Hartree calculation of local magnetic fields in an anyon superconductor

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We use a modified Hartree approximation to calculate the charge and current distributions induced by the presence of a positive point charge above a plane of positively charged superconducting anyons, at zero temperature. The anyons are taken to have semion statistics, and are treated within a simple two-species effective-mass model. We also consider a geometry where the point charge is partially screened by a second, metallic, plane that does not contain anyons. Applying the results of muon-spin-relaxation experiments in $YBa_2Cu_3O_7$ and $Bi_2Sr_2CaCu_2O_8$, we find that, if the CuO_2 planes have charge carriers obeying semion statistics, then the magnetic field at the muon site should be in the range 0–25 G, with the actual value depending on the muon location in the crystal.

I. INTRODUCTION

The idea that the charge carriers in high-temperature superconductors may have fractional, or anyon, statistics has been suggested by Kalmeyer and Laughlin.[1,2] Since then, this proposal has led to much theoretical work[3] and several attempts at experimental verification.[4–8] The existence of fractional statistics requires violation of the time-reversal and reflection symmetries.[9,10] If anyons are present in the copper oxide superconductors, then these symmetries must be spontaneously broken, which should result in unusual magnetization properties.[9,11,12] In particular, we may expect that any density inhomogeneities in the anyon carriers will be accompanied by local magnetic fields.

The muon-spin-relaxation (μSR) experiments that have been performed on these materials[4,5] provide a highly sensitive probe of local magnetic fields. In essence, spin-polarized positive muons are injected into the sample and are used to measure the magnitude and direction of the magnetic field at the sites where they come to rest inside the material. The muon charge must be screened by the surrounding carriers, and the nearby CuO_2 planes should account for at least some of the screening. If the relevant carriers in the planes are anyons, then, according to the general principles discussed by Halperin, March-Russell, and Wilczek,[12] it follows that the resulting density perturbation must be accompanied by a magnetization gradient and, hence, by local electric currents. Thus we expect the muon to be subject to a magnetic field, even if there were none in its absence. The magnitude of this magnetic field has been previously estimated to be on the order of 10 G.[12,13] However, the experiments performed on samples of $YBa_2Cu_3O_7$ and $Bi_2Sr_2CaCu_2O_8$ (Refs. 4 and 5) place an upper limit of $\approx$0.8 G on any magnetic field of nonnuclear origin at the muon position(s). In light of the theoretical predictions, these experimental results are certainly not encouraging. However, we feel that a more careful and refined estimate of the expected magnetic field is necessary to see to what extent the negative experimental results may rule out the existence of anyons in the cuprate materials.

It is the purpose of this and the preceding companion paper[14] (hereafter referred to as I) to examine in greater detail the formation of an orbital magnetic moment in a layer of anyons under external perturbation and to obtain a more careful estimate of the magnetic field that the perturbing muon would be subject to. In a two-dimensional system with broken time-reversal and reflection symmetries, density variations of small amplitude and long wavelength should be accompanied by magnetization current that is directly proportional to the density gradient. For dilute anyons in an effective-mass model, the relevant proportionality constant γ may be obtained exactly at zero temperature.[12] In our case, however, the size of the screening hole induced by the muon is comparable to the spacing between the carriers. Moreover, if the muon is close to the layer, then the induced density variation will be comparable to the average density, placing us well into the nonlinear regime (even the electrostatics becomes nonlinear if the charge density is fully depleted). This indicates that a linear-response treatment should be inadequate and that a Hartree-type calculation is required.

In I we reviewed the formalism for the Hartree approximation appropriate for a dilute anyon system and also established a number of general results for the magnetization of an anyon system with arbitrary density variations. In this paper we continue the analysis with an explicit calculation of the self-consistent density and current distributions in an isolated layer of anyons screening a muon positioned nearby. In addition, as a way of estimating the screening effect of the CuO chains in $YBa_2Cu_3O_7$, we briefly consider a model where the muon charge is partially screened by a second parallel metallic plane located behind the muon (see Fig. 4). We treat the anyons in the effective-mass approximation, which involves introducing multiple species of anyons to account for the multiple minima in the Brillouin zone (see Sec. IV B). We restrict our calculation to the most physically relevant case of semion statistics ($\theta=\pi/2$) and, correspondingly, two anyon species. Although the single-particle states are calculated in the Hartree approximation, where exchange interactions are neglected, the final current distribution is obtained by calculating the expectation value for the Slater determinant of these single-particle states; i.e., exchange terms are included in evaluating the expectation value of the two-body term $\mathbf{a}(\mathbf{r})\rho(\mathbf{r})$, which is the diamagnetic part of the current operator in an anyon system. As discussed in Sec. IV of I, this approach is guaranteed to give the correct value of the coefficient γ.

