Stable hc/e vortices in a gauge theory of superconductivity in strongly correlated systems

Subir Sachdev
Center for Theoretical Physics, P.O. Box 6666, Yale University, New Haven, Connecticut 06511
(Received 15 May 1991)

A phenomenological model F of the superconducting phase of electronic systems with strong short-range repulsive interactions is studied. Fluctuations of the connection $\mathbf{a}$ of an internal U(1) gauge symmetry suppress local charge fluctuations. Above H_{c1}, magnetic flux can pierce the superconductor in vortices with flux $hc/2e$ throughout the superconducting phase, but regimes are found in which the lowest-energy configuration has vortices with flux hc/e. Experiments by Little and Parks and others, which examine periodicities as a function of a *varying* magnetic field, always observe a period in external flux of $hc/2e$. The low-energy properties of a symplectic large-N expansion of a model of the CuO_2 layers of the cuprate superconductors are shown to be well described by F. This analysis and some normal-state properties of the cuprates suggest that hc/e vortices should be stable at the lowest dopings away from the insulating state at which superconductivity first occurs, unless the superconductor-normal transition is strongly first order.

I. INTRODUCTION

Much attention has recently focused on the anomalous normal-state properties of the cuprate superconductors.[1,2] A promising description of the high-temperature state of these materials has emerged from recent gauge theories.[3] These theories[2-4] began by assuming a separation of the spin and charge degrees of freedom, but nevertheless found that they were strongly coupled by an internal U(1) gauge symmetry [hereafter denoted $U_I(1)$] and the associated gauge connection $\mathbf{a}$. In contrast, few unusual properties of the superconducting state have been suggested. Experimentally, too, the superconducting state appears rather conventional, with most of its properties well described within the usual phenomenological Landau-Ginzburg framework of a charge $2e$ order parameter. This paper will address the issue of whether the unusual normal-state properties of these systems have any remnant in the superconducting phase. We will do this within the framework of a phenomenological model F which incorporates the $U_I(1)$ gauge symmetry; a closely related phenomenological free energy for superconductors with broken time-reversal invariance has been discussed by Wen and Zee[4]—time-reversal invariance will be assumed to be preserved in this paper. Our main result will be the existence of parameter regimes in which the lowest-energy mechanism for magnetic flux to pierce the system is with vortices carrying flux hc/e: the cores of these vortices lose superconducting coherence without a decrease in antiferromagnetic correlations. Magnetic flux can also penetrate in vortices with flux $hc/2e$, but such configurations are not always globally stable. A previous symplectic large-N expansion.[5,6] of a model of the CuO_2 layers[7] suggested that the region of stability of the hc/e vortices is the low-doping boundary of the superconducting state, i.e., the superconducting region closest to the half-filled insulating state. We will argue that this conclusion is also supported by differences between NMR experiments on the normal state in the small- and large-doping regions.[8] However, a strong first-order superconductor-normal transition could preempt the existence of stable hc/e vortices. Flux decoration experiments of these "low" T_c superconductors will therefore be of great interest.

An important property of the model of this paper is that the preference for hc/e vortices is purely energetic. The fundamental "flux-quantum" remains at $hc/2e$. In particular, experiments which examine periodicities as a function of a *varying* magnetic field observe a period in total magnetic flux of $hc/2e$ throughout the superconducting phase (see Sec. II B). One such experiment is that of Little and Parks[9] which measures shifts in T_c of a thin-walled superconducting cylinder in an axial magnetic field.

The crucial ingredient of the gauge theories[1-3] of the normal state is the representation[10,11] of the creation operator $d_{i\alpha}^\dagger$ for holes on the Cu d orbital in the following form:

$$d_{i\alpha}^\dagger = f_{i\alpha}^\dagger b_i \ , \tag{1.1}$$

where i is a site label, $\alpha = \uparrow, \downarrow$ is the spin index, f is a fermion annihilation operator, and b a boson annihilation operator. The local constraint

$$b_i^\dagger b_i + f_{i\alpha}^\dagger f_i^\alpha = 1 \tag{1.2}$$

projects out states with two holes on a Cu orbital; $b^\dagger b$ is the number operator for sites with no holes. It was argued in Ref. 3 that at sufficiently high temperature the b and f^α quanta behave like independent excitations, leading to a separation of the spin and charge degrees of freedom.

At lower temperatures we expect the spin-singlet pairing correlations induced by the antiferromagnetic exchange interactions to become important. We therefore introduce the pairing amplitude

$$\Delta_{ij} = \langle \varepsilon^{\alpha\beta} f_{i\alpha}^\dagger f_{j\beta}^\dagger \rangle \ , \tag{1.3}$$

where i,j are nearby sites. The antiferromagnetic exchange interaction between the Cu orbitals will promote the existence of large values of Δ. A nonzero local value of Δ measures the amplitude for the existence of a singlet bond, but does not imply the existence of superconductivity—this point will be discussed in greater detail below. Superconductivity requires in addition the condensation of b. In contrast to earlier assertions[12] we have shown elsewhere[7] the condensation of single b quanta (and not just pairs of b quanta) occurs in the presence of incommensurate spin correlations—incommensurate correlations have recently been observed in neutron scattering experiments.[13] The phenomenological free energy F will therefore be expressed in terms of the condensates Δ and b.

The form of the phenomenological free-energy-controlling fluctuations of the fields Δ and b is essentially dictated by gauge invariance. In the continuum limit, it is easily shown that the action must be invariant under

$$\begin{aligned} &f^\dagger \to f^\dagger \exp(i\chi - ie\nu\omega) , \\ &\Delta \to \Delta \exp(2i\chi - 2i\nu\omega) , \\ &b \to b \exp[-i\chi - ie(1-\nu)\omega] , \\ &d^\dagger \to d^\dagger \exp(-ie\omega) , \\ &\mathbf{a} \to \mathbf{a} - \nabla\chi , \quad \mathbf{A} \to \mathbf{A} - \nabla\omega . \end{aligned} \tag{1.4}$$

Here χ generates the internal $U_I(1)$ gauge symmetry introduced by the decomposition (1.1) and $\mathbf{a}$ is the associated vector potential. The electromagnetic gauge symmetry $U_{em}(1)$ is generated by ω and its vector potential is $\mathbf{A}$. We have absorbed a factor of $1/(\hbar c)$ in the magnitude of e. The d fermion carries electromagnetic charge e; we have placed νe of this charge on the f fermion and the remainder $(1-\nu)e$ on the b boson. At this point the parameter ν is arbitrary and the properties of the theory should be independent of its value. The physical superconducting order parameter $\Psi_{ij}^{SC} = \langle \varepsilon^{\alpha\beta} d_{i\alpha}^\dagger d_{j\beta}^\dagger \rangle$ is of course invariant under $U_I(1)$ and transforms like a charge $2e$ scalar field $\Psi^{SC} \to \Psi^{SC} \exp(-2ie\omega)$ under $U_{em}(1)$.

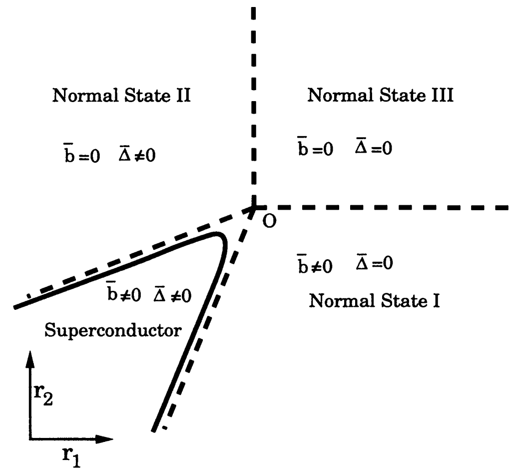

FIG. 1. Mean-field phase diagram of F as a function of r_1 and r_2 for $v>0$ and $v^2<u_1u_2$. The point O is $r_1=0$ and $r_2=0$. The mean-field phase transitions are shown by dashed lines. The boundaries of the superconducting phase are given by $\bar{r}_1=r_1-(r_2v)/u_2=0$ and $\bar{r}_2=r_2-(r_1v)/u_1=0$. The expected location of the superconductor-normal transition in the presence of fluctuations is shown by the solid line; the various normal states only have quantitative differences in their properties and are expected to be connected by smooth crossovers in $d=2$. The region of stability of the hc/e vortices is close to the superconductor–normal-state II phase boundary and well away from the superconductivity–normal-state I boundary.

