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Critical exponents in $Cd_{1-x}Mn_xTe$ spin glass

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We argue that the conventional dynamic scaling expressions in spin glasses are relevant only above the temperature at which a maximum of the imaginary part of the ac magnetic susceptibility is observed. Then different methods to derive the critical exponent $z\nu$ in $Cd_{0.6}Mn_{0.4}Te$ independently from the spin-glass temperature give the same result $z\nu \approx 10$. We also discuss the values of the critical scaling exponents β and γ.

I. INTRODUCTION

The $Cd_{1-x}Mn_xTe$ semimagnetic semiconductor is the archetype of Heisenberg frustrated disordered antiferromagnets, for which the nature of the spin freezing has been subject to debate in the past. In a recent paper,[1] however, Geschwind *et al.* have shown that critical dynamics in $Cd_{1-x}Mn_xTe$ are inconsistent with the activated scaling they had previously suggested,[2] and now agree with the prior works[3-5] that the conventional power-law scaling applies to this material. As a consequence, there is now an overall agreement concerning the fundamental property that $Cd_{1-x}Mn_xTe$ and related compounds undergo a spin-glass phase transition at finite temperature T_c, characterized by critical exponents.

The numerical values of the dynamic and static critical exponents $z\nu$, and β, γ, however, are still subject to controversies; they have been found larger by Geschwind, Huse, and Delvin[1,6] than by other authors.[7] Still their accurate knowledge is needed, for example, to investigate the existence of universality classes among spin glasses. It is thus desirable to determine the origin of the scattering in the values reported so far, and to define how the scaling analysis of the data must be achieved to obtain reliable values for the critical exponents. In the present work, we emphasize and discuss basic features which must be kept in mind in any attempt of scaling analysis.[8] The paper is divided as follows: in Sec. II, we suggest that the dynamic scaling holds true only at temperatures $T > T_p$, where T_p stands for the temperature of the peak of the imaginary part χ'' of the ac magnetic susceptibility. In Sec. III, we apply to $Cd_{1-x}Mn_xTe$ ($x \approx 0.4$) a method of independently determining $z\nu$ and T_c, which is different and complementary to that of Ref. 1.

II. DYNAMIC SCALING

A. Derivation of the basic equations

Let us explicitly show how the dynamic scaling expressions are derived for the magnetic susceptibility $\chi = \chi' - i\chi''$. The magnetization $\langle M \rangle$ resulting from a magnetic-field perturbation h applied to the system is $\langle M \rangle = \mathrm{tr}[\rho(t)M]$, where $\rho(t)$ is the density operator in the Heisenberg representation. The system is supposed to be at thermal equilibrium before introduction of the perturbation, i.e.,

$$\rho(-\infty) = e^{-\beta H_0}, \tag{1}$$

with H_0 the unperturbed Hamiltonian. At observation time t, the perturbation will induce a departure from thermal equilibrium by $\rho(t) - \rho(-\infty)$ in a power series of h. When truncated to the first-order term in h, this series reduces to[9]

$$\rho(t) = e^{-\beta H_0} + \frac{i}{\hbar}\int_{-\infty}^{t} [M(t')h(t'), e^{-\beta H_0}]dt'. \tag{2}$$

$M(t')$ stands for the magnetization operator expressed in the interaction representation, and the square bracket is the commutator. This approximation, however, explicitly supposes that the departure from thermal equilibrium is small at the observation time t. Only when this condition is fulfilled, the substitution of Eq. (2) into the expression of M is allowed, which is needed to obtain the expression of χ under the form explicited by Eqs. (4) and (5) of Ref. 8. Then, in this prior work, we derive the scaling relations on χ', χ'' [Eqs. (9) and (10) in Ref. 8] which can be cast in the form of Ref. 1:

$$\chi''(\omega)T/\omega^{\beta/z\nu} = f(t/\omega^{1/z\nu}), \tag{3}$$

where t is the reduced temperature $(T-T_c)/T$. The critical exponents $z\nu$ and β have their usual meaning.

