PHYSICAL REVIEW B VOLUME 45, NUMBER 7 15 FEBRUARY 1992-I

Scattering of a scalar beam from a two-dimensional randomly rough hard wall: Enhanced backscattering

P. Tran and A. A. Maradudin

Department of Physics and Institute for Surface and Interface Science, University of California, Irvine, California 92717

(Received 12 August 1991)

We have calculated numerically the scattering of a scalar beam incident normally on a large-amplitude, large-rms-slope, two-dimensional, randomly rough, hard wall (Dirichlet boundary condition). Our result is equivalent to a fourth-order Kirchhoff approximation, and displays a well-defined peak in the retroreflection direction in the angular dependence of the intensity of the waves scattered incoherently.

Enhanced backscattering in the scattering of classical waves by a randomly rough surface is manifested by a well-defined peak in the retroreflection direction in the angular dependence of the intensity of the diffuse component of the scattered waves. In the great majority of the theoretical studies of this effect to date the random surfaces involved have been one dimensional.[1-16] In the comparatively few studies of enhanced backscattering (of light) from two-dimensional random surfaces, whether by perturbation theory[17] or by numerical simulation methods,[18,19] in which the effect was treated as a multiple-scattering phenomenon, the surfaces involved were very weakly corrugated, and supported surface electromagnetic waves (SEW).[20] The mechanism responsible for enhanced backscattering in this case is the coherent interference of each multiply scattered light plus SEW path with its time-reversed partner.

In this paper we study the scattering of a scalar beam from a large rms slope, two-dimensional, randomly rough, hard wall (Dirichlet boundary condition). The method of calculation employed is based on writing the integral equation for the normal derivative of the total field on the random surface, in terms of which the scattered field is expressed, in the form of an inhomogeneous Fredholm equation of the second kind, and solving the latter by iteration. The results obtained are applicable to the elastic scattering of neutral atoms from a randomly rough surface, if the length scales of the roughness are comparable to the de Broglie wavelength of the atoms, and to the scattering of acoustic waves from a pressure-release surface.

The system we consider consists of vacuum in the region $z>\zeta(\mathbf{R})$, where $\mathbf{R}=(x,y)$, and an impenetrable medium in the region $z<\zeta(\mathbf{R})$. The surface profile function $\zeta(\mathbf{R})$ is assumed to be a stationary, Gaussian process, defined by the properties

$$\langle\zeta(\mathbf{R})\rangle=0\,, \tag{1}$$

$$\langle\zeta(\mathbf{R})\zeta(\mathbf{R}')\rangle=\sigma^2\exp[-(\mathbf{R}-\mathbf{R}')^2/a^2]\,. \tag{2}$$

The angle brackets in these equations denote an average over the ensemble of realizations of the surface profile, σ is the rms height of the surface, and a is the transverse correlation length of the surface roughness.

The field in the vacuum region $\psi(\mathbf{r})$ is given by the Helmholtz-Kirchhoff theorem[21] together with the Dirichlet boundary condition,

$$\psi(\mathbf{r})=\psi_{\rm in}(\mathbf{r})-\frac{1}{4\pi}\int d^2S'\,G(\mathbf{r},\mathbf{r}')\frac{\partial\psi(\mathbf{r}')}{\partial n'},\quad z>\zeta(\mathbf{R})\,, \tag{3}$$

where $\psi_{\rm in}(\mathbf{r})$ is the incident field, $\partial/\partial n$ is the derivative along the normal to the surface directed into the vacuum region, and the integration is over the random surface $z'=\zeta(\mathbf{R}')$. $G(\mathbf{r},\mathbf{r}')$ is a Green's function that obeys the equation

$$[\nabla^2+(\omega/c)^2]G(\mathbf{r},\mathbf{r}')=-4\pi\delta(\mathbf{r}-\mathbf{r}')\,, \tag{4}$$

where ω is the angular frequency of the field, and c is its speed. [In the case of the scattering of a particle of mass m from the impenetrable medium, $(\omega/c)^2$ is given by $2mE/\hbar^2$, where $E(>0)$ is the energy of the particle.] The real-space representation of $G(\mathbf{r},\mathbf{r}')$ is

$$G(\mathbf{r},\mathbf{r}')=\frac{e^{i(\omega/c)|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}\,. \tag{5}$$

