Perturbation of γ-ray angular correlation by an Ornstein-Ulhenbeck process

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We derive explicitly the effect of Ornstein-Ulhenbeck noise on the perturbed angular correlation. Numerical results of the derived expression are also presented graphically.

I. INTRODUCTION

The theory of perturbed angular correlation (PAC) with a first-order treatment of random time-dependent interaction was put forward by Abragam and Pound.[1] Since then it has been used in a number of experimental situations with reasonable success. A major breakthrough occurred with the appearance of a paper[2] in which the case of very strong interactions was treated. The need for refinement of Abragam and Pound's first-order theory with a proper consideration of the perturbing environment has then become inevitable and has gained much theoretical and experimental interest in recent years.[3–6]

In the PAC experiment, the nucleus, having emitted its first γ ray, is under the influence of its surrounding before emission of the second γ ray. The large surrounding (having a large number of degrees of freedom) affects the system (the nucleus) in a very complicated way. In the experimental time scale of interest, what we really observe is the average global effect of its surrounding on the system. Since the detailed mechanism of the interaction of the surrounding is not of primary concern, the proper way to incorporate its effect is by modeling its characteristics by a specified random noise. This is the famous Langevin approach.

Upon reflection, one immediately realizes that the perturbing field, being a vector (with a dipole approximation of the nucleus), can be specified by its magnitude or strength and its direction. Since we model the field as random, the probability distribution for both the magnitude as well as the direction are to be specified in order to represent its global effect on the angular correlation. Moreover, there could be situations where the axis of perturbing field flips in different directions within the lifetime of the intermediate state of the nucleus. This is important because the nucleus would then be subjected to different strengths at different time points, depending on the angle between the nuclear dipole axis and the axis of the interacting field.

An attempt was made towards this goal by Bosch and Spehl[4] when they specified the distribution of the strength of the perturbing field by a purely Gaussian and a uniform angular distribution, although no consideration was given towards the flip of the axis of interaction during the lifetime of the intermediate state. Obviously, this model is applicable if the average duration for which the direction of the perturbing field does not change appreciably (the correlation time for direction distribution) is of the order of the lifetime of the intermediate state of the nucleus. That is, to the nucleus in the intermediate state the direction of the perturbing field appears fixed. This is known as the fixed-orientation Gaussian approximation (FOGA) model. It is expected to fail when the correlation time of the "direction" distribution is small compared with the lifetime of the intermediate state. In this case, the nucleus will be affected by perturbing field acting in various directions as time proceeds. Experimentally what one observes is the average effect of such different orientations. Since one does not have detailed information about the mechanism of change in the direction of the surrounding field, which is an impossible proposition, one again invokes, as argued previously, the idea of a probability distribution of a number of different directions at different time points within the lifetime of the intermediate state. Following this approach, Bosch and Spehl published a subsequent paper[6] where the probability distribution of a number of different directions has been modeled as a Poisson distribution. They carried out an average over the ensemble of a class of nuclei in the presence of a number of flips of the direction of the perturbing field. The question, therefore, naturally arises at what point of time the nucleus is affected by the flip and the duration for which it observes the specified orientation of the field. The specified orientation results in only a fixed value of the strength. Apart from the probabilistic approach towards the strength, the length of the time interval during which the specified strength acts on the nucleus is important since the phase change of the state is proportional to this interval. Because of this extremely complicated mechanism, here one again does not know the exact point of time at which the perturbing field flips. Consequently one further invokes the probability of getting different time points at which the flips occur for each member of the ensemble, characterized by a fixed number of flips. Since the flips can occur at any instant of time, it is natural to choose a uniform probability of selecting time points with a note of caution of ordered time

points—the flips being associated with the transition of the phase of nuclear states cannot be permuted among themselves.

Having clarified the issue of handling the flipping of the axis of the perturbing field, it can be mentioned, however, that Bosch and Spehl have assumed the purely Gaussian distribution of the strength of interaction at each point of time without any correlation. Therefore in their model they have not included any memory effect of the past. This is, of course, a mathematical idealization of the actual state of affairs. In a real situation, no distribution is free from its autocorrelation. A particularly simple way to incorporate memory effect is to model the noise as an Ornstein-Ulhenbeck process (OUP). The need to consider such colored noise has been demonstrated recently[7,8] to get a closer approximation to the experimental results. Therefore we have modified the calculation of Bosch and Spehl by introducing an exponential autocorrelation function in the distribution of the perturbing field. This clearly brings out the special features about the dependence of PAC on the characteristic property of the surrounding media.

