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Dynamics of the normal to vortex-glass transition: Mean-field theory and fluctuations

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A mean-field theory is developed for the recently proposed normal to superconducting vortex-glass transition. Using techniques developed to study the critical dynamics of spin glasses, we calculate the mean-field vortex-glass phase boundary $T_g(H)$, and find that the dynamic critical exponent $z=4$ in mean-field theory. In addition, we find that the fluctuation conductivity in the vicinity of the transition has the form $\sigma \sim \xi^{(2-d+z)}$, in agreement with recent scaling theories of the transition. The extension of these results beyond the mean-field regime is also discussed.

Understanding the dynamic properties of the mixed state in type-II superconductors is theoretically challenging due to the competition between collective intervortex interactions and vortex pinning. As discussed by Larkin and Ovchinnikov,[1] pinning destroys the translational long-range order of the Abrikosov flux lattice. Recently, one of us (M.P.A.F.) argued that pinning and collective effects conspire to produce a vortex-glass phase at sufficiently low temperatures.[2,3] In this phase the vortices are frozen into an equilibrium configuration characterized by a type of "spin-glass" order, rather than the translational long-range order of the flux lattice.[4] As a result, the linear resistance $R_L \equiv \lim_{I\to 0} V/I$ is identically *zero* in this phase, in contradistinction to the Anderson-Kim model of flux creep,[5] which predicts that $R_L \neq 0$ throughout the entire mixed state. Tentative experimental evidence for the vortex-glass phase in the high-temperature superconductor Ya-Ba-Cu-O has been given by Koch *et al.*,[6] and Gammel, Schneemeyer, and Bishop,[7] who have used a recently developed scaling theory of the conductivity[3] to interpret the nonlinear *I-V* characteristics in terms of the vortex-glass model. Theoretical evidence for the transition consists of the existence of a vortex-glass phase in a two-dimensional toy model of a vortex glass,[2,3] and numerical simulations on a simplified three dimensional model of a vortex glass.[8]

In this paper we consider a mean-field theory for the dynamics of the normal to superconducting vortex-glass transition. This allows us to calculate the mean-field phase boundary for a realistic model of a vortex glass in three dimensions. In addition, we determine the mean-field static and dynamic critical exponents for the transition. Gaussian fluctuations about the mean-field theory lead to a linear conductivity which diverges at the vortex-glass phase boundary, in agreement with expectations based on the scaling theory.[3] Finally, we construct an effective field theory for the vortex-glass order-parameter fields, which allows for the calculation of the dynamic critical exponent using renormalization-group methods. Our work draws heavily on studies of the critical dynamics of spin glasses.[9]

The Ginzburg-Landau free-energy functional for a superconductor in an external magnetic field $\mathbf{H}=\nabla\times\mathbf{A}$ is

$$\mathcal{H}=\int d\mathbf{r}\left\{\frac{\hbar^2}{2m}|[\nabla-i(2e/\hbar c)\mathbf{A}]\psi|^2+[a+a_1(\mathbf{r})]|\psi|^2+\frac{b}{2}|\psi|^4-h^*\psi-h\psi^*\right\}, \tag{1}$$

where m is the mass of a Cooper pair (assumed to be isotropic), h and h^* are conjugate fields which are introduced to generate response and correlation functions, and where the terms for the magnetic-field energy have been dropped. For simplicity we ignore fluctuations in the electromagnetic field, and we are thus modeling an extreme type-II superconductor (with an infinite penetration length). The quenched disorder has been incorporated by defining a random T_c $[a=a_0(T/T_0-1)]$, which is assumed to have Gaussian white-noise correlations, so that $\overline{a_1(\mathbf{r})}=0$, $\overline{a_1(\mathbf{r})a_1(\mathbf{r}')}=4\Delta\delta(\mathbf{r}-\mathbf{r}')$, where the overbar denotes an ensemble average over disorder. This spatial variation in T_c simulates vortex pinning since regions of the sample which have a locally lower T_c, and, consequently, a higher free energy, will tend to attract the normal cores of the vortices. We will assume relaxational dynamics for the nonconserved order parameter (model A),[10]

