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Conservation of momentum, and its consequences, in interband resonant tunneling

E. E. Mendez and J. Nocera
IBM Research Division, Thomas J. Watson Research Center, P.O. Box 218, Yorktown Heights, New York 10598

W. I. Wang
Department of Electrical Engineering, Columbia University, New York, New York 10027

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We have observed a drastic change with temperature (300–4 K) in the line shape of the current-voltage characteristics of GaSb-AlSb-InAs-AlSb-GaSb heterostructures, which below $\simeq 50$ K develop a kink at low biases that is most pronounced at 4 K. This behavior is a direct consequence of the conservation of parallel momentum of electrons and holes participating in interband resonant tunneling, a conservation law that is apparent only at low temperatures because of the small hole Fermi energy. Confirmation of this interpretation is provided by experiments under a magnetic field perpendicular to the tunnel current that show a large peak in the conductance at 6 T. This is the field at which the hole distribution has gained enough parallel momentum to equal the Fermi momentum of electrons.

Resonant tunneling in semiconductor heterostructures, like $Ga_{1-x}Al_xAs$-GaAs-$Ga_{1-y}Al_yAs$ between heavily doped GaAs electrodes, is governed by the conservation of total energy and parallel (to the interfaces) momentum of the carriers—either electrons or holes—participating in the process. These laws are responsible for the line shape of the current-voltage (I-V) characteristics: Energy conservation determines the onset voltage of the tunneling current, while the conservation of momentum leads to current cutoff and the emblematic negative differential resistance of resonant tunneling.[1] Thus, once temperature is decreased and thermionic currents become negligible, quasitriangular line shapes are observable without any significant change in the 77–4 K temperature range.

Recently, there have been reports of resonant interband tunneling experiments, at room temperature and 77 K (Refs. 2 and 3), simultaneously involving electrons in the conduction band of InAs and holes from the valence band of GaSb. In that temperature interval, current flows resonantly between two electrodes (e.g., GaSb) via electronic states in a thin quantum well (e.g., InAs) as soon as a voltage difference is applied to them. Tunneling stops, and negative differential resistance occurs, at a bias such that the valence-band edge of GaSb is in the band gap of InAs. Energy considerations are then essential to the I-V characteristics of resonant interband tunneling, but the conservation of momentum does not seem to play any conspicuous role in the process.

In this paper we show that, contrary to this simple conclusion valid at least down to 77 K, momentum conservation is manifested directly in a peculiar line shape of the low-temperature ($T \leq 30$ K) I-V characteristics, and in the tunnel conductance under a magnetic field transverse to the current. In spite of the similarities of type-II interband resonant tunneling with conventional p-n tunneling, these effects are unique to the former because the process takes place between a quasi-three-dimensional (3D) electrode and a two-dimensional (2D) region.

Interband tunneling through a "barrier" material (such as AlSb) occurs when the top of the valence band of a semiconductor (let us say, GaSb) on one side of the barrier is higher than the bottom of the conduction band of a semiconductor on the other side (for example, InAs). By combining and varying the order of the components of this building block numerous configurations are possible, and several of them have been implemented already.[2-4] Here, for simplicity, we will focus on GaSb-AlSb-InAs-AlSb-GaSb, whose band structure in the neighborhood of the Fermi level is shown in Fig. 1, for 40-Å AlSb barriers and a 150-Å InAs quantum well.[5]

A charge-transfer process from the valence band of GaSb to the conduction band of InAs gives rise to the accumulation of electrons in the InAs quantum well and holes at the two GaSb interfaces. Ideally the number of holes in each interface is half the amount of electrons in the quantum well. In practice, however, slight differences between the two barriers, residual doping, interface states, etc. can introduce departures from that ideal charge distribution. Previous magnetotunneling experiments[5] yield-