In Sec. II the statement of the problem with a brief account of the underlying physical structure is made in mathematical terms. The statistics used in the derivation are discussed in Sec. III. Section IV deals with the actual derivation of the attenuation coefficient along with the results of the numerical calculation. Finally, we offer a few concluding remarks in Sec. V.

II. STATEMENT OF THE PROBLEM

A nucleus decays from the initial level i with spin I_i to the intermediate level with spin I with the emission of radiation in the direction $\mathbf{k}_1$. If the nucleus is not known to be in any definite quantum state we have to talk in terms of a mixed state defined through the density matrix operator

$$\hat{\rho}_i=\sum_i |m_i\rangle g_i \langle m_i| \ , \tag{1}$$

where $|m_i\rangle$ are the magnetic states corresponding to spin I_i and g_i are the statistical weight factors. The probability of observing the nucleus in any state $|l\rangle$ is

$$\langle l|\hat{\rho}_i|l\rangle=\sum_i g_i |\langle l|m_i\rangle|^2 \ . \tag{2}$$

The operator $\hat{H}_1$ induces the transition accompanied by radiation in the direction $\mathbf{k}_1$. With the first-order treatment of interaction, the density matrix for the intermediate state would be

$$\hat{\rho}'=\sum_i \hat{H}_1 |m_i\rangle g_i \langle m_i|\hat{H}_1 \ . \tag{3}$$

The presence of extra nuclear perturbation (surrounding field) causes a change of phase to the intermediate state of the nucleus. This is accounted for by a unitary time-development operator $\hat{\Lambda}(t,t')$. The operator $\hat{\Lambda}$ satisfies the following Schrödinger equation in the interaction representation:

$$i\hbar\frac{\partial}{\partial t}\hat{\Lambda}(t,t')=\hat{K}\hat{\Lambda}(t,t') \ , \tag{4}$$

where the operator $\hat{K}$ refers to the interaction of nuclear intermediate state with the surrounding perturbing field.

If the time scale is initialized at the formation of the intermediate state, after some time, let us say, t, the density matrix would be

$$\begin{aligned}\hat{\rho}'(t)&=\hat{\Lambda}(t)\hat{\rho}'(0)\hat{\Lambda}^{\dagger}(t)\\ &=\sum_i \hat{\Lambda}(t)\hat{H}_1|m_i\rangle g_i \langle m_i|\hat{H}_1\hat{\Lambda}^{\dagger}(t) \ .\end{aligned} \tag{5}$$

Suppose at time t, the nucleus emits another radiation in the direction $\mathbf{k}_2$ accompanied by a transition from the intermediate state of spin I to the final state of spin I_f. If $\hat{H}_2$ is the corresponding interaction operator that induces such transition, then the density matrix for the final state takes the form

$$\begin{aligned}\hat{\rho}_f&=\hat{H}_2\hat{\rho}'(t)\hat{H}_2\\ &=\sum_i \hat{H}_2\hat{\Lambda}(t)\hat{H}_1|m_i\rangle g_i \langle m_i|\hat{H}_1\hat{\Lambda}^{\dagger}(t)\hat{H}_2 \ .\end{aligned} \tag{6}$$

Therefore the probability of finding the system in the state $|m_f\rangle$ at time t accompanied by two successive transitions in the directions $\mathbf{k}_1$ and $\mathbf{k}_2$ is

$$\begin{aligned}\langle m_f|\hat{\rho}_f|m_f\rangle=\sum_i &\langle m_f|\hat{H}_2\hat{\Lambda}(t)\hat{H}_1|m_i\rangle\\ &\times g_i\langle m_i|\hat{H}_1\hat{\Lambda}^{\dagger}(t)\hat{H}_2|m_f\rangle \ .\end{aligned} \tag{7}$$