$$\Gamma_0^{-1}\partial_t\psi(\mathbf{r},t)=-\frac{\delta\mathcal{H}}{\delta\psi^*(\mathbf{r},t)}+\zeta(\mathbf{r},t), \tag{2a}$$

$$\langle\zeta^*(\mathbf{r},t)\zeta(\mathbf{r}',t')\rangle=2k_BT\Gamma_0^{-1}\delta(\mathbf{r}-\mathbf{r}')\delta(t-t'), \tag{2b}$$

where the angular brackets denote a noise average. Equations (1) and (2) define our model. The response and correlation functions for the order parameter ψ are

$$R(\mathbf{r},t;\mathbf{r}',t')=\frac{\delta\langle\psi(\mathbf{r},t)\rangle}{\delta h^*(\mathbf{r}',t')}, \tag{3}$$

$$C(\mathbf{r},t;\mathbf{r}',t')=\langle\psi(\mathbf{r},t)\psi^*(\mathbf{r}',t')\rangle. \tag{4}$$

These are the response and correlation functions for a particular realization of the disorder; the disorder averaged response and correlation functions will be denoted by $\bar{R}$ and $\bar{C}$. We also need to define an appropriate order parameter for the vortex-glass phase; this is the Edwards-Anderson (EA) order parameter,[4] defined by $q_{\rm EA}=\overline{|\langle\psi(\mathbf{r})\rangle|^2}$. In order to study spatial and temporal fluctuations, following Sompolinsky and Zippelius[9] we introduce the vortex-glass propagators

$$G^{(1)}(\mathbf{k},\omega,\omega')=\int d(\mathbf{r}-\mathbf{r}')\times e^{i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')}\overline{R(\mathbf{r},\mathbf{r}';\omega)R^*(\mathbf{r},\mathbf{r}';\omega')}, \tag{5}$$

$$G^{(3)}(\mathbf{k},\omega,\omega')=\int d(\mathbf{r}-\mathbf{r}')\times e^{i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')}\overline{C(\mathbf{r},\mathbf{r}';\omega)C^*(\mathbf{r},\mathbf{r}';\omega')}. \tag{6}$$

Using the fluctuation-dissipation theorem, $C(\mathbf{r},\mathbf{r}';\omega)=2k_BT\,\mathrm{Im}R(\mathbf{r},\mathbf{r}',\omega)/\omega$, we may express $G^{(3)}$ in terms of $G^{(1)}$:

$$G^{(3)}(\mathbf{k},\omega,\omega')=\frac{(k_BT)^2}{\omega\omega'}[G^{(1)}(\mathbf{k},\omega,-\omega')+G^{(1)}(\mathbf{k},-\omega,\omega')-G^{(1)}(\mathbf{k},\omega,\omega')-G^{(1)}(\mathbf{k},-\omega,-\omega')]. \tag{7}$$

We also introduce a dynamic vortex-glass susceptibility, $\chi_{\rm VG}(\mathbf{k},\omega)=(k_BT)^2G^{(1)}(\mathbf{k},\omega,\omega)$. The static vortex-glass susceptibility, $\chi_{\rm VG}(\mathbf{k},0)$, is the Fourier transform of the equilibrium vortex-glass correlation function[3] $G_{\rm VG}(\mathbf{r}-\mathbf{r}')=\overline{|\langle\psi(\mathbf{r})\psi^*(\mathbf{r}')\rangle|^2}$. This correlation function will approach a nonzero constant as $|\mathbf{r}-\mathbf{r}'|\to\infty$ in the vortex-glass phase, and in the vortex-fluid phase it will fall off exponentially with the vortex-glass correlation length, $\xi_{\rm VG}$.[3] As a result, the long wavelength vortex-glass susceptibility, $\chi_{\rm VG}(0,0)$, should diverge at the vortex-glass transition. The propagator $G^{(3)}$ will be important in our discussion of the fluctuation conductivity as the vortex-glass phase boundary is approached from above (see below).