In Sec. II below we describe our model in more detail and outline the salient features of our calculational method. In Sec. III we present and discuss the numerical results. For the simple case of a muon near an isolated anyon plane, there are only two dimensionless parameters in our model, $\tilde{d}\equiv d/r_0$ and $r_s\equiv r_0/a_H$, where d is the distance of the muon from the CuO_2 plane, r_0 is the Wigner-Seitz radius for the anyons, which is related to the density ρ_0 of the anyons in the plane by $\pi r_0^2=\rho_0^{-1}$, and a_H is the effective Bohr radius, determined by the anyon mass m^* and the background dielectric constant ϵ. With this in mind we may write the magnetic field at the muon site in the form

$$B=\frac{\mu_B^*}{r_0^3}\beta(\tilde{d},r_s)\ , \tag{1}$$

where $\beta(\tilde{d},r_s)$ is a dimensionless function and $\mu_B^*=e\hbar/2m^*c$ is the effective Bohr magneton for the anyons. We find that the magnetic field at the muon site is not very sensitive to the value of r_s in the range of interest; i.e., $\beta(\tilde{d},r_s)$ is only weakly dependent on r_s. For values of d large compared with r_0, the value of B becomes essentially independent of all material parameters other then the effective mass m^* and is given by the asymptotic formula $B\sim\mu_B^*/16d^3$.

In order to apply our results to $YBa_2Cu_3O_7$, we choose an effective mass of $m^*=3m_e$ and a carrier density ρ_0 that is equivalent to 0.33 carriers per copper atom in each CuO_2 plane. This gives us $r_0=3.8$ Å and $\mu_B^*/r_0^3=56$ G. If, for example, the muon is 1 Å away from a CuO_2 plane (e.g., it forms an OH bond with one of the oxygens in this plane), then we find a magnetic field of ≈ 24 G at the muon site. On the other hand, Nishida and Miyatake[5] have suggested a possible muon site which is 3.15 Å away from the nearest CuO_2 plane and 1 Å away from a CuO chain. If we ignore the screening effect of the CuO chains and simply consider a point charge 3.15 Å away from an isolated anyon plane, we get a magnetic field of ≈ 5 G at the muon site. However, if we take into account screening by the array of CuO chains, modeling this effect by placing a metallic plane 1 Å behind the muon (Fig. 4), then the magnetic field is reduced to ≈ 3 G. Of course, if the muon is located at a symmetric site midway between two anyon planes and if the sign of the broken time-reversal symmetry is opposite in the two planes ("antiferromagnetic stacking"), then the magnetic field at the muon site should be precisely zero.

In Sec. IV we discuss further some of the approximations inherent in applying the simple dilute anyon model to the actual copper oxide superconductors, and we speculate about the features of the real system which could reduce the magnetic field for a given muon site below the values obtained in our model. We also compare our results with some other theoretical estimates of the magnetic field at a muon site.

Our overall conclusion is that, taken by themselves, the existing μSR results are highly discouraging, but not definitive as a test for the existence of the broken time-reversal symmetry necessary for anyons in the high-T_c materials. If the muon is located in a suitable asymmetric position, close to a single CuO_2 plane, it is difficult to imagine that effects left out of our model could reduce the calculated field of ≈ 20 G to a value of under 1 G, as required by the experiments. However, if the muon is located at an unfavorable site, relatively far from the nearest CuO_2 plane and close to a screening chain or close to the midpoint between two CuO_2 planes, then it would not be hard to imagine that effects left out of our model could reduce the field below the level observable in experiments. Clearly, it would be very desirable to obtain more information on the location of the muon site or sites.

In our paper the calculations have been carried out for a positive muon. In principle, analogous experiments can also be performed using negative muons, which come to rest in orbits very close to one of the nuclei in the crystal. A negative muon which binds to a spinless nucleus, such as ^{16}O, retains a portion of its initial polarization during the trapping process and, hence, can be used as a probe of the local magnetic field at the binding site. A negative-muon experiment has the advantage that there are only a small number of possible binding sites which can contribute to the signal, and it is likely that a reasonable proportion of the contributing muons will be located on an oxygen nucleus in a CuO_2 plane.[15] The polarization signal in a negative-muon experiment is typically much weaker than in a positive-muon experiment, however.

Our calculations can be readily extended to the case of a negatively charged probe. Although we have not done this, we would expect that, when the charge is close to the anyon plane, the magnetic field would be somewhat greater than for a positive muon, but that the field strength should be independent of the sign of the charge when the charge is far away from the plane.

The most sensitive alternative approach that has been used to search for broken time-reversal symmetry in high-temperature superconductors involves optical experiments designed to look for a spontaneous Faraday effect on transmission through a thin sample or for analogous effects on reflection.[6–8,16] The experimental situation here is very confused. Weber *et al.*[8] have reported strong rotational effects in single crystals of $YBa_2Cu_3O_x$ and $Bi_2Sr_2CaCu_2O_8$ whose sign could be controlled by cooling in an external field of ≈ 200 G. If correct, these results would imply large ferromagnetic domains with a substantial magnetic moment.[17] However, the highly sensitive experiments of Spielman *et al.*[7] have not shown any evidence of broken time-reversal symmetry. Earlier experiments by Lyons *et al.*[6] showed optical anomalies which were consistent with broken time-reversal symmetry, but might also admit an alternative explanation.

