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Phenomenological electrodynamics of two-dimensional Coulomb systems

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We construct a phenomenological electrodynamics for plane monolayer plasmas at the interface of dielectric materials. The presence of externally applied dc magnetic fields can be assumed. The Maxwell equations and fluctuation-dissipation relations are formulated exclusively in terms of surface quantities. The conceptual notion of the surface dielectric-response tensor and its relation to the customary susceptibility and current-current correlation tensors in the presence of a dielectric environment is clarified.

I. INTRODUCTION

The concepts of the frequency- and wave-vector-dependent dielectric-response function $\epsilon(\mathbf{k}\omega)$ and of its generalization, the dielectric response tensor, are among the most fruitful and widely used ideas of many-body physics. They have also been used,[1–5] although more sporadically, in the analysis and description of two-dimensional (2D) and quasi-two-dimensional periodic electronic structures (monolayers and superlattices). Nevertheless, the concept of the dielectric-response tensor as related to the behavior of 2D (as opposed to 3D) structures has never been erected from the precise foundations of phenomenological plasma electrodynamics. The principal thrust of theoretical works in this area has been directed at the understanding of the collective mode structure of these two-dimensional plasmas. Models of increasing complexity have been considered over the past two decades. Stern[6] found the plasmon dispersion relation in the nonretarded limit. Chiu, Quinn, Lee, and Eguiluz[7] studied the effect of the dielectric environment on the dynamics of the plasma layer. The effect of an external magnetic field was considered by Chiu and Quinn,[8] whose dispersion relation in the random-phase approximation is the most general to date. Bonsall and Maradudin[9] analyzed the dispersion properties of a two-dimensional Wigner lattice in a magnetic field and arrived at a result formally analogous to that of Chiu and Quinn.[8]

Central to the description of the plasma layer are its dielectric susceptibility $\overleftrightarrow{\xi}(\mathbf{k}\omega)$ or the density and current response functions $\chi_L(\mathbf{k}\omega)$ and $\chi_T(\mathbf{k}\omega)$, defined below, in terms of which the dispersion relation can be expressed. In many-body theory the dispersion relation is conveniently determined from the zeros of the determinant of the dispersion tensor $\overleftrightarrow{\Delta}(\mathbf{k}\omega)$, which is constructed out of the dielectric tensor $\overleftrightarrow{\epsilon}(\mathbf{k}\omega)$. For three-dimensional Coulomb systems the relationship between $\overleftrightarrow{\epsilon}(\mathbf{k}\omega)$ and $\overleftrightarrow{\xi}(\mathbf{k}\omega)$ is trivially simple. This is, however, not the case for two-dimensional plasma systems: the difference is due to (i) the "retardation effect," which manifests itself as a frequency-dependent effective potential, and (ii) to the influence of the dielectric properties of the surrounding medium. Thus, the knowledge of $\overleftrightarrow{\xi}(\mathbf{k}\omega)$ in these systems is not automatically equivalent to the knowledge of $\overleftrightarrow{\epsilon}(\mathbf{k}\omega)$; neither is the information provided by the dispersion relation sufficient to reconstruct the structure of $\overleftrightarrow{\Delta}(\mathbf{k}\omega)$. Yet, the identification of the surface dielectric tensor and the derivation of the dispersion relations through the use of this quantity have a number of advantages. First and the most immediate is the possibility of adopting a formalism that is completely analogous to the formalism of the three-dimensional phenomenological electrodynamics and to describe the surface layer in terms of physical quantities *defined on the surface* only. Second, and more importantly, $\overleftrightarrow{\epsilon}(\mathbf{k}\omega)$ contains, in addition to the dispersion relation, significant further physical information; in particular, via the fluctuation-dissipation theorem, it determines the density and current-density fluctuation spectra of the system.

