c/a=10.25$) where the average drift vanishes. As mentioned earlier, the flat-band condition differs between electric and magnetic modulation. Such a difference can be confirmed by measuring the positions of ρ_{xx} minima.

Hall conductivity can be expressed[5] as

$$\sigma_{yx}=\frac{i\hbar e^2}{\Omega}\sum_{\zeta\neq\zeta'}f_\zeta(1-f_{\zeta'})\langle\zeta|v_x|\zeta'\rangle \times\langle\zeta'|v_y|\zeta\rangle\frac{1-e^{-\beta(E_{\zeta'}-E_\zeta)}}{(E_\zeta-E_{\zeta'})^2}. \qquad (17)$$

In a magnetically modulated 2DEG system, we obtain σ_{yx} to the leading order in B_1:

$$\sigma_{yx}=\frac{e^2}{ha}\sum_n\int_0^a dx_0\frac{(f_{nx_0}-f_{n+1x_0})[n+1+(B_1/B_0)e^{-u/2}L_n^1(u)\cos Kx_0]}{[1+(\hbar\omega_c)^{-1}(V_{n+1}-V_n)\cos Kx_0]^2}. \qquad (18)$$

The above σ_{yx} differs substantially from that of an electrically modulated 2DEG system due to the addition of an extra term $(B_1/B_0)e^{-u/2}L'_n(u)\cos Kx_0$. Using $\rho_H=1/\sigma_{yx}$, we have calculated $\Delta\rho_H=\rho_H(B_1)-\rho_H(B_1=0)$, i.e., the difference in ρ_H between field-modulated and non-modulated 2DEG. The result of $\Delta\rho_H$ is presented in Fig. 5 as a function of B_0. The general features of $\Delta\rho_H$ are similar to ρ_{xx} with in-phase oscillations. The slowly varying envelope is due to the bandwidth oscillation, while the short-period oscillation is SdH-type. $\Delta\rho_H$ can be best determined by taking the derivative $d\rho_H/dB_0$ during measurement.

Our results are valid for small B_1, which enabled us to proceed analytically. There do exist linear effects with B_1 in σ_{xx} and σ_{yx}, as can be seen from Eqs. (11) and (18). The linear effect in σ_{yy} is smaller, implying that a better treatment on the full Hamiltonian is needed. It needs to be pointed out that a similar perturbation approach has been applied to 2DEG with electric modulation (Refs. 1–6), and results obtained are consistent with experiment. This lends validity to our calculation of the magnetic modulation effect, which in the perturbation limit is a generalization of electric modulation.

In conclusion, we have studied the electron energy spectra in magnetically modulated 2DEG's and have calculated the resistivity tensor. In the low-field limit ($B_0<1$ T), ρ_{xx} and ρ_{yy} behave similarly to their counterparts in the electrically modulated 2DEG, exhibiting beating both as a function of B_0 and n_s. This behavior is the result of the resonance of the cyclotron diameter with the modulation period and with the Fermi wavelength. The classical picture of electron motion is different from the electrically modulated case; the guiding-center drift is caused by the spatial variation in cyclotron frequency. Consequently, the flat-band condition or the positions of the ρ_{xx} minima differ from those in the electrically modulated system. In addition, the Hall conductivity acquires a new term that is absent in the electric modulation case.

We would like to thank N. Krendera, P. Xiong, and N. S. Rebello for assistance in some numerical calculations. This work was supported in part by the National Science Foundation through Grant No. DMR-9024402. One of us (G.X.) wishes to acknowledge financial support from the A. P. Sloan Foundation.

*Present address: Department of Electrical Engineering, Queen's University, Kingston, Ontario, Canada K7L 3N6.

[1]D. Weiss, C. Zhang, R. R. Gerhardts, K. v. Klitzing, and G. Weimann, Phys. Rev. B **39**, 13 020 (1989).

[2]R. R. Gerhardts, D. Weiss, and K. v. Klitzing, Phys. Rev. Lett. **62**, 1173 (1989).

[3]R. W. Winkler, J. P. Kotthaus, and K. Ploog, Phys. Rev. Lett. **62**, 1177 (1989).

[4]D. Weiss, K. v. Klitzing, K. Ploog, and G. Weimann, in *High Magnetic Fields in Semiconductor Physics II,* edited by G. Landwehr (Springer-Verlag, Berlin, 1989), p. 357.
[5]P. Vasilopoulos and F. M. Peeters, Phys. Rev. Lett. **63**, 2120 (1989).
[6]Chao Zhang and R. R. Gerhardts, Phys. Rev. B **41**, 12 850 (1990).
[7]P. H. Beton *et al.,* Phys. Rev. B **42**, 9229 (1990).
[8]P. H. Beton *et al.,* Phys. Rev. B **42**, 9689 (1990).
[9]S. J. Bending, K. v. Klitzing, and K. Ploog, Phys. Rev. Lett. **65**, 1060 (1990); Phys. Rev. B **42**, 9859 (1990).
[10]*Tables of Integrals, Series, and Products,* edited by I. S. Gradshteyn and I. M. Ryzhik (Academic, San Diego, 1980).
[11]M. Charbonneau, K. M. van Vliet, and P. Vasilopoulos, J. Math. Phys. **23**, 318 (1982).
[12]P. Vasilopoulos, Phys. Rev. B **32**, 771 (1985).
[13]C. W. J. Beenakker, Phys. Rev. Lett. **62**, 2020 (1989).

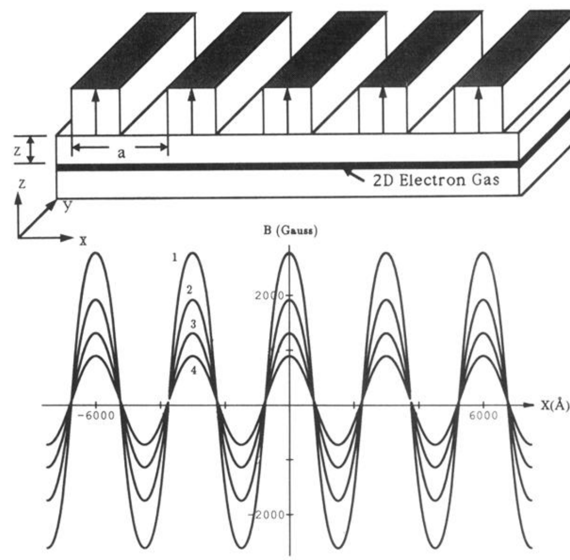

FIG. 1. Proposed 1D periodic magnetic grating structure to generate field modulation. The magnetic lines fabricated lithographically serve as local micromagnets. The lower graph is the calculated magnetic field at a 2DEG that is Z distance away from the bottom of the magnetic line (1500×1500 Å^2, $a = 3000$ Å, material: iron). Note: for curve 1, $Z = 400$ Å; 2, 600 Å; 3, 800 Å; 4, 1000 Å.