The optical experiments are complementary to the

μSR experiments in several respects. Although optical experiments, such as those carried out by Spielman *et al.* have a very high intrinsic sensitivity, this sensitivity is degraded considerably if the stacking of layers is antiferromagnetic or if the intrinsic domain size is very small compared with the size of the illuminated spot. Also, the expected magnitudes of the anomalous optical effects in an anyon superconductor are highly model dependent, so that the theoretical significance of a null result is even harder to quantify here than in the case of the μSR experiments.[17] Nevertheless, if it should turn out that continued refinements of the optical experiments, such as, for example, the use of very small samples with an odd number of layers, should fail to reveal clear evidence of broken time-reversal symmetry, then the evidence against the existence of anyons in the copper oxide superconductors would have to be considered very strong.

II. MODEL

We consider a collection of N positively charged anyons in the x-y plane that are characterized by the statistical angle $\theta=\pi(1-p^{-1})$ and the number of species n_a. The actual calculation will be restricted to the case $p=n_a=2$. To take advantage of the cylindrical symmetry, we assume the neutralizing background to be a disk of radius R and charge density $-e\rho_0$, with the positive muon positioned a distance d above the disk center. The many-body Hamiltonian for our model, written in the fermion gauge, is setting ($\hbar=m^*=1$)

$$H=\sum_{i=1}^{N}\left[\frac{1}{2}(\mathbf{p}_i-\mathbf{a}_i)^2-\rho_0\int_{\text{disk}}d^2r'\frac{e^2/\epsilon}{|\mathbf{r}_i-\mathbf{r}'|}+U_\mu(r_i)\right]+\frac{1}{2}\sum_{(i\neq j)}\frac{e^2/\epsilon}{|\mathbf{r}_i-\mathbf{r}_j|}\ , \tag{2}$$

$$\mathbf{a}_i=\frac{1}{p}\sum_{(j\neq i)}\frac{\hat{\mathbf{z}}\times(\mathbf{r}_i-\mathbf{r}_j)}{|\mathbf{r}_i-\mathbf{r}_j|^2}\ , \tag{3}$$

where $\mathbf{a}_i$ is the statistical vector potential, the second term in (2) is due to the negative background, and $U_\mu(r)=e^2/\epsilon(r^2+d^2)^{1/2}$ is the electrostatic potential of the muon. For the parameters we have chosen to model $YBa_2Cu_3O_7$, we have $r_0=3.8$ Å, while d is in the range $1<d<4$ Å. If we use an estimate $\epsilon\approx3$, we find that a_H is not far from its free-electron value of 0.53 Å, so that $r_s\approx7.2$.

Following the derivation in I, we write the self-consistent Hartree Hamiltonian

$$\hat{H}=\tfrac{1}{2}[\mathbf{p}-\langle a(r)\rangle\hat{\boldsymbol{\theta}}]^2+U_j(r)+U_c(r)+U_\mu(r)\ , \tag{4}$$

where $\hat{\boldsymbol{\theta}}$ is a unit vector in the azimuthal direction (clockwise). Since $\hat{H}$ is cylindrically symmetric, the self-consistent energy levels will have the form $\phi_{nl}(r,\theta)=f_{nl}(r)e^{il\theta}$, where n and l are the radial and orbital quantum numbers. Each level can accommodate n_a anyons. The anyon density in the Hartree approximation is thus $\langle\rho(r)\rangle=n_a\sum_{nl}|f_{nl}(r)|^2$, where the sum is over the occupied energy levels. The mean-field vector potential $\langle a(r)\rangle\hat{\boldsymbol{\theta}}$ has the magnitude

$$\langle a(r)\rangle=\frac{1}{p}\left|\int d^2r'\frac{\hat{\mathbf{z}}\times(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^2}\langle\rho(r')\rangle\right| =\frac{2\pi}{pr}\int_0^r dr'\,r'\langle\rho(r')\rangle\ . \tag{5}$$

The Hartree Coulomb potential is

$$U_c(r)=\frac{e^2}{\epsilon}\int_{\text{disk}}d^2r'\frac{\langle\rho(r')\rangle-\rho_0}{|\mathbf{r}-\mathbf{r}'|} =\frac{e^2}{\epsilon}\int_0^R dr'\,F(r,r')[\langle\rho(r')\rangle-\rho_0]\ , \tag{6}$$

where the kernel $F(r,r')$ obtained by angular integration is

$$F(r,r')=\frac{4r'}{r+r'}K\left[\frac{2\sqrt{rr'}}{r+r'}\right]\ , \tag{7}$$

with $K(x)$ denoting a complete elliptic integral of the first kind.

