Quasiclassical calculation of magnetoresistance oscillations of a two-dimensional electron gas in an anharmonic lateral superlattice potential

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A classical approach, relating magnetoresistance oscillations of a two-dimensional electron gas (2DEG) in a weak lateral superlattice potential to the guiding-center drift of cyclotron orbits, is extended to periodic potentials of general shape, and justified from the quantum-mechanical Kubo formula. Several reasonable model potentials in a (gate) plane at some distance from the 2DEG are investigated in order to study two competing effects: (1) the suppression of higher harmonics with increasing distance according to Poisson's equation and (2) the relative enhancement of higher harmonics due to the (Thomas-Fermi) screening by the 2DEG itself. An experimental result by Winkler, Kotthaus, and Ploog, showing magnetoresistance oscillations with a rich structure, is closely reproduced by the calculations.

I. INTRODUCTION

The two-dimensional electron gas (2DEG) in a high-mobility GaAs-$Al_xGa_{1-x}As$ heterostructure subject to both a perpendicular magnetic field and a weak lateral superlattice potential $V(x)$ with period $a \sim 100-500$ nm shows interesting magnetoresistance oscillations, first observed by Weiss *et al.*[1] At liquid He temperatures, $T \approx 4$ K, these oscillations are seen most clearly at low magnetic fields ($B \sim 0.1-0.6$ T) where the usual Shubnikov–de Haas (SdH) oscillations (which are well visible for $B \geq 0.5$ T) are not yet resolved due to thermal broadening. Similar to the SdH oscillations, the oscillations are periodic in $1/B$ with a period

$$\Delta \left[\frac{c}{B} \right] = \frac{ae}{2\hbar k_F} \tag{1}$$

depending on the period a of the superlattice and on the mean density $N_s = k_F^2/2\pi$ of the 2DEG. Weiss *et al.*[1,2] found the largest oscillations in the resistivity component ρ_{xx}, measured when the current flows perpendicular to the equipotential lines of the modulation potential $V(x)$, and subsequent experimental investigations[3,4] have focused on this situation. The large-amplitude oscillations of ρ_{xx} are attributed to the occurrence of a "band conductivity"[3] $\Delta\sigma_{yy}$ originating from the fact that the superlattice potential broadens the Landau levels (LL's) into bands that yield a finite group velocity in the y direction. In this picture[2,3] the "Weiss oscillations" originate from the oscillatory dependence of the superlattice-induced width of the Landau bands on the Landau quantum number.

The group velocity of the Landau bands is, however, just the quantum analog of the classical Hall drift of the guiding centers of cyclotron orbits in the periodic superlattice-electric field. Beenakker[5] has pointed out that, in the interesting low-B regime, $\Delta\sigma_{yy}$ can already be calculated within a quasiclassical approach without taking into account any energy quantization. He thus argues that the Weiss oscillations are of a classical nature. Later experiments by Weiss on samples with a 2D superlattice[6–8] questioned this point of view: an additional periodic modulation in the second lateral direction dramatically suppressed the band conductivity. This was not expected within the quasiclassical approach, but can be understood (qualitatively) within a quantum theory based on the correct Hofstadter-type energy spectrum and taking into account the interplay of collision broadening and subband splitting of this spectrum.[8,9]

It should be realized, however, that all the theoretical investigations of the Weiss oscillations which have been published so far are based on the simple harmonic ansatz

$$V(x,y) = V_x \cos(2\pi x/a_x) + V_y \cos(2\pi y/a_y) \tag{2}$$

for the superlattice potential (with $V_y = 0$ in the 1D case). It is not obvious that this is a realistic description of the experimental situation with a 2D superlattice. Moreover, strong effects of the third harmonic $[\propto \cos(6\pi x/a)]$ have been reported by Winkler, Kotthaus, and Ploog[3] for 1D superlattices generated by a microstructured gate.

