Excitation of Alfvén waves at the difference frequency of two microwave beams in a highly collisional magnetoactive compensated semiconductor

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A theoretical investigation has been carried out on the excitation of electromagnetic Alfvén waves at the difference frequency of two microwave beams propagating in a highly collisional magnetoactive compensated semiconductor plasma, viz., germanium. The kinetic theory, viz., the Boltzmann-transport equation and the Krook-model solution, has been employed to find the nonlinear response of electrons and holes in a highly collisional compensated semiconductor plasma in the presence of an external static magnetic field. We show that for typical plasma parameters in compensated Ge, two microwave beams of equal power density 1 MW cm^{-2} can excite electromagnetic Alfvén waves of power density 100 kW cm^{-2}. The power carried by the excited Alfvén wave gradually decreases with the electron-phonon collision frequency in the collision-dominated semiconductor.

I. INTRODUCTION

The use of microwave techniques for reliable diagnostics and the study of various optical properties in metals, semiconductors, and semimetals has been of considerable interest in recent years. In the presence of an external magnetic field the microwaves, whose frequencies are less than the electron plasma frequency and the electron cyclotron frequency, may propagate either as an Alfvén wave mode or a helicon wave mode in the semiconductor plasma.[1-8] Usually the electromagnetic microwaves at the Alfvén and helicon wave frequencies are generated in solid-state devices using the negative-differential-conductivity (NDC) properties of semiconductors. This is called microwave generation by the Gunn effect. With the dc (bias) field lies in the NDC regime, the semiconductor specimen is unstable for space-charge-density perturbation, whose growth rate falls into the microwave range of frequencies. This instability is the basic principle underlying present-day solid-state microwave sources.[9,10]

The nonlinear excitation of various types of electrostatic and/or electromagnetic waves at the difference frequency of two wave beams is being investigated extensively[11-16] because of its potential role in the collective plasma-laser accelerators, cascade plasma heating, laser-pellet fusion research, and plasma density diagnostics.[7] Recently, Maheshwari and Tarey[17] have studied the resonant excitation of helicon waves by two microwave beams in a solid-state plasma. They employed the fluid model and did not consider collisional effects fully, and they also neglected the hole effect. To the best of our knowledge, no attempt has been made to study the excitation of electromagnetic Alfvén waves by the beating of two microwave beams in a collision-dominated compensated semiconductor plasma. In this paper, we make a rigorous theoretical investigation of the excitation of electromagnetic Alfvén waves by two microwave beams propagating in a magnetoactive compensated semiconductor plasma.

In Sec. II we study the nonlinear response of electrons and holes at the difference frequency of two microwave beams propagating in a collision-dominated semiconductor plasma in the presence of an external static magnetic field. In Sec. III we derive the expression for the power carried by the excited electromagnetic Alfvén waves. The numerical applications of the results are presented in the same section. Finally, a brief discussion is given in Sec. IV.

II. KINETIC ANALYSIS FOR GENERATED WAVE CURRENT DENSITY

We consider the propagation of two microwave beams of frequencies ω_1 and ω_2 and propagation vectors $\mathbf{k}_1$ and $\mathbf{k}_2$ in a homogeneous, highly collisional compensated semiconductor plasma (Ge) immersed in an external static magnetic field $\mathbf{B}_s \| \mathbf{k}_1 \| \mathbf{k}_2 \| \hat{\mathbf{z}}$. The electric and magnetic fields $\mathbf{E}_{1,2}$ and $\mathbf{B}_{1,2}$ of these incident electromagnetic microwaves (the homogeneous pump waves) are described by

$$\mathbf{E}_{1,2}=\mathbf{E}'_{1,2}\exp[-i(\omega_{1,2}t-k_{1,2}z)] ,$$
$$\mathbf{B}_{1,2}=c\mathbf{k}_{1,2}\times\mathbf{E}_{1,2}/\omega_{1,2} , \tag{1}$$
$$k_{1,2}=\frac{\omega_{1,2}}{c}\left[\epsilon_L-\frac{\omega_P^2}{\omega_{1,2}(\omega_{1,2}-\omega_c)}-\frac{\omega_{\rm ph}^2}{\omega_{1,2}(\omega_{1,2}+\omega_{\rm ch})}\right] .$$