The magnetization current induced by the muon will be in the azimuthal direction, by symmetry. We define the Hartree current as

$$j_H(r)=\langle j_P(r)\rangle-\langle\rho(r)\rangle\langle a(r)\rangle\ , \tag{8}$$

where the paramagnetic current operator $\mathbf{j}_P(\mathbf{r})$ is

$$\mathbf{j}_P(\mathbf{r})=\tfrac{1}{2}\sum_i\{\mathbf{p}_i,\delta(\mathbf{r}-\mathbf{r}_i)\}\ , \tag{9}$$

and its expectation value has magnitude

$$\langle j_P(r)\rangle=\frac{n_a}{r}\sum_{nl}l|f_{nl}(r)|^2\ . \tag{10}$$

Finally, the term $U_j(r)$ in the single-particle Hartree Hamiltonian is given by

$$U_j(r)=\frac{1}{p}\int d^2r'\frac{\hat{\mathbf{z}}\times(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^2}\cdot\mathbf{j}_H(\mathbf{r}) =\frac{2\pi}{p}\int_0^r dr'\,j_H(r')\ . \tag{11}$$

The current operator that corresponds to our many-body Hamiltonian (1) is

$$\mathbf{j}(\mathbf{r})=\mathbf{j}_P(\mathbf{r})-\rho(\mathbf{r})\mathbf{a}(\mathbf{r})\ . \tag{12}$$

The Hartree current $j_H(r)$ is the mean value of this current operator calculated within the Hartree approximation. However, the diamagnetic current $-\rho(\mathbf{r})\mathbf{a}(\mathbf{r})$ is a two-body operator, in an anyon system, and therefore its expectation value is directly affected by two-particle correlations in the wave function. In particular, it was shown in I that the exchange correlations are very important in determining the current and that $j_H(r)$ does not give the correct value of γ and, hence, does not give the correct current in the long-wavelength limit. Therefore, to extract the physical current from the Hartree approximation, it is better to take the expectation value of the current operator in the Slater determinant of the occupied single-particle levels $\phi_{nl}(r,\theta)$. At the very least, this ensures that we get the correct value of γ (see Sec. IV of I). We define the antisymmetrized Hartree current, in the azimuthal direction, as

$$j_{HA}(r)=\langle j_P(r)\rangle - \langle \rho(r)a(r)\rangle_A \,, \tag{13}$$

where $\langle \cdots \rangle_A$ denotes expectation value in the Slater determinant. Specifically,

$$\langle \rho(r)a(r)\rangle_A=\langle \rho(r)\rangle\langle a(r)\rangle-\frac{n_a}{p}\sum_{nl,n',l'}\int d^2r'\frac{r-\hat{\mathbf{r}}\cdot\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^2}\phi^*_{nl}(\mathbf{r})\phi^*_{n'l'}(\mathbf{r}')\phi_{n'l'}(\mathbf{r})\phi_{nl}(\mathbf{r}')\,, \tag{14}$$

where the sum is over the occupied states of the Hartree Hamiltonian (4). When we calculate the magnetic field at the muon site, we use $j_{HA}(r)$ as its source.

The inhomogeneous Hartree problem outlined above has to be solved numerically. A numerical self-consistent calculation is limited to a relatively small number of states, and since the cylindrical geometry precludes the use of periodic boundary conditions, we have to be careful to avoid unwanted edge effects. In an analogous three-dimensional geometry, $\langle \rho(r)\rangle$ would deviate from ρ_0 only close to the surface, healing exponentially over a length of a few r_0. However, for a two-dimensional system with three-dimensional Coulomb forces, any charge imbalance at the edge will set up a long-range electric field, which falls off only as power of the distance from the edge. This field will tend to distort the self-consistent density and current throughout the system. In principle, to recover the thermodynamic limit of constant density and zero current, we have to make the system very large on the scale of r_0 ($R \gg r_0$) and, in addition, restrict ourselves to the region $r \ll R$. In this region, in the absence of the perturbation due to the muon, we indeed have $\langle \rho(r)\rangle=\rho_0$ and $j(r)=0$, and hence the vector potential is $\langle a(r)\rangle=\frac{1}{2}\bar{b}r$, where $\bar{b}=2\pi\rho_0/p$. The single-particle eigenstates $\phi^{(0)}_{nl}$ are simply the radial Landau states. For $n_a=p=2$ the ground state is obtained by completely filling the lowest Landau-level eigenstates $f^{(0)}_{0l}(r)\propto r^l e^{-r^2/4r_0^2}$, where for large l each is localized near $(2l)^{1/2}r_0$.

Introducing the muon above $r=0$ will create a screening hole with total change in particle number $\delta N=-1$, since at long distance the muon must be completely screened. The size of the screening hole will be approximately $r_1 \approx \max[d,r_0]$. In the region $r_1 \ll r \ll R$, the density will still be ρ_0 and the scalar potentials will be constant. Eigenstates localized in this region will be affected only by the change in the vector potential, $\langle \delta a(r)\rangle=\delta N/pr$ for $r \gg r_1$. One can readily show that these eigenstates become $f_{0l}(r)\propto r^{l-\delta N/p}e^{-r^2/4r_0^2}$; i.e., they are basically shifted outward. Thus the self-consistent eigenstates localized outside the screening hole are known and only the states within the hole need to be calculated by numerical iteration. In practice, we find it adequate to confine our numerical calculation to the region $r<8r_1$, which means that we calculate explicitly about 30 angular momentum states.

