Superconductors in strong magnetic fields: de Haas–van Alphen effect

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We consider superconductors in strong magnetic fields in the mixed state when the orbital quantization of the electrons is important. By averaging over the Abrikosov vortex lattice, solutions are obtained for the Green's function and gap parameter in the quantum case. These solutions are appropriate to describe phenomena on a scale large compared with the vortex-line separation and are independent of the detailed vortex configuration. These results are used to calculate the de Haas–van Alphen oscillations in the free energy in the mixed state. The effects of spin splitting and a layered structure are discussed.

I. INTRODUCTION

The properties of superconductors in the mixed state in large magnetic fields is a subject of considerable current interest. In the high-T_c materials it is possible to attain fields where the quantization of the orbits of the electrons in the field becomes important. The condition we find for the observation of effects associated with quantization of the orbits is $Rl/\xi^2<1$, where R is the cyclotron-resonance radius of an electron in the magnetic field, $l=(c/eH)^{1/2}$ is the magnetic length and $\xi=v_F/\Delta$ is the coherence length in the field. Close to H_{c2} the gap parameter Δ is depressed and ξ becomes long and this condition can be realized. This condition can also be written $\Delta^2<\sqrt{\mu\omega_c^3}$, where μ is the Fermi energy, ω_c is the cyclotron frequency, and Δ is the gap parameter in the field. These conditions are discussed further in Sec. V. Low temperatures such that $\omega_c\sim kT$ are also important because otherwise the quantization effects are smeared out. Previous work on this subject includes a study of the effect of quantization on the upper critical field $H_{c2}(T)$ by Gruenberg and Gunther[1] and by Tešanović *et al.*[2] Possible pairing schemes for electrons in quantized orbits have been discussed by Markiewicz *et al.*[3] Quantization of the orbits of the electrons in the field leads to quantum oscillations in the magnetization of normal metals [de Haas–van Alphen effect (dHvA)] and provides valuable information on the Fermi surface. It is clearly of interest to carry out such experiments on the high-T_c superconducting oxides. The low temperatures needed and the fact that the upper critical field in these materials at low temperatures is extremely large requires that such experiments be carried out in the mixed state. The dHvA effect in the mixed state of $2H$-$NbSe_2$ has been previously reported.[4] The condition that $\omega_c\tau>1$, where τ is the collision time, makes these experiments difficult in the mixed state of type-II materials as they are frequently alloys. This is presumably the reason for the lack of further experimental results. Recently two theoretical studies of the quantization of quasiparticle orbits in the presence of superconducting pairing have appeared[5,6] and been applied to calculate dHvA oscillations in the quasiparticle magnetization. The present author[5] considered the extreme quantum limit and Maki[6] used a method of quantizing the semiclassical approximation. Both authors found that the gap in the mixed state behaved as an effective temperature and reduced the amplitude of the dHvA oscillations.

In this paper we obtain solutions to the Gorkov equations for the Green's functions and gap parameter in the mixed state when the orbits of the electrons in the applied field are quantized. It is argued that a number of properties of the mixed state are independent of the detailed structure of the Abrikosov vortices which allow us to average over this structure and leads to considerable simplifications. We thus obtain a theory of the superconducting state in strong fields when the electron orbits are quantized, which is useful for describing properties of the mixed state which are independent of the detailed vortex lattice structure. One such property is the dHvA effect. The cyclotron-resonance radius R_c of an electron with the Fermi energy is generally much larger than the vortex spacing d. Thus, $R_c^2/d^2\sim\mu/2\pi\omega_c$ and is large except in the extreme quantum limit $\omega_c\sim\mu$. These results are then used to calculate the dHvA effect in the mixed state. These results extend those previously obtained. We also discuss the effect of spin splitting and the two- or three-dimensional crossover for layered materials.