Here, $\omega_p=(4\pi n_0^0 e^2/m)^{1/2}$ is the electron plasma frequency, e is the electronic charge, n_0^0 is the equilibrium density of electrons, m is the effective mass of an electron,

$\omega_c = eB_s/mc$ is the electron cyclotron frequency, c is the speed of light in vacuum, ϵ_L is the lattice dielectric constant, and $\omega_{\rm ph}$, $\omega_{\rm ch}$, and m_h are the corresponding quantities for the hole.

The two beams $(\omega_{1,2}, k_{1,2})$ interact in the semiconductor and generate a nonlinear pondermotive force at the beat frequency $\omega = \omega_1 - \omega_2$. This nonlinear force develops a nonlinear velocity in the electrons and holes that interacts further with the external static magnetic field to excite electromagnetic Alfvén waves, which satisfy the dispersion relation

$$k = \frac{\omega}{v_A}(1 + v_A^2/c^2)^{1/2} , \tag{2}$$

where $v_A = B_s/(4\pi n_0^0 m_h)^{1/2}$ is the Alfvén speed.

In the presence of the incident pump waves $(\omega_{1,2}, k_{1,2})$ and the excited Alfvén wave $(\omega, \mathbf{k})$, the nonlinear response of electrons in a semiconductor plasma may be described by the Boltzmann-transport equation

$$\frac{\partial f^T}{\partial t} + \mathbf{v}\cdot\nabla_v f^T - \frac{e}{m}\left[\mathbf{E} + \frac{1}{c}\mathbf{v}\times\mathbf{B}\right]\cdot\nabla_v f^T = \left[\frac{\partial f^T}{\partial t}\right]_{\rm collision} . \tag{3}$$

Here, the superscript T refers to the total quantity involved, and the total velocity distribution function f^T can be expressed as

$$f^T = f_0^0 + f + f_1 + f_2 , \tag{4}$$

where $f_{1,2}$ are the distribution functions corresponding to the pump $(\omega_{1,2}, \mathbf{k}_{1,2})$, f is the distribution function corresponding to the generated Alfvén wave, and f_0^0 is the equilibrium Maxwellian distribution function at temperature T_e:

$$f_0^0 = n_0^0 \left[\frac{m}{2\pi k_B T_e}\right]^{3/2} \exp\left[-\frac{mv^2}{2k_B T_e}\right] , \tag{5}$$

where k_B is the Boltzmann constant and v is the random speed of electrons.

By using the Krook-model solution, the collision term in Eq. (3) can be written as[18]

$$\left[\frac{\partial f^T}{\partial t}\right]_{\rm collision} = -\nu(f^T - f_0^0) , \tag{6}$$

where ν is the average electron-phonon collision frequency in the semiconductor plasma.

III. DERIVATION OF THE EXPRESSION FOR POWER CARRIED BY EXCITED ELECTROMAGNETIC ALFVÉN WAVES

The linear response of electrons due to the incident waves $(\omega_{1,2}, \mathbf{k}_{1,2})$ and the excited Alfvén wave $(\omega, \mathbf{k})$ is obtained by solving the Boltzmann-transport equation, Eq. (3). To include the first-order effect of the external magnetic field in the distribution functions of the incident waves, we take the distribution functions in the zeroth-order approximation of the unmagnetized plasma as $-ef_0^0\mathbf{E}_{1,2}\cdot\mathbf{v}/[k_B T_e(\nu - i\omega_{1,2} + i\mathbf{k}_{1,2}\cdot\mathbf{v})]$. Thus, the linear distribution functions corresponding to the incident waves $(\omega_{1,2}, \mathbf{k}_{1,2})$ and the excited Alfvén wave $(\omega, \mathbf{k})$ including the effect of the external magnetic field in the plasma can be written from the Boltzmann-transport equation, Eq. (3), as