III. RESULTS

Not surprisingly, we find that the anyon layer has screening properties similar to those of the two-dimensional electron gas, at least in the Hartree approximation. In the context of the Hartree approximation, we define the electrostatic limit as the limit of large r_s. In this limit the charge distribution $\langle \rho(r)\rangle$ is dominated by the Coulomb forces and will be given by continuum electrostatics. The same limit is approached at long wavelengths, i.e., for $\tilde{d}\to\infty$ at fixed r_s.

For $\tilde{d}^2>\frac{1}{2}$, screening in the electrostatic limit is exactly like that of a metallic plate; i.e., the anyon layer is an equipotential. The corresponding density distribution is

$$\rho_{es}(r)=\rho_0\left[1-\frac{\tilde{d}}{2(\xi^2+\tilde{d}^2)^{3/2}}\right]\,, \tag{15}$$

where $\xi=r/r_0$. For finite r_s and $\tilde{d}$, the quantum-mechanical degeneracy pressure flattens the screening hole, raising the density in the center above the electrostatic limit (Fig. 1).

For $\tilde{d}^2<\frac{1}{2}$, screening in the electrostatic limit is quite different. The anyon layer can no longer be an equipotential near the center since there the charge density is depleted. The size of the anyon-free region depends on the external perturbation and the electrostatics is therefore highly nonlinear. For finite r_s the density never reaches

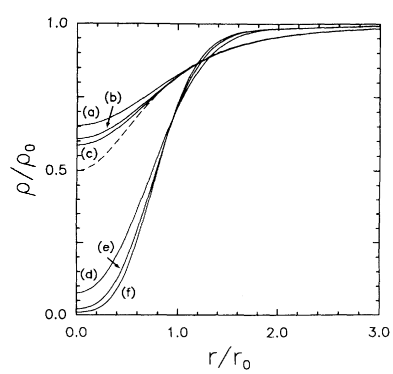

FIG. 1. Self-consistent anyon density distribution $\langle \rho(r)\rangle$. Lines (a)–(c) are for $\tilde{d}=1$ and $r_s=4,8,12$, respectively. Lines (d)–(f) are for $\tilde{d}=0.25$ and $r_s=4,8,12$, respectively. The dashed line gives the density in the electrostatic limit for $\tilde{d}=1$ [Eq. (15)].

zero due to effects of kinetic energy (Fig. 1). Note that for any fixed value of $\tilde{d}$ the density profile is not very sensitive to the value of r_s.

The antisymmetrized Hartree current $j_{HA}(r)$ induced by the muon is shown in Fig. 2, for a range of parameters. Note that for fixed $\tilde{d}$ its magnitude scales roughly as r_0^{-3}. We obtain the magnetic field at the muon site from

$$B=\mu_B^* \int_0^\infty dr \frac{4\pi r^2 j_{HA}(r)}{(r^2+d^2)^{3/2}} , \tag{16}$$

where $\mu_B^*=e\hbar/2m^*c$ (the field points in the positive $\hat{\mathbf{z}}$ direction, for our sign convention). Scaling out the dimensional quantities, we write

$$B=b_0\beta(\tilde{d},r_s) , \tag{17}$$

$$\beta(\tilde{d},r_s)=\int_0^\infty d\xi \frac{4\pi\xi^2 r_0^3 j_{HA}(r_0\xi)}{(\xi^2+\tilde{d}^2)^{3/2}} , \tag{18}$$

where $b_0=\mu_B^*/r_0^3$. We find that $\beta(\tilde{d},r_s)$ depends only weakly on r_s and is mainly a function of $\tilde{d}$, as shown in Fig. 3. An important consequence of this is that the magnetic field is essentially independent of the uncertain bulk dielectric constant ϵ. For the parameters we use to model $YBa_2Cu_3O_7$, we have $b_0=56$ G. For example, if the muon is 1 Å from the plane, then we obtain $B\approx 24$ G.

In the long-wavelength, small-amplitude limit, the current is proportional to the density gradient

$$j_M(r)=-\frac{\gamma}{2}\frac{d\rho}{dr} , \tag{19}$$

where $\gamma=\frac{1}{4}$ for $n_a=p=2$. In our geometry we approach this limit for large $\tilde{d}$. In the same limit, the screening density should approach that of a metallic plate [Eq. (15)]. To obtain the magnetic field, we substitute $\rho_{es}(r)$ into (19) and obtain

$$B\sim\frac{|\gamma|}{4}\frac{\mu_B^*}{d^3} . \tag{20}$$

From Fig. 3 we see that this asymptotic form gives a good approximation to the magnetic field for $\tilde{d}>1$.

The actual magnetic field at the muon site may be lowered appreciably by the other antiferromagnetically aligned CuO_2 layers or by the screening due to other

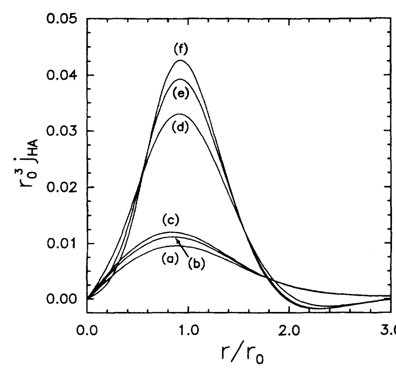

FIG. 2. Antisymmetrized Hartree current $j_{HA}(r)$ scaled by r_0^3 Lines (a)–(c) are for $\tilde{d}=1$ and $r_s=4,8,12$, respectively. Lines (d)–(f) are for $\tilde{d}=0.25$ and $r_s=4,8,12$, respectively.