II. GREEN'S FUNCTIONS IN STRONG FIELDS

The properties of a superconductor are contained in the ordinary and anomalous Green's functions, G_s and F_s, which satisfy the Gorkov equations[7]

$$\left[-i\omega+\frac{1}{2m}\left[\mathbf{p}_1-\frac{e}{c}\mathbf{A}_1\right]^2-\mu\right]G_s(12\omega)-\Delta(1)F_s^+(12\omega)=-\delta(1-2)\,, \tag{1}$$

$$\left[-i\omega-\frac{1}{2m}\left[\mathbf{p}_1+\frac{e}{c}\mathbf{A}_1\right]^2+\mu\right]F_s^+(1,2,\omega)-\Delta^+(1)G_s(1,2,\omega)=0\,. \tag{2}$$

$\mathbf{A}=(0,Hx,0)$ is the vector potential in the Landau gauge and the field is assumed uniform. The variations in the field due to the vortex structure are of order Δ^2 and neglected as small. μ is the Fermi energy and $\omega=(2n+1)\pi/\beta$ are the Matsubara frequencies. The spatial coordinates $\mathbf{r}_1\equiv 1$, etc. We can rewrite the equation for G_s as an integral equation

$$G_s(1,1',\omega)=G_0(1,1',\omega) - \int d2\,d3\,G_0(1,2,\omega)\Delta(2)G_0(3,2,-\omega) \times\Delta(3)G_s(3,1',\omega)\ , \tag{3}$$

where G_0 is the normal Green's function. In terms of the Landau level wave functions

$$G_0(12\omega)=\sum_{n,q,k}\frac{\phi_{nqk}(1)\phi^*_{nqk}(2)}{i\omega-\varepsilon_{nk}}\ , \tag{4}$$

where $\varepsilon_{nk}=(n+\frac{1}{2})\omega_c+k^2/2m-\mu$ and

$$\phi_{nqk}=N_n e^{ikz+iqy}e^{-(x-ql^2)^2/2l^2}H_n((x-ql^2)/l)\ , \tag{5}$$

where k is the momentum along the field, $l=\sqrt{c/eH}$ is the magnetic length, H_n is a Hermite polynomial and N_n is a normalization constant.

In the mixed state of a type-II superconductor, there will be localized excitations in the cores of the vortex lines and excitations outside the cores. When the coherence length is small, the localized excitations would not be expected to be appreciably affected by the magnetic field and further near H_{c2} where the cores almost overlap most excitations will be delocalized. The delocalized excitations would be expected to have some kind of Landau level structure which will lead to de Haas–van Alphen oscillations in their magnetization. The cyclotron radius R_c of an electron with the Fermi energy will be much larger than the vortex line spacing d ($R_c^2/d^2\sim\mu/\omega_c$), where ω_c is the cyclotron frequency except for extremely large fields such that all the electrons are the lowest Landau level, a situation we do not consider. It is then appropriate to average over the vortex lines and the appropriate average required in (3) is

$$V(r_1-r_2)=\langle e^{i\phi(r_1,r_2)}\Delta(r_1)\Delta^*(r_2)\rangle\ , \tag{6}$$

where the phase factor

$$\phi(r_1,r_2)=(x_2+x_1)(y_2-y_1)/l^2$$

in the Landau gauge. The form of this average does not depend sensitively on the distribution of the vortex lines and should be the same whether we have a vortex lattice or a disordered arrangement of vortex lines due to pinning or melting of the vortex lattice. We first consider V for a square Abrikosov vortex lattice[8] where

$$\Delta(\mathbf{r})\sim\sum_n e^{iq_0ny}\exp\left[-\left[x-\frac{nq_0l^2}{2}\right]^2\frac{1}{l^2}\right]\ . \tag{7}$$

Including the phase factor the expression in the brackets on the right-hand side of (6) is periodic in the variable $\mathbf{R}=(\mathbf{r}_1+\mathbf{r}_2)/2$ and the averaging consists in integrating $\mathbf{R}$ over a unit cell. This gives[9]

$$V(r)=\Delta^2e^{-r^2/2l^2}\ , \tag{8}$$

where Δ is the magnitude of the gap parameter and is determined below. Thus, in the vortex lattice the average order-parameter correlations are short-range perpendicular to the field and determined by the magnetic length.