In, or close to, the superconducting phase we expect that the fermions can be safely integrated out and the system described solely in terms of Δ, b and $\mathbf{a}$. The invariances (1.4) dictate that their action for static fluctuations be of the following form:

$$\frac{F}{k_BT} = \int d^2r \left[|(\nabla + 2i\mathbf{a} - 2ie\nu\mathbf{A})\Delta|^2 + r_1|\Delta|^2 + \frac{u_1}{2}|\Delta|^4 + |[\nabla - i\mathbf{a} - ie(1-\nu)\mathbf{A}]b|^2 + r_2|b|^2 + \frac{u_2}{2}|b|^4 + v|b|^2|\Delta|^2 + \frac{1}{8\pi}(\nabla\times\mathbf{A})^2 + \frac{\sigma}{2}(\nabla\times\mathbf{a})^2 + \cdots \right] \tag{1.5}$$

with u_1, $u_2>0$ and $v^2<u_1u_2$. The fields Δ and b have been rescaled to make the coefficients of their gradient terms unity. We expect the cross coupling $v>0$ because the constraint (1.2) suggests that $|\Delta|$ becomes larger when $|b|$ becomes small and vice versa. The parameters r_1, r_2, u_1, u_2, v, and $1/\sigma$ all have the dimensions of $1/L^2$ (L is the unit of length) and are expected to be of roughly the same order of magnitude; an exception to this is the region close to the superconductor-normal phase boundary when a combination determining the superconducting coherence length (see Sec. II) will become large. The electric charge e has dimensions of $1/L$; we will study only strong type-II superconductors, in which case $4\pi e^2 \ll u_1, u_2, 1/\sigma$. It is the inequality $4\pi e^2 \ll 1/\sigma$ which distinguishes the role of $U_{em}(1)$ and $U_I(1)$: it implies that the fluctuations of $\mathbf{A}$ are almost pure gauge while the flux $\nabla\times\mathbf{a}$ is strongly fluctuating. The connection between F and a symplectic large-N expansion[7] on a realistic microscopic model of the CuO_2 layers will be discussed in Sec. IV; this analysis will give some information on the variation of the parameters in F with temperature and doping.

A cross term $(\nabla\times\mathbf{a})\cdot(\nabla\times\mathbf{A})$ in F is also permitted by the gauge symmetries of (1.4). However we can use the freedom in the value of ν to adjust the coefficient of this cross term to 0. Application of this criterion to the large-N expansion in Sec. IV yields the value $\nu=0$; this corresponds to complete spin-charge separation as the electromagnetic charge is carried completely by the b quanta. However, it is clear that the more general context of Landau theory also permits partial separation of spin and charge with the values $0<\nu<1$.

We also introduce the gauge invariant currents

$$\mathbf{J}_\Delta = \frac{1}{i}\,\mathrm{Im}[\Delta^*(\nabla+2i\mathbf{a}-2ie\nu\mathbf{A})\Delta]\ ,$$
$$\mathbf{J}_b = \frac{1}{i}\,\mathrm{Im}\{b^*[\nabla-i\mathbf{a}-ie(1-\nu)\mathbf{A}]b\}\ . \tag{1.6}$$

Upon examining variations of F with respect to $\mathbf{A}$, the electromagnetic supercurrent is easily seen to be $\mathbf{J}_{\mathrm{em}}=-e(1-\nu)\mathbf{J}_b-2e\nu J_\Delta$. Stationarity of F with respect to variations in $\mathbf{a}$ leads to the condition

$$-2\mathbf{J}_\Delta+\mathbf{J}_b=\sigma\nabla\times(\nabla\times\mathbf{a})\ . \tag{1.7}$$

This equation can be interpreted as the consequence of the local constraint (1.2); the terms on the left-hand side represent the current of pairs of f^α fermions and the boson current, respectively, while the right-hand side is the current of the single f^α fermions which have been integrated out.

We now discuss qualitative features of the phase of F in the simplest mean-field theory which ignores the fluctuations of the gauge fields. The results of a minimization of F with respect to the mean-field values $\Delta=\bar{\Delta}$ and $b=\bar{b}$ are shown in Fig. 1 as a function of r_1 and r_2. At the mean-field level, the point $r_1=0$, $r_2=0$ behaves like a tetracritical point[14] with four regions converging upon it. These four regions are characterized by finite or zero values of $|\bar{\Delta}|$ and $|\bar{b}|$; the existence of these four regions was also noted by Wen and Zee.[4] We discuss the four regions, and the nature of the boundaries between them, in turn.

(i) Superconductor. Only the region in which both $|\bar{\Delta}$ and $|\bar{b}|$ are nonzero is superconducting as $\Psi^{SC}\sim\bar{\Delta}\bar{b}^2$. All other regions are "normal" and do not display a Meissner effect for $\mathbf{A}$. The boundaries between the superconductor and its neighboring regions are thus true phase transitions. The normal phases however display important quantitative differences in their properties.

(ii) Normal-state I. This is the region with only $\bar{b}$ nonzero and is most like a conventional Fermi liquid. We will show in Sec. II C that the transition between this region and the superconducting phase is well described by the fluctuations of a scalar, Ψ_{2e}, which is invariant under $U_I(1)$ and carries electromagnetic charge $2e$.

(iii) Normal-state II. Here only $\bar{\Delta}$ is nonzero. This is expected to lead to a pseudogap feature in the f^α fermion spectrum and a suppression of the spin susceptibility. The transition between this phase and the superconductor is shown in Sec. II D to be controlled by the fluctuations of a scalar, Ψ_e, which is invariant under $U_I(1)$ and carries electromagnetic charge e. We expect F to display a smooth crossover in the superconductor-normal transition between regimes dominated by fluctuations of scalars with charge e and $2e$ as one passes from normal-state II to normal-state I.

(iv) Normal-state III. Now both the mean-field values $\bar{b}$ and $\bar{\Delta}$ are zero. The novel properties of this region have already been examined by Nagaosa and Lee and Ioffe and Weigmann.[3]

The consequences of gauge-field fluctuations upon the transitions between normal-states I, II, and III are expected to be significantly different from those between the superconductor and the normal states. The superconducting order will be coherent between the CuO_2 layers and the critical fluctuations near the superconductor-normal transition will be three dimensional. In contrast the $U_I(1)$ gauge connection can only be defined within each layer; fluctuations between the normal states are therefore described by a two-dimensional Abelian Higgs model which is expected to possess a smooth crossover and not a phase transition.[15] The nonlocal order parameter construction,[16] which demands the existence of a phase transition between the Higgs and normal phases, fails in $d=2$. Of course, none of the above considerations rule out a first-order transition between the normal states.

NMR data of the Cu Knight shift in $YBa_2Cu_3O_{6.5+\delta}$ for $\delta\sim0.1$ (Ref. 8) shows a strong temperature dependent suppression of the spin susceptibility at temperatures above the superconducting T_c. This is consistent with these compositions and temperatures being identified as normal-state II. At larger dopings near $\delta\sim0.5$, the spin susceptibility of the nonsuperconducting phase is temperature independent, consistent with the properties of normal-state I. I am grateful to A. Millis for drawing my attention to this data. Finally, the normal-state-III region is expected to appear at higher temperatures at all doping concentrations.

These assignments are also consistent with the results of a previous microscopic large-N calculation on a three-band model of the CuO_2 layers;[7] this calculation is reviewed in Sec. IV and the results are summarized in Fig. 2. Note that the overall topology of the phases is consistent with the Landau theory results summarized in Fig. 1; the control parameters r_1r_2 have now been replaced by the temperature T and the doping δ. Just as was argued in the previous paragraph, we find normal-state I in the small doping region, normal-state II in the large doping region, and normal-state III at high temperatures. The transition between normal-state II and the superconductor is found to be first order at the lowest temperatures in the large-N limit (Fig. 2).