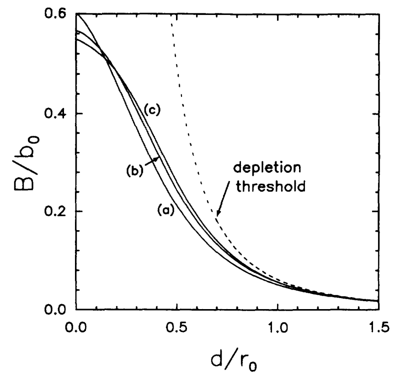

FIG. 3. Magnetic field at the muon site as a function of the muon distance. Solid lines (a)–(c) give the Hartree approximation for $r_s=3.58, 7.16, 10.74$, respectively. The dashed line gives the simple asymptotic form of magnetic field [Eq. (20)] that is valid for large $\tilde{d}$ and is derived using the linear electrostatic limit (15) for the charge density and the long-wavelength limit (19) for the current. Arrow shows distance below which charge depletion invalidates the linear electrostatic approximation (15) for the charge density.

mobile charges in the system. Thus, if the muon is located close to the metallic CuO chains, these chains will screen part of the muon charge. They will also screen the Coulomb interactions between the anyons in the nearby CuO_2 layer. We may obtain a rough model of these effects by replacing the chains with a metallic plane (Fig. 4). The electrostatic potentials will get modified by the appropriate image charges. Thus, in addition to the external potential $U_\mu(r)$ due to the muon, we will have the potential $U_\mu^*(r)=-e^2/\epsilon[r^2+(2d_p+d)^2]^{1/2}$ due to the muon's image. Also, the Coulomb interactions between the anyons in the plane get modified by the appropriate image contributions:

$$\frac{1}{|\mathbf{r}_i-\mathbf{r}_j|}\to\frac{1}{|\mathbf{r}_i-\mathbf{r}_j|}-\frac{1}{[|\mathbf{r}_i-\mathbf{r}_j|^2+4(d+d_p)^2]^{1/2}} . \tag{21}$$

Nishida and Miyatake[5] suggest that in $YBa_2Cu_3O_7$ the muon binds to the O(1) oxygen of a CuO chain and is located 3.15 Å above the CuO_2 layer, with the chain layer 1 Å behind it. If we neglect chain screening, we obtain $B\approx 5$ G for this muon position. Introducing the screening metallic plate as described above, with $d=3.15$ Å and $d_p=1$ Å, lowers the field to ≈ 3 G.

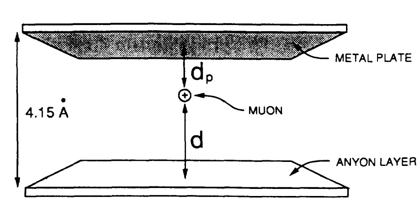

FIG. 4. Geometry of model in which the screening effect of the CuO chains is represented by a metallic plane at a distance d_p behind the muon.

We have attempted to take account of the surrounding medium by introducing the bulk dielectric constant ϵ and treating screening by the metallic chains explicitly. But, in addition, we may expect the individual ions of the lattice, such as Y^{3+} and Ba^{2+}, to perturb the anyon gas. Hence local magnetic fields may be present even in the absence of the muon. If these magnetic fields were of the same magnitude as the field induced by the muon, then the total field acting on the latter could turn out to be rather small because of cancellation. However, for a sheet of periodically arranged ions, the parallel component of the electric field falls off very rapidly with distance, $E_\parallel \propto e^{-2\pi z/a}$, where a is the lattice constant. This will significantly reduce the perturbative effect of the ions that do not lie in the CuO_2 plane itself. The nearest such charges, the yttrium ions, are located ≈ 1.7 Å away from the plane and are attenuated by a factor of ≈ 0.06. The Cu^{2+} and O^{-2} ions of the CuO_2 plane will not be subject to such attenuation; however, they can no longer be treated as point charges. Their core states are now part of the band structure and will have the effect of limiting the density variation of the anyon gas. We further note that any currents that are induced in the plane by a sheet of ions will also be periodic, and so the resulting magnetic field will suffer a similar attenuation, $\propto e^{-2\pi z/a}$, as a function of distance from the plane. Hence, if the muon is not too close to the CuO_2 plane, the periodic ion contribution to the magnetic field should be quite small.

If the muon is located close to a CuO_2 plane, then there could be a significant contribution to the magnetic field at the muon site arising from the periodic modulation of anyon density by the Cu^{2+} and O^{2-} ions in the plane. In this case, however, the magnetic field produced by the muon perturbation is also quite large, and so it is very unlikely that there could be a cancellation such as to reduce the total magnetic field below the threshold of experimental sensitivity.