We first sketch the derivation of the vortex-glass susceptibility.[11] First, we rewrite the equation of motion, Eq. (2), as a functional integral, and introduce a set of auxiliary fields to facilitate the calculation of the response and correlation functions. This functional is then averaged over the Gaussian disorder, thereby generating a spatially uniform but temporally nonlocal quartic interaction.[9,12] We decouple the disorder-induced quartic term by introducing vortex-glass fields $Q_{\alpha\beta}$, as in Sompolinsky and Zippelius,[9] and we treat the quartic interaction term $|\psi|^4$ using a self-consistent Hartree approximation. Evaluating the resulting functional by the saddle-point method leads to the following self-consistent equations for the disorder-averaged order-parameter response and correlation functions (for $T>T_g$):

$$\left[-i\Gamma_0^{-1}\omega-\Delta\bar{R}(\mathbf{r},\mathbf{r};\omega)-\frac{\hbar^2}{2m}\left(\nabla-i\frac{2e}{\hbar c}\mathbf{A}\right)^2+a+k_BTb\bar{R}(\mathbf{r},\mathbf{r};0)\right]\bar{R}(\mathbf{r},\mathbf{r}';\omega)=\delta(\mathbf{r}-\mathbf{r}'), \tag{8}$$

$$\bar{C}(\mathbf{r},\mathbf{r}';\omega)=2k_BT\Gamma_0^{-1}\int d\mathbf{r}_1\bar{R}^*(\mathbf{r},\mathbf{r}_1;\omega)\bar{R}(\mathbf{r}',\mathbf{r}_1;\omega)+\Delta\int d\mathbf{r}_1\bar{R}^*(\mathbf{r},\mathbf{r}_1;\omega)\bar{R}(\mathbf{r}',\mathbf{r}_1;\omega)\bar{C}(\mathbf{r}_1,\mathbf{r}_1;\omega). \tag{9}$$

We note that (1) these equations are equivalent to the "ladder summation" approximation employed by Ma and Rudnick[13] to study the zero-field random T_c model and (2) in the absence of disorder ($\Delta=0$) these equations reduce to the Hartree approximation for a superconductor in a magnetic field, which has been used by several authors to study the transport properties[14] of a superconductor in a magnetic field in the absence of pinning. The vortex-glass propagator $G^{(1)}$ may be calculated in a similar fashion, by summing a ladder of disorder-induced vertex corrections, which is equivalent to keeping Gaussian fluctuations about the saddle point:

$$G^{(1)}(\mathbf{k},\omega,\omega')=I(\mathbf{k},\omega,\omega')[1-\Delta I(\mathbf{k},\omega,\omega')]^{-1}, \tag{10}$$

where

$$I(\mathbf{k},\omega,\omega')=\int d(\mathbf{r}-\mathbf{r}')e^{i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')}\times\bar{R}(\mathbf{r},\mathbf{r}';\omega)\bar{R}^*(\mathbf{r},\mathbf{r}';\omega'). \tag{11}$$

Equations (8)–(11) are the main results of our mean-field approach. If desired, this approach may be systematized into a controlled large N expansion (N ψ fields), with the saddle-point solution as the exact $N=\infty$ solution and the Gaussian fluctuations giving $O(1/N)$ corrections.