We finally turn to a discussion of the structure of votices of F in the superconducting phase. It is of course important to characterize the vortex by gauge-invariant quantities. Far from the core of the vortex, finiteness of the energy demands the configuration

$$\Delta(\mathbf{r})=\bar{\Delta}\exp[i\phi_1(\mathbf{r})]\ ,$$
$$b(\mathbf{r})=\bar{b}\exp[i\phi_2(\mathbf{r})]\ . \tag{1.8}$$

The values of the phases ϕ_1,ϕ_2 are non-gauge-invariant, but the integers n_i

$$n_i=\frac{1}{2}i\oint_C\nabla\phi_i\cdot d\mathbf{r} \tag{1.9}$$

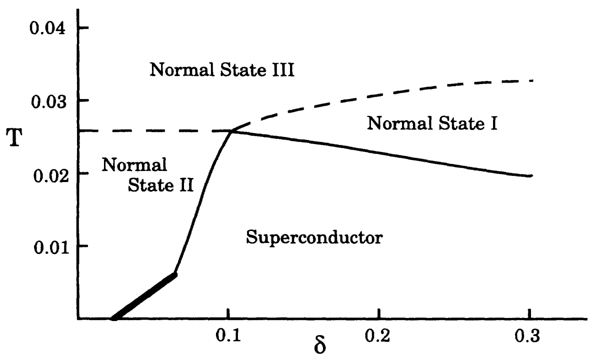

FIG. 2. Phase diagram from Ref. 7 of the symplectic large-N calculation on a model of the CuO_2 layers. The axes are the temperature T and the hole concentration δ. Notice the similarity in the topology of this figure and the Landau theory results of Fig. 1. The solid line is the only true phase transition and separates the normal states from the superconductor. The thick line at small doping denotes a first-order transition; elsewhere it is second order. The region of stability of the hc/e vortices is expected to be the superconductor closest to the normal-state II phase.

(where $i=1,2$ and the contour C encircles the core of the vortex) are invariant under nonsingular gauge transformations of both $U_I(1)$ and $U_{em}(1)$. Each pair of integers (n_1,n_2) thus defines a topologically distinct vortex configuration. The existence of a two-parameter family of vortices has already been pointed out by Wen and Zee.[4] To determine the values of the fluxes of $\mathbf{a}$ and $\mathbf{A}$, we apply the usual argument[17] for the finiteness of the vortex energy to the two gradient terms in F. This yields the constraints (after restoring factors of $\hbar c$)

$$-2\int d^2r(\nabla\times\mathbf{a})_z+\frac{2ev}{\hbar c}\int d^2r(\nabla\times\mathbf{A})_z=2\pi n_1\ ,$$
$$\int d^2r(\nabla\times\mathbf{a})_z+\frac{e(1-\nu)}{\hbar c}\int d^2r(\nabla\times\mathbf{A})_z=2\pi n_2\ , \qquad (1.10)$$

for a vortex in the x,y plane. Solving for the total electromagnetic flux we find

$$\int d^2r(\nabla\times\mathbf{A})_z=\frac{hc}{2e}(n_1+2n_2) \qquad (1.11)$$

which is independent of ν. Note that, in contrast to the conventional Abrikosov theory, the electromagnetic flux does not uniquely define a vortex configuration; there is an infinite number of choices of the integers n_1,n_2 for a given em flux.

For a vortex with flux $hc/2e$, the lowest energy configuration is obviously $n_1=1$, $n_2=0$. The existence of a nontrivial winding in the phase of Δ now demands that $|\Delta|$ vanish at the core of the vortex. A standard estimate shows that the energy cost of this core will scale linearly with $|\tilde{r}_1|$; here $\tilde{r}_1=r_1-(r_2v)/u_2$ is the renormalized "mass" of the Δ field (Sec. II C) and equals the horizontal distance to the superconductor-normal-state I boundary in Fig. 1.

For vortices with flux hc/e, the choice for the lowest energy is not *a priori* obvious: there are two reasonable candidated $n_1=2$, $n_2=0$ and $n_1=0$, $n_2=1$. The first choice is however expected, by a standard argument,[17] to be unstable to splitting into two $hc/2e$ vortices and will therefore not be considered further. The situation for the second configuration $n_1=0$, $n_2=1$ is completely different. The nontrivial phase winding now occurs solely in the phase of b; by an argument parallel to that of the previous paragraph, $|b|$ vanishes at the core and the core-energy cost will now scale linearly with $|\tilde{r}_2|$ [again $\tilde{r}_2=r_2-(r_1v)/u_1$ is the renormalized $|b|$ "mass" and equals the vertical distance to the superconductor–normal-state-II phase boundary]. In Sec. III A we will estimate the energy contribution of the region outside the core but within the London penetration depth (this a large region for a strong type-II superconductor) for both the $n_1=1$, $n_2=0$ and $n_1=0$, $n_2=1$ vortices. We find that the energy from this region is proportional to the physical electromagnetic superfluid stiffness in both cases, with the contribution to the $n_1=0$, $n_2=1$ hc/e vortex being four times larger than that to the $n_1=1$, $n_2=0$ $hc/2e$ vortex. The superfluid stiffness is in turn shown to be approximately proportional to the *smaller* of $|\tilde{r}_1|$ and $|\tilde{r}_2|$.

The estimates of the last two paragraphs suggest that a remarkable situation can develop in the parameter regime $|\tilde{r}_1|>|\tilde{r}_2|$. The energy of the $hc/2e$ vortex is dominated by the core contribution and scales linearly with $|\tilde{r}_1|$. In contrast the energy of the $(n_1=0,\ n_2=1)hc/e$ vortex scales linearly with $|\tilde{r}_2|$. It is thus entirely possible that if $|\tilde{r}_1|$ is sufficiently larger than $|\tilde{r}_2|$, then the energy of a hc/e vortex can become smaller than twice the energy of a $hc/2e$ vortex. If so, the vortices forced into the superconductor by an external magnetic field above H_{c1} will carry flux hc/e.[17] The above scenario has been confirmed by detailed numerical calculations on vortex solutions of F. Details will be discussed in Sec. III B. An experimental test of this scenario will clearly be useful. It is of course necessary to search for a regime in which $|\tilde{r}_1|>|\tilde{r}_2|$. *This is most likely in the region closest to the superconductor–normal-state-II phase boundary and well away from the superconductor–normal-state-I phase boundary.* From Figs. 1 and 2 and our earlier discussion of the NMR experiments and the large-N expansion, we conclude that the most favorable regime is near the low doping onset of superconductivity as one moves away from the insulating state. A strong first-order transition between the superconductor and normal-state II could however prevent the existence of a region in which $|\tilde{r}_1|/|\tilde{r}_2|$ is large enough; the large-N calculation did find this transition to be first order at the very lowest temperatures (Fig. 2).

The outline of the rest of the paper is as follows. In Sec. II we will examine the properties of homogeneous phases described by F. We will determine the London penetration depth in the superconducting phase and the nature of the critical fluctuations near the phase transition between the superconductor and the normal states. We will also examine the consequences of a Little-Parks[9] experiment on a system described by F and find that the results have a period in external flux of $hc/2e$ (Sec. II B).

Section III will turn to an examination of vortex solutions of F. Some exact analytic results will be obtained in the London model of the region well outside the core in Sec. III A. The results of numerical calculations are presented in Sec. III B. Section IV will review the large N calculations of Ref. 7 and outline a microscopic derivation of the action F. Finally Sec. V will conclude and recapitulate the central results.

II. PROPERTIES OF THE SUPERCONDUCTING PHASE

In this section we examine the properties of the action F [Eq. (1.5)] in or close to the superconducting phase. We will focus in particular on the response of the system to an external magnetic field and the magnitude of the London penetration depth. In Sec. II A we study the bulk response of the system in the region deep within the superconducting phase. In Sec. II B we consider the multiply connected Little-Parks[9] configuration of an axial magnetic field piercing a thin-walled superconducting cylinder. The nature of the phase transition between the superconductor and normal-states I and II will be examined in Secs. II C and II D, respectively.