In the measurement process, we generally deal with successive radiations emitted by the nucleus without reference to any particular $|m_f\rangle$. Therefore the contributions to all $|m_f\rangle$'s are present in the experiment. Thus it is meaningful to talk about the total probability taking into consideration of contributions from all $|m_f\rangle$'s. This probability refers to two radiations in the specified directions. It gives the notion of some kind of joint probability distributions of emission of radiations in two specified directions; the members of the ensemble being characterized by the direction of emissions. The joint probability cannot be thought of as a product of independent probabilities of two events (emissions of two radiations) because the occurrence of the second must be associated with the first and hence there exist some correlations between the two events. Thus we can attach the notion of correlation to the total probability. Therefore the perturbed angular correlation function is defined as

$$W(\mathbf{k}_1,\mathbf{k}_2,t)=\sum_{m_f}\langle m_f|\hat{\rho}_f|m_f\rangle \ . \tag{8}$$

As there is no restriction over different $|m_i\rangle$ states we can very well assume that they are equally populated with the weight factor

$$g_i=(2I_i+1)^{-1} \ .$$

Employing the completeness of $|m\rangle$ states we can express the correlation function $W(\mathbf{k}_1,\mathbf{k}_2,t)$ in the more usual form

$$W(\mathbf{k}_1,\mathbf{k}_2,t)=\sum_{\substack{m_f,m_i,\\ m_b,m_a,\\ m_b',m_a'}}\langle m_f|\hat{H}_2|m_b\rangle\langle m_b|\hat{\Lambda}(t)|m_a\rangle\langle m_a|\hat{H}_1|m_i\rangle(2I_i+1)^{-1}\langle m_i|\hat{H}_1|m_a'\rangle\langle m_a'|\hat{\Lambda}^{\dagger}(t)|m_b'\rangle\langle m_b'|\hat{H}_2|m_f\rangle\ . \tag{9}$$

The matrix element $\langle m|\hat{H}_1|m_i\rangle$ stands for $\langle I,m,\mathbf{k}_1,\sigma_1|H_1|I_i,m_i\rangle$; the polarization σ_1 is given by the component of the spin in the direction of propagation $\mathbf{k}_1$. The factor $(2I_i+1)^{-1}$ is a constant depending only on the magnitude of the initial spin. It would not affect the correlation properties in any way. Therefore we redefine the correlation function $W(\mathbf{k}_1,\mathbf{k}_2,t)$ without this multiplying factor,

$$W(\mathbf{k}_1,\mathbf{k}_2,t)=\sum_{\substack{m_i,m_f,\\ m_a,m_b,\\ m_a',m_b'}}\langle m_f|\hat{H}_2|m_b\rangle\langle m_a|\hat{H}_1|m_i\rangle\langle m_i|\hat{H}_1|m_a'\rangle\langle m_b'|\hat{H}_2|m_f\rangle\langle m_b|\hat{\Lambda}(t)|m_a\rangle\langle m_a'|\hat{\Lambda}^{\dagger}(t)|m_b'\rangle\ . \tag{10}$$

This is the underlying physical structure of angular correlation theory. In the angular correlation experiment one is concerned about the direction of the emitted radiation. Therefore it is convenient to express the function explicitly in terms of the characteristics of the emitting radiation. This has been done[9] and we only quote the results.

$$W(\mathbf{k}_1,\mathbf{k}_2,t)=\sum_{\substack{k_1,k_2,\\ N_1,N_2}}A_{k_1}(1)A_{k_2}(2)G_{k_1k_2}^{N_1N_2}(t)[(2k_1+1)(2k_2+1)]^{-1/2}Y_{k_1}^{N_1*}(\theta_1,\phi_1)Y_{k_2}^{N_2}(\theta_2,\phi_2)\ , \tag{11}$$

where the perturbation factor is given by

$$G_{k_1k_2}^{N_1N_2}(t)\sum_{m_a,m_b}(-1)^{2I+m_a+m_b}[(2k_1+1)(2k_2+1)]^{1/2}\begin{pmatrix} I & I & k_1\\ m_a' & -m_a & N_1\end{pmatrix}\begin{pmatrix} I & I & k_2\\ m_b' & -m_b & N_2\end{pmatrix}$$
$$\times\langle m_b|\hat{\Lambda}(t)|m_a\rangle\langle m_a'|\hat{\Lambda}^{\dagger}(t)|m_b'\rangle\ . \tag{12}$$