$$f_1^L = -\frac{ef_0^0}{k_B T_e(\nu - i\omega_1 + i\mathbf{k}_1\cdot\mathbf{v})} \times \left[\mathbf{E}_1\cdot\mathbf{v} + \frac{\mathbf{E}_1\cdot\mathbf{v}\times\boldsymbol{\omega}_c}{(\nu - i\omega_1 + i\mathbf{k}_1\cdot\mathbf{v})}\right] ,$$
$$f_2^L = -\frac{ef_0^0}{k_B T_e(\nu - i\omega_2 + i\mathbf{k}_2\cdot\mathbf{v})} \times \left[\mathbf{E}_2\cdot\mathbf{v} + \frac{\mathbf{E}_2\cdot\mathbf{v}\times\boldsymbol{\omega}_c}{\nu - i\omega_2 + i\mathbf{k}_2\cdot\mathbf{v})}\right] , \tag{7}$$
$$f^L = -\frac{ef_0^0}{k_B T_e(\nu - i\omega + i\mathbf{k}\cdot\mathbf{v})}\left[\mathbf{E}\cdot\mathbf{v} + \frac{\mathbf{E}\cdot\mathbf{v}\times\boldsymbol{\omega}_c}{(\nu - i\omega + i\mathbf{k}\cdot\mathbf{v})}\right] .$$

In the presence of the external magnetic field, the interaction of the two electromagnetic microwave beams $(\omega_{1,2}, \mathbf{k}_{1,2})$ in the semiconductor gives rise to a nonlinear response of electrons at the frequency of the generated Alfvén wave $(\omega, \mathbf{k})$ which can be obtained from the Boltzmann-transport equation, Eq. (3), as

$$f^{\rm nl} = \frac{e}{2m(\nu - i\omega + i\mathbf{k}\cdot\mathbf{v})} \times \left[\mathbf{E}_1\cdot\nabla_v f_2^{L*} + \mathbf{E}_2^*\cdot\nabla_v f_1^L \frac{1}{c}\mathbf{v}\times\mathbf{B}_1\cdot\nabla_v f_2^{L*} + \frac{1}{c}\mathbf{v}\times\mathbf{B}_2^*\cdot\nabla_v f^L\right] , \tag{8}$$

where the asterisk denotes the complex conjugate of the quantity involved. Under the approximations $\nu > \omega_{1,2}$, $\mathbf{k}_{1,2}\cdot\mathbf{v}$ and $\nu > \omega$, $\mathbf{k}\cdot\mathbf{v}$, Eq. (8) reduces to

$$f^{\rm nl} = -\frac{2e^2 f_0^0 E_{1x}E_{2x}^*}{mk_B T_e \nu^2} \times \left\{1 - \frac{1}{2}\left[\left[\frac{k_1}{\omega_1} + \frac{k_2}{\omega_2}\right] + \frac{i\omega_c}{\nu}\left[\frac{k_1}{\omega_1} - \frac{k_2}{\omega_2}\right]\right]v_z - \frac{m}{2k_B T_e}(v_x^2 + v_y^2)\right\} . \tag{9}$$

Using the relation for the current density, we obtain the nonlinear current density due to the motion of electrons for the generated Alfvén wave $(\omega, \mathbf{k})$ as

$$\mathbf{J}_e^{\rm nl} = -e\int \mathbf{v} f^{NL} d\mathbf{v} = -\frac{e^3 n_0^0 E_{1x}E_{2x}^*}{m^2\nu^2}\left[\frac{k_1}{\omega_1} + \frac{k_2}{\omega_2} + \frac{i\omega_c}{\nu}\left[\frac{k_1}{\omega_1} - \frac{k_2}{\omega_2}\right]\right]\hat{\mathbf{z}} . \tag{10}$$

Similarly, the nonlinear current density due to the motion of holes for the generated Alfvén wave is given by

$$\mathbf{J}_h^{\rm nl}=\frac{e^3 n_0^0 E_{1x}E_{2x}^*}{m_h^2\nu^2}\left[\frac{k_1}{\omega_1}+\frac{k_2}{\omega_2}-\frac{i\omega_c}{\nu}\frac{m_e}{m_h}\left[\frac{k_1}{\omega_1}-\frac{k_2}{\omega_2}\right]\right]\hat{\mathbf{z}}\ . \tag{11}$$