Thus the purpose of the present paper is to provide a foundation anchored in two-dimensional phenomenological electrodynamics, for the construction of the surface dielectric tensor $\overleftrightarrow{\epsilon}(\mathbf{k}\omega)$. There are several advantages to this approach: (1) The Maxwell equations can be formulated exclusively in terms of surface quantities even in the presence of dielectric media in which the layer is embedded; (2) the effects of the surrounding media on the dielectric response can be identified; (3) the "retardation effect" can be described in terms of a frequency-dependent effective interaction potential; (4) the dispersion relation, both for longitudinal and transverse modes, can be expressed in a form analogous to the 3D dispersion relation; and (5) one can derive the relationship between "total" and "external" response functions, and relate density and current fluctuation spectra via the fluctuation-dissipation theorem to the latter.

The model analyzed in this paper is that of a 2D layer of electrons in a compensating background, sandwiched between two—in general, different—dielectric materials. In Sec. II we formulate the 2D phenomenological electrodynamics. In Sec. III we construct the dielectric-response tensor; we make contact with other works by showing that the ensuring dispersion relation, albeit seemingly different, is identical to that existing in the literature; it also contains the dispersion relation for surface waves on the interface of two dielectric media. In Sec. IV we identify the external response functions and establish the fluctuation-dissipation theorem.

II. 2D FIELD EQUATIONS

Let the plasma particles occupy the large but bounded area S in the plane $z=0$ of a Cartesian coordinate system. The electric displacement and *total* electric field are linked through the usual constitutive relation

$$\mathbf{D}(z,\mathbf{k}\omega)=[\epsilon_1\Theta(z)+\epsilon_2\Theta(-z)]\mathbf{E}(z,\mathbf{k}\omega)\ , \tag{1}$$

where $\epsilon_1=1+\alpha_1$ and $\epsilon_2=1+\alpha_2$ are the dielectric constants of the surrounding media, $\Theta(z)$ and $\Theta(-z)$ are unit step functions, and $\mathbf{k}$ is the 2D wave vector conjugate to the 2D position vector $\mathbf{r}_\perp=\hat{\mathbf{e}}_x x+\hat{\mathbf{e}}_y y$.

Let the system be perturbed by weak external charge and current-density sources $\hat{\rho}(z,\mathbf{k}\omega)$ and $\hat{\mathbf{J}}(z,\mathbf{k}\omega)$ which, for the sake of mathematical convenience, are to be confined to the plane $z=0$, viz.,

$$\begin{aligned}\hat{\rho}(z,\mathbf{k}\omega)&=\hat{\rho}(\mathbf{k}\omega)\delta(z)\ ,\\ \hat{\mathbf{J}}(z,\mathbf{k}\omega)&=\hat{\mathbf{J}}(\mathbf{k}\omega)\delta(z)\ ,\\ \omega\hat{\rho}(\mathbf{k}\omega)&=\mathbf{k}\cdot\hat{\mathbf{J}}(\mathbf{k}\omega)\ .\end{aligned} \tag{2}$$

The average charge- and current-density responses of the monolayer plasma particles are given by $\rho(\mathbf{k}\omega)\delta(z)$ and $\mathbf{J}(\mathbf{k}\omega)\delta(z)$. The Fourier-transformed Maxwell equations relating the various field quantities to the free charge and current density can then be stated as follows:

$$\left[i\mathbf{k}+\hat{\mathbf{e}}_z\frac{d}{dz}\right]\times[\mathbf{B}_\perp(z,\mathbf{k}\omega)+\hat{\mathbf{e}}_z B_z(z,\mathbf{k}\omega)]=\frac{4\pi}{c}\mathbf{J}^f(\mathbf{k}\omega)\delta(z)-\frac{i\omega}{c}\mathbf{D}(z,\mathbf{k}\omega)\ , \tag{3}$$

$$\left[i\mathbf{k}+\hat{\mathbf{e}}_z\frac{d}{dz}\right]\times[\mathbf{E}_\perp(z,\mathbf{k}\omega)+\hat{\mathbf{e}}_z E_z(z,\mathbf{k}\omega)]=\frac{i\omega}{c}\mathbf{B}(z,\mathbf{k}\omega)\ , \tag{4}$$

$$i\mathbf{k}\cdot\mathbf{B}_\perp(z,\mathbf{k}\omega)+\frac{d}{dz}B_z(z,\mathbf{k}\omega)=0\ , \tag{5}$$

$$i\mathbf{k}\cdot\mathbf{D}_\perp(z,\mathbf{k}\omega)+\frac{d}{dz}D_z(z,\mathbf{k}\omega)=4\pi\rho^f(\mathbf{k}\omega)\delta(z)\ , \tag{6}$$

where

$$\begin{aligned}\rho^f(\mathbf{k}\omega)&=\hat{\rho}(\mathbf{k}\omega)+\rho(\mathbf{k}\omega)\ ,\\ \mathbf{J}^f(\mathbf{k}\omega)&=\hat{\mathbf{J}}(\mathbf{k}\omega)+\mathbf{J}(\mathbf{k}\omega)\ .\end{aligned} \tag{7}$$