We now argue that the same result (8) also applies for a disordered vortex lattice. We can construct a disordered lattice in the same manner as Abrikosov by writing $\Delta(r)$ as a superposition of states from the lowest Landau level

$$\Delta(r)\sim\sum_p C_p e^{ipy}\exp\left[-\left[x-\frac{pl^2}{2}\right]^2/l^2\right]\ , \tag{9}$$

where C_p are random variables with $\langle C_pC_p^*\rangle=C^2\delta_{pp'}$. This leads immediately to form (8) for the order-parameter correlation function. The trial order parameter (9) leads to a higher free energy than the Abrikosov lattice (the Abrikosov parameter $\beta_A=2$). This higher free energy could be offset by pinning.

It is difficult to interpret the order parameter (9) physically so we consider another method of constructing a disordered vortex lattice. Using a symmetric gauge, we can write trial solution for the order parameter

$$\Delta(r)\sim e^{-|z|^2/2l^2}\Pi_\nu(z-z_\nu) \tag{10}$$

when $z=x+iy$ and z_ν are the positions of the vortex lines. This form for the order parameter has been used by Brandt,[10] Kogan,[11] and others. We assume that these lines are distributed at random and calculate the average in (2.6) by averaging independently over the position of each vortex line. Thus,

$$\langle\Delta(r_1)\Delta^+(r_2)\rangle\sim e^{-(|z_1|^2+|z_2|^2)/2l^2}(z_1z_2^*+\langle|z_\mu|^2\rangle)^N\ , \tag{11}$$

where N is the number of lines. The radius of the system is $\sqrt{N^2l^2}$ so we take $\langle|z_\nu|^2\rangle=Nl^2$ and for large N we obtain

$$\langle\Delta(r_1)\Delta^*(r_2)\rangle\sim e^{-(|z_1|^2+|z_2|^2-2z_1z_2^*)/2l^2}\ , \tag{12}$$

which reduces to (8) when the appropriate phase factor for the symmetrical gauge is inserted. These arguments indicate that (8) should be correct under most conditions. In the Appendix we show that (8) can also be obtained from the Ginzburg-Landau equations.

We now solve (3) by treating Δ as a random variable with a correlation function given by (8). The problem is analagous to that of an electron in a random potential and, since Δ^2 is small, we use the coherent-potential approximation (CPA). The following result makes Eq. (3) easy to solve:

$$\langle\phi_{nqk}(1)|V(1-2)e^{-i\phi(12)}G_0(2,1,-\omega)|\phi_{n'q'k'}(2)\rangle = -\Delta^2\delta_{nqk,n'q'k'}\sum_{n_1}\frac{I_{nn_1}}{i\omega+\varepsilon_{n_1k}}\ , \tag{13}$$

where

$$I_{nn_1}=\frac{(n+n_1)!}{n!n_1!2^{n+n_1+1}} . \tag{14}$$

This result shows that the average of G_s, which we denote by G, is diagonal when expanded in Landau states (5) and has the form

$$G(12)=\sum_{n,q,k}\frac{\phi_{nqk}(1)\phi^*_{nqk}(2)}{i\omega-\varepsilon_{nk}-\Sigma_{nk}(\omega)} , \tag{15}$$

where self-consistency requires

$$\Sigma_{nk}(\omega)=\Delta^2\sum_{n_1}\frac{I_{nn_1}}{i\omega+\varepsilon_{n_1k}+\Sigma_{n_1k}(-\omega)} . \tag{16}$$

Before looking at solutions of this equation we obtain the equivalent of the gap equation for Δ^2. From (2) the equation for the gap is

$$\Delta^+(2)=\frac{\lambda}{\beta}\sum_{\omega}\int G_0(3,2,-\omega)\Delta^+(3)G(3,1,\omega) , \tag{17}$$

where λ is the interaction. We multiply this by $\Delta(1)e^{i\phi(1,2)}$ and average both sides as in (6) again using the CPA approximation. This gives the equation for V

$$V(1,2)=\frac{\lambda}{\beta}\sum_{\omega}\int G_0(3,2,-\omega)V(1,3)\times e^{i[\phi(1,2)-\phi(1,3)]}G(3,1,\omega) . \tag{18}$$