A. Electromagnetic response in the bulk superconductor

Deep within the superconducting phase, it is permissible to replace Δ and b by the mean-field values $\bar{\Delta}$ and $\bar{b}$ which minimize F:

$$|\bar{\Delta}|^2=-\frac{r_1u_2-r_2v}{u_1u_2-v^2},\quad |\bar{b}|^2=-\frac{r_2u_1-r_1v}{u_1u_2-v^2}\ . \tag{2.1}$$

Inserting this into F [Eq. (1.5)], the resulting action for $\mathbf{a}$ and $\mathbf{A}$ takes the form

$$F_a=\int d^2r\left[[\mathbf{a}+e(1-v)\mathbf{A}]^2|b|^2+4(\mathbf{a}-ev\mathbf{A})^2|\bar{\Delta}|^2+\frac{1}{8\pi}(\nabla\times\mathbf{A})^2+\frac{\sigma}{2}(\nabla\times\mathbf{a})^2\right]\ . \tag{2.2}$$

We may now integrate our the massive $\mathbf{a}$ fluctuations and obtain the following effective action for the electromagnetic field for small e^2:

$$F_{\text{em}}=\int d^2r\left[\frac{1}{8\pi}\left[(\nabla\times\mathbf{A})^2+\frac{1}{\lambda^2}\mathbf{A}^2\right]\right]\ . \tag{2.3}$$

The London penetration depth λ is found to be independent of v and is given by

$$\frac{1}{\lambda^2}=8\pi e^2\frac{1}{1/|\bar{b}|^2+1/|2\bar{\Delta}|^2}\ . \tag{2.4}$$

Notice the inverse-square London penetration depth, or equivalently the superfluid density is approximately proportional to the *smaller* of $|\bar{b}|^2$ and $\bar{\Delta}|^2$. In the event that either of them vanishes, so does the superfluid density and the Meissner response.

B. Little-Parks experiment

In this section we determine the value of the "flux-quantum," as determined by a Little-Parks experiment.[18] We will find that it takes the value $hc/2e$ throughout the superconducting phase.

Consider a thin-walled superconducting cylinder of radius R with electromagnetic flux $\tilde{\Phi}_A$ along the axis of the cylinder. Ignoring the radial dependencies, we expect that the fields will take the value

$$\Delta=\bar{\Delta}\exp(in_1\theta),\quad b=\bar{b}\exp(in_2\theta)\ ,$$
$$A_\theta=\frac{\tilde{\Phi}_A}{2\pi R},\quad a_\theta=\frac{\tilde{\Phi}_a}{2\pi R}\ , \tag{2.5}$$

where θ is the angular coordinate, and the integers n_1,n_2 and the real number $\tilde{\Phi}_a$ must be chosen to minimize the value of F in the presence of the electromagnetic flux $\tilde{\Phi}_A$. Inserting (2.5) into F we find that the free-energy density F_R is

$$F_R=\frac{|2\bar{\Delta}|^2}{4\pi^2R^2}\left[\pi n_1+\tilde{\Phi}_a-\frac{ev}{\hbar c}\tilde{\Phi}_A\right]^2+\frac{|\bar{b}|^2}{4\pi^2R^2}\left[2\pi n_2-\tilde{\Phi}_a-\frac{e(1-v)}{\hbar c}\tilde{\Phi}_A\right]^2+\cdots\ , \tag{2.6}$$

where we have reinserted factors of $\hbar c$ and the omitted terms are independent of the fluxes and the phase windings n_1,n_2. Finally, we minimize F_R with respect to $\tilde{\Phi}_a$ and obtain

$$F_R=\frac{\pi^2e^2}{h^2c^2R^2}\frac{1}{1/|\bar{b}|^2+1/|2\bar{\Delta}|^2}\left[\tilde{\Phi}_A-\frac{hc}{2e}(n_1+2n_2)\right]^2\ . \tag{2.7}$$

Two important features of F_R are immediately apparent: (i) the minimum value of F_R over the set of integers n_1,n_2 is a periodic function of $\tilde{\Phi}_A$ with period $hc/2e$; (ii) the amplitude of the oscillation is proportional to the superfluid stiffness, or equivalently, the inverse London penetration depth squared [see Eq. (2.4)].

C. Superconductor – normal-state-I phase transition

As this region is well away from the region in which $|b|$ vanishes ($r_2 \ll 0$) it should be permissible to neglect magnitude fluctuations in $|b|$. We therefore fix $|b|^2$ at the value $-(r_2+v|\Delta|^2)/u_2$ which minimizes F and parametrize

$$b=\left[-\frac{r_2+v|\Delta|^2}{u_2}\right]^{1/2}e^{i\phi_2}\ . \tag{2.8}$$

Inserting this into the expression for F, and keeping only lowest order terms we obtain the following effective action:

$$F_{\Delta a}=\int d^2r\left[|(\nabla+2i\mathbf{a}-2ie\nu\mathbf{A})\Delta|^2+\tilde{r}_1|\Delta|^2+\frac{\tilde{u}_1}{2}|\Delta|^4+\frac{|r_2|}{u_2}[\nabla\phi_2-\mathbf{a}-e(1-\nu)\mathbf{A}]^2+\frac{1}{8\pi}(\nabla\times\mathbf{A})^2+\frac{\sigma}{2}(\nabla\times\mathbf{a})^2\right], \tag{2.9}$$

where $\tilde{r}_1=r_1-(r_2v)/u_2$ and $\tilde{u}_1=u_1-v^2/u_2$. The **a** field is now massive and can be integrated out. For large $|r_2|$ this now produces

$$F_\Delta=\int d^2r\left[|(\nabla+2i\nabla\phi_2-2ie\,\mathbf{A})\Delta|^2+\tilde{r}_1|\Delta|^2+\frac{\tilde{u}_1}{2}|\Delta|^4+\frac{1}{8\pi}(\nabla\times\mathbf{A})^2\right], \tag{2.10}$$

where the ν dependence has dropped out. Finally, we introduce the field

$$\Psi_{2e}=\Delta e^{2i\phi_2}. \tag{2.11}$$

Notice from the gauge transformation in Eq. (1.4) that Ψ_{2e} is invariant under $U_I(1)$ and carries electromagnetic gauge charge $2e$. In terms of Ψ_{2e}, the effective action controlling fluctuations near the superconductor–normal-state I boundary is finally

$$F_{2e}=\int d^2r\left[|\nabla-2ie\,\mathbf{A})\Psi_{2e}|^2+\tilde{r}_1|\Psi_{2e}|^2+\frac{\tilde{u}_1}{2}|\Psi_{2e}|^4+\frac{1}{8\pi}(\nabla\times\mathbf{A})^2\right] \tag{2.12}$$

which is identical to the usual Ginzburg-Landau functional for a conventional superconductor. In mean-field theory the location of the phase boundary is determined by the change of sign of $\tilde{r}_1$, which is identical with the results quoted in Sec. I and Fig. 1.

D. Superconductor–normal-state-II phase transition

The analysis is complementary to that of Sec. II C, with Δ and b interchanging roles. We neglect magnitude fluctuations in Δ ($r_1 \ll 0$) and parametrize

$$\Delta=\left[-\frac{r_1+v|b|^2}{u_1}\right]^{1/2}e^{i\phi_1}. \tag{2.13}$$

Inserting this into the expression for F, and keeping only lowest order terms, we obtain the following effective action:

$$F_{ba}=\int d^2r\left[|[\nabla-i\mathbf{a}-ie(1-\nu)\mathbf{A}]b|^2+\tilde{r}_2|b|^2+\frac{\tilde{u}_2}{2}|b|^4+\frac{|r_1|}{u_1}(\nabla\phi_1+2\mathbf{a}-2e\nu\mathbf{A})^2+\frac{1}{8\pi}(\nabla\times\mathbf{A})^2+\frac{\sigma}{2}(\nabla\times\mathbf{a})^2\right], \tag{2.14}$$

where $\tilde{r}_2=r_2-(r_1v)/u_1$ and $\tilde{u}_2=u_2-v^2/u_1$. As before, the **a** field is massive and can be integrated out. For large $|r_1|$ this now produces

$$F_b=\int d^2r\left[|(\nabla+i\nabla\phi_1/2-ie\,\mathbf{A})b|^2+\tilde{r}_2|b|^2+\frac{\tilde{u}_2}{2}|b|^4+\frac{1}{8\pi}(\nabla\times\mathbf{A})^2\right]. \tag{2.15}$$

Finally, we introduce the field

$$\Psi_e=be^{i\phi_1/2}. \tag{2.16}$$

Notice from the gauge transformation in Eq. (1.4) that Ψ_e is invariant under $U_I(1)$ but carries electromagnetic gauge charge e. In terms of Ψ_e, the effective action controlling fluctuations near the superconductor–normal-state II boundary is finally

$$F_e=\int d^2r\left[(\nabla-ie\,\mathbf{A})\Psi_e|^2+\tilde{r}_2|\Psi_e|^2+\frac{\tilde{u}_2}{2}|\Psi_e|^4+\frac{1}{8\pi}(\nabla\times\mathbf{A})^2\right] \tag{2.17}$$

which is identical to the usual Ginzburg-Landau functional for a conventional superconductor apart from the important difference that the scalar Ψ_e carries charge e. The location of the phase transition in the mean field is determined by the change of sign of $\tilde{r}_1$ (Fig. 1). As stated earlier, we expect the full action F to display a smooth crossover between the regimes described in Secs. II C and II D. Simplification of F is not possible in this crossover regime.