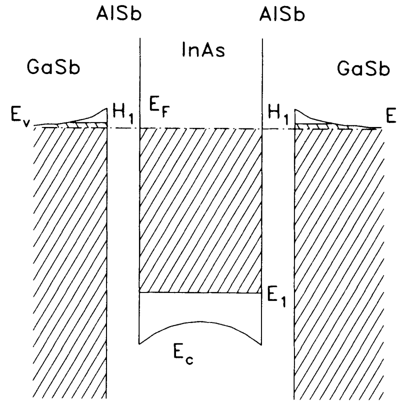

FIG. 1. Energy-band profile of a GaSb-AlSb-InAs-AlSb-GaSb heterostructure in the proximity of the Fermi level. The profile, for a 150-Å InAs quantum well and 40-Å AlSb barriers, was calculated solving Poisson's equation. The energy position of the ground state in the conduction band of InAs and in the accumulation layer of the valence band of GaSb, E_1 and H_1, respectively, is only schematic.

ed a 2D electron density N_s of 1.2×10^{12} for the structure depicted in Fig. 1. Although measurements under hydrostatic pressure suggest that the number of holes per interface is somewhat less than half the total number of electrons,[6] in the subsequent analysis we will assume ideal conditions, which simplifies the arguments and still provides valid conclusions. Taking into account the nonparabolicity of the conduction band in InAs, treating the effects of band bending by perturbation theory, and using standard materials parameters,[7] a simple calculation yields the following values for the relevant energies: 29 meV for the ground-state energy E_1 in the InAs quantum well, relative to the conduction-band edge at its center; 95 meV for the Fermi energy E_F relative to E_1; and 4.4 meV for the quantized heavy-hole level H_1, with respect to the Fermi level.

In principle, the holes in GaSb should be considered strictly 2D, but since their quantization energy is very small, for purposes of conservation of momentum only we will treat them as 3D. This assumption is justified, at least from the experimental point of view: If the behavior of those holes were 2D the I-V characteristic would have a delta-function-like shape, in contrast with the observed shape (see below). The dispersion relation of holes, along with that of electrons, is shown in Fig. 2(a). It is important to note that because of the double degeneracy of the holes (since there are two identical interfaces), their Fermi wave vector, k_{FH}, is smaller than that of electrons, k_{FE}. (For strictly 2D holes it would be $k_{FH}=k_{FE}/\sqrt{2}$.)

It follows from this fact that (at $T=0$ K) when a small bias is applied between the GaSb electrodes and the emitter goes down in energy relative to the InAs well, tunneling is not possible if parallel momentum, $k_\parallel$, is to be conserved, since the maximum momentum of the holes is significantly less than the parallel momentum of electrons with the same total energy. With a further increase in bias, tunneling is still prevented until holes at the quasi-Fermi level have the same energy as electrons with the hole Fermi wave vector [condition 1 in Fig. 2(a)]. From then on, 3D holes can tunnel through 2D electron states provided their total momentum is larger than the momentum of electrons with the same energy. ($k_\parallel$ is conserved since holes can "accommodate" the difference as momentum in the tunneling direction.) The current increases monotonically with increasing voltage, as the number of available hole states [indicated by the thick-line regions between the dotted curves in Fig. 2(a)] increases. The maximum tunnel current is reached when the quasi-Fermi level of the holes is aligned with the bottom of the E_1 subband. Any subsequent increase in voltage leads to a monotonic reduction of the current until it vanishes when H_1 reaches E_1 and no electron and hole states have the same energy. A sketch of the expected I-V characteristics is shown in Fig. 2(b).

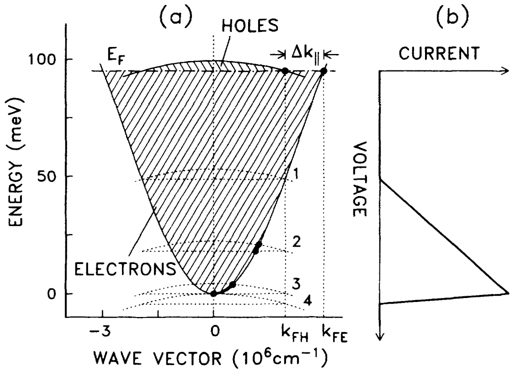

FIG. 2. (a) Energy dispersion relations for electrons in the InAs quantum well and holes in the accumulation region of the GaSb electrodes, for the structure of Fig. 1. In the figure, nonparabolicity was included to draw the dispersion of electrons; for simplicity, however, the calculations described in the text were carried out with parabolic bands. $\Delta k_\parallel$ represents the difference between the electron and hole Fermi wave vectors at $V=0$. The dotted parabolas correspond to the dispersion of holes under different biases between the electrode and the quantum well. (b) Schematic representation of the expected current-voltage characteristics, taking into account the conservation of parallel momentum of holes during tunneling.