The phase boundary. We first consider the solution of the saddle-point equations for $k=\omega=0$, in order to determine the mean-field phase boundary. First, note from Eqs. (10) and (11) that the vortex-glass susceptibility $\chi_{\rm VG}(0,0)=G^{(1)}(0,0,0)$ diverges when $\Delta I(0,0,0)=1$. Next, we set $\omega=0$ in the self-consistent equation for the response function, Eq. (8), and solve the resulting equation by expanding the response function in a basis consisting of Landau levels in the x-y plane and plane waves along the z axis. The result is

$$\tilde{a}=a+(k_BTb-\Delta)\frac{m\omega_0}{2\pi\hbar}\left(\frac{m}{2\hbar^2}\right)^{1/2}\times\sum_{n=0}^{N}[\tilde{a}+\hbar\omega_0(n+\tfrac{1}{2})]^{-1/2}, \tag{12}$$

where N is a cutoff, $\tilde{a}$ is the renormalized value of a, and $\omega_0=2eH/mc$. Stability considerations require that $k_BTb>\Delta$. The vortex-glass susceptibility will diverge at a certain value $\tilde{a}_g$ of $\tilde{a}$ given by

$$1=\frac{\Delta}{2}\frac{m\omega_0}{2\pi\hbar}\left(\frac{m}{2\hbar^2}\right)^{1/2}\sum_{n=0}^{\infty}[\tilde{a}_g+\hbar\omega_0(n+\tfrac{1}{2})]^{-3/2}. \quad (13)$$

We must solve Eqs. (12) and (13) simultaneously to determine $T_g(H)$. This is rather difficult, in general; however, in the limit of high magnetic fields or weak disorder it can be shown that the solution is dominated by the lowest Landau level ($n=0$).[11] Then by evaluating Eq. (12) at $T=T_g(H)$, we may eliminate $\tilde{a}_g$ from Eqs. (12) and (13) to obtain the following expression for $T_g(H)$ [recall that $a=a_0(T/T_0-1)$]:

$$T_g(H)=\frac{T_{c2}(H)+3T_0(\delta h)^{2/3}}{1+(2k_BT_0b/\Delta)(\delta h)^{2/3}}, \quad (14)$$

where $T_{c2}(H)=T_0(1-\hbar\omega_0/2a_0)$ is the mean-field Abrikosov flux lattice transition temperature. We have introduced the dimensionless parameters $h=H/H_{c2}(0)$, $\delta=(m^2\xi_0/2\pi\hbar^4)\Delta$, with $H_{c2}(0)=\hbar/2e\xi^2(0)$ the zero-temperature critical field and $\xi(0)=\hbar/(2ma_0)^{1/2}$ the zero-temperature coherence length. Equation (14) is correct in the limit $\delta^{1/3}\ll h^{1/6}$.[11] Under some circumstances we find that $T_{c2}-T_g\propto h^{2/3}$, reminiscent the vortex-glass phase boundary determined by Koch *et al.*,[6] but for other choices of the parameters the phase boundary has a curvature which is in disagreement with the experiments (i.e., $T_g>T_{c2}$).

Mean-field critical exponents. We now turn to the critical dynamics in mean-field theory, in order to determine the structure of the dynamic vortex-glass susceptibility. Since we are primarily interested in the long-wavelength, low-frequency response near the transition, we can expand Eq. (11) for small k, ω, and $t\equiv(T-T_g)/T_g$, to obtain the vortex-glass susceptibility [for $T>T_g(H)$]

$$\chi_{VG}(\mathbf{k},\omega)\approx A[k_\perp^2+\gamma^2k_z^2+\xi_{VG}^{-2}-i\omega/\Gamma(\omega)]^{-1} \quad (15)$$

where

$$-i\omega/\Gamma(\omega)=-\xi_{VG}^{-2}[1-(1-i\omega/\Omega)^{1/2}], \quad (16)$$

where A is a constant, γ is an anisotropy parameter, $\xi_{VG}\propto t^{-1/2}$ is the vortex-glass correlation length, and $\Omega\propto\xi_{VG}^{-4}$. By a suitable rescaling of the momenta, we see that the vortex-glass transition is *isotropic*. Therefore, near the vortex-glass transition, the susceptibility assumes the scaling form,