It is not difficult to show that (8) is a solution of (18) with Δ^2 being determined by

$$\Delta^2=\frac{-\lambda}{2\pi l^2\beta L}\sum_{n,k}\sum_{\omega}G(n,k,\omega)\Sigma(n,k,\omega) . \tag{19}$$

We obtain the equation for the transition temperature by letting $\Delta^2\to 0$ on both sides of (19) which gives

$$1=\frac{\lambda}{2\pi l^2\beta L}\sum_{nkn',\omega}I_{nn'}G_0(n,k,\omega)G_0(n',k,-\omega) , \tag{20}$$

which is identical with that obtained by Gruenberg and Gunther.[1]

We now examine the solutions of (16). For large quantum numbers, the case of interest when $\mu>\omega_c$, we can approximate $I_{nn_1}\sim e^{-(n-n_1)^2/4n}/\sqrt{4\pi n}$ and thus $|n-n_1|\sim\sqrt{n}\sim(\mu/\omega_c)^{1/2}$. We then consider two cases.

(1) Weak fields $kT>|n_1-n|\omega_c\sim\sqrt{\mu\omega_c}$. In this case, in the denominator of (16) we can replace ε_{n_1k} by ε_{nk} and neglect Σ and then

$$\Sigma_{nk}(\omega)=\frac{\Delta^2}{i\omega+\varepsilon_{nk}} \tag{21}$$

and

$$G(nk\omega)=\frac{-(i\omega+\varepsilon_{nk})}{\omega^2+\varepsilon_{nk}^2+\Delta^2} , \tag{22}$$

which is of the BCS form and the quasiparticles have a Landau level structure. It should be noted that this result also applies at low temperatures and low fields $\Delta^2>\sqrt{\mu\omega_c}$ as it goes over into the zero-field BCS form. It also applies in the extreme quantum limit when effectively only one term contributes in (16).

(2) Strong fields $kT<\sqrt{\mu\omega_c}$. We replace the sum on n_1 in (16) by an integral, neglect the Σ in the denominator, and obtain

$$\Sigma=\frac{\Delta^2}{(4\pi n)^{1/2}\omega_c}\int_{-\infty}^{\infty}dy\frac{e^{-y^2/4n\omega_c^2}}{i\omega+\varepsilon_{nk}+y} . \tag{23}$$

After introducing a new variable $y'=y+\varepsilon_{nk}$ and evaluating the integral in the low-temperature limit, we find

$$\Sigma=\frac{\Delta^2}{(4\pi n)^{1/2}\omega_c}\left[\frac{-i\pi\omega}{|\omega|}+\left[\frac{\pi}{n}\right]^{1/2}\frac{\varepsilon_{nk}}{\omega_c}\right] . \tag{24}$$

The first term in (24) corresponds to a scattering of the excitations by the vortex lines with a scattering rate

$$\frac{1}{\tau_v}=\frac{\Delta^2}{\omega_c}\sqrt{\pi/n}\simeq\Delta^2\left[\frac{\pi}{\mu\omega_c}\right]^{1/2} . \tag{25}$$

The second term renormalizes the energy and is unimportant.

With these results we can now evaluate the gap equation. We do this in the semiclassical limit, i.e., sums on n are replaced by integrals. The calculations are very similar to those of Gruenberg and Gunther,[1] who determined $T_c(H)$ from (20), and so we omit the details and just quote the results.

(a) Weak fields $kT>\sqrt{\mu\omega_c}$. This is the situation near T_{c0}, the zero-field transition temperature. When (19) is expanded in Δ^2 using (22) we find

$$\tfrac{7}{8}\zeta(3)\left[\frac{\Delta}{\pi kT}\right]^2=\frac{T_{c0}-T}{T_{c0}}-\frac{7}{12}\frac{\mu\omega_c}{(\pi kT)^2}\zeta(3) , \tag{26}$$

where ζ is the Riemann ζ function. The critical H_{c2} is determined by the vanishing of the right-hand side of (26) and gives the well-known result of Abrikosov.[6] $\Delta^2\sim\mu\omega_{c2}(1-H/H_{c2})$ as found by Abrikosov but does not depend on the vortex structure because we have averaged over this structure.