Therefore, it is of considerable interest to investigate the effect of higher potential harmonics on the Weiss oscillations. This is what we will do in the following for several typical examples of 1D superlattices and of 2D superlattices with rectangular symmetry. In Sec. II we extend a simplified quasiclassical approach, which was suggested and, for the most interesting range of magnetic fields, justified by Beenakker,[5] to a general potential of the form

$$V(x,y) = \sum_{\mathbf{g}} V_{\mathbf{g}} e^{i\mathbf{g}\cdot\mathbf{r}}, \quad \mathbf{g} = 2\pi \left[\frac{n_x}{a_x}, \frac{n_y}{a_y} \right], \tag{3}$$

with n_x and n_y integers and $V_{-\mathbf{g}} = V_{\mathbf{g}}^*$, i.e., $V(x,y)$ real. In the Appendix we show how, for the 1D superlattice, the same result can be derived from the quantum-mechanical Kubo formula. Some special applications are discussed in Sec. III, and a brief summary is given in Sec. IV.

II. QUASICLASSICAL APPROACH

Following Beenakker,[5] we calculate the drift velocity

$$\mathbf{v}^D = \frac{c}{B^2}\langle \mathbf{E}\rangle_{\mathbf{r}_0,R} \times \mathbf{B} \tag{4}$$

of the guiding center $\mathbf{r}_0$ of a cyclotron orbit with radius R in the magnetic field $\mathbf{B}=B\hat{\mathbf{z}}$ by taking the average of the local electric field $\mathbf{E}(x,y)=\nabla(V/e)$ over the orbit. This assumes that the cyclotron orbits are not changed by the weak superlattice potential and is reasonable for sufficiently large values of the relevant cyclotron energy ($\frac{1}{2}m\omega_c^2R^2 \gg |V|$), where $\omega_c = eB/mc$ is the cyclotron frequency and m the effective mass of 2DEG. Moreover, it assumes implicitly that cyclotron orbits are well completed before the electrons are scattered, i.e., $\omega_c\tau \gg 1$, where τ is the transport relaxation time. Inserting the potential $V(x,y)$ from Eq. (3), this yields

$$\mathbf{v}^D = \frac{ic}{eB}\sum_{\mathbf{g}} V_{\mathbf{g}}(\mathbf{g}\times\hat{\mathbf{z}})e^{i\mathbf{g}\cdot\mathbf{r}_0}J_0(gR) \;, \tag{5}$$

where $g=|\mathbf{g}|$ and J_0 is the Bessel function of order zero with the asymptotic expansion

$$J_0(gR) \approx \left[\frac{2}{\pi gR}\right]^{1/2}\cos\left[gR-\frac{\pi}{4}\right] \tag{6}$$

for $gR \gg 1$. The contribution of the guiding-center drift to the conductivity tensor $\sigma_{\mu\nu}$ is then given as the average

$$\Delta\sigma_{\mu\nu} = \frac{\tau e^2 m}{\pi\hbar^2}\langle v_\mu^D v_\nu^D\rangle_{\mathbf{a}} \tag{7}$$

over the unit cell of the superlattice.[5] This yields for the "band conductivity" tensor

$$\Delta\sigma_{\mu\nu} = \frac{\tau e^2}{\pi m(\hbar\omega_c)^2}\sum_{\mathbf{g}} q_{\mu\nu}|V_{\mathbf{g}}|^2[J_0(gR)]^2 \tag{8}$$

with $q_{xx}=g_y^2$, $q_{yy}=g_x^2$, and $q_{xy}=-g_xg_y$. This result can also be obtained from the quantum-mechanical Kubo formula in the limit of large Landau quantum numbers, if the internal energy band structure of the Landau levels is neglected, i.e., for not too large B values, $\hbar\omega_c \ll k_BT$. This is shown in the Appendix for 1D superlattices. The proof for 2D superlatices is given elsewhere.[10] If the potential has a reflection symmetry $V(-x,y)=V(x,y)$ or $V(x,-y)=V(x,y)$, the nondiagonal part vanishes, $\Delta\sigma_{xy}=\Delta\sigma_{yx}=0$. If the potential is of the form $V(x,y)=V_x(x)+V_y(y)$, i.e., $V_{\mathbf{g}}=0$ if $g_xg_y\neq 0$, then $\Delta\sigma_{yy}$ is independent of $V_y(y)$ and depends only on $V_x(x)$, etc.