IV. UNCERTAINTIES IN THE APPLICABILITY OF THE MODEL

The applicability of our model calculations to the actual high-temperature superconductors is affected by the uncertainties in the model parameters and the approximations inherent in the model itself. Here we discuss some of these considerations. We also compare our results with other estimates of the magnetic field at the muon site.

A. Values of the parameters

The uncertainty in the magnetic field acting on the muon due to the uncertainty in its position has already been discussed. In addition, there is also some uncertainty in the carrier density and effective mass. Within the context of an effective-mass approximation, the value of m^* is to a great extent constrained by the measured value of the London penetration depth λ_L. For magnetic fields perpendicular to the CuO_2 planes, the zero-temperature value $\lambda_L \approx 1400$ Å, reported for $YBa_2Cu_3O_{7-x}$ determines the zero-temperature value of ρ_s/m^*, summed over CuO chains and two CuO_2 planes. If we assume, somewhat arbitrarily, that a chain layer contributes the same amount to the supercurrent flow as each of the CuO_2 planes, then the value of ρ_s/m^* for each CuO_2 layer is approximately equal to $0.11 m_e^{-1}$ per unit cell. Although there are no anyons in the chain layer, there would presumably be superconductivity induced in the chains, which would lead to nonzero ρ_s. Our estimate for ρ_0 of 0.33 holes per unit cell in each layer comes from assuming, similarly, that the formal valence charge of one hole per unit cell is distributed equally over the three copper ions. This gives us the effective mass of $3m_e$ that we have used in our estimates. Note that the value of the magnetic field at the muon site does not strongly depend on the choice of ρ_0, if the ratio ρ_0/m^* is fixed.

If instead of assuming equal contributions from chains and planes one assumes that the CuO chain layers make no contribution to the value of the London penetration depth at $T=0$, then the value of ρ_s/m^* for the CuO_2 plains must be increased to $0.16 m_e^{-1}$ per unit cell. This leads to the effective mass $m^*=2m_e$, if the same value of ρ_0 is employed, which would lead to an increase in magnetic field at the muon site by a factor of ≈ 1.5 over the values quoted in Sec. III.

For a model of weakly coupled planes of dilute anyon gas, it is reasonable to estimate the superconducting transition temperature from the unrenormalized Kosterlitz-Thouless temperature $T^0_{KT}=\pi\hbar\rho_0/(2m^*p^2)$, as discussed in I. The values of ρ_0 and m^* used by us in Sec. III lead to a value of $T^0_{KT}\approx 250$ K, which is well above the value of T_c for Y-Ba-Cu-O. This suggests, of course, that the dilute anyon model does not give a consistent parametrization of the superconducting properties.

B. Filling of the Brillouin zone

As was noted by Kalmeyer and Laughlin, the Brillouin zone for spin excitations (spinons) in the insulating chiral spin-liquid state has only half the area of the Brillouin zone of the lattice.[12,18] The reason for this is that the operators T_a and T_b, which translate a spinon by one lattice constant in either of the two different directions within the plane, obey the relation $T_aT_b=-T_bT_a$. A similar relation is obeyed by the translation operators for the charge carriers (holons), which may be considered to be a bound state of an ordinary hole and a spinon. This commutation relation is identical to that of the translation operators for a charged particle on a lattice with half a flux quantum per unit cell, and the reduction of the Brillouin zone occurs for the same reasons as in that case. The commutation relation also requires that within the reduced zone there be an additional degeneracy so that every feature of the energy spectrum is repeated at least twice. This is the reason that the energy spectrum must have at least two minima in the reduced Brillouin zone, with identical effective mass, which gives rise to two distinguishable semion species in our effective-mass approximation. An obvious problem with the effective-mass approximation is that the concentration of carriers in $YBa_2Cu_3O_7$ or $Bi_2Sr_2CaCu_2O_8$ is actually rather large. A concentration of $\frac{1}{3}$ holes per unit cell of the CuO_2 plane means that the Brillouin zone for anyons is actually $\frac{2}{3}$ full.

C. Internal structure of the anyon

Other possible problems in the applicability of our calculations arise from the possibility of a complicated internal structure of the anyon. If the anyons have a finite size, then they may overlap when the concentration is not very small, and the interaction between them may not be well represented by a structureless Coulomb repulsion. Moreover, the interaction may become temperature-dependent, even at temperatures below the superconducting transition.

Even in the dilute limit, where T_c is very small, anyons may have nonzero intrinsic magnetic moment, which we have thus far ignored in our discussion. As in Sec. II of I, we may write the intrinsic moment, pointing along the $\hat{z}$ axis, as $\mu_z = \pm g^* \mu_B^* /4$, so that the value of γ at $T=0$ is replaced by

$$\gamma = \mp \frac{\hbar}{4}(1-g^*) . \tag{21}$$

For a microscopic model with $g^*=1$, the coefficient γ would vanish at $T=0$, and the magnetic field at the muon site would likewise be greatly reduced.