We consider only the magnetic interaction between the nucleus and the surrounding perturbing field. Assuming the nucleus to be a magnetic dipole having moment $\boldsymbol{\mu}$ and the surrounding field to be approximated by a magnetic field $\mathbf{B}$, the interaction Hamiltonian is $\hat{H}=-\boldsymbol{\mu}\cdot\mathbf{B}$. Then the time-development operator would be

$$\hat{\Lambda}(t)=\exp\left[-\frac{i}{\hbar}\hat{H}t\right]\ . \tag{13}$$

As has been mentioned in the Introduction we have considered the family of the nuclei having faced different numbers of orientation of the perturbing field, each member being classified by a fixed number of orientations. For each member the correlation function $W(\mathbf{k}_1,\mathbf{k}_2,t)$ is to be averaged over the joint probability distribution of strength, $\omega=\boldsymbol{\mu}\cdot\mathbf{B}/h$ and direction Ω of the surrounding perturbing field. The averaged quantity should then be multiplied by the probability of getting that particular member out of the family. Thus the total probability will be

$$\langle G_{k_1k_2}^{N_1N_2}(t)\rangle=E\{G_{k_1k_2}^{N_1N_2}|n=0\}P(n=0)+E\{G_{k_1k_2}^{N_1N_2}|n=1\}P(n=1)+\cdots+E\{G_{k_1k_2}^{N_1N_2}|n\}P(n)+\cdots\ , \tag{14}$$

where $P(n)$ is the probability of having n flips in time t and

$$E\{G_{k_1k_2}^{N_1N_2}|n\}=\int\int_{\substack{\cdots\\ 0\le t_1\le t_2\cdots\le t_n\le t}}\{G_{k_1k_2}^{N_1N_2}\}_{t_1,t_2,\ldots,t_n}W(\omega_0,\Omega_0,0;\ \omega_1,\Omega_1,t_1;\ \ldots;\ \omega_n,\Omega_n,t_n)d\omega_0d\Omega_0\prod_{i=1}^{n}d\omega_id\Omega_idt_i\ , \tag{15}$$

with $\{G^{N_1N_2}_{k_1k_2}\}_{t_1,t_2,\ldots,t_n}$ indicating that the flips have taken place at times $t_1,t_2,\ldots,t_n$ where $0\le t_1\le t_2\ldots\le t_n\le t$. The importance of selection of different time points is clarified in the Introduction. Since the exact time points at which the flips occur are not known, the probability distribution of choosing ordered time points is to be invoked.

III. THE STATISTICS USED

The joint distribution W is modeled as

$$W(\omega_0,\Omega_0,0;\ \omega_1,\Omega_1,t_1;\ \ldots;\ \omega_n,\Omega_n,t_n)$$
$$=P(\omega_0,\Omega_0|0;\ \omega_1,\Omega_1|t_1;\ \ldots;\omega_n,\Omega_n|t_n)$$
$$\times W(t_1,t_2,\ldots,t_n)\ , \tag{16}$$

where P is the conditional probability of strength and direction of the surrounding field given the specific ordered time points $t_1,t_2,\ldots,t_n$, and $W(t_1,t_2,\ldots,t_n)$ is the probability of selecting "n" ordered time points.

$W(t_1,t_2,\ldots,t_n)$ is assumed to be some constant. This constant can be obtained from the normalization of W, i.e.,

$$\int_0^t\int_0^{t_n}\cdots\int_0^{t_2}W(t_1,t_2,\ldots,t_n)dt_n dt_{n-1}\ldots dt_1=1\ ,$$

which yields

$$W(t_1,t_2,\ldots t_n)=n!/t^n\ .$$

The conditional probability P is assumed to be the product of the conditional probabilities of strength and the direction of the perturbing field assuming these random variables are independent at any time.

$$P(\omega_0,\Omega_0|0;\ \omega_1,\Omega_1|t_1;\ \ldots;\omega_n,\Omega_n|t_n)$$
$$=P(\omega_0|0;\ \omega_1|t_1;\ \ldots;\ \omega_n|t_n)$$
$$\times P(\Omega_0|0;\ \Omega_1|t_1;\ldots;\Omega_n|t_n)\ . \tag{17}$$