The scattered field is given by the second term on the right-hand side of Eq. (3). In order to evaluate it we need to know the normal derivative of the total field on the surface. To obtain the equation satisfied by this quantity we note that for points below the surface we have the extinction theorem,[22]

$$0=\psi_{\rm in}(\mathbf{r})-\frac{1}{4\pi}\int d^2S'\,G(\mathbf{r},\mathbf{r}')\frac{\partial\psi(\mathbf{r}')}{\partial n'},\quad z<\zeta(\mathbf{R})\,. \tag{6}$$

If we evaluate the normal derivative of Eq. (3) at $z=\zeta(\mathbf{R})+\epsilon$, evaluate the normal derivative of Eq. (6) at $z=\zeta(\mathbf{R})-\epsilon$, where ϵ is a positive infinitesimal, and add the resulting two equations, we obtain the equation for $\partial\psi(\mathbf{r})/\partial n$ on the surface $z=\zeta(\mathbf{R})$,[23]

$$\frac{\partial\psi(\mathbf{r})}{\partial n}=2\frac{\partial\psi_{\rm in}(\mathbf{r})}{\partial n}-\frac{1}{2\pi}\mathrm{P}\int d^2S'\,\frac{\partial G(\mathbf{r},\mathbf{r}')}{\partial n}\frac{\partial\psi(\mathbf{r}')}{\partial n'}\,, \tag{7}$$

where P denotes the principal part, and the limit $\epsilon\to 0^+$ has been taken.

To solve Eq. (7) we first replaced integration over the random surface by integration over the xy plane with the aid of the relation

$$d^2S = [1 + (\nabla\zeta)^2]^{1/2} d^2R \,. \tag{8}$$

We then limited the integration to the region $-L/2 < x$, $y < L/2$, which is much larger than the illuminated area, and discretized it to obtain a matrix equation,

$$\frac{\partial\psi(\mathbf{r})}{\partial n} = 2\frac{\partial\psi_{\rm in}(\mathbf{r})}{\partial n} - \frac{L^2}{2\pi N^2}\sum_{\mathbf{R}'(\neq\mathbf{R})}\{1+[\nabla\zeta(\mathbf{R}')]^2\}^{1/2} \times \frac{\partial G(\mathbf{r},\mathbf{r}')}{\partial n}\frac{\partial\psi(\mathbf{r}')}{\partial n'} \,, \tag{9}$$

where $N \times N$ is the number of points within the region $-L/2 < x$, $y < L/2$. Equation (9) can be solved by a linear-system-solver algorithm, which requires N^6 computer operations, or by iteration, which requires N^4 operations times the number of iterations. In the present calculations N was chosen to be 64. Consequently, the former approach would have been very expensive computationally. We therefore used the latter approach. An additional advantage of the iterative method is that the nth term in the resulting expansion for $\partial\psi(\mathbf{r})/\partial n$ on the surface describes an n-fold scattering of the field from the surface. We can therefore examine the contribution to the differential reflection coefficient from single-, double-, etc., scattering processes individually.

Once $\partial\psi(\mathbf{r})/\partial n$ on the surface has been found we used Eq. (3) to obtain the scattered field, which can be written as

$$\psi_{\rm sc}(\mathbf{r}) = \sum_{\mathbf{K}} A(\mathbf{K}) e^{i\mathbf{K}\cdot\mathbf{R}} e^{ipz} \,. \tag{10}$$

Here $\mathbf{K}$ and p are the components of the wave vector of the scattered wave parallel and perpendicular to the mean surface $z=0$, respectively ($K^2+p^2=\omega^2/c^2$).