Our first objective is to reformulate the Maxwell equations solely in terms of quantities unambiguously defined in the plasma plane. The surface current and charge densities already satisfy these conditions; out of the family of field components, $\mathbf{E}_\perp(0,\mathbf{k}\omega)\equiv\mathscr{E}(\mathbf{k}\omega)$ and $B_z(0,\mathbf{k}\omega)\equiv\mathscr{B}(\mathbf{k}\omega)$ are the appropriate choices. For the characterization of the plasma, one can use the density and current response function $\chi_L(\mathbf{k}\omega)$ and $\chi_T(\mathbf{k}\omega)$, defined through the constitutive relations

$$\begin{aligned}n(\mathbf{k}\omega)&=-\chi_L(\mathbf{k}\omega)e\Phi(\mathbf{k}\omega)\ ,\\ \mathbf{j}(\mathbf{k}\omega)&=-\chi_T(\mathbf{k}\omega)ec\,\mathbf{A}(\mathbf{k}\omega)\ ;\end{aligned} \tag{8}$$

$n(\mathbf{k}\omega)$ and $\mathbf{j}(\mathbf{k}\omega)$ are the particle density and particle current density, respectively; $\Phi(\mathbf{k}\omega)$ and $\mathbf{A}(\mathbf{k}\omega)$ are the *total* scalar and vector potentials, respectively, evaluated at $z=0$. Or, one can use the susceptibility tensor $\overleftrightarrow{\xi}(\mathbf{k}\omega)$ defined through

$$\mathbf{J}(\mathbf{k}\omega)=-i\omega\overleftrightarrow{\xi}(\mathbf{k}\omega)\cdot\mathscr{E}(\mathbf{k}\omega)\ , \tag{9}$$

where $\mathbf{J}(\mathbf{k}\omega)=-e\,\mathbf{j}(\mathbf{k}\omega)$. This latter has the advantage of being gauge invariant, an especially useful feature in the presence of an external magnetic field: for this reason we use $\overleftrightarrow{\xi}(\mathbf{k}\omega)$ in the following.

The 2D *total* susceptibility tensor $\overleftrightarrow{\xi}(\mathbf{k}\omega)$, by definition, portrays the linear response of the monolayer plasma to the *total* field perturbation $\mathscr{E}(\mathbf{k}\omega)$. Of the three response tensors to be discussed in this paper, $\overleftrightarrow{\xi}(\mathbf{k}\omega)$ is the only one that, in the absence of particle correlations, is determined by the properties of the plasma, unaffected by the environment—be it dielectric or vacuum—above and below the monolayer; it is the customary response tensor frequently encountered in the literature. For example, it was calculated by Chiu and Quinn[8] in the random-phase approximation (RPA) for the two-dimensional electron gas, and by Bonsall and Maradudin[9] in the harmonic approximation for the 2D hexagonal Wigner crystal.

There are equation-of-motion approximation schemes [e.g., the quasilocalized-charge (QLC) model of Kalman and Golden[10]], which, for the sake of clarity in presentation of the physical concepts, are structured so as to explicitly display not the *total* field $\mathscr{E}(\mathbf{k}\omega)$, but rather the *external* field $\hat{\mathbf{E}}_\perp(0,\mathbf{k}\omega)\equiv\hat{\mathscr{E}}(\mathbf{k}\omega)$ as the driving perturbation. This latter is understood to be the electric field at the interface of the two dielectrics in the *absence* of the plasma layer [but in the presence of the surrounding dielectric media; see Eqs. (33) and (27)]. Within the framework of such approximation schemes, one therefore calculates the so-called 2D *external* susceptibility tensor $\hat{\overleftrightarrow{\xi}}(\mathbf{k}\omega)$ defined through the constitutive relation

$$\mathbf{J}(\mathbf{k}\omega)=-i\omega\hat{\overleftrightarrow{\xi}}(\mathbf{k}\omega)\cdot\hat{\mathscr{E}}(\mathbf{k}\omega)\ . \tag{10}$$

More will be said about this latter response tensor in Sec. IV.