Now, substituting the expression for nonlinear current density into the wave equation for the generated Alfvén wave

$$\overleftrightarrow{\mathbf{D}}\cdot\mathbf{E}=\frac{4\pi i\omega}{c^2}(\mathbf{J}_e^{\rm nl}+\mathbf{J}_h^{\rm nl})\ , \tag{12}$$

where $\overleftrightarrow{\mathbf{D}}$ is the dispersion tensor defined as

$$\overleftrightarrow{\mathbf{D}}=k^2\overleftrightarrow{\mathbf{I}}-\mathbf{k}\mathbf{k}-\frac{\omega^2}{c^2}\overleftrightarrow{\epsilon}\ , \tag{13}$$

$\overleftrightarrow{\mathbf{I}}$ is the unit tensor of rank 2, and $\overleftrightarrow{\epsilon}$ is the linear dielectric tensor for the excited Alfvén wave in a compensated semiconductor, which is, for approximations $\nu \gg \omega$ and $\omega_c \gg \omega$, given by

$$\overleftrightarrow{\epsilon}=\begin{bmatrix}\epsilon_L+\dfrac{\omega_p^2}{\omega_c^2}\left[1+\dfrac{m_h}{m}\right] & 0 & 0\\ 0 & \epsilon_L+\dfrac{\omega_p^2}{\omega_c^2}\left[1+\dfrac{m_h}{m}\right] & 0\\ 0 & 0 & \epsilon_L+\dfrac{i\omega_p^2}{\omega\nu}\left[1+\dfrac{m}{m_h}\right]\end{bmatrix}, \tag{14}$$

we obtain the electric field E associated with the excited Alfvén wave as

$$\mathbf{E}=\frac{e\omega_p^2E_{1x}E_{2x}^*k_1}{m\nu^2\omega_1\omega(\epsilon_L^2+R^2)}[(\epsilon_LQ-RS)-i(RQ+\epsilon_LS)]\hat{\mathbf{z}}\ , \tag{15}$$

where

$$Q=\frac{\omega_c}{\nu}\left[1-\frac{k_2}{k_1}\frac{\omega_1}{\omega_2}\right]\left[1+\frac{m^3}{m_h^3}\right],$$

$$R=\frac{\omega_p^2}{\nu\omega}\left[1+\frac{m}{m_h}\right], \tag{16}$$

$$S=\left[1+\frac{k_2}{k_1}\frac{\omega_1}{\omega_2}\right]\left[1-\frac{m^2}{m_h^2}\right].$$

The power carried by the excited Alfvén wave is given by[19]

$$P_A=\frac{1}{8\pi}\left[\frac{\partial\omega}{\partial k}\right]\left[\frac{\partial(\omega\epsilon)}{\partial\omega}\mathbf{E}\cdot\mathbf{E}^*+\mathbf{B}_f\cdot\mathbf{B}_f^*\right], \tag{17}$$

where $\mathbf{B}_f$ is the fluctuating magnetic field of the generated wave obtained by using Maxwell's equation, $\nabla\times\mathbf{E}=-(1/c)(\partial\mathbf{B}/\partial t)$,

$$\mathbf{B}_f\cdot\mathbf{B}_f^*=\frac{2c^2k^2}{\omega^2}(\mathbf{E}\cdot\mathbf{E}^*)\ . \tag{18}$$

Using Eqs. (2), (14), (15), (17), and (18), we obtain

$$P_A=\frac{e^2\omega_p^4k_1^2\epsilon_LC_AXY}{8\pi m^2\omega_1^2\omega^2\nu^4Z}|E_{1x}|^2|E_{2x}|^2\ , \tag{19}$$

where

$$C_A=v_A/(1+v_A^2/c^2)^{1/2}\ ,$$

$$X=1+2c^2k^2/\omega^2\ ,$$

$$Y=(\epsilon_L^2+R^2)(Q^2+S^2)\ ,$$

$$Z=(\epsilon_L^2+R^2)^2\ .$$

In order to have some numerical appreciation of the results, we have made calculations for the power carried