Since only the modulus $|V_{\mathbf{g}}|$ of the potential Fourier coefficients enters Eq. (8), it is in general not possible to reconstruct the form of the superlattice potential from the magnetic-field dependence of the band conductivity.

III. SOME EXAMPLES

A. 1D superlattices

We first consider a lateral modulation only in the x direction, i.e., $V_{\mathbf{g}}=0$ if $g_y\neq 0$. With $\mathbf{g}=(n2\pi/a,0)$, $V_{\mathbf{g}}\equiv V_n$, and the asymptotic form (6) we obtain

$$\Delta\sigma_{yy} = \frac{e^2}{2\pi\hbar}\frac{16\tau}{\hbar m\omega_c^2 aR_c}\sum_{n=1}^{\infty} n|V_n|^2\cos^2\left[2\pi n\frac{R_c}{a}-\frac{\pi}{4}\right] . \tag{9}$$

For the simple cosine potential $V(x)=V_x\cos(2\pi x/a)$, i.e., $V_1=\frac{1}{2}V_x$, $V_n=0$ if $n>1$, this reduces to the known result,[2,3,5] where $R_c=v_F/\omega_c$ is the radius of a cyclotron orbit with Fermi energy, $E_F=\frac{1}{2}mv_F^2$. In the following we add different harmonics to this fundamental cosine potential and plot the rescaled band conductivity

$$(\Delta\sigma_{yy})^{\rm res} = B^{\rm res}\sum_{n=1}^{\infty} n|V_n/V_1|^2\cos^2[\pi(n/B^{\rm res}-\tfrac{1}{4})] \;, \tag{10}$$

as indicated in the figures by "arb. units," versus the rescaled magnetic field $B^{\rm res}=a/2R_c$. Assuming the Drude results $\sigma_{yx}=e^2N_s/m\omega_c$ and $\rho_0=1/\sigma_0=m/(e^2N_s\tau)$ for the unmodulated system, one obtains, with the approximation $\Delta\rho_{xx}\approx\Delta\sigma_{yy}/\sigma_{yx}^2$, the relative band resistivity $\Delta\rho_{xx}/\rho_0$ if one multiplies $\Delta\sigma_{yy}^{\rm res}$ with the B-independent dimensionless factor $\pi(4\tau|V_1|/\pi\hbar)^2/a^2N_s$. For the simple cosine potential and apart from notation, this result agrees with Eq. (1) of Ref. 5. Since our evaluation of Eqs. (4) and (7) is limited to the lowest order in the small parameter $1/\omega_c\tau$, we have to omit contributions to ρ_{yy} and ρ_{xy} containing $\Delta\sigma_{yy}$, which formally arise in our approximation and are of the order $(\omega_c\tau)^{-2}$. As has been discussed by Beenakker,[5] these contributions are spurious and do not occur in a more rigorous treatment based on an exact solution of Boltzmann's equation in the relaxation-time approximation. This treatment also does not require the present restrictions on the magnetic field. However, for the experimental situations we have in mind,[2,3] the most interesting range of magnetic fields is $0.1\leq B\leq 0.5$ T and our approximations are sufficient.

Note that the cosine in Eq. (10) vanishes for values of its argument satisfying

$$2nR_c/a=\lambda-\tfrac{1}{4} \;,\quad \lambda=1,2,\ldots \;. \tag{11}$$

For $n=1$, this is the commensurability condition obtained previously,[1-3] leading to the period given in Eq. (1).

Figure 1(a) shows results for the model potential $V(x)=V_1\cos(2\pi x/a)+V_2\cos(4\pi x/a)$ and several values of the ratio V_2/V_1. With increasing V_2/V_1 the maxima of $\Delta\sigma_{yy}$ shift slightly to higher values of the magnetic field, whereas the minima shift to lower B values, i.e., from $B^{\rm res}=1/(\lambda-\frac{1}{4})$ towards $B^{\rm res}=1/(\lambda-\frac{1}{8})$ for $\lambda=1,2,\ldots$. Simultaneously, shoulders appear below $B^{\rm res}=1/(\lambda+\frac{3}{8})$ which develop into side maxima for $V_2/V_1>\frac{1}{2}$. [To obtain these $B^{\rm res}$ values, the prefactor $B^{\rm res}$ in (10) was omitted.]