The conditional probability for directions $\{\Omega_i\}$ are assumed to be

$$P(\Omega_0|0;\ \Omega_1|t_1;\ \ldots;\ \Omega_n|t_n)=\prod_{i=0}^{n}P(\Omega_i|t_i)$$
$$=\left[\frac{1}{4\pi}\right]^{n+1}\ . \tag{18}$$

The variable ω is modeled as an Orstein-Ulhenbeck process (OUP).

$$P(\omega_0|0;\ \omega_1|t_1;\ \ldots;\ \omega_n|t_n$$
$$=\prod_{i=1}^{n}P_i(\omega_i;t_i|\omega_{i-1};t_{i-1})P_0(\omega_0;0)\ , \tag{19}$$

where

$$P_i(\omega_i;t_i|\omega_{i-1};t_{i-1})=[2\pi\sigma^2(1-e^{-2\gamma(t_i-t_{i-1})})]^{-1/2}$$
$$\times\exp\left[-\frac{(\omega_i-e^{-\gamma(t_i-t_{i-1})}\omega_{i-1})^2}{2\sigma^2(1-e^{-2\gamma(t_i-t_{i-1})})}\right] \tag{20}$$

and

$$P_0(\omega_0;0)=(2\pi\sigma^2)^{-1/2}\exp(-\omega_0^2/2\sigma^2)\ . \tag{20a}$$

σ^2 is the variance of the distribution and γ is the correlation time of O-U-P.

The probability of having exactly "n" points in the time interval t is assumed to be given by the Poisson distribution

$$P(n)=\frac{(\lambda t)^n}{n!}e^{-\lambda t}\ , \tag{21}$$

where λ is the mean number of points per unit time.

IV. DETERMINATION OF THE ATTENUATION COEFFICIENT

The $(n+1)$th term of $\langle G^{N_1N_2}_{k_1k_2}(t)\rangle$ in Eq. (14) corresponding to n flips is

$$e^{-\lambda t}\lambda^n\int_0^t dt_n\int_0^{t_n}dt_{n-1}\cdots\int_0^{t_2}dt_1X(0,t_1,t_2,\ldots,t_n)\ ,$$

with

$$X(0,t_1,t_2,\ldots,t_n)=\left[\frac{1}{4\pi}\right]^{n+1}\int\cdots\int\{G^{N_1N_2}_{k_1k_2}\}_{t_1,t_2,\ldots,t_n}\left[\prod_{i=1}^{n}P_i(\omega_i,t_i|\omega_{i-1};t_{i-1})d\omega_i\right]P_0(\omega_0;0)d\omega_0\times\left[\prod_{i=0}^{n}d\Omega_i\right]\ ,$$

where $\{G^{N_1N_2}_{k_1k_2}\}_{t_1,t_2,\ldots,t_n}$ is given by Eq. (15). The specification in the subscript refers to the calculation of the matrix elements of the time evolution operator in $G^{N_1N_2}_{k_1k_2}$. that is,

$$\hat{\Lambda}(t)=\hat{\Lambda}(t-t_n)\hat{\Lambda}(t_n-t_{n-1})\cdots\hat{\Lambda}(t_2-t_1)\hat{\Lambda}(t_1)\ , \tag{22}$$

where each individual $\hat{\Lambda}(t_i-t_{i-1})$ should be calculated separately and during the time (t_i-t_{i-1}) the interaction Hamiltonian remains unchanged, remembering at every point t_j, as the field axis flips the interaction Hamiltonian changes. The interaction Hamiltonian $\hat{H}$ is

$$\hat{H}=-g_I\mu_N\hat{I}_{z'}B=-\hbar\omega\hat{I}_{z'} \tag{23}$$

where g_I is the Lande g factor, μ_n is the Bohr magneton, ω refers to the strength of the perturbing field and it is a

random variable, and $I_{z'}$ refers to the projection of I along **B**.