This qualitative description of the tunneling process can be expressed analytically for $T=0$ K using a very simple model in which both the electron and hole dispersion relations are considered parabolic. After a little algebra it is easy to show[8] that the tunnel current density J as a function of the voltage drop V between an electrode and the quantum well can be expressed by

$$J=\begin{cases}0, & 0\le eV\le eV_{\rm th}\,,\\ A(m_e/m_h)(eV-eV_{\rm th}), & eV_{\rm th}\le eV\le E_F\,,\\ A(\Delta-eV), & E_F\le eV\le\Delta\,,\\ 0, & \Delta\le eV\,,\end{cases}\tag{1}$$

where

$$V_{\rm th}=E_F-(\Delta-E_F)m_h/m_e\,,$$

$$A=eT_0^2\Gamma\mu/2\pi\hbar^3\,,$$

$$\frac{1}{\mu}=\frac{1}{m_e}+\frac{1}{m_h}\,,$$

and $\Delta=E_F+H_1$; m_e and m_h are the effective masses of electrons and holes, respectively, T_0 is the transmission coefficient at resonance and Γ the resonance width, and e and $\hbar$ are fundamental constants.

Equation (1) shows that beyond a threshold voltage $V_{\rm th}$ the current indeed increases linearly with voltage up to a maximum value $J_{\rm max}=A(\Delta-E_F)$ at a peak voltage $V_p=E_F/e$, and then decreases monotonically to zero in the interval between V_p and the cutoff voltage $V_{\rm off}=\Delta/e$. From the values calculated for the heterostructure described in Fig. 1, we obtain $V_{\rm th}=32$ mV, $V_p=95$ mV, and $V_{\rm off}=99$ mV. Since these quantities are measurable and they are related to basic band parameters of the InAs-GaSb system, in principle the I-V characteristics should provide direct information about those parameters. We note that any voltages derived from the experimental I-V curves should be halved before they can be related to the above. (This assumes, of course, a symmetrical double-barrier structure and no voltage drops at the electrodes.)

Departures from the ideal conductions of the model may affect drastically its predictions. First, since the heavy-hole state H_1 is close in energy to the Fermi level (4.4 meV in the calculations outlined above) a moderate temperature will give rise to a Fermi-Dirac distribution of holes with an appreciable number of them having energies well above the Fermi level and, therefore, with wave vectors even larger than k_{FE}. These holes will have enough momentum to tunnel through the quantum well and will contribute to the current even at very small voltages. The threshold voltage can then be smeared out by temperature and will disappear at a characteristic T_c such that $kT_c = \Delta E$, where ΔE is the incremental energy that a hole at the Fermi level needs to acquire a momentum equal to k_{FE}. Second, a difference between the total number of electrons and holes may alter the threshold voltage V_{th}. If, for example, the "extrinsic" number of holes in the GaSb electrode is substantial, so that the unbalance between electrons and holes is large, the Fermi wave vector of the latter may be comparable or even larger than that of the former, thus reducing or even eliminating completely the threshold voltage.

Experimentally the I-V curves of type-II interband tunneling structures have not shown any evidence of a threshold behavior at either 300 or 77 K; at 4 K, however, a kink in their I-V characteristics has recently been observed.[5] A systematic study of the temperature dependence, summarized in Fig. 3, illustrates the transition between the two behaviors. The curves of Fig. 3 correspond to a heterostructure consisting of a 150-Å InAs quantum well between 40-Å AlSb barriers, clad by thick GaSb electrodes p-type doped gradually from 1×10^{17} to 2×10^{18} cm^{-3}. (More details about this structure can be found in Ref. 5.)