III. SURFACE DIELECTRIC-RESPONSE TENSOR

Consider now the domain $z\neq0$. Eliminating $\mathbf{B}$ between (3) and (4), one readily obtains

$$i\mathbf{k}\left[i\mathbf{k}\cdot\mathbf{E}_\perp(z,\mathbf{k}\omega)+\frac{d}{dz}E_z(z,\mathbf{k}\omega)\right]+\left[\beta_m^2-\frac{d^2}{dz^2}\right]\mathbf{E}_\perp(z,\mathbf{k}\omega)=0\ , \qquad (11)$$

$$E_z(z,\mathbf{k}\omega)=-\frac{1}{\beta_m^2}\frac{d}{dz}i\mathbf{k}\cdot\mathbf{E}_\perp(z,\mathbf{k}\omega)\ , \qquad (12)$$

where $\beta_m^2\equiv k^2-\epsilon_m\omega^2/c^2$ $(m=1,2)$ is the familiar attenuation constant that shows up in the following solutions to (11) and (12):

$$\mathbf{E}_\perp(z,\mathbf{k}\omega)=\mathcal{E}(\mathbf{k}\omega)\times\begin{cases}e^{-\beta_1 z}\ , & z>0\\ e^{\beta_2 z}\ , & z<0,\end{cases} \qquad (13)$$

$$E_z(z,\mathbf{k}\omega)=i\mathbf{k}\cdot\mathcal{E}(\mathbf{k}\omega)\times\begin{cases}\frac{1}{\beta_1}e^{-\beta_1 z}, & z>0\\ -\frac{1}{\beta_2}e^{\beta_2 z}, & z>0\ .\end{cases} \qquad (14)$$

Equations (13) and (14), when substituted into (4), give

$$\mathbf{B}_\perp(z>0,\mathbf{k}\omega)=\frac{i\epsilon_1\omega}{c\beta_1}\hat{\mathbf{e}}_z\left[\left[\frac{\beta_1^2c^2}{\epsilon_1\omega^2}\overleftrightarrow{\mathrm{T}}(\mathbf{k})-\overleftrightarrow{\mathrm{L}}(\mathbf{k})\right]\cdot\mathcal{E}(\mathbf{k}\omega)\right]e^{-\beta_1 z}\ ,$$

$$\mathbf{B}_\perp(z<0,\mathbf{k}\omega)=-\frac{i\epsilon_2\omega}{c\beta_2}\hat{\mathbf{e}}_z\left[\left[\frac{\beta_2^2c^2}{\epsilon_2\omega^2}\overleftrightarrow{\mathrm{T}}(\mathbf{k})-\overleftrightarrow{\mathrm{L}}(\mathbf{k})\right]\cdot\mathcal{E}(\mathbf{k}\omega)\right]e^{\beta_2 z}\ , \qquad (15)$$

$$\hat{e}_zB_z(z,\mathbf{k}\omega)=\frac{c}{\omega}\mathbf{k}\times\mathbf{E}_\perp(z,\mathbf{k}\omega)\ ; \qquad (16)$$

$\overleftrightarrow{\mathrm{L}}(\mathbf{k})\equiv\mathbf{kk}/k^2$ and $\overleftrightarrow{\mathrm{T}}(\mathbf{k})\equiv\overleftrightarrow{\mathrm{I}}-\overleftrightarrow{\mathrm{L}}(\mathbf{k})$ are notationally convenient longitudinal and transverse projection tensors.