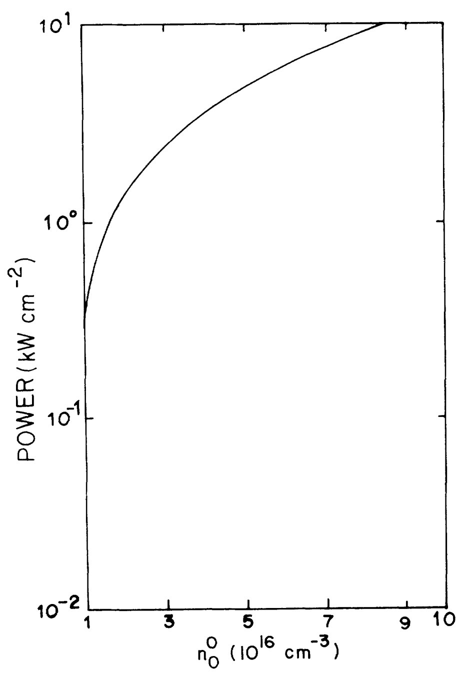

FIG. 1. The variation of the power carried by the excited Alfvén wave P_A as a function of the equilibrium electron/hole density n_0^0 for the following plasma parameters in compensated Ge: $\epsilon_L=16$ at 77 K, $B_s=10$ kG, $\nu=10^{11}$ rad sec^{-1}, $m=0.1m_e$, $m_h=3m$, $\omega_1=2.1\times10^{10}$ rad sec^{-1}, $\omega_2=2\times10^{10}$ rad sec^{-1}, $(c/8\pi)\mathbf{E}_1\cdot\mathbf{E}_1^*=(c/8\pi)\mathbf{E}_2\cdot\mathbf{E}_2^*=1$ MW cm^{-2}, $c=3\times10^{10}$ cm sec^{-1}.

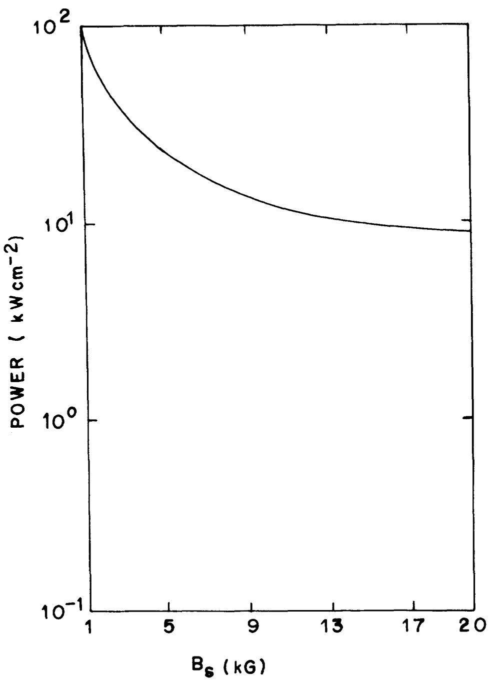

FIG. 2. The variation the power P_A as a function of the external static magnetic field $\mathbf{B}_s$ for $n_0^0 = 10^{17}$ cm^{-3}. The other parameters and specifications are the same as in Fig. 1.

by the excited Alfvén wave for parallel propagation with respect to the external magnetic field. The calculations are made for the following typical parameters in compensated Ge: $\epsilon_L = 16$ at 77 K, $n_0^0 = 10^{16} - 10^{17}$ cm^{-3}, $B_s = 1-20$ kG, $\nu = 10^{11} - 10^{12}$ rad sec^{-1}, $m = 0.1 m_e$ (m_e is the mass of a free electron), $m_h = 3m$, $\omega_1 = 2.1 \times 10^{10}$ rad sec^{-1}, $\omega_2 = 2 \times 10^{10}$ rad sec^{-1}, $c = 3 \times 10^{10}$ cm sec^{-1}, $(c/8\pi)\mathbf{E}_1 \cdot \mathbf{E}_1^* = (c/8\pi)\mathbf{E}_2 \cdot \mathbf{E}_2^* = 1$ MW cm^{-2}.