The magnetic states $|m\rangle$ for which the matrix element is to be evaluated refer to our fixed coordinate system. But $I_{z'}$ or H is diagonal in the system where the z' axis is parallel to field axis **B**. Hence one has to incorporate different complete substates corresponding to different orientations of **B**, which will be eigenstates of $I_{z'}$ at different times. This would naturally bring various Wigner coefficients depending on the Eulerian angles between space fixed axis and field axis. Integration over the angles can be easily performed using the properties of Wigner D functions. The tricks involved in the above procedure are by now quite standard. We would therefore omit these steps. For $n=0$, i.e., when during time t there has been no flip, $X(0)$ turns out to be

$$X(0)=\int \sum_{p,p'} \begin{bmatrix} I & I & k \\ p' & -p & N \end{bmatrix}^2 e^{-i(p-p')\omega_0 t} P_0(\omega_0;0)\,d\omega_0 \,. \tag{24}$$

$\{p\}$'s refer to magnetic quantum numbers associated with the states in the frame containing the field axis as one of its axes. After substitution of $P_0(\omega_0,0)$ from Eq. (20a), $X(0)$ turns out to be

$$X(0)=\sum_{p_1,p_1'=0}^{k} \begin{bmatrix} I & I & k \\ p_1' & -p_1 & N \end{bmatrix}^2 e^{-(p_1-p_1')^2\sigma^2 t^2/2} \,. \tag{25}$$

Similarly we can calculate $X(0,t_1)$ for the case when one flip occurs during time $t(n=1)$ and it becomes

$$X(0,t_1)=\sum_{\substack{p_1,p_1',\\ p_2,p_2'}} \begin{bmatrix} I & I & k \\ p_1' & -p_1 & s_1 \end{bmatrix}^2 \begin{bmatrix} I & I & k \\ p_2' & -p_2 & s_2 \end{bmatrix}^2 \exp\left[\left(-\frac{\sigma^2}{2}\right)[\{(p_2-p_2')(t-t_1)+(p_1-p_1')t_1 e^{-\gamma t_1}\}^2 + (1-e^{-2\gamma t_1})(p_1-p_1')^2 t_1^2]\right] . \tag{26}$$

Continuing the procedure for $n=2$, $X(0,t_1,t_2)$ turns out to be

$$X(0,t_1,t_2)=\sum_{\substack{p_1,p_1',\\ p_2,p_2',\\ p_3,p_3'}} \begin{bmatrix} I & I & k \\ p_1' & -p_1 & s_1 \end{bmatrix}^2 \begin{bmatrix} I & I & k \\ p_2' & -p_2 & s_2 \end{bmatrix}^2 \begin{bmatrix} I & I & k \\ p_3' & -p_3 & s_3 \end{bmatrix}^2 \times \exp\left[\left(-\frac{\sigma^2}{2}\right)(\{(p_3-p_3')(t-t_2)+e^{-\gamma(t_2-t_1)}[(p_2-p_2')(t_2-t_1)+(p_1-p_1')t_1 e^{-\gamma t_1}]\}^2 + (1-e^{-2\gamma(t_2-t_1)})[(p_2-p_2')(t_2-t_1)+(p_1-p_1')t_1 e^{-\gamma t_1}]^2 + (1-e^{-2\gamma t_1})(p_1-p_1')^2 t_1^2)\right] , \tag{27}$$

what we are attempting is to get a closed expression for $X(0,t_1,t_2,\ldots,t_r)$. Inspired by the systematics found in the expression we can say that

$$X(0,t_1,t_2,\ldots,t_r)=\sum_{\substack{p_i,p_i' \\ (i=1,2,\ldots r+1)}} \prod_{i=1}^{r+1} \left[\begin{bmatrix} I & I & k \\ p_i' & -p_i & s_i \end{bmatrix}^2 \right] e^{-(\sigma^2/2)\bar{A}_r} , \tag{28}$$

where

$$\bar{A}_r = A_{r+1,r}^2 + \sum_{i=r}^{1} [1-e^{-2\gamma(t_i-t_{i-1})}] A_{i,i-1}^2 \tag{29}$$

and $A_{i,i-1}$ satisfies a difference equation

$$A_{i,i-1}=(p_i-p_i')(t_i-t_{i-1})+e^{-\gamma(t_{i-1}-t_{i-2})}A_{i-1,i-2} \,, \tag{30}$$