$$\chi_{VG}(\mathbf{k},\omega)=A\xi_{VG}^{2-\eta}f(k\xi_{VG},\omega/\Omega), \quad (17)$$

where $\xi_{VG}\sim|T-T_{VG}(H)|^{-\nu}$, with $\nu=\frac{1}{2}$, $\eta=0$, and $\Omega\propto\xi_{VG}^{-z}$, with $z=4$. These are the same mean-field critical exponents which have previously obtained for the critical dynamics of the Ising spin glass.[4] Using Eq. (7), we see that the propagator $G^{(3)}$ also has a scaling form,

$$G^{(3)}(\mathbf{k},\omega,\omega')=A\xi^2\Omega^{-2}g^{(3)}(k\xi_{VG},\omega/\Omega,\omega'/\Omega), \quad (18)$$

where $g^{(3)}(\bar{k},\bar{\omega},\bar{\omega}')$ is a scaling function.

The fluctuation conductivity. What measurement will probe the vortex-glass correlations discussed above? Since the equilibrium vortex-glass correlation function involves a disorder average of four order-parameter fields $\psi(\mathbf{r})$, one might expect that the vortex-glass correlations could be observed by measuring a suitable four-point function. One experimentally accessible four-point function is the conductivity; one implication of the vortex-glass instability is that the conductivity diverges at $T_{VG}(H)$. To show this, we first relate the real part of the conductivity to the current-current correlation function via the Kubo formula:

$$\sigma'_{\mu\nu}(\omega)=\frac{1}{2k_BTV}\int dt\int d\mathbf{r}_1d\mathbf{r}_2e^{-i\omega t}\overline{\langle j_\mu(\mathbf{r}_1,t)j_\nu(\mathbf{r}_2,0)\rangle}$$
$$=\frac{k_BT}{2}\frac{\hbar e}{mi}\int\frac{d\omega'}{2\pi}\frac{1}{\omega_+\omega_-}\int d\mathbf{r}_1(\tilde{\partial}_{1\mu}-\tilde{\partial}^*_{2\mu})[-\Pi_\nu(\mathbf{r}_1,\mathbf{r}_2;\omega_+,\omega_-)+\Pi_\nu(\mathbf{r}_1,\mathbf{r}_2;\omega_+,-\omega_-)$$
$$+\Pi_\nu(\mathbf{r}_1,\mathbf{r}_2;-\omega_+,\omega_-)-\Pi_\nu(\mathbf{r}_1,\mathbf{r}_2;-\omega_+,-\omega_-)]|_{2=1}, \quad (19)$$

where $\Pi(\mathbf{r}_1,\mathbf{r}_2;\omega_1,\omega_2)$ is the vector vertex function, $\omega_\pm\equiv\omega'\pm\omega/2$, V is the system volume, and $\tilde{\partial}_\mu\equiv\partial_\mu-i(2e/\hbar)A_\mu$. To evaluate the vertex function we use the ladder approximation, and keep only the noncrossing impurity ladders. In this regard, the calculation of the fluctuation conductivity is similar in spirit to the calculation of the conductivity for noninteracting electrons in a random potential.[15] The ladder approximation leads to the following integral equation for the vertex function:

$$\Pi_\nu(\mathbf{r},\mathbf{r}';\omega_1,\omega_2)=\frac{\hbar e}{mi}\int d\mathbf{r}_1(\tilde{\partial}_{1\nu}-\tilde{\partial}^*_{2\nu})\bar{R}(\mathbf{r},\mathbf{r}_1;\omega_1)\bar{R}^*(\mathbf{r}',\mathbf{r}_2;\omega_1)|_{2=1}+\Delta\int d\mathbf{r}_1\bar{R}(\mathbf{r},\mathbf{r}_1;\omega_1)\bar{R}^*(\mathbf{r}',\mathbf{r}_1;\omega_2)\Pi_\nu(\mathbf{r}_1,\mathbf{r}_1;\omega_1,\omega_2). \quad (20)$$