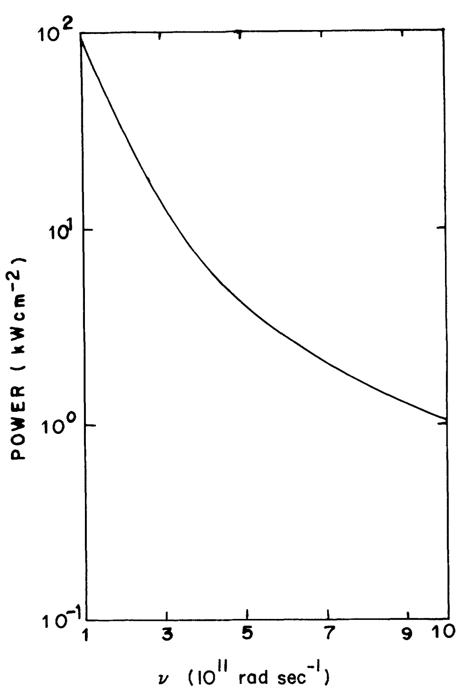

FIG. 3. The variation of the power P_A as a function of the electron-phonon collision frequency ν for $n_0^0 = 10^{17}$ cm^{-3} and $B_s = 1$ kG. The other parameters and specifications are the same as in Fig. 1.

The results of calculations are displayed in the form of curves in Figs. 1–3.

Figure 1 shows the variation of the power (P_A) carried by the excited Alfvén wave as a function of the equilibrium density of electrons or holes (n_0^0) for different plasma parameters of interest. The power (P_A) increases with n_0^0.

Figure 2 shows the variation of the power (P_A) as a function of the external magnetic field ($\mathbf{B}_s$) for different plasma parameters of interest. The power P_A decreases with B_s.

Figure 3 shows the variation of P_A as a function of the effective electron-phonon collision frequency (ν) for typical plasma parameters. The power P_A decreases sharply with increasing ν.

IV. DISCUSSION

The beating of the two microwave beams propagating in a highly collisional semiconductor plasma in parallel with an external static magnetic field applied to it (semiconductor) excites electromagnetic Alfvén wave at their (two microwave beams) difference frequency efficiently. The kinetic equation, viz., the Boltzmann-transport equation and the Krook-model solution, have been employed to find the nonlinear response of electrons and holes in the semiconductor. Here it can be noted that the beating of two incident microwave beams each of power density 1 MW cm^{-2} excites Alfvén waves of power density 100 kW cm^{-2} for the following plasma parameters in Ge: $\epsilon_L = 16$ at 77 K, $m = 0.1 m_e$ (m_e is the effective mass of a free electron), $m_h = 3m$, $\nu = 10^{11}$ rad sec^{-1}, $n_0^0 = 10^{17}$ cm^{-3}, $B_s = 1$ kG, $\omega_1 = 2.1 \times 10^{10}$ rad sec^{-1}, and $\omega_2 = 2 \times 10^{10}$ rad sec^{-1}. The power carried by the excited Alfvén wave P_A decreases gradually with the increase of the effective electron-phonon collision frequency ν. The power P_A decreases with the external magnetic field B_s, but increases with equilibrium electron and/or hole density n_0^0.

From this theoretical study, it may be suggested that an experiment may be designed in the laboratory to investigate the excitation of Alfvén waves through the parametric instability, and the results could be compared to those of the conventional methods in the bulk semiconductor. The details of the various aspects of the parametric excitation might also be verified in the semiconductor plasma, where the plasma parameters could be varied over a wide range of values without much difficulty.

It may be added here that the inhomogeneities of the self-generated magnetic field and the saturation of the parametric excitation by considering relativistic, ultrarelativistic, and extreme-relativistic effects are also problems of great importance, but are beyond the scope of the present paper.

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