To obtain the differential reflection coefficient, we write the scattered flux crossing a horizontal plane as

$$\begin{aligned} j_{\rm sc} &= \frac{\hbar}{m}{\rm Im}\int d^2R\, \psi^*_{\rm sc}(\mathbf{r})\frac{\partial\psi_{\rm sc}(\mathbf{r})}{\partial z} \\ &= \frac{\hbar}{m}L^2\left(\frac{L}{2\pi}\right)^2\int_{K<\omega/c} d^2K\, p|A(\mathbf{K})|^2 \\ &= \frac{\hbar}{m}L^2\left(\frac{L}{2\pi}\right)^2\int d\Omega\,\frac{\omega}{c}p^2|A(\mathbf{K})|^2 \,, \end{aligned} \tag{11}$$

where $d\Omega$ is the element of solid angle and we have used the result that $K = (\omega/c)\sin\theta$, where the scattering angle θ is measured from the normal to the mean surface. The contribution to the ensemble-averaged differential reflection coefficient from the field scattered incoherently is obtained by normalizing the integrand in Eq. (11) by the incident flux $j_{\rm in}$, and is

$$\left\langle\frac{\partial R}{\partial\Omega}\right\rangle_{\rm incoh} = \frac{\hbar}{m}L^2\left(\frac{L}{2\pi}\right)^2\frac{\omega}{c}\frac{p^2}{j_{\rm in}} \times [\langle|A(\mathbf{K})|^2\rangle - |\langle A(\mathbf{K})\rangle|^2] \,, \tag{12}$$

where the second term is the contribution from the field scattered coherently. For the incident field we used a sum of plane waves with a Gaussian weight function,

$$\psi_{\rm in}(\mathbf{r}) = \frac{2\pi W^2}{L^2}\sum_{\mathbf{K}} e^{i\mathbf{K}\cdot\mathbf{R}} e^{-ipz}\exp[-(\mathbf{K}-\mathbf{K}_0)^2W^2/2] \,, \tag{13}$$

where the sum is over the region $|\mathbf{K}| \le \omega/c$. For $W \gg c/\omega$ we can obtain an approximate analytic expression for $\psi_{\rm in}(\mathbf{r})$ by expanding p in a Taylor series for $\mathbf{K}$ around $\mathbf{K}_0$ and keeping terms to first order in $(\mathbf{K}-\mathbf{K}_0)$. The result is an incident beam which at the mean surface $z=0$ is a plane wave modulated by a Gaussian envelope of half-width W centered at $\mathbf{R}=0$. The area of the beam spot is $\simeq \pi W^2$, and as long as the area of the plane $z=0$ covered by the random surface, L^2, is much larger than πW^2 we can restrict the integration in Eq. (7) to the region $-L/2 < x, y < L/2$, as we have done in obtaining Eq. (9). However, rather than using this approximate analytic expression, which is valid only for large W, we used Eq. (13) and evaluated the sum numerically. Although it takes more computer time, this way of evaluating $\psi_{\rm in}(\mathbf{r})$ enables us to use an incident field with an arbitrary spot size. This is important because for a fixed L we can choose W to satisfy the condition $\pi W^2 \ll L^2$, while to use the approximate analytic expression we need also to satisfy the condition $W \gg c/\omega$, which in many cases cannot be done.

The total incident flux crossing a horizontal plane is

$$j_{\rm in} = \frac{\hbar}{m}{\rm Im}\int d^2R\, \psi^*_{\rm in}(\mathbf{r})\frac{\partial\psi_{\rm in}(\mathbf{r})}{\partial z} \,. \tag{14}$$

On substituting Eq. (13) into Eq. (14), we obtain

$$j_{\rm in} = \frac{\hbar}{m}W^4\int_{K<\omega/c} d^2K\, p\exp[-(\mathbf{K}-\mathbf{K}_0)^2W^2] \,. \tag{15}$$