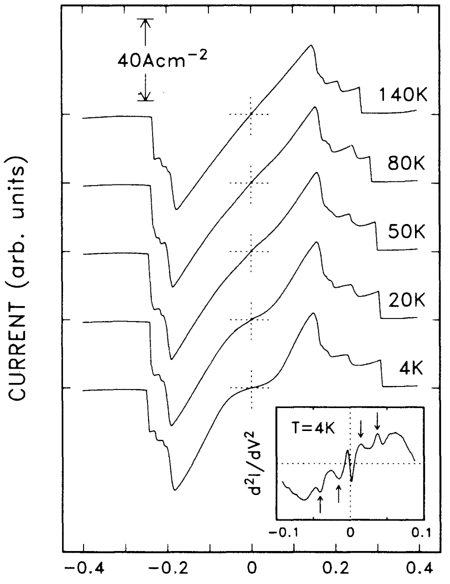

FIG. 3. Experimental current-voltage characteristics for several temperatures, of a heterostructure like the one described in Fig. 1. Note the kink that develops at $\simeq$0.05 V at temperatures below 50 K, and which represents the onset of momentum-conserving tunneling. Below 20 K, the conductance exhibits unexplained fine structure at low bias, as it is apparent in the inset of the figure, which show the second derivative of the current.

At high temperatures (140 K and above) the current increases linearly with voltage and then drops abruptly at a bias that varies slightly with temperature and bias polarity, but in all cases is between 0.15 and 0.20 V. As the temperature decreases, the conductance at small biases decreases gradually and at about 30 K the I-V characteristics develop a kink at $\simeq$0.04 V, which is most apparent at 4 K. (No significant change was observed between 4 and 0.4 K.) Beyond the negative-differential-resistance region the current remains low as the voltage is increased beyond 0.3 V, before it finally takes over at very high biases. The low current levels are particularly noticeable below 140 K, with an overall peak-to-valley current ratio exceeding 100 to 1 at 4 K. (At 300 K that ratio was 8 to 1.)

We assign the formation at low temperature of the kink in the I-V characteristics to the conservation of parallel momentum of the holes during the tunneling process, as discussed in detail above. At high T, the thermal energy available is sufficient to provide to the holes the additional momentum they need to tunnel through electron states in InAs. When T falls below a characteristic temperature that energy is not available anymore and momentum-conserving tunneling cannot occur at small biases. From the gradual transition observed, we find T_c to be between 50 and 30 K, which agrees quite well with our estimate of 40 K for a 4.4-meV hole Fermi energy. It is the smallness of this energy that limits the manifestation of the conservation of momentum to relatively low temperatures.

The low-temperature conductance near $V=0$ is not zero, however, and is T independent below $\simeq$8 K. This finite conductance, even at 0.4 K, suggests the presence of some inelastic tunneling. Possible sources of the latter are the various material interfaces and phonon emission. This residual current will deserve a more detailed look in the future; within the scope of this work it is at least worth mentioning the presence of some distinct features in the second derivative of the current with respect to the applied voltage, as shown in the inset of Fig. 3. These features, which appear at about the same bias for both polarities, are only observable below 20 K. Their origin is not clear, but, in any case, it seems difficult to correlate the low voltages at which they occur with any electronic-related process, especially in view of the fact that the application of hydrostatic pressure did not modify substantially either their strength or their voltage position. The corresponding energies, 8 and 19 meV (assuming that the voltage is divided equally between the two AlSb barriers and that the voltage drop at each electrode is negligible), are more typical of phonons, be they acoustical or optical. In this regard, it may be more than a simple coincidence that, e.g., the energy of an AlSb acoustical phonon at the X-point zone boundary is 8 meV, especially since AlSb, the barrier material, is an indirect semiconductor whose conduction-band minimum occurs at the X point.

A comparison between the calculated threshold and peak voltages and the experimental results is in order. The absence of a sharp turn-on voltage in the I-V characteristics, even at the lowest temperature, makes it difficult to determine precisely the threshold voltage. Thus, if we extrapolate the linear portion of the characteristics to zero current we get a value of 0.05 V, but if the threshold is defined by the onset of the linear region then we obtain 0.07 V. These values, after being halved, are comparable to the predicted threshold voltage of 32 mV. The experimental peak voltage, in the range 0.15–0.20 V, agrees reasonably well with the calculated voltage of 95 mV.