To solve Eq. (20) we first set $\mathbf{r}'=\mathbf{r}$; the kernel in the resulting integral equation is only a function of $\mathbf{r}-\mathbf{r}_1$, and the integral equation can be solved using Fourier transforms. The resulting expression for the vertex function is then substituted into the conductivity, Eq. (19). Near the vortex-glass transition temperature the conductivity simplifies, and we find

$$\sigma'_{\mu\nu}(\omega)=B_{\mu\nu}\int\frac{d\omega'}{2\pi}\int\frac{d^dk}{(2\pi)^d}G^{(3)}(k,\omega_+,\omega_-)=\xi_{VG}^{2-d+z}S'_{\mu\nu}(\omega\xi_{VG}^z), \quad (21)$$

where the second line is obtained by using the scaling form of $G^{(3)}$ in Eq. (18), and an appropriate rescaling of the momentum and frequency variables. In Eq. (21) $B_{\mu\nu}$ is a (magnetic-field dependent) constant, and $S'_{\mu\nu}(x)$ is a scaling function. Thus, the linear conductivity assumes the scaling form recently proposed by Fisher, Fisher, and Huse.[3] At the transition, the scaling function is of the form $S'(x) \sim x^{-(2-d+z)/z}$. In addition, causality dictates that at the transition the real and imaginary parts be related by $\sigma''(\omega)/\sigma'(\omega) = \tan[\pi(2-d+z)/2z]$, so that the phase angle between the real and imaginary parts of the conductivity is universal.[16] We have explicitly confirmed these expectations within our saddle-point approximation. Recent measurements of the ac impedance of Y-Ba-Cu-O thin films appear to support these predictions.[17] Note that in the absence of any pinning ($\Delta = 0$), the conductivity does not diverge due to flux flow.[14]

Critical fluctuations. Finally, we study the fluctuations about the saddle-point solution, in order to calculate the corrections to our saddle-point approximation. By expanding the functional integral about its saddle point, we find that the first nonlinear term in the effective theory is of order Q^3. Power counting arguments then indicate that $d=6$ is the upper critical dimension for the transition, which is the same as for conventional spin glasses.[4] By studying the critical dynamics for $d<6$ in an $\epsilon = 6-d$ expansion, we find that the vortex-glass kinetic coefficient is not renormalized to first order in ϵ, in agreement with the results of Zippelius.[9] Therefore van Hove scaling, $z=2(2-\eta)$, is correct to $O(\epsilon)$; however, this relationship may break down at higher order in ϵ.[18] A replica calculation[19] gives $\eta = -\epsilon/6 + O(\epsilon^2)$, indicating a dynamic exponent $z \approx 5$ in three dimensions, although the extrapolation to $\epsilon = 3$ should be only viewed as a guide to the magnitude of z. The precise value of z in $d=3$ is best determined by detailed numerical simulation.[8]

To summarize, we have introduced a random T_c model to study the effects of vortex pinning in the mixed state of type-II superconductors. We have shown that a mean-field approximation which has been employed in the study of spin glasses causes an appropriately defined vortex-glass susceptibility to diverge at the vortex-glass phase boundary $T_g(H)$. The mean-field critical exponents are the same as for Ising spin glasses, $\nu = \frac{1}{2}$ and $z=4$. We have also calculated the electrical conductivity for $T > T_g(H)$, and have shown that it diverges with an exponent which is in agreement with a recently developed scaling theory of the conductivity.[3] This is in contrast with the conductivity in the absence of any vortex pinning, which is always finite due to flux flow.[14] Finally, in the critical regime we find that the dynamic critical exponent $z=2(2-\eta)$, in agreement with the results of Zippelius[9] for the Ising spin glass.

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[1] A. I. Larkin and Yu. N. Ovchinnikov, J. Low Temp. Phys. **34**, 409 (1979).