with $t_0=0$, $A_{0,-1}=0$, $t_{-1}=0$, $t_{r+1}=t$. Defining

$$A_{i,i-1}=y_{i+1},\quad e^{-\gamma(t_{i-1}-t_{i-2})}=\bar{A}_i$$

and

$$(p_i-p_i')(t_i-t_{i-1})=R_i \,,$$

the difference equation can be expressed in a simpler form

$$y_{i+1}=\tilde{A}_i y_i+R_i \ ,$$

with

$$A_{0,-1}=y_1=0 \ .$$

Solution of this equation is simple. Employing the initial condition one arrives

$$y_i=\tilde{A}_1\tilde{A}_2\ldots\tilde{A}_{i-1}\sum_{p=1}^{i-1}\frac{R_p}{\tilde{A}_1\tilde{A}_2\ldots\tilde{A}_p} \ . \tag{31}$$

Substituting the values of $\tilde{A}_i$ and R_i, y_{i+1} simplifies to

$$y_{i+1}=A_{i,i-1}=e^{-\gamma t_{i-1}}\sum_{q=1}^{i}(p_q-p_q')(t_q-t_{q-1})e^{\gamma t_{q-1}} \ . \tag{32}$$

Our objective is to calculate the closed form expression for A_r in terms of known quantities like p_i, t_i. We first note that Eq. (29) can be rewritten in the following form:

$$\overline{A}_r=A_{1,0}^2+\sum_{i=1}^{r}[\,A_{i+1,i}^2-e^{-2\gamma(t_i-t_{i-1})}A_{i,i-1}^2\,] \ . \tag{33}$$

From Eq. (30) it is clear that

$$\begin{aligned}A_{i+1,i}^2&-e^{-2\gamma(t_i-t_{i-1})}A_{i,i-1}^2\\&=[(p_{i+1}-p_{i+1}')(t_{i+1}-t_i)]^2\\&\quad+2(p_{i+1}-p_{i+1}')(t_{i+1}-t_i)e^{-\gamma(t_i-t_{i-1})}A_{i,i-1} \ .\end{aligned} \tag{34}$$

Therefore $\overline{A}_r$ in Eq. (33) is simplified to

$$\begin{aligned}\overline{A}_r=&\sum_{i=0}^{r}[(p_{i+1}-p_{i+1}')(t_{i+1}-t_i)]^2\\&+2\sum_{i=1}^{r}(p_{i+1}-p_{i+1}')(t_{i+1}-t_i)e^{-\gamma(t_i-t_{i-1})}A_{i,i-1} \ .\end{aligned} \tag{35}$$

Employing the value of $A_{i,i-1}$, $\overline{A}_r$ takes the final form

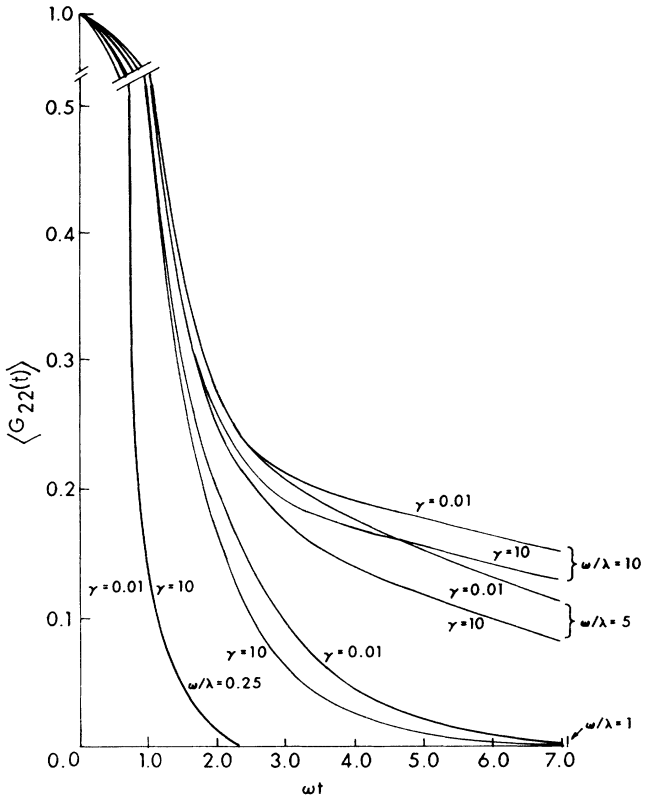

FIG. 1. Attenuation factors are plotted against (ωt) for different ratios of (ω/λ) with two extreme values of correlation times (γ) of Ornstein-Ulhenbeck noise; $\gamma=10$ corresponds to the Gaussian white noise.