We have carried out calculations of $\langle\partial R/\partial\Omega\rangle_{\rm incoh}$ using three iterations in the solution of Eq. (9), which corresponds to including quadruple-scattering processes. We generated 100 random surfaces possessing the properties (1) and (2) by an extension to two-dimensional surfaces of the method of García and Stoll.[24] The scattered field was obtained for each surface, and the averages in Eqs. (12) computed. The surface roughness was characterized by the parameters $a=2\lambda$ and $\sigma=\lambda$, where λ is the wavelength of the incident beam. The parameters of the incident field were $W=2\lambda$ and $\mathbf{K}_0=0$ (normal incidence). In solving Eq. (9) we used a square grid with 64 points per side and $L=16\lambda$. The results for in-plane scattering are shown in Fig. 1, where the contributions from the pure n-fold scattering processes are plotted for $n=1$, 2, 3, 4, together with the total. Two sets of displayed data are the results as they came from the computer [Fig. 1(a)] and the results obtained from the latter by averaging the values for the scattering angles θ and $-\theta$ [Fig. 1(b)]. These two sets of results are not seen to differ significantly. They show that the dominant contribution to $\langle\partial R/\partial\Omega\rangle_{\rm incoh}$ is from the double-scattering process. We also see a well-defined peak in the retroreflection direction in the double-scattering contribution, a weaker and broader peak in the triple-scattering contribution, but no

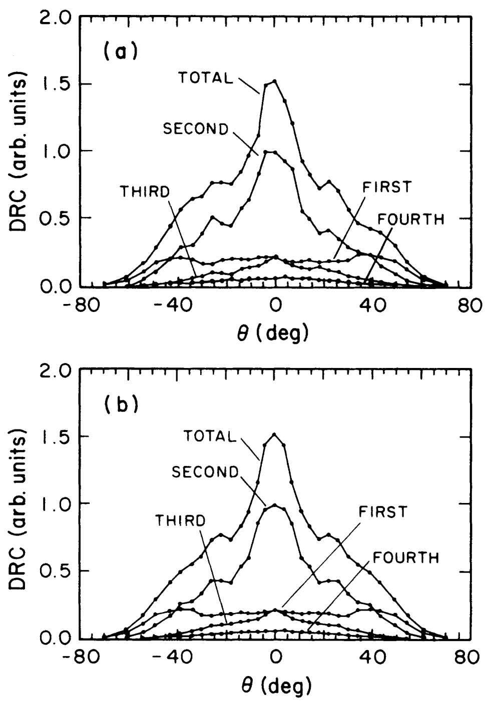

FIG. 1. The contribution to the mean differential reflection coefficient from the incoherent component of the scattered field, with the contributions from the individual scattering processes indicated. (a) Unsymmetrized results; (b) symmetrized results.

peak in the single- and quadruple-scattering contributions. The large angular width of the enhanced backscattering peak is due to the spread in **K** of our incident beam. For the case under consideration the spread corresponds to an angular width of 9°. Subsidiary maxima are also seen on both sides of this peak. However, the present results do not confirm their existence because their amplitudes are within the statistical fluctuations of our results.

In summary, we have carried out a multiple-scattering calculation of the interaction of a scalar beam with a two-dimensional, randomly rough hard wall. The results predict enhanced backscattering, and suggest the presence of subsidiary maxima in the differential scattering cross section. The mechanism responsible for both features is believed to be the coherent interference of each multiply scattered beam path with its time-reversed partner. In this context the absence of a peak in the backscattering direction in the quadruple-scattering contribution to the mean differential reflection coefficient is of interest since such a peak is expected in the contribution from each n-fold scattering process for $n \geq 2$. However, from our results it seems as if the width of the enhanced backscattering peak increases with the order of the scattering process, so that its width in the case of quadruple scattering could be great enough for it to be indistinguishable from the background.

The two ingredients in the present calculation that have contributed to its success are the use of a narrow incident beam, made possible by the use of Eq. (13), and the use of an iterative approach to the solution of Eq. (9), which made it possible to solve the large (4096×4096) matrix equations involved with a reasonable expenditure of computer time. However, it should be noted that although the results presented in Fig. 1 show that the contributions to the mean differential reflection coefficient from the successive terms in the iterative solution of Eq. (9) are monotonically decreasing past the double-scattering contribution, we have not addressed the question of the convergence of the iterative expansion. That is left to subsequent work. Nevertheless, the results presented here indicate that the approach used here is a viable one for the study of the scattering of a scalar beam from a large-rms-height and large-rms-slope, two-dimensional, random hard wall.