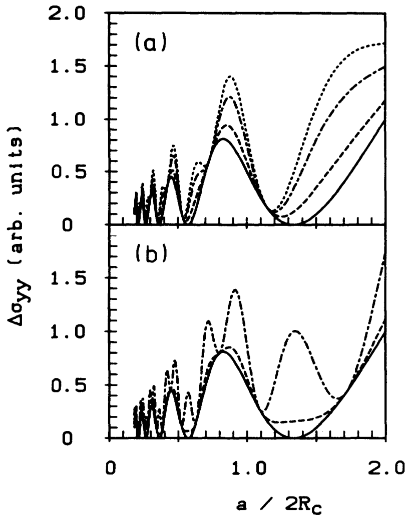

FIG. 1. Rescaled band conductivity, Eq. (10), vs rescaled magnetic field $B^{res}=a/2R_c$ for the simple one-dimensional superlattice potentials $V(x)=V_1\cos(2\pi x/a)+V_k\cos(k2\pi x/a)$; (a) $k=2$ and $|V_2/V_1|=0.0$ (solid line), 0.3 (dashed line), 0.5 (dashed-dotted line), and 0.6 (dotted line); (b) $k=3$ and $|V_3/V_1|=0.0$ (solid line), 0.2 (dashed line), and 0.5 (dashed-dotted line).

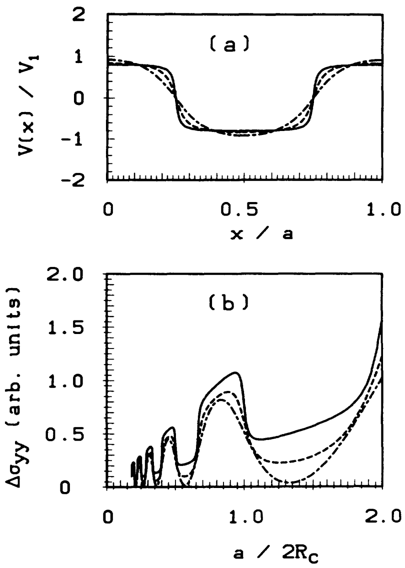

FIG. 2. (a) Effective potential according to Poisson's equation and (b) rescaled band conductivity created by the step-function potential (12) with aspect ratio $d/a=0.5$ at a distance z from the 2DEG for $z/a=0.01$ (solid lines), 0.03 (dashed lines), and 0.1 (dashed-dotted lines).

If the second harmonic vanishes, for instance due to symmetry reasons, the third harmonic may become important. For the model potential $V(x)=V_1\cos(2\pi x/a)-V_3\cos(6\pi x/a)$ and for three values of the ratio V_3/V_1, Fig. 1(b) shows the rescaled $\Delta\sigma_{yy}$. For small V_3/V_1 ($<\sqrt{3}/9$) the result is similar to that of a small second harmonic: the maxima shift to higher and the minima to lower B values. For $V_3/V_1>\sqrt{3}/9$ [to obtain this value again the prefactor B^{res} in Eq. (10) was disregarded] a triple structure evolves from each fundamental peak. Near the maxima the fundamental peaks appear to be split into doublets, and additional side peaks appear at the minima of the fundamental structure. In this regime the structures due to the third harmonics are significantly different from those produced by a second harmonic.

We now consider—as a still very crude approximation to the potential created by a metallic grating gate at a distance z from the 2DEG—a periodic step potential with steps of height $V(x)=\tilde{V}$ and width d, alternating with steps of height $V(x)=0$ and width $a-d$. Taking the center of a $\tilde{V}$ step at $x=0$, one obtains the Fourier coefficients

$$V_0=\tilde{V}d/a\ ,$$
$$V_n=\tilde{V}\sin(\pi nd/a)/\pi n \quad \text{for} \quad n=\pm1,\pm2,\ldots . \qquad (12)$$