(b) Strong fields $kT<\sqrt{\mu\omega_c}$. The upper critical field at low temperatures $H_{c2}(0)$ can be expressed in terms of the zero-field, zero-temperature gap Δ_0 by

$$\ln\left[\frac{\Delta_0^2}{16\mu\omega_{c2}(0)}\right]+2+\gamma=0 , \tag{27}$$

where γ is Euler's constant. Then, by expanding (19) in powers of Δ^2 and using (24), we find the gap parameter at zero temperature near $H_{c2}(0)$ is given by

$$\frac{(\pi\Delta)^2}{8\omega_c\mu}\left[1+\frac{4}{\pi}+\frac{\gamma}{\pi}+\frac{2}{\pi}\ln\left[\frac{\omega_D}{\mu\omega_c}\right]\right]=\ln\left[\frac{H_{c2}(0)}{H}\right] \tag{28}$$

and the gap is of order $\Delta^2\sim\mu\omega_{c2}(0)[1-H/H_{c2}(0)]$. ω_D is the Debye frequency. From a knowledge of the form of the Green's functions we can evaluate various properties and in the next section we consider the de Haas-van

Alphen oscillations in the magnetization in the mixed state.

III. de HAAS–van ALPHEN OSCILLATIONS

In this section we evaluate the oscillatory terms in the free energy in the weak- and strong-field limits. The oscillatory terms in the free energy are easily calculated by introducing the integrated density of states determined by Dingle[12]

$$Z(\varepsilon)=\int_{-\infty}^{\varepsilon} d\varepsilon' \int_{-\infty}^{\varepsilon'} d\varepsilon'' \frac{1}{2\pi l^2 L} \sum_{nk} \delta(\varepsilon''-\varepsilon_{nk}) . \quad (29)$$

The oscillatory part of the density of states is

$$Z_0=-\sum_{p=1}^{\infty} C_{pd} \cos\left[\frac{2\pi p}{\omega_c}(\varepsilon+\mu)-\alpha_d\right] , \quad (30)$$

where d is the dimensionality and the coefficients

$$\begin{aligned} C_{p2}&=(-1)^p \frac{\omega_c}{4\pi^3 p^2 l^3} , \quad \alpha_2=0 , \\ C_{p3}&=(-1)^p \frac{\omega_c}{8\pi^4 p^{5/2} l^3} , \quad \alpha_3=\frac{\pi}{4} . \end{aligned} \quad (31)$$

In deriving these results for $d=3$ it is assumed that the kinetic energy of an electron along the field is $k^2/2m$. In layered materials this may not be appropriate and it may be more realistic to take

$$\varepsilon_{nk}=(n+\tfrac{1}{2})\omega_c - t\cos(ka)-\mu \quad (32)$$

so that the hopping between the planes has a bandwidth $2t$. In this case

$$C_{p3}=(-1)^p \left[\frac{\omega_c}{4\pi^3 p^2 l^2 a}\right] J_0\left[\frac{2\pi p t}{\omega_c}\right] , \quad \alpha_3=0 , \quad (33)$$

where J_0 is a Bessel function. This leads to an increase in the magnitude of the oscillations by a factor l/a and also introduces extra Fourier components through the Bessel function. In the limit $t<\omega_c$ we cross over to the $d=2$ result.