In order to arrive at the desired 2D formulation, we integrate Eqs. (3) and (6) over all z values, making use of (13), (15), and (16). The resulting relationships are

$$i\mathbf{k}\times\mathcal{B}(\mathbf{k}\omega)+\frac{i\omega}{c}[1+\bar{\alpha}(k\omega)]\mathcal{E}(\mathbf{k}\omega)=\frac{2\pi}{c}\bar{\beta}(k\omega)\mathbf{J}^f(\mathbf{k}\omega)\ , \qquad (17)$$

$$\mathbf{k}\times\mathcal{E}(\mathbf{k}\omega)-\frac{\omega}{c}\mathcal{B}(\mathbf{k}\omega)=0\ , \qquad (18)$$

$$i[1+\bar{\alpha}(k\omega)]\mathbf{k}\cdot\mathcal{E}(\mathbf{k}\omega)=2\pi\bar{\beta}(k\omega)\rho^f(\mathbf{k}\omega)\ . \qquad (19)$$

Here

$$\bar{\alpha}(k\omega)=\frac{\beta_1(k\omega)\alpha_2+\beta_2(k\omega)\alpha_1}{\beta_1(k\omega)+\beta_2(k\omega)} \qquad (20)$$

is the effective polarizability of the dielectric 1–dielectric 2 interface and

$$\bar{\beta}(k\omega)=2\frac{\beta_1(k\omega)\beta_2(k\omega)}{\beta_1(k\omega)+\beta_2(k\omega)}\ . \qquad (21)$$

Note that in a 2D space, the $\mathbf{k}\times\mathcal{E}$ vector product is a (pseudo)scalar and the $\mathbf{k}\times\mathcal{B}$ vector multiplication with a (pseudo)scalar is a vector.

The full surface dielectric tensor $\overleftrightarrow{\epsilon}(\mathbf{k}\omega)$ now incorporating the effect of the plasma sources is

$$\overleftrightarrow{\epsilon}(\mathbf{k}\omega)=\overleftrightarrow{\mathrm{I}}[1+\bar{\alpha}(k\omega)]+\overleftrightarrow{\alpha}(\mathbf{k}\omega)\ ,$$
$$\overleftrightarrow{\alpha}(\mathbf{k}\omega)=2\pi\bar{\beta}(k\omega)\overleftrightarrow{\xi}(\mathbf{k}\omega)\ . \qquad (22)$$

The dielectric-response tensor $\overleftrightarrow{\epsilon}(\mathbf{k}\omega)$ of the plasma monolayer, in contrast to the susceptibility $\overleftrightarrow{\xi}(\mathbf{k}\omega)$, exhibits a marked dependence on the dielectric constants (of the materials in which the monolayer is embedded) in the effective propagation constant. With the aid of $\overleftrightarrow{\epsilon}(\mathbf{k}\omega)$, (17) and (19) can be recast in the form resembling the standard 3D formulation:

$$i\mathbf{k}\times\mathcal{B}(\mathbf{k}\omega)+\frac{i\omega}{c}\overleftrightarrow{\epsilon}(\mathbf{k}\omega)\cdot\mathcal{E}(\mathbf{k}\omega)=\frac{2\pi}{c}\bar{\beta}(k\omega)\hat{\mathbf{J}}(\mathbf{k}\omega)\ , \qquad (23)$$

$$i\mathbf{k}\cdot\overleftrightarrow{\epsilon}(\mathbf{k}\omega)\cdot\mathcal{E}(\mathbf{k}\omega)=2\pi\bar{\beta}(k\omega)\hat{\rho}(\mathbf{k}\omega)\ . \qquad (24)$$

Equations (23) and (18) can be combined into the wave equation. This latter can be derived, along with the Poisson equation (24), more directly by integrating (4) and (6) through the plasma layer only, i.e., from $z=0^-$ to $z=0^+$. One obtains

$$\overleftrightarrow{\Delta}(\mathbf{k}\omega)\cdot\mathcal{E}(\mathbf{k}\omega)=\frac{2\pi i}{\omega}\bar{\beta}(k\omega)\hat{\mathbf{J}}(\mathbf{k}\omega)\ , \qquad (25)$$

with

$$\overleftrightarrow{\Delta}(\mathbf{k}\omega)=n^2\overleftrightarrow{\mathrm{T}}(\mathbf{k})-\overleftrightarrow{\epsilon}(\mathbf{k}\omega) \qquad (26)$$

($n=kc/\omega$ is the 2D index of refraction).