III. VORTICES IN THE SUPERCONDUCTING PHASE

This section will examine in detail the structure of single vortex solutions of F. It will, of course, not be possible to obtain an exact analytic solution. We will begin in Sec. III A by examining the region well outside the core: analytic progress can be made here using the analog of the London equations. The complete solution for the structure and energy of the vortices will be examined numerically in Sec. III B.

A. Solution in the London model

To simplify the somewhat cumbersome algebra, we will restrict the analysis in this subsection to the case $\nu=0$. We will only be interested in the leading contributions, for small e^2, to the energy of the region outside the core: it is not difficult to show that this leading term is independent of the value of ν.

In a strong type-II superconductor, the magnitude of the order parameter is essentially constant outside the core of the vortex. We expect similar behavior here at distances away from the core greater than ξ, which is roughly

$$\xi=\max\left[\frac{1}{\sqrt{\tilde{r}_1}},\frac{1}{\sqrt{\tilde{r}_2}}\right] . \tag{3.1}$$

We therefore use the parametrizations in Eq. (1.8) in the expression for F [Eq. (1.5)]:

$$F_a'=\int d^2r\left[(\nabla\phi_2-\mathbf{a}-e\,\mathbf{A})^2|\bar{b}|^2+(\nabla\phi_1+2\mathbf{a})^2|\bar{\Delta}|^2 + \frac{1}{8\pi}(\nabla\times\mathbf{A})^2+\frac{\sigma}{2}(\nabla\times\mathbf{a})^2\right] . \tag{3.2}$$

Using an analysis very similar to that reviewed by Fetter and Hohenberg in Ref. 17, the equations implying stationarity of F_a' with respect to variations in $\mathbf{A}$ and $\mathbf{a}$ and the line integrals (1.9) yield

$$\begin{aligned}&-\frac{1}{8\pi e}\nabla^2B+|\bar{b}|^2(h+eB)=2\pi|\bar{b}|^2n_2\delta^2(\mathbf{r}) ,\\ &-\frac{\sigma}{2}\nabla^2h+4|\bar{\Delta}|^2h+|\bar{b}|^2(h+eB)\\ &\qquad\qquad=2\pi(-2n_1|\bar{\Delta}|^2+n_2|\bar{b}|^2)\delta^2(\mathbf{r}) ,\end{aligned} \tag{3.3}$$

where $B=(\nabla\times\mathbf{A})_z$ and $h=(\nabla\times\mathbf{a})_z$. These are the analogs of the usual London equations governing the decay of the magnetic fields away from the core of the vortex. By inserting the solutions of Eqs. (3.3) into (3.2), and simplifying using the method Ref. 17, we obtain the contribution of the region outside the core to the energy of the vortex

$$F_a'=\frac{n_1+2n_2}{8e}B(r\sim\xi)-\frac{\pi n_2\sigma}{2}h(r\sim\xi) . \tag{3.4}$$

It is straightforward, though tedious, to obtain the exact solution to Eqs. (3.3) by a Fourier transform. We present only the leading contribution to the result in strong type-II limit $4\pi e^2\ll u_1,u_2,1/\sigma$

$$\begin{aligned}B&=\frac{n_1+2n_2}{2e\lambda^2}K_0(r/\lambda) ,\\ h&=-\left[\frac{|\bar{b}|^2}{|\bar{b}|^2+|2\bar{\Delta}|^2}\right]\frac{n_1+2n_2}{2\lambda^2}K_0(r/\lambda) ,\end{aligned} \tag{3.5}$$

where λ is the London penetration depth given by Eq. (2.4) and K_0 is a Hankel function of imaginary argument and zero order. Inserting the results (3.5) into (3.4) we obtain the following vortex energy for small e^2.

$$\begin{aligned}F_a'&=\frac{(n_1+2n_2)^2}{16e^2\lambda^2}\ln\left[\frac{\lambda}{\xi}\right]\\ &=2\pi(n_1+2n_2)^2\frac{|\bar{\Delta}|^2|\bar{b}|^2}{|\bar{b}|^2+4|\bar{\Delta}|^2}\ln\left[\frac{\lambda}{\xi}\right] ,\end{aligned} \tag{3.6}$$

where we have used Eq. (2.4) and the fact that $K_0(x)\sim-\ln x$ for small x. The first of these expressions is identical to the energy of a vortex with flux $(n_1+2n_2)(hc/2e)$ in the usual Ginzburg-Landau functional.[17] Thus at distances greater than ξ away from the center of the vortex, the energy is almost entirely carried in the kinetic energy of the electromagnetic supercurrent and the electromagnetic field. Currents associated with $\mathbf{a}$ are largely screened out. In this region the properties of F are almost indistinguishable from conventional superconductors; a substantial difference will of course appear when effects at length scales shorter than ξ are considered in Sec. III B.

A crucial feature of the result (3.6) is that the prefactor of the logarithm is *independent* of e^2. As claimed in Sec. I, the prefactor is roughly proportional to the smaller of $|\tilde{r}_1|,|\tilde{r}_2|$. It is thus possible for the core contributions the vortex energy to be of the same order or larger than those of Eq. (3.6).

B. Numerical results

A complete solution of the vortex configurations of F requires numerical analysis. Here we will present numerical results for single-vortex solutions which are axially symmetric about the origin. Such configurations are completely specified by the pair of integers (n_1,n_2) defined by Eqs. (1.8) and (1.9).

Using the axial symmetry, F can be expressed as a functional of $|\Delta(r)|$, $|b(r)|$, $B(r)$, and $h(r)$ where r is the radial coordinate. The functional was discretized on a line of up to 4400 points and then minimized by a conjugate-gradient algorithm. The minimization procedure was found to be much more robust than the alternative of solving the coupled differential equations obtained from variational derivates of F. While the latter procedure was faster, it was highly unstable due to the possibility of mixing in exponentially growing solutions.