We now use these results to evaluate the oscillating terms in the free energy. The interesting case is that of strong fields where $\omega_c > kT$ and Σ can be approximated by (24). At low temperature it is sufficient to calculate the energy per unit volume which is given by

$$E=\frac{2}{2\pi l^2 \beta L} \sum_{n,k} \sum_{\omega} [\varepsilon_{nk}+\Sigma_{nk}(\omega)] G(n,k,\omega)+\frac{\Delta^2}{\lambda} . \quad (34)$$

We will omit the last term in (34) and approximate Σ_{nk} in (24) by replacing $n\omega_c$ by $\mu+\varepsilon_{nk}$. This is exact in $d=2$ and a good approximation in $d=3$ when the bandwidth along the z direction is small. The reason for this approximation is to express (34) in terms of ε_{nk} so that we can use (30). Then

$$\Sigma_{nk}(\omega)\simeq \frac{\Delta^2}{\sqrt{4\pi\omega_c(\mu+\varepsilon_{nk})}} \left[-\frac{i\pi\omega}{|\omega|}+\left[\frac{\pi}{\mu\omega_c}\right]^{1/2}\varepsilon_{nk}\right] . \quad (35)$$

The last term in (35) to first order in ε_{nk} leads to an equal shift in the cyclotron frequency and Fermi energy and thus does not have an important effect and will be omitted. Then using (29) to evaluate the oscillatory terms in (34) we find

$$E_{\rm osc}=\sum_{p=1}^{\infty} C_{pd} e^{-\pi p/\omega_c \tau_v} \cos\left[\frac{2\pi p\mu}{\omega_c}(1+\delta)-\alpha_d\right] , \quad (36)$$

where $\delta=\pi\Delta^4/8\omega_c\mu^3$ is a small modulation of the frequency and τ_v is given by (25). The amplitude is reduced by the scattering of the excitations by the vortex lines. The amplitude in (35) is the same as that found by Maki[6] but the frequency shift is smaller.

IV. EFFECTS OF SPIN

The Zeeman energy of an electron in the field is $\pm h=\pm g\mu_B H$. In a normal metal the spin splitting of the Landau levels leads to two sets of oscillating terms in the density of states (30), i.e., C_{pd} is replaced by $C'_{pd}=C_{pd}\cos(2\pi hp/\omega_c)$. In a superconductor in the high-temperature case $kT>\sqrt{\mu\omega_c}$ the quasiparticle energies are $E_{nk\pm}=\sqrt{\varepsilon_{nk}^2+\Delta^2}\pm h$.

In strong fields $\sqrt{\mu\omega_c}>kT$ when the Zeeman energy included the self-energy (24) is replaced by

$$\Sigma_{nk\pm}(\omega)=\frac{\Delta^2}{\sqrt{4\pi n}\,\omega_c}\left[\frac{-\pi\omega}{|\omega|}+\left[\frac{\pi}{n}\right]^{1/2}\varepsilon_{nk\mp}\Big/\omega_c\right] , \quad (37)$$

where $\varepsilon_{nk\pm}=\varepsilon_{nk}\pm h$. When this result is substituted in the Green's function $G_{\pm}^{-1}=i\omega-\varepsilon_{nk\pm}-\Sigma_{nk\pm}$, we see that the quasiparticle energies have energies $\varepsilon_{nk}\pm\tilde{h}$, where $\tilde{h}=h(1-\Delta^2/\omega_c\mu)$. We thus obtain two sets of oscillatory terms in place of (36) with $\mu\pm\tilde{h}$ replacing μ.

V. CONCLUSION

In order to observe dHvA oscillations in the mixed state, a number of conditions have to be satisfied. Since type-II superconductors are generally alloys, an important condition which applies both in the normal and superconducting states is (1) $\omega_c\tau>1$, where τ is the quasiparticle collision time. The results we have given are for the clean case. In the dirty case, the sum in (36) should include an extra term $e^{-\pi p/\omega_c\tau}$ which leads to an additional decrease in amplitude. The second condition which applies both in the normal and superconducting states is (2) $kT<\omega_c$, because otherwise the Landau level structure is smeared out. The third condition involves the superconducting gap parameter.