$$\overline{A}_r=\sum_{q=0}^{r}\Phi_q^2+2\sum_{q=1}^{r}e^{-\gamma t_q}\Phi_q\sum_{l=0}^{q-1}e^{\gamma t_l}\Phi_l \ , \tag{36}$$

where

$$\Phi_q=(p_{q+1}-p_{q+1}')(t_{q+1}-t_q) \ .$$

Thus the attenuation coefficient takes the closed form expression

$$\langle G_{kk}(t)\rangle=e^{-\lambda t}\sum_{\substack{p_i,p_i'\\(i=1,2,\ldots,n+1)}}\prod_{i=1}^{n+1}\begin{bmatrix}I & I & k\\ p_i' & -p_i & s_i\end{bmatrix}^2\int_0^t dt_n\int_0^{t_n}dt_{n-1}\cdots\int_0^{t_2}dt_1 e^{-(\sigma^2/2)\overline{A}_n} \ . \tag{37}$$

It is interesting to observe that the angular integration leads to nontrivial results only for $k_1=k_2=k$. This is due to the fact that the angular average is taken over a uniform distribution, which relates to the consequence of the isotropy of the model. Noting that $\overline{A}_n$ is a function of $\{(p_i-p_{i'})\}$ and $\{t_i\}$, we can carry out the partial sum over the $3-j$ symbol and the attenuation coefficient in Eq. (37) becomes

$$\langle G_{kk}(t)\rangle=e^{-\lambda t}\sum_{\{N_i\}=-k}^{k}\frac{1}{(2k+1)^{n+1}}\int_0^t dt_n\int_0^{t_n}dt_{n-1}\cdots\int_0^{t_2}dt_1\exp\left[-\frac{\sigma^2}{2}\overline{A}_n\right] \ , \tag{38}$$

where $\overline{A}_n$ is a function of $\{N_i\}$ and $\{t_i\}$.

Calculated attenuation factors obtained by numerical methods for different (ω/λ) and γ values are shown graphically in Fig. 1. The case of Gaussian white noise has been obtained for a large γ value. We took approximately $\gamma = 10$.

V. CONCLUDING REMARKS

Various approximations that have entered into the formulation of the theory of the perturbed angular correlation at different levels are clearly shown in this paper. This work will allow further improvement of the theory in a more systematic way. We have considered magnetic interactions of the nucleus with its surrounding. The electric interactions are also another important aspect to consider. The nucleus is assumed to be a dipole in this work. Considering it as a distributed charge, we obtain a picture of several order multipoles emerging and the corresponding higher-order tensorial interaction with different derivatives of the surrounding field should also be included in the theory. In these situations, one has to invoke modeling of various coupling strengths by suitable stochastic processes. In this work, we have accounted for correlations in the strength of the dipole-field interaction. As argued in the preceding text, no real process is devoid of correlations. In this respect, it is worthwhile to consider further correlation in the angular distribution. For simplicity we have assumed that the distribution of the strength and directions of the field are independent at all instants. In a real situation, it need not be so. One may incorporate a further correlation between them. Our consideration of correlations in the strength of the field has clearly demonstrated a dependence of the attenuation coefficient on the finite correlation time. The inclusion of the auto-correlation function invokes an experimental time scale of interest. In order to observe the memory effect, the experimental time scale must be less than the autocorrelation time. Experimental verification of the derived results which requires a very sensitive time-resolving instrument, is to be encouraged. However, if the fluctuating field possesses a very small autocorrelation time, the effect of which cannot be detected by the resolution time of the instrument, the result should agree instead with a model making no consideration of autocorrelation time (the Scherer-Blume and the Bosch-Spehl model). This fact has clearly been shown in the figure with $\gamma = 10$.

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