An important dimensionless parameter which must be varied is the Ginzburg-Landau parameter κ. Unfortunately, as there is more than one length scale determining the variation of the fields Δ and b, there is no simple choice for the length ξ. Close to the superconductor–normal-state phase boundary there are, however, natural choices. Near the superconductor–normal-state-I phase boundary we use the effective action F_{2e} in Eq. (2.12) to define $\xi=1/\sqrt{\tilde{r}_1}$. Using the value of λ in Eq. (2.4) we obtain

$$\kappa=\frac{\lambda}{\xi}\approx\frac{1}{\sqrt{8\pi e^2}}\frac{\sqrt{\tilde{u}_1}}{2} . \tag{3.7}$$

Similarly near the superconductor–normal-state-II phase boundary we use the action F_e in Eq. (2.17) to define $\xi=1\sqrt{\tilde{r}_2}$ and

TABLE I. Energies of the vortices ε_{n_1,n_2} with flux $(n_1+2n_2)(h/2e)$ as a function of r_2 with $u_1=1.0$, $u_2=1.0$, $v=0.5$, $\sigma=0.5$, $\nu=0.8$, $\bar{\kappa}=10$, and $r_1=-1.0$. Also shown are the values of the renormalized masses $\tilde{r}_2=r_2-(vr_1)/u_1$ and $\tilde{r}_1=r_1-(vr_2)/u_2$. By interpolation we conclude that hc/e vortices are stable in the region $-0.47<\tilde{r}_2<0$.

r_2	$\tilde{r}_2$	$\tilde{r}_1$	$\varepsilon_{1,0}/(2\pi)$	$\varepsilon_{0,1}/(4\pi)$
−0.7	−0.2	−0.65	0.775	0.354
−0.8	−0.3	−0.60	0.796	0.521
−0.9	−0.4	−0.55	0.801	0.682
−1.0	−0.5	−0.50	0.793	0.834
−1.1	−0.6	−0.45	0.771	0.977
−1.2	−0.7	−0.40	0.736	1.108
−1.3	−0.8	−0.35	0.688	1.223

$$\kappa=\frac{\lambda}{\xi}\approx\frac{1}{(8\pi e^2)^{1/2}}(\tilde{u}_2)^{1/2} . \tag{3.8}$$

In the following we will simply present the solutions as a function of the compromise choice $\bar{\kappa}\equiv 1/\sqrt{8\pi e^2}$.

The results presented here are with the choices $u_1=1.0$, $u_2=1.0$, $v=0.5$, $\sigma=0.5$, $\nu=0.8$, and $r_1=-1.0$. For a variety of values of $\bar{\kappa}$, the parameter r_2 was then varied to approach the superconductor–normal-state-II phase boundary. It is near this boundary that we expect hc/e vortices to become stable.

The results for the energy of the $hc/2e$ vortex with $(n_1=1,\ n_2=0)$, $\epsilon_{1,0}$ are tabulated in Table I as a function of r_2 for $\bar{\kappa}=10$. Also listed are the renormalized masses $\tilde{r}_2$, which vanish at the superconductor–normal-state-II mean-field phase boundary, and $\tilde{r}_1$. The energy, $\varepsilon_{0,1}$, of a hc/e vortex with $(n_1=0,\ n_2=1)$ is in the final column of Table I. The condition for hc/e vortices to appear in the presence of an external magnetic field is[17]

$$\varepsilon_{1,0}>\frac{\varepsilon_{0,1}}{2} . \tag{3.9}$$

We notice that this happens at a value $\tilde{r}_2=\tilde{r}_2^c=-0.47$. The hc/e vortices are favored in the region $\tilde{r}_2^c<\tilde{r}_2<0$.

The configuration of the fields in the $(n_1=1,\ n_2=0)$ vortex for $r_2=-0.8$ and $\bar{\kappa}=10$ are shown in Figs. 3 and 4, and those for the $(n_1=0,\ n_2=1)$ vortex in Figs. 5 and 6. We show magnitudes of the order parameters $|\Delta(r)|$ and $|b(r)|$; these approach $|\bar{\Delta}|$ and $|\bar{b}|$ as $r\to\infty$. The gauge fields A, a are specified by plots of the total fluxes Φ_A,Φ_a within a radius r:

$$\Phi_A(r)=\frac{e}{2\pi}\int_0^r 2\pi\rho\, d\rho\, B(\rho) ,$$
$$\Phi_a(r)=\frac{1}{2\pi}\int_0^r 2\pi\rho\, d\rho\, h(\rho) . \tag{3.10}$$

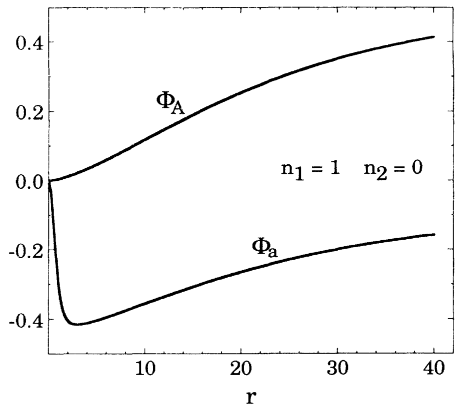

FIG. 4. The fluxes Φ_A,Φ_a with a radius r associated with the A, a gauge fields [Eq. (3.10)] for the vortex of Fig. 3. Notice the compression of the scale of the x axis from that of Fig. 3. From Eq. (3.11) these fluxes satisfy $\Phi_A\to 0.5$ and $\Phi_a\to -0.1$ as $r\to\infty$.

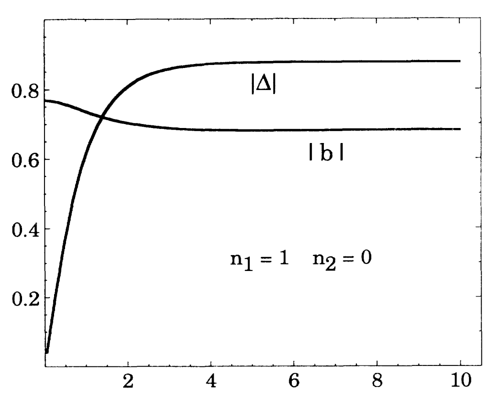

FIG. 3. Values of the fields $|\Delta|$ and $|b|$ as a function of the radial coordinate r for a vortex solution of F. The winding in the phase of $|\Delta|(b)$ is $2\pi n_1(2\pi n_2)$; we have $n_1=1$ and $n_2=0$. The parameters in F have the values $u_1=1.0$, $u_2=1.0$, $v=0.5$, $\sigma=0.5$, $r_2=0.8$, $\bar{\kappa}=10$, $\nu=0.8$, and $r_1=-1.0$.

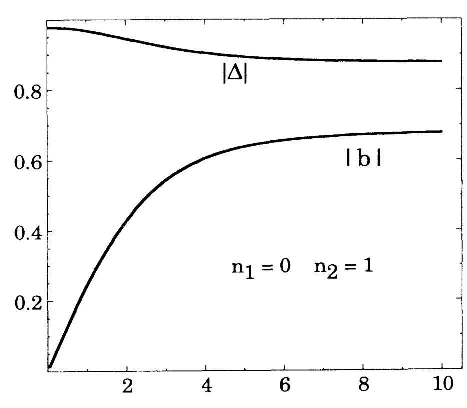

FIG. 5. As in Fig. 3 but for the vortex with $n_1=0$ and $n_2=1$.

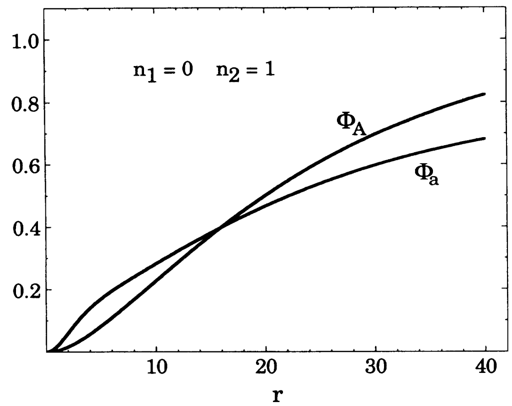

FIG. 6. As in Fig. 4 for the $n_1=0$, $n_2=1$ vortex of Fig. 5. The fluxes satisfy $\Phi_A \to 1.0$ and $\Phi_a \to 0.8$ as $r \to \infty$.

It is easy to show that these fluxes satisfy

$$\lim_{r\to\infty} \Phi_A(r) = \frac{n_1+2n_2}{2} ,$$
$$\lim_{r\to\infty} \Phi_a(r) = \frac{(\nu-1)n_1+2\nu n_2}{2} . \qquad (3.11)$$

Finally Table II has results for the critical value $\tilde{r}_2^c$ at which Eq. (3.9) is first satisfied, as a function of $\bar{\kappa}$. As a consequence of the logarithmic dependence upon $\bar{\kappa}$ in Eq. (3.6) of the contribution of the outer regions we expect that

TABLE II. The critical value of $\tilde{r}_2^c$ as a function of $\bar{\kappa}$ at which the condition (3.9) is first satisfied. Vortices with flux hc/e are stable in the region $\tilde{r}_2^c < \tilde{r}_2 < 0$. Other parameters in F have been chosen as in Table I.