Recently, Wiegmann[19] has calculated the random-phase-approximation (RPA) density-current response function for a particular microscopic model, where anyons are obtained from a gauge interaction between holons and spinons, and has found a vanishing value of the coefficient γ at $T=0$, analogous to the occurrence of $g^*=1$. However, we have not been able to follow Wiegmann's analysis, and we have not been able to reproduce his results. The model considered by Wiegmann, as we understand it, assumes, in the spirit of the Kalmeyer-Laughlin model, that there is already a chiral energy gap E_g in the spinon spectrum in the absence of doping and the Fermi level remains in this gap in the presence of the self-consistent gauge field generated by doping. We find in this model that the density-current response of the spinon system is properly described by a local Chern-Simons term in the effective Lagrangian for the gauge field, at any wavelength large compared with $(W/E_g)^{1/2}$, where W is the spinon bandwidth. If the doping is small, so that r_0 for the holons is large compared with this length, then RPA analysis similar to Wiegmann's should give simply the value $\gamma = \mp \hbar/2$, which is expected in the Hartree approximation for the holon system, with no cancellation due to spinons. On the other hand, if the doping becomes large, so that r_0 is comparable to $(W/E_g)^{1/2}$, there could be a reduction of γ similar to that caused by a nonzero value of g^*. Further analysis of this model would be worthwhile.

D. Other estimates of local magnetic fields

Recently, McMullen, Jena, and Khanna[20] have carried out a calculation of the magnetic field at the muon site within a linear-response approximation. Like us, they consider screening of a positive muon by an isolated plane of anyons. However, they use an effective-mass model with a *single* species of semion and calculate charge and current distributions using the RPA density-current response function of Fetter, Hanna, and Laughlin[21] (with Coulomb interactions added in). For a single species of semion, RPA gives $|\gamma|=1$.[11] The exact value for the single-species model, obtained when exchange terms are included, is $|\gamma|=\frac{3}{4}$.[12] Our calculation gives, as it must, $|\gamma|=\frac{1}{4}$, which is the exact value for two species of semion. Hence, in the long-wavelength (large $\tilde{d}$) limit McMullen, Jena, and Khanna obtain a magnetic field that exceeds our estimate by a factor of 4. When the muon is located close to the anyon plane, nonlinear effects, included in our Hartree calculation, should become important, and the difference in γ is no longer sufficient to account for the difference between our results and those of McMullen, Jena, and Khanna. In the range $0.1 < \tilde{d} < 1$, McMullen, Jena, and Khanna obtain magnetic fields that are 2–8 times greater than the corresponding fields given by our calculation.

Hetrick, Hosotani, and Lee[22] have analyzed the bulk magnetic properties of an anyon superconductor, at $T=0$ and at finite temperature, using a self-consistent-field approximation. They find a nonzero internal magnetic field, whose magnitude they estimate to be of order 10^{-4} G, at $T=100$ K. They have implicitly assumed a ferromagnetic arrangement of the layers, and the system is not superconducting at 100 K in their analysis. Their result is in conflict with our own analysis and that of Ref. 12, which predicts a substantial magnetic field, on the order of 10 G, for the ferromagnetic stacking of anyon layers, in the normal state.

We believe that the result of Hetrick, Hosotani, and Lee is unphysical and reflects the fundamental weakness of the Hartree approximation at finite temperatures, which was noted in Sec. III of I. Since the Hartree approximation does not include the effects of the logarithmic interaction between the thermally excited vortices, it incorrectly predicts a finite density of free vortices at any temperature $T>0$ (recall that excited particles and holes are to be identified with vortices in an anyon superconductor). The density of vortices obtained by Hetrick, Hosotani, and Lee at $T=100$ K is extremely small, however, so that the system, although technically not superconducting, has a very large diamagnetic susceptibility. We believe that this may be the reason for the very small values of the equilibrium internal magnetic field that they obtain. Hetrick, Hosotani, and Lee do not calculate the local magnetic fields that would be produced by microscopic inhomogeneities in the charge density. In their approximation, such fields would be much larger than the uniform field.

Lederer, Poinblanc, and Rice[23] have estimated the spatially varying magnetic field in a "commensurate flux phase," which is related to, but somewhat different from, the Kalmeyer-Laughlin picture of anyon superconductivity. In particular, the charge density and current density in the commensurate flux phase have a lower translational symmetry than the original lattice. Lederer, Poinblanc, and Rice attain local magnetic fields somewhat higher than our estimates, reaching values of 70 G or larger at some points of the extended unit cell.

Hsu, Marston, and Affleck[24] have calculated the local magnetic fields that would be expected in the so-called staggered flux phase, which has a locally broken time-reversal symmetry. They obtain field values up to about 10 G, depending on the muon site, which is not very different from the values that we obtain in the present paper for the dilute anyon model.

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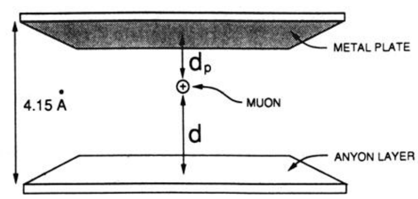

FIG. 4. Geometry of model in which the screening effect of the CuO chains is represented by a metallic plane at a distance d_p behind the muon.