B. Range of validity

Let $g(\tau)$ be the distribution of relaxation times in the system, characterized by a width τ_w. The time-Fourier transform of $g(\tau)$ will be a distribution $g(\omega)$ of typical width τ_w^{-1}. As in the previous section, the scaling law on $\chi''(\omega)$ (and the same holds true for χ') is valid only if the system is close to equilibrium at the observation time t. This condition is fulfilled only if the system is observed at long enough time, typically $t > \tau_w$. To make contact with experiments, let us consider the situation where the observation time $t = 1/\omega$ is fixed for every temperature. Then, the condition $t > \tau_w$ can be written $T > T_p$, with T_p the temperature at which $\omega = \tau_w^{-1}$. The thermal dependence of χ'' is related to that of the spin autocorrelation function $q(\omega)$ by the Kubo theorem:

$$\chi''(\omega) = \frac{\omega}{2k_B T} q(\omega) . \quad (4)$$

This expression can be viewed as the Fourier transform of Eqs. (4) and (8) in Ref. 8. Since the temperature dependence of $q(\omega)$ is critical at the finite temperature T_c, we can neglect the T^{-1} dependence of the prefactor in Eq. (4), and assimilate the temperature dependence of $\chi''(\omega)$ to that of $q(\omega)$ near T_c. In the limit $\omega < \tau_w^{-1}$, we can make the approximation

$$q(\omega) \approx q(0) \text{ for } T > T_p . \quad (5)$$

As T decreases and approaches T_c from above, τ_w increases (and eventually diverges like $t^{-z\nu}$ at T_c), so the width τ_w^{-1} of the $g(\omega)$ or $q(\omega)$ spectra decreases. Note, however, that the sum rule

$$\int_{-\infty}^{+\infty} q(\omega')d\omega' = \langle S^2(t=0)\rangle = S(S+1) \quad (6)$$

implies that the integral of the $q(\omega')$ peak is a constant, whatever the temperature is. Therefore, the shrinking of the $q(\omega')$ peak, as T decreases, results in an increase of $q(0)$. Taking Eqs. (4) and (5) into account, it follows that $\chi''(\omega)$ increases upon cooling for $T > T_p$. A decrease of the temperature below T_p results in an additional increase of τ_w; hence, $\omega > \tau_w^{-1}$. We thus conclude that the temperature T_p is nearly that of the maximum of $\chi''(\omega)$. This demonstration is not rigorous since the decrease of $\chi''(\omega)$ upon cooling at $T < T_p$ has been derived within the linear-response formalism which is precisely not valid in this temperature range. It is believed, however, that the error on χ'' introduced by the nonlinear effects is quantitative [i.e., generates deviations with respect to Eq. (3) crucial for any scaling attempt], but not qualitative. Indeed, the fact that the experimental curve $\chi''(\omega)$ actually goes through a maximum at temperature $T_p(\omega)$ supports our model.

Note our conclusion disagrees with the claim of Ref. 1 that the scaling plot according to Eq. (3) may be extended even to $t < 0$. The condition $T > T_p$ is indeed more stringent than $t > 0$, except in the static limit $\omega \rightarrow 0$.

C. Discussion

Various attempts of dynamic scaling have been performed for semimagnetic materials, such as $Cd_{1-x}Mn_xTe$, $Hg_{1-x}Mn_xTe$.[3–5] In Refs. 4 and 5 the relation $\phi = \chi''/\chi' = \omega\tau_{av} \propto \omega t^{-z\nu}$, only valid in the limit of a small value of ϕ,[10] has been used to derive $z\nu$ from a straight-line plot of $\ln(t)$ vs $\ln(\omega)$ at fixed ϕ. The criterion of a small ϕ, i.e., that ϕ/ω does not depend on ω for any given temperature T, has been shown to be valid in our scaling $(\phi \approx 10^{-3})$.[5,11] Note that only data points corresponding to $T > T_p$ have been considered in the scaling analyses of Refs. 3–5, which all lead to $z\nu = 9.5 \pm 0.5$. Finally, a new method has been reported in Ref. 1 to determine $z\nu$ independently from T_c. When only data points above T_p are taken into account (see curve $0.3R$–$0.8R$ in notations of Ref. 1), the result displayed in Fig. 2(b) of Ref. 1 gives $z\nu = 10$, in agreement with prior works.