$\bar{\kappa}$	$\tilde{r}_2^c$
2	−0.58
10	−0.47
20	−0.43
40	−0.39

$$\tilde{r}_2^c \sim -\frac{1}{\ln\bar{\kappa}} . \qquad (3.12)$$

IV. LARGE-N EXPANSION FOR CuO_2 LAYERS

We have already mentioned in Sec. I the results of a previous large-N expansion[7] based upon the symplectic groups Sp(N) [Sp(1)≅SU(2)] on a model Hamiltonian of the CuO_2 layers. Here we will briefly review this analysis and outline the extensions necessary to derive the effective action F from this model.

The model Hamiltonian considered in Ref. 7 is composed of three parts:

$$H = H_K + H_C + H_J . \qquad (4.1)$$

The first term H_K is the kinetic energy of holes moving on the Cu $d_{x^2-y^2}$ and the O p_x, p_y orbitals on a square lattice of N_s sites

$$H_K = (\varepsilon_d^0 - \mu)\sum_i d_{i\alpha}^\dagger d_i^\alpha + (\varepsilon_p - \mu)\sum_k p_{k\alpha}^\dagger p_k^\alpha + \mu N_s N(1+\delta) - \frac{t_{pd}}{\sqrt{N}} \sum_{(i,k)} \eta_{ik} d_{i\alpha}^\dagger p_k^\alpha + \text{H.c.} - t_{pp} \sum_{(k,l)} \eta_{kl} p_{k\alpha}^\dagger p_l^\alpha . \qquad (4.2)$$

We use indices i,j (k,l) to denote sums over the vertices (links) of a square lattice. The Sp(N) index $\alpha=1,\ldots,2N$, the η_{ik} are phase factors arising from the spatial form of the d and p orbitals, μ is the chemical potential which fixes an average of $N(1+\delta)$ holes per unit cell, ε_d^0 and ε_p are on-site energies for the Cu and O sites, and t_{pd} and t_{pp} are hopping matrix elements.

The second term H_c represents the Coulomb interactions among the holes. Following previous work,[19,20] only an infinite on-site repulsion between d holes was included. This is implemented by inserting the decomposition (1.1) of $d_{i\alpha}^\dagger$ into H_K. The constraint (1.2) is generalized to

$$b_i^\dagger b_i + f_{i\alpha}^\dagger f_i^\alpha = N . \qquad (4.3)$$

Finally H_J is the nearest-neighbor exchange interaction among the d holes

$$H_J = -\frac{J}{2N}\sum_{(i,j)} (\mathcal{J}^{\alpha\beta} f_{i\alpha}^\dagger f_{j\beta}^\dagger)(\mathcal{J}_{\gamma\delta} f_j^\delta f_i^\gamma) , \qquad (4.4)$$

which is included explicitly following Ref. 20. The antisymmetric tensor $\mathcal{J}$ is an Sp(N) invariant and a generalization of the totally antisymmetric tensor ϵ of SU(2); $\mathcal{J}^{\alpha\alpha'} = \mathcal{J}_{\alpha\alpha'} = \delta^{mm'}\epsilon^{\sigma\sigma'}$, $\alpha \equiv (m,\sigma)$, $m=1,\ldots,N$, $\sigma=\uparrow,\downarrow$.

The large-N expansion performed in Ref. 7 begins by decoupling H_J by a Hubbard-Stratonovich field Δ_{ij} into

$$\sum_{(i,j)} \left[\frac{N|\Delta_{ij}|^2}{J} + \Delta_{ij}\mathcal{J}^{\alpha\beta} f_{i\alpha}^\dagger f_{j\beta}^\dagger + \text{H.c.} \right] . \qquad (4.5)$$

The constraint (4.3) is implemented by a Lagrange multiplier Λ. Upon integrating out the fermions f_α and p_α and parametrizing the boson $b=\sqrt{N}\,\rho$ it was found that the only N dependence of the resulting effective action for Δ, Λ, and ρ was a prefactor N. In the large-N limit one may then use a saddle-point expansion of this action. The results of such a calculation[7] are summarized in Fig. 2.

In this paper we extend the above analysis by expanding H to $H' = H_K + H_C + H_J + H_{dd}$ where the last term is a direct d-d hopping term

$$H_{dd} = -\frac{t_{dd}}{N} \sum_{(i,j)} e^{ieA_{ij}} d^{\dagger}_{i\alpha} d^{\alpha}_{j}$$

$$= -\frac{t_{dd}}{N} \sum_{(i,j)} e^{ieA_{ij}} f^{\dagger}_{i\alpha} f^{\alpha}_{j} b^{\dagger}_{j} b_i \ . \qquad (4.6)$$

We have explicitly displayed the coupling to an external electromagnetic vector potential A, as it will be needed in the analysis below. This can be decoupled by a Hubbard-Stratonovich field Q_{ij} into

$$\sum_{(i,j)} \left[\frac{N|Q_{ij}|^2}{t_{dd}} + Q_{ij} f^{\dagger}_{i\alpha} f^{\alpha}_{j} + Q^{*}_{ij} e^{-ieA_{ji}} b^{\dagger}_{j} b_i \right] . \qquad (4.7)$$

The previous[7] method of integrating out the fermions f_α and p_α and parametrizing the boson $b = \sqrt{N}\,\rho$ also generates a large-N expansion of H' where the effective action is now a functional of Δ, Λ, ρ, and Q.

It is best to consider the functional integral over the complex field Q as two functional integrals over the real field Q^R and Q^I with $Q = Q^R + iQ^I$. It is, however, expected that the saddle-point values of Q^R and Q^I will be complex, leading to different mean-field hopping amplitudes $\bar{Q}^b$ and $\bar{Q}^f$ for the bosons and fermions, respectively. Upon considering the fluctuations of Q^R and Q^I about this saddle point, we expect that the phases of the effective hopping amplitudes are the only modes which can possibly lead to singular effects; there phases transform like a vector potential under $U_I(1)$ and their long-wavelength fluctuations can cost very little energy. We therefore parametrize the fluctuations of H_{dd} about the saddle point as

$$\sum_{(i,j)} (\bar{Q}^f_{ij} e^{-ia_{ij}} f^{\dagger}_{i\alpha} f^{\alpha}_{j} + \bar{Q}^b_{ij} e^{-ia_{ji} - ieA_{ij}} b^{\dagger}_{j} b_i) \ . \qquad (4.8)$$

In the continuum limit, the field a_{ij} becomes the field $\mathbf{a}$ considered earlier and transforms under $U_I(1)$ as in Eq. (1.4). The effective action for the fields, Λ, Δ, b, and $\mathbf{a}$ obtained after integrating out f^α and p^α is strongly constrained by the gauge invariances in (1.4) (Λ transforms like the time component of a vector potential.[7]) It is inevitable that its low-order terms have the form of F [Eq. (1.5)] for steady-state configurations.

Several important technical points about this large-N expansion should be noted.

(i) The expansion could formally have been carried out without introducing the Hubbard-Stratanovich decoupling field Q_{ij} in Eq. (4.7). It is easy to show that just by parametrizing $b_i = \sqrt{N}\,\rho_i$ and integrating out the fermions, one obtains a large-N saddle point completely equivalent to the one discussed above. However a difference does arise upon considering the structure of the fluctuations in the long-wavelength limit. The expansion without the Q_{ij} fields will only include long wavelengths in the single boson field b_i. In the presence of the Q_{ij}, long wavelengths in both the single boson b_i and bilinear boson fields $b^{\dagger}_i b_j$ will be present. The latter procedure is clearly more complete and is the one followed here.

(ii) There is no saddle point with $\langle b \rangle = 0$ and either $\langle f^{\dagger}_{i\alpha} f^{\alpha}_{j} \rangle \neq 0$ or $\langle b^{\dagger}_i b_j \rangle \neq 0$ for i,j nearest neighbors on the square lattice. Therefore, the gauge-field $\mathbf{a}$ cannot be defined in the normal-state II phase. This can easily by remedied by introducing the additional term

$$-\frac{\tilde{J}}{N} \sum_{(i,j)} (f^{\dagger}_{i\alpha} f^{\alpha}_{j})(f^{\dagger}_{j\beta} f^{\beta}_{i}) \ , \qquad (4.9)$$

which can be considered to have been generated by integrating out the short-wavelength fluctuations of the b quanta. For appropriate values of $\tilde{J}$ it is now possible to have $\langle f^{\dagger}_{i\alpha} f^{\alpha}_{j} \rangle \neq 0$ in all the phases.