This research was supported in part by Army Research Office Grant No. DAAL-88-K-0067. It was also supported by the University of California, Irvine, through an allocation of computer time.

[1] A. R. McGurn, A. A. Maradudin, and V. Celli, Phys. Rev. B **31**, 4866 (1985).
[2] V. Celli, A. A. Maradudin, A. M. Marvin, and A. R. McGurn, J. Opt. Soc. Am. A **2**, 2225 (1985).
[3] M. Nieto-Vesperinas and J. M. Soto-Crespo, Opt. Lett. **12**, 979 (1987).
[4] P. Tran and V. Celli, J. Opt. Soc. Am. A **5**, 1635 (1988).
[5] A. A. Maradudin, E. R. Méndez, and T. Michel, Opt. Lett. **14**, 151 (1989).
[6] T. Michel, A. A. Maradudin, and E. R. Méndez, J. Opt. Soc. Am. B **6**, 2438 (1989).
[7] M. Nieto-Vesperinas and J. M. Soto-Crespo, Phys. Rev. B **39**, 8193 (1989).
[8] A. A. Maradudin, E. R. Méndez, and T. Michel, in *Scattering in Volumes and Surfaces*, edited by M. Nieto-Vesperinas and J. C. Dainty (North-Holland, Amsterdam, 1990), p. 157.
[9] A. A. Maradudin, T. Michel, A. R. McGurn, and E. R. Méndez, Ann. Phys. (N.Y.) **203**, 255 (1990).
[10] Jun Q. Lu, A. A. Maradudin, and T. Michel, J. Opt. Soc. Am. B **8**, 311 (1991).
[11] A. R. McGurn, A. A. Maradudin, and R. F. Wallis, Waves in Random Media **1**, 43 (1991).
[12] A. A. Maradudin, Jun Q. Lu, T. Michel, Zu-Han Gu, J. C. Dainty, A. J. Sant, E. R. Méndez, and M. Nieto-Vesperinas, Waves in Random Media **1**, S129 (1991).
[13] Jun Q. Lu, A. A. Maradudin, and R. F. Wallis, Waves in Random Media **4**, 309 (1991).
[14] M. Nieto-Vesperinas, J. A. Sánchez-Gil, and A. A. Maradudin, Waves in Random Media **1**, S157 (1991).
[15] C. Macaskill, J. Opt. Soc. Am. A **8**, 88 (1991).
[16] A. R. McGurn and A. A. Maradudin, J. Opt. Soc. Am. B **4**, 910 (1987).
[17] V. Celli, P. Tran, A. A. Maradudin, Jun Lu, T. Michel, and Zu-Han Gu, in *Laser Optics of Condensed Matter Vol. 2*,

edited by E. Garmire, A. A. Maradudin, and K. K. Rebane (Plenum, New York, 1991), p. 315.

[18]P. Tran, A. A. Maradudin, and V. Celli, J. Opt. Soc. Am. B **8**, 1526 (1991).

[19]T. A. Antsygina, V. D. Freylikher, S. A. Gredeskul, L. A. Pastur, and V. A. Slyusarev, J. Electr. Waves Appl. **5**, 873 (1991).

[20]A study of the enhanced backscattering of light from a two-dimensional, random metal surface that is free from these restrictions is the double Kirchhoff approximation calculation by Ya-Qiu Jin and M. Lax, Phys. Rev. B **42**, 9819 (1990).

[21]M. Born and E. Wolf, *Principles of Optics*, 2nd ed. (Macmillan, New York, 1964), p. 377.

[22]F. Toigo, A. Marvin, V. Celli, and N. R. Hill, Phys. Rev. B **15**, 5618 (1977).

[23]A. W. Maue, Z. Phys. **126**, 601 (1949).

[24]N. García and E. Stoll, Phys. Rev. Lett. **52**, 1798 (1984).