Unfortunately, all the other scaling attempts in Ref. 1 involve data points below T_p and are consequently irrelevant. In fact, the inclusion of data points at temperatures smaller and smaller below T_p affects the quality of the scaling and leads to larger and larger overestimations of $z\nu$, well evidenced in Fig. 1(b) of Ref. 1. Inclusion of data points slightly below T_p ($0.92L$ in notations of Ref. 1) is sufficient to raise $z\nu$ to 11.4, whereas $z\nu$ reaches 12.3 if more data points below T_p ($0.8L$) are also introduced. These deviations with respect to $z\nu = 10$ are indicative of the errors that can be done on $z\nu$ by including, in the scaling analysis, data points for which Eq. (3) does not apply.

III. RESULTS ON $Cd_{0.6}Mn_{0.4}Te$

We have already pointed out that it is difficult to determine unambiguously both T_c and the critical exponents from the conventional dynamic scaling plot of χ'' (Ref. 4) or from the static scaling plot of the nonlinear magnetization M_{nl}.[12] It is thus necessary to determine T_c independently from any scaling attempt. This is discussed below for $Cd_{1-x}Mn_xTe$ where different samples with nominal concentration $x \approx 0.4$ have been studied to elucidate the critical behavior.[1,4,12]

In Ref. 1, and in agreement with Eq. (3), $T_c = 12.3$ K is determined as the temperature where the $\chi''(T)$ curves cross together, for any frequency. Unfortunately, this crossing occurs at a temperature which is smaller than T_p at finite frequency, for which we have argued that Eq. (3) is no longer valid. This is, in our view, the reason why the experimental $\chi''(T)$ curves do not cross together at a well-defined temperature (see Fig. 3 in Ref. 1); definitely, T_c cannot be estimated by this method.

A procedure to independently determine T_c and $z\nu$ has been proposed by Soulétie and Tholence[13] and successfully applied to spin glasses.[13,14] In the case of the power law $\phi = \omega\tau_0 t^{\beta - z\nu}$ (Ref. 8) valid for small ϕ, the method consists in plotting $P_{\tau'} = -\partial\ln(T)/\partial\ln(\tau')$ vs T: one eliminates the parameter τ_0 in the differentiation, and obtains a straight line which intersects the temperature axis at T_c, and the $T=0$ axis at $1/(\beta - z\nu)$. The notation τ' has been used so that no confusion with the relaxation time defined by $\tau = \tau_0 t^{-z\nu}$ in the scaling hypothesis occurs.[8] (The ap-

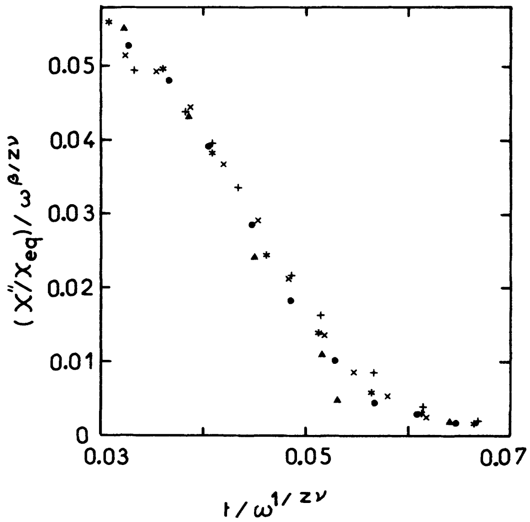

FIG. 1. Scaling of χ''/χ_{eq} according to the data of Fig. 4 in Ref. 1 above the temperature of the peak of χ'', for $z\nu=10$, $\beta=0.7$, and $T_c=12.4$ K. The symbols Δ, ∗, ○, ×, and + denote frequencies between 9.75 Hz and 97.5 kHz, in decade steps.