Due to Poisson's equation the harmonics fall off exponentially away from the gate and we must replace V_n in the plane of the 2DEG by $V_n(z)=V_n\exp(-n2\pi z/a)$, if we assume no charges between the gate and the 2DEG and absorb a (homogeneous) background dielectric constant in $\tilde{V}$. In Fig. 2 the effective potential and the resulting $\Delta\sigma_{yy}$ are shown for three values of z/a and an aspect ratio $d/a=0.5$, where only Fourier coefficients V_n with odd values of n occur. For realistic values of z/a, i.e., $z/a>0.1$,[3] only the fundamental period survives. Although not of practical importance, it is interesting to note that for a bare step-function potential ($z/a=0$) Eq. (10) yields a divergent result for $\Delta\sigma_{yy}$. This is an artifact of our evaluation of the drift velocity according to Eq. (4), which yields a divergent result for cyclotron orbits having a line of potential discontinuity as a tangent. A direct evaluation for this situation yields $v_y^D\propto\tilde{V}^{1/2}$, clarifying the problem with a perturbative calculation to lowest order in $\tilde{V}$. In a quantum treatment this problem does not occur, as is shown in the Appendix.

So far we have not discussed the screening of the superlattice potential by the 2DEG itself. This, however, provides an effective mechanism for the suppression of the long-wavelength contributions in favor of the higher harmonics. Using the Thomas-Fermi approximation, one obtains for the dielectric function of the 2DEG $\epsilon(g)=1+2/ga_B$, with $a_B=9.79$ nm the effective Bohr radius of GaAs.[11,12] This means a reduction of $V_n(z)$ by a factor $\epsilon(2\pi n/a)=1+a/(\pi a_B n)$. For $a=500$ nm,[3] this reduces $V_1(z)$ by a factor ≈17 and $V_3(z)$ by only 6.3, which means a relative enhancement of the third harmonics in $\Delta\sigma_{yy}$ by roughly an order of magnitude. Figure 3 includes this screening effect due to the 2DEG. Now the effect of higher harmonics survives to much larger z/a values than in the approximation of Fig. 2.

The analogs of Figs. 2 and 3 have been calculated for the aspect ratio $z/a=0.3$ (and the equivalent value $z/a=0.7$), and are shown in Figs. 4(a) and 4(b), respectively. They are dominated by the second harmonic instead of the third one (Fig. 1), and the peaks in Fig. 4(a) show double-step profiles instead of single-step profiles in Fig. 2(b).

It is very interesting to note that the solid curve of Fig.

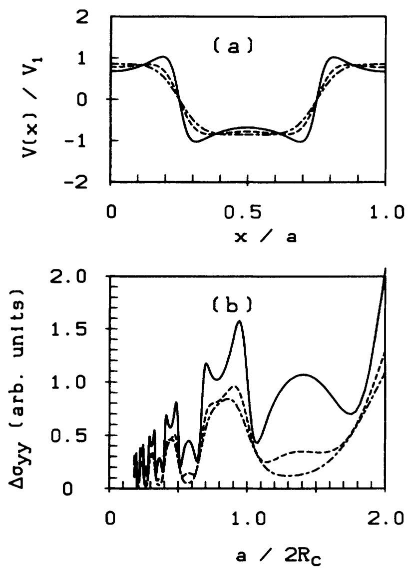

FIG. 3. Same as Fig. 2 but with Thomas-Fermi screening by the 2DEG included. The model parameters are $a/\pi a_B=16.0$ and $z/a=0.05$ (solid lines), 0.09 (dashed lines), and 0.13 (dashed-dotted lines).

3(b) for $a/2R_c \geq 0.3$ nicely reproduces all the structures seen in the curve measured by Winkler, Kotthaus, and Ploog[3] for a gate voltage of $+150$ mV [lowest curve of Fig. 1(b) in Ref. 3], of course apart from the SdH oscillations seen in the experiment at $B>0.5$ T and without the superimposed structure at low magnetic fields which is related to positive magnetoresistance and magnetic breakdown.[13] In view of the experimental value $z/a \leq 0.2$,[3] however, this cannot be considered as satisfactory agreement. On the other hand, it may well be that our assumption of vanishing space charge between the gate and the 2DEG is not satisfied in the experiment.[3] If a redistribution of charges in the Si-doped $Al_xGa_{1-x}As$ layer, partially screening the periodic gate voltage, were somehow frozen in, this could easily lead to the relative enhancement of the third harmonic observed in experiment.