[2] M. P. A. Fisher, Phys. Rev. Lett. **62**, 1415 (1989).

[3] D. S. Fisher, M. P. A. Fisher, and D. A. Huse, Phys. Rev. B **43**, 130 (1991).

[4] For a review of spin glasses, see K. Binder and A. P. Young, Rev. Mod. Phys. **58**, 801 (1986).

[5] P. W. Anderson and Y. B. Kim, Rev. Mod. Phys. **36**, 39 (1964).

[6] R. H. Koch, V. Foglietti, W. J. Gallagher, G. Koren, A. Gupta, and M. P. A. Fisher, Phys. Rev. Lett. **63**, 1511 (1989); R. H. Koch, V. Foglietti, and M. P. A. Fisher, Phys. Rev. Lett. **64**, 2586 (1990).

[7] P. L. Gammel, L. F. Schneemeyer, and D. J. Bishop, Phys. Rev. Lett. **66**, 953 (1991).

[8] D. A. Huse and H. S. Seung, Phys. Rev. B **42**, 1059 (1990); J. D. Reger, T. A. Tokuyasu, A. P. Young, and M. P. A. Fisher, *ibid.* **44**, 7147 (1991); M. J. P. Gingras (unpublished); K. H. Lee and D. Stroud, Phys. Rev. B **44**, 9780 (1991).

[9] H. Sompolinsky and A. Zippelius, Phys. Rev. B **25**, 6860 (1982); Phys. Rev. Lett. **50**, 1297 (1983); A. Zippelius, Phys. Rev. B **29**, 2717 (1984).

[10] P. C. Hohenberg and B. I. Halperin, Rev. Mod. Phys. **49**, 435 (1977).

[11] Details of the calculation will be provided in a future publication; A. T. Dorsey and M. P. A. Fisher (unpublished).

[12] C. De Dominicis, Phys. Rev. B **18**, 4913 (1978).

[13] S. k. Ma and J. Rudnick, Phys. Rev. Lett. **40**, 589 (1978). The ladder approximation used in this paper incorrectly predicts the existence of a spin-glass phase for the zero-field random T_c model, as shown by D. Sherrington, Phys. Rev. B **22**, 5553 (1980). Here, however, the external magnetic field modulates the order parameter and provides a source of frustration which is absent in the zero-field case. This fact, along with the existing numerical evidence for a vortex-glass transition (see Ref. 8), would indicate that the ladder approximation is at least qualitatively correct for the vortex-glass transition.

[14] R. Ikeda, T. Ohmi, and T. Tsuneto, J. Phys. Soc. Jpn. **58**, 1377 (1989); R. Ikeda, T. Ohmi, and T. Tsuneto (unpublished); S. Ullah and A. T. Dorsey, Phys. Rev. Lett. **65**, 2066 (1990); S. Ullah and A. T. Dorsey, Phys. Rev. B **44**, 262 (1991).

[15] See, for instance, A. A. Abrikosov, L. P. Gorkov, and I. E. Dzyaloshinski, *Methods of Quantum Field Theory in Statistical Physics* (Prentice-Hall, Englewood Cliffs, NJ, 1963), Sect. 39.

[16] A. T. Dorsey, Phys. Rev. B **43**, 7575 (1991).

[17] H. K. Olsson, R. H. Koch, W. Eidelloth, and R. P. Robertazzi, Phys. Rev. Lett. **66**, 2661 (1991).

[18] A breakdown of the van Hove result is suggested by Monte Carlo simulations of the $d=3$ Ising spin glass by A. T. Ogielski, Phys. Rev. B **32**, 7384 (1985); and D. A. Huse, *ibid.* **40**, 304 (1989), which indicate that $(2-\eta)/z \approx 0.39$.

[19] A. Houghton and M. A. Moore, Phys. Rev. B **38**, 5045 (1988); M. A. Moore and S. Murphy, *ibid.* **42**, 2687 (1990).