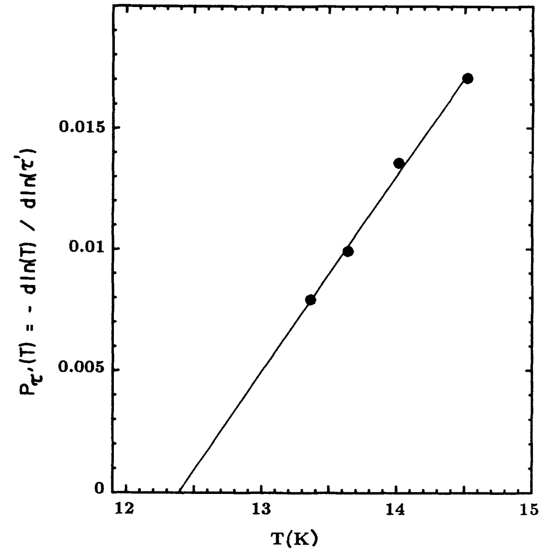

FIG. 2. Plots of $\partial \ln(T)/\partial \ln(\omega)$ as a function of T, with T the temperature defined by $|\chi''/\chi'|=10^{-3}$, after the data of Ref. 1.

proximation $\tau=\tau'$ has been made in the earlier works reported in Refs. 4, 5, and 13.) The value of the critical exponent β will be discussed in the next paragraph. In a first stage, it is sufficient to note that it is small ($\beta<1$). Such a plot of $P_{\tau'}$ vs T is reported in Fig. 1 for $Cd_{0.6}Mn_{0.4}Te$: the data are from Ref. 1, and we have chosen to determine the temperature dependence of τ' by using the criterion $\phi(T)=10^{-3}$, justified earlier in the text.[11] The low-frequency (0.975 Hz) curve is doubtful and does not scale with the higher-frequency data, in contrast with other experimental results obtained on similar samples.[3-6] Thus we have rejected the lower-frequency data in the scaling analysis. We then find $T_c=12.4$ and $z\nu-\beta\approx 9.7$. Note this value of $z\nu$ is in agreement with the result $z\nu=10$, reported from another method allowing an independent determination of $z\nu$ and T_c [curve $0.3R$-$0.8R$ in Fig. 2(b) of Ref. 1], already discussed in Sec. II. Taking $z\nu=10$, another estimation of T_c can be deduced from the dynamic scaling given by Eq. (3) which involves the two parameters T_c and β. The rescaling of the data of Ref. 1 for $T>T_p$ is best achieved for $\beta=0.07\pm0.2$ and $T_c=12.4$ K, as is illustrated in Fig. 2. Therefore, we find a value $T_c=12.4$ K for the sample of Ref. 1, compared with 12.13 K used in the scaling analysis of Eq. (3) by the authors of Ref. 1. We thus find a real difference of 0.4 K in the values of T_c for the samples studied in Refs. 1 and 4, certainly due to a small difference in Mn concentrations: indeed, the peak in χ'(97.5 Hz) appears at 13.48 K for their sample[1] compared with 13.85 K for our sample.[4]

Finally, let us discuss our static scaling analysis for another $Cd_{1-x}Mn_xTe$ sample.[12] In this case, T_c has been calibrated from quasistatic experiments with respect to the sample used in Ref. 4: the temperature at which the slow relaxation of the field-cooled magnetization changes sign with step cooling of 0.05 K is 12.9 K for the sample used in Ref. 4, and 0.43 K lower for the sample of Ref. 12. This difference should reflect on the spin-glass temperature, hence $T_c=12.8-0.43=12.37$ K for the sample of Ref. 12. This is the value of T_c which has been used in the scaling analysis of the nonlinear magnetization, leading to the exponents $\gamma=3.3\pm0.3$ and $\beta=0.9\pm0.2$ [see Ref. 4, and also Fig. 1(a) of Ref. 6]. Different exponents can be obtained naturally if a wrong value of T_c is artificially injected in the scaling attempt [Fig. 1(b) in Ref. 6].

IV. CONCLUSION

We conclude that the critical exponents in the semimagnetic spin glasses are those reported in Ref. 7. Although the discussion has been focused on $Cd_{0.6}Mn_{0.4}Te$ in the present work, the values of these critical exponents are not specific to this particular material and have been determined on other compounds of the same family at different magnetic ion concentrations, and other systems as well.[7] Different values of the exponents reported by Geschwind, Huse, and Delvin[1,6] are attributable to the underestimation of T_c in the static scaling analysis, or (and) the inclusion of data points at too low temperaures ($T<T_p$) in the scaling analysis.

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