The equivalent of (26) for the dielectric media (in the absence of the plasma layer) is

$$\overleftrightarrow{\bar{\Delta}}(\mathbf{k}\omega)=n^2\overleftrightarrow{\mathrm{T}}(\mathbf{k})-\overleftrightarrow{\mathrm{I}}[1+\bar{\alpha}(k\omega)]\ . \qquad (27)$$

The surface dielectric-response tensor $\overleftrightarrow{\epsilon}(\mathbf{k}\omega)$ contains information about the dispersion and damping of the collective modes, i.e., about the complex eigenfrequencies $\omega(\mathbf{k})$ of the plasma monolayer. The formulation of the dispersion relation for the calculation of $\omega(\mathbf{k})$ is straightforward

$$\mathrm{Det}\overleftrightarrow{\Delta}(\mathbf{k}\omega)=0 \qquad (28)$$

and is a consequence of setting $\hat{\mathcal{J}}(\mathbf{k}\omega)=0$ for $\mathcal{E}(\mathbf{k}\omega)\neq0$ in Eq. (25).

In the absence of an external magnetic field, the system is isotropic and the response tensor are diagonal with a longitudinal (L) and a transverse (T) diagonal element. In this case it is profitable to trade the susceptibility $\overleftrightarrow{\xi}$ for the density and current response functions χ_L and χ_T; ϵ_L and ϵ_T can then be expressed as

$$\epsilon_L(\mathbf{k}\omega)=1+\bar{\alpha}(k\omega)-\psi(k\omega)\chi_L(\mathbf{k}\omega)\ , \qquad (29)$$

$$\epsilon_T(\mathbf{k}\omega)=1+\bar{\alpha}(k\omega)+n^2\psi(k\omega)\chi_T(\mathbf{k}\omega)\ , \qquad (30)$$

through the effective potential

$$\psi(k\omega)=\frac{2\pi e^2}{k^2}\bar{\beta}(k\omega) . \quad (31)$$

In this situation, for the study of the transverse dispersion relation, the form of the dispersion tensor,

$$\begin{aligned}\overleftrightarrow{\Delta}(\mathbf{k}\omega)=&[(n^2-\epsilon_1)^{1/2}(n^2-\epsilon_2)^{1/2}\\&-n^2\psi(k\omega)\chi_T(\mathbf{k}\omega)]\overleftrightarrow{\mathrm{T}}(\mathbf{k})\\&-[1+\bar{\alpha}(k\omega)-\psi(k\omega)\chi_L(\mathbf{k}\omega)]\overleftrightarrow{\mathrm{L}}(\mathbf{k}) ,\end{aligned} \quad (32)$$

is more useful: it clearly exhibits the coupling caused by the plasma monolayer between the two electromagnetic waves propagating in medium 1 and 2.

IV. FLUCTUATION-DISSIPATION RELATION

In this section we establish the fluctuation-dissipation theorem for the fluctuations of the surface current densities. To this end, we consider first the relationship between the total and external susceptibility tensors defined by Eqs. (9) and (10). The straightforward derivation is carried out first by combining (25) with the *dielectric* electrodynamic equation

$$\hat{\mathbf{J}}(\mathbf{k}\omega)=-\frac{i\omega}{2\pi\bar{\beta}}\overleftrightarrow{\bar{\Delta}}(\mathbf{k}\omega)\cdot\hat{\mathscr{E}}(\mathbf{k}\omega) . \quad (33)$$

We obtain

$$\mathscr{E}(\mathbf{k}\omega)=\overleftrightarrow{\Delta}^{-1}(\mathbf{k}\omega)\cdot\overleftrightarrow{\bar{\Delta}}(\mathbf{k}\omega)\cdot\hat{\mathscr{E}}(\mathbf{k}\omega) . \quad (34)$$

The desired relation

$$\overleftrightarrow{\hat{\xi}}(\mathbf{k}\omega)=\xi(\mathbf{k}\omega)\cdot\overleftrightarrow{\Delta}^{-1}(\mathbf{k}\omega)\cdot\overleftrightarrow{\bar{\Delta}}(\mathbf{k}\omega) \quad (35)$$

then follows from (13), (9), and (10). In the absence of the dielectrics (i.e., $\epsilon_1=1$, $\epsilon_2=1$, $\beta_1=\beta_0$, $\beta_2=\beta_0$), (35) is identical to its three-dimensional counterpart,[11] as it should be.