(iii) The field whose phase defines the vector potential $\mathbf{a}$ (e.g., Q) also breaks the staggered component of the internal $U_I(1)$ gauge symmetry.[7] A theory with a gauge connection $\mathbf{a}$ for the uniform part of $U_I(1)$ and gapless gauge-field fluctuations associated with an unbroken staggered symmetry is therefore not consistent.

(iv) The fluctuations of the Lagrange multiplier Λ are associated with gapless modes in the normal-state III which are important in their transport properties.[3] However in both the low-temperature phases (normal-state II and the superconductor), the $U_I(1)$ gauge symmetry is broken and the fluctuations of Λ become massive.[7] Thus, they are not expected to be relevant for the steady-state configurations of the superconducting phase considered in this paper. [The case of normal-state I is more subtle: although the $U_I(1)$ gauge symmetry is formally broken,[11] the presence of gapless fermion excitations makes the propagator of the longitudinal part of a a singular function of momentum and frequency.] The saddle-point value of Λ will of course be space dependent for the vortex configuration,[21] with space dependence chosen to precisely satisfy the constraint (1.2) at all points in space. The length scale associated with these variations will be roughly $1/(\tilde{r}_1)^{1/2}$, which is smaller than the superconducting coherence length $1/(\tilde{r}_2)^{1/2}$ near the superconductor–normal-state II boundary. The consequences of the variations in Λ can therefore be safely subsumed into renormalizations of the parameters of F.

(v) The nature of the coupling of the f^α to $\mathbf{a}$ in (4.8) shows that no term like $(\nabla \times \mathbf{a}) \cdot (\nabla \times \mathbf{A})$ is obtained. Thus the particular large-N expansion described leads naturally to the choice $\nu = 0$. However, in the more general spirit of Landau theory we expect the coefficient of $(\nabla \times \mathbf{a}) \cdot (\nabla \times \mathbf{A})$ to vanish for a value of ν of order unity.

V. CONCLUSIONS

This paper has examined a phenomenological model of superconductivity in strongly correlated electronic systems.[1,3,4] The local constraints associated with the strong repulsive interactions are implemented by the decomposition (1.1) of the physical fermion operators. This introduces a local $U_I(1)$ gauge symmetry which is crucial in restricting the form of the phenomenological free energy. The presence of the local gauge symmetry also implies that simple gradient terms of the f^α and b operators in (1.4) are forbidden. The motion of these quanta can only be facilitated by a gauge connection $\mathbf{a}$ which transforms under $U_I(1)$. Such a gauge connection had been introduced earlier in the context of an analysis of the normal-state properties of these strongly correlated systems.[2,3] With the introduction of a pairing field Δ for the f^α fermions, a gauge-invariant free energy F [Eq. (1.5)] of the superconducting phase was obtained.

An attempt was then made to determine if F displayed any measurable difference from the usual Landau-Ginzbrug free energy of a conventional superconductor. If was found that over a large portion of the superconducting phase, the two approaches were essentially indistinguishable. One striking difference did, however, appear in the response of the superconductor to an external magnetic field. Near the phase boundary in F between the superconductor and normal-state II (see Fig. 1 and Sec. I) vortices with flux hc/e generically become the optimum way for the magnetic field to pierce the system. These vortices are stabilized because their cores lose superconducting coherence without a significant decrease in antiferromagnetic correlations; in contrast the cores of the $hc/2e$ vortices lose both superconducting and antiferromagnetic correlations. In the cuprate superconductors, the most favorable region for the stability of hc/e vortices is the lowest doping concentrations at which superconductivity first occurs. *The hc/e vortices also become increasingly likely as the field goes from* H_{c1} *to* H_{c2}: this is because the vortex core energy contributes the largest fraction of the total energy at H_{c2}. An experimental search for such vortices will be quite useful. An important caveat is that the hc/e vortices could be preempted by a strong first-order transition between the superconductor and the normal state. In either case, an experimental test of the scenario of this paper is available.

Some additional theoretical issues on the properties of F and related models remain open. A complete study of the transport and thermodynamic properties of normal-state II should be performed. This will complement the studies of the anomalous properties of normal-state III in Ref. 3. Monte Carlo simulations on the phase transitions displayed by F will also be of great interest. In particular, a detailed understanding of the crossover in the superconductor-normal phase transition from an order parameter with charge $2e$ to charge e should be obtained. Finally the nature of the interlayer coupling and the crossover to three-dimensional behavior should be explored: in particular anomalous fluctuation effects may exist between the superconductor and normal-state II.

ACKNOWLEDGMENTS

I thank D. Fisher, A. Millis, and J. Ye for useful discussions. I am also grateful to A. Zee for pointing out the relevance of Ref. 4. I have also received a copy of unpublished work from N. Nagaosa and P. A. Lee which also addressed the structure of vortex solutions of a model closely related to F. This work was supported by the A. P. Sloan Foundation and by the National Science Foundation PYI Grant No. DMR 8857228.

[1]G. Baskaran, Z. Zou, and P. W. Anderson, Solid State Commun. **63**, 973 (1987); G. Baskaran and P. W. Anderson Phys. Rev. B **37**, 580 (1987).

[2]L. B. Ioffe and A. I. Larkin, Phys. Rev. B **39**, 8988 (1989).

[3]N. Nagaosa and P. A. Lee, Phys. Rev. Lett. **64**, 2450 (1990); L. B. Ioffe and P. B. Wiegmann, *ibid.* **65**, 653 (1990); L. B. Ioffe and G. Kotliar, Phys. Rev. B **42**, 10 348 (1990).

[4]X. G. Wen and A. Zee, Phys. Rev. Lett. **62**, 2873 (1989).

[5]N. Read and S. Sachdev, Phys. Rev. Lett. **66**, 1773 (1991).

[6]S. Sachdev and N. Read, Int. J. Mod. Phys. B **5**, 219 (1991).

[7]Jinwu Ye and S. Sachdev, Phys. Rev. B (to be published).

[8]R. E. Walstedt *et al.*, Phys. Rev. B **41**, 9574 (1990).

[9]W. A. Little and R. D. Parks, Phys. Rev. Lett. **9**, 9 (1962).

[10]S. E. Barnes, J. Phys. F **6**, 1375 (1976); N. Read and D. M. Newns, J. Phys. C **16**, 3273 (1983); P. Coleman, Phys. Rev. B **29**, 3035 (1984).

[11]M. Grilli and B. G. Kotliar, Phys. Rev. Lett. **64**, 1170 (1990).

[12]L. B. Ioffe and A. I. Larkin, Phys. Rev. B **40**, 6941 (1989).

[13]S.-W. Cheong, G. Aeppli, T. E. Mason, H. Mook, S. M. Hayden, P. C. Canfield, Z. Fisk, K. N. Clausen, and J. L. Martinez, Phys. Rev. Lett. **67**, 1791 (1991).

[14]K.-S. Liu and M. E. Fisher, J. Low Temp. Phys. **10**, 655 (1973).

[15]B. I. Halperin and D. R. Nelson, J. Low Temp. Phys. **36**, 599 (1979).

[16]T. Kennedy and C. King, Phys. Rev. Lett. **55**, 776 (1985).

[17]A. L. Fetter and P. C. Hohenberg, in *Superconductivity,* edited by R. D. Parks (Marcel Dekker, New York, 1969), Vol. 1.

[18]See, e.g., M. Tinkham, *Introduction to Superconductivity* (McGraw-Hill, New York, 1975).

[19]G. Kotliar, P. A. Lee, and N. Read, Physica C **153-155**, 538 (1988).

[20]C. Castellani and G. Kotliar, Phys. Rev. B **39**, 2876 (1989); C. Castellani, M. Grilli, and G. Kotliar, *ibid.* **43**, 8000 (1991).

[21]G. Murthy and S. Sachdev, Nucl. Phys. **B344**, 557 (1990).