From (36) we require $\omega_c\tau_v>1$ and from (25) this condition can be written

$$\frac{\Delta^2}{\omega_c\sqrt{\mu\omega_c}}<1 , \quad (38)$$

where Δ^2 is the gap parameter in the magnetic field. From (28), $\Delta^2\sim\mu\omega_c(1-H/H_{c2})$ near H_{c2} so that (38) becomes

$$\left[\frac{\mu}{\omega_c}\right]^{1/2}\left[1-\frac{H}{H_{c2}}\right]<1\ . \tag{39}$$

We can interpret this condition by putting $\sqrt{n_0}=H_{c2}/(H_{c2}-H)$. The number of observable quantized levels is of order n_0 and a dHvA effect is only observable if $n_0>n=\mu/\omega_c$. At low temperatures the zero field gap Δ_0 is of order $\sqrt{\mu\omega_{c2}}$ so that (39) can also be written

$$\frac{\Delta_0}{\omega_c}\left[1-\frac{H}{H_{c2}}\right]<1\ , \tag{40}$$

which is similar to the condition given by Maki.[6] For the high-T_c materials this appears to restrict the dHvA effects to a narrow region near H_{c2} or require an extremely large field. The situation is much more favorable in the low-temperature superconductors.

It has been suggested by a number of authors that, in the high-temperature superconductors, the Fermi-liquid behavior breaks down. This will lead to a smearing out of the Fermi surface and reduce the amplitude of the dHvA oscillations. (Temperature and scattering also smear the Fermi surface.) For example, the anomalous normal-state properties of the high-temperature superconductors can be understood with the hypothesis of a marginal Fermi liquid[13] in which the electron self-energy is frequency dependent

$$\Sigma(\omega)=i\lambda\omega\left[\ln\left[\frac{\omega_0}{|\omega|}\right]+\frac{i\pi}{2}\mathrm{sgn}\omega\right]\ , \tag{41}$$

where λ is a coupling constant and ω_0 some cut-off frequency. At low temperatures $kT<\omega_c$, the first term on the right hand side of (41) leads to a reduction in the amplitude of the dHvA oscillations with integer p by a factor $[1+\lambda\log(2\pi p\omega_0/\omega_c)]^{-1}$ if $\omega_0>\omega_c$. The second term on the right in (41) affects the phase of the oscillations.

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APPENDIX

In this appendix we show that the order-parameter correlation function (8) can be obtained from the Ginzburg-Landau (GL) equation and that the average current is zero. The GL equation for the gap parameter is[7]

$$\frac{1}{4m}\left[\nabla_1-\frac{2ie}{c}\mathbf{A}_1\right]^2\Delta(r_1)+\frac{1}{\beta_L}\frac{T_c-T}{T_c}\Delta(r_1)-B|\Delta(r_1)|^2\Delta(r_1)=0\ , \tag{A1}$$

where $\beta_L=[7\zeta(3)/6(\pi T_c)^2]\mu$ and $B=\frac{3}{4}\mu$.

We multiply this by $\Delta^*(r_2)$ and average supposing that Δ is a Gaussian random variable which allows us to factor the quartic term. This leads to an equation for the correlation function $C(r_1r_2)=\langle\Delta^*(r_2)\Delta(r_1)\rangle$ which is

$$\left[\frac{1}{4m}\left[\nabla_1-\frac{2ie}{C}A_1\right]^2+\frac{1}{\beta_L}\frac{T_c-T}{T_c}-2B\Delta^2\right]C(r_1r_2)=0\ , \tag{A2}$$

where $\Delta^2=C(r_1r)$ a constant. The solution is easily shown to be

$$C(r_1r_2)=\Delta^2e^{i\phi(21)}e^{-(r_1-r_2)^2/2l^2} \tag{A3}$$

in agreement with (8) with Δ^2 being determined by (25).

The current is given by

$$\mathbf{J}(r)\sim\left[\frac{ie}{m}(\nabla_2-\nabla_1)\Delta^*(2)\Delta(1)\right]_{r_1=r_2=r}-\frac{4e^2}{m}|\Delta(r)|^2\mathbf{A}(r)\ . \tag{A4}$$

When this is averaged supposing $\mathbf{A}$ is due to a uniform field $\langle\mathbf{J}\rangle=0$. This justifies the neglect of variations in the field and the approximation of $\mathbf{A}$ by the vector potential of a uniform field.

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