A more useful version of (35) that does not involve $\overleftrightarrow{\epsilon}(\mathbf{k}\omega)$ is

$$\begin{aligned}&\overleftrightarrow{\xi}^{-1}(\mathbf{k}\omega)-\overleftrightarrow{\hat{\xi}}^{-1}(\mathbf{k}\omega)\\&=2\pi\bar{\beta}(k\omega)\left[\frac{1}{n^2-1-\bar{\alpha}(k\omega)}\overleftrightarrow{\mathrm{T}}(\mathbf{k})-\frac{1}{1+\bar{\alpha}(k\omega)}\overleftrightarrow{\mathrm{L}}(\mathbf{k})\right] .\end{aligned} \quad (36)$$

The principal usefulness of $\overleftrightarrow{\hat{\xi}}$ lies in the fact that its anti-Hermitian part can be directly related to the surface current-current fluctuations in the layer through the 2D fluctuation-dissipation theorem

$$\begin{aligned}\hat{\xi}^{AH}_{\nu\mu}(\mathbf{k}\omega)&=\tfrac{1}{2}[\hat{\xi}_{\nu\mu}(\mathbf{k}\omega)-\hat{\xi}^{+}_{\nu\mu}(\mathbf{k}\omega)]\\&=\frac{ie^2n}{\hbar\omega^2}\tanh\frac{\beta\hbar\omega}{2}C_{\mu\nu}(\mathbf{k}\omega) ,\end{aligned} \quad (37)$$

$$C_{\mu\nu}(\mathbf{k}\omega)=\frac{1}{2N}\int_{-\infty}^{\infty}dt\, e^{i\omega t}\langle\{j_{\mathbf{k}\nu}(t),j_{-\mathbf{k}\mu}(0)\}\rangle , \quad (38)$$

which can be derived from straightforward statistical-mechanical linear-response theory;[12] $n=N/S$ is the average areal density,

$$\mathbf{j}_{\mathbf{k}}=\tfrac{1}{2}\sum_{i=1}^{N}\{\mathbf{v}_i,e^{-i\mathbf{k}\cdot\mathbf{x}_i}\} \quad (39)$$

is the local particle current density operator in the equilibrium (unperturbed) system, and $\langle\ \rangle$ denotes ensemble averaging over the equilibrium system. [Note that some definitions of the current correlation tensor in the literature differ from definition (38) by a factor of 2π.] Equations (37) and (35) provide a simple way to calculate the important surface current-current correlations in terms of the accessible dielectric susceptibility tensor $\overleftrightarrow{\xi}(\mathbf{k}\omega)$. Note, however, that the role of the surrounding dielectric media enters in this relationship in a nontrivial way.

V. CONCLUSIONS

The purpose of this paper has been to establish a two-dimensional phenomenological electrodynamics, in terms of surface quantities only, for a two-dimensional electron layer embedded between two dielectric media. The correct construction of the dielectric-response tensor $\overleftrightarrow{\epsilon}(\mathbf{k}\omega)$ with the aid of the surface susceptibility tensor $\overleftrightarrow{\xi}(\mathbf{k}\omega)$ and the dielectric constants of the surrounding dielectrics has been established: this formalizes and generalizes the work of Refs. 7 and 8: these latter papers correctly derive the dispersion relation both in the isotropic[7] and anisotropic[8] situations, but do not attempt to identify $\overleftrightarrow{\epsilon}$ in terms of $\overleftrightarrow{\xi}$, α_1, and α_2 [of Eqs. (20–(22)] and the related external response functions, which are instrumental in establishing the fluctuation-dissipation theorem. Having established this theorem, one can determine the effect of the surrounding dielectric media on the correlations of the surface current densities (or on the important dynamical-structure function) as expressed in terms of the susceptibility tensor of the layer.

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