Another important consequence of the conservation of parallel momentum is revealed in tunneling measurements under a magnetic field H, perpendicular to the current; that is, parallel to the material interfaces. Under these conditions the peak current and the peak voltage decrease with increasing magnetic field, as reported before.[9] We focus here, however, on a drastic change with field in the conductance at very small biases. For example, at $V=0$ (see Fig. 4) the conductance increases quite abruptly above 4 T and peaks at $\simeq 6$ T, a field beyond which it decreases monotonically. Similar behavior is observed for small, finite biases.

This result can be easily understood in the framework of the momentum-conservation law. As discussed above, at $V=0$ and $H=0$ the holes at the emitter electrode do not have enough $k_\parallel$ to tunnel through the electron states in the well, and therefore the conductance is small (ideally zero). However, a transverse field provides additional parallel momentum $\Delta k_\parallel$ and the hole distribution shifts to the right in Fig. 2, so that beyond a threshold field, $H_{\rm th}$, some holes will have a $k_\parallel$ equal to or larger than $k_{\rm FE}$. $H_{\rm th}$ can be estimated using a simple semiclassical argument. Since the transverse momentum gained by a charged particle in a field H along a path of length Δl is $\Delta p = eH\Delta l$, it is $H_{\rm th} = \hbar(k_{\rm FE} - k_{\rm FH})/e\Delta l$, where Δl is the distance from the "center" of the accumulation layer in the emitter to the center of the well.

Using appropriate values for the heterostructure discussed here, one obtains $H_{\rm th} = 3.9$ T, in good agreement with the experimental threshold around 4 T. With increasing $H > H_{\rm th}$ the conductance increases, as more holes participate in tunneling, until it reaches a maximum when the center of the hole distribution (with $k_\parallel = 0$ at $H=0$) reaches $k_\parallel = k_{\rm FE}$. From then on, the conductance is expected to decrease monotonically with increasing H. This behavior is observed experimentally in Fig. 2, although the conductance peak occurs at a field lower than predicted. Departures from ideality in the number of holes, and, more important, the failure of a semiclassical analysis (or of a perturbation approximation in quantum mechanics) when large changes of momentum are involved are probably at the core of the quantitative discrepancy.

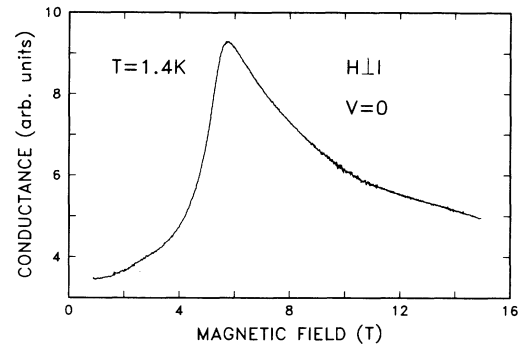

FIG. 4. Zero-bias conductance as a function of the strength of a magnetic field perpendicular to the direction of tunneling, for the heterostructure of Fig. 1. The conductance shows a prominent maximum at $\simeq 6$ T, as a result of the gain in parallel momentum of the holes by the presence of the field. The subsequent decrease in conductance is a consequence of the decrease in the number of holes that can participate in the tunneling process.

We have seen the important, although sometimes subtle, role that the conservation of parallel momentum plays in interband resonant tunneling. The momentum that holes in the electrodes of GaSb-AlSb-InAs-AlSb-GaSb heterostructures need to acquire to tunnel resonantly through the InAs quantum well can be supplied by an external voltage (giving rise to threshold-bias effects), by temperature (leading to triangular line shapes above a characteristic temperature), or by a transverse magnetic field (producing a sharp increase in the conductance beyond a critical field). A simple model is able to explain reasonably well the experiments that manifest the conservation of momentum. Undoubtedly, a more complete theory, which we hope this work will stimulate, would shed additional light to the details of the tunneling process.

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[7] We have used the following values throughout the text: for InAs, $0.023m_0$ (m_0 is the free-electron mass) for the conduction-edge effective mass, 0.41 eV for the energy gap, 0.38 eV for the spin splitting of the valence band, and 14.6 for the dielectric constant; for GaSb $0.33m_0$ for the heavy-hole effective mass and 15.7 for the dielectric constant.

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