It is very plausible that higher harmonics do not appear in the experiments by Weiss *et al.*[1,2] First, the charged donor distribution created in the Si-doped (AlGa)As layer by holographic illumination and the resulting electrostatic potential are, probably, already very cosinelike with little content of higher harmonics. Second, the period a is considerably smaller, $a \sim 300$ nm, so that z/a is not very small, and the screening by the 2DEG is not so very different for the fundamental and for the higher harmonics, as in Ref. 3.

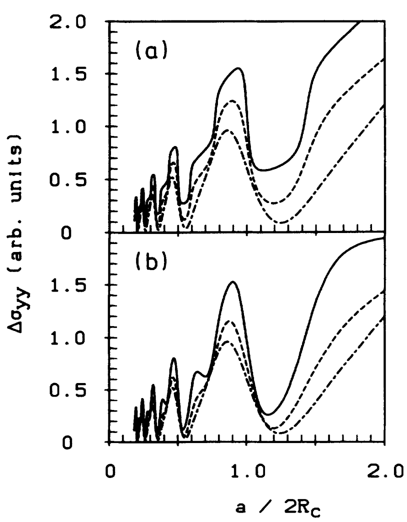

FIG. 4. (a) Same as Fig. 2(b), but with aspect ratio $d/a=0.3$ and with $z/a=0.01$ (solid line), 0.03 (dashed line), and 0.10 (dashed-dotted line); (b) same as Fig. 3(b), but with $d/a=0.3$ and with $z/a=0.09$ (solid line), 0.14 (dashed line), and 0.20 (dashed-dotted line).

B. 2D superlattices

The dramatic suppression of the band conductivity observed in holographically modulated samples with 2D superlattices[6–8] has so far been discussed on the basis of the additive model potential of Eq. (2). For this model, the classical result (8) for the band conductivity $\Delta\sigma_{yy}$ is independent of the modulation in the y direction, and the total suppression must be ascribed to a quantum effect.[8,9] In view of the experimental modulation process, this additive model seems, however, not to be realistic. In the first illumination step, a 1D modulation pattern is created where essentially no donors are ionized in the dark stripes and essentially all donors are ionized in the stripes with maximum light intensity. In the second illumination step, after rotation of the sample by 90°, only those donors can be ionized that have not been ionized in the first step. The maxima and the minima of the ionized donor concentration have the same values for both the 1D and the 2D superlattice. Therefore, it seems to be more realistic to describe the potential of the 2D modulation by a multiplicative than by an additive ansatz. Similar arguments apply to gated samples, where a metal gate is deposited on a holographically structured photoresist. We thus consider the model

$$\begin{aligned} V(x,y) &= \tilde{V}\tfrac{1}{2}(1+\cos Kx)\tfrac{1}{2}(2-\alpha+\alpha\cos Ky) \\ &= V_{0,0}+2V_{1,0}\cos Kx+2V_{0,1}\cos Ky \\ &\quad +4V_{1,1}\cos Kx\,\cos Ky\;, \end{aligned} \tag{13}$$

with $0 \leq \alpha \leq 1$, and we have introduced the Fourier coefficients of Eq. (3) with the notation $V_{\mathbf{g}}=V_{n_x,n_y}$ for $\mathbf{g}=2\pi(n_x,n_y)/a$, so that $V_{\pm1,0}=(1-\alpha/2)\tilde{V}/4$ and $V_{0,\pm1}=2V_{\pm1,\pm1}=(\alpha/2)\tilde{V}/4$. If we deliberately omit the mixed term, $V_{1,1}=0$, we see that the second modulation reduces $V_{1,0}$ from the 1D value ($\alpha=0$) to one-half of that value for the lattice with square symmetry ($\alpha=1$). This "classical" effect reduces the band conductivity by a factor $\frac{1}{4}$, which is already considerable. It is, however, by far not sufficient to explain the dramatic suppression of $\Delta\sigma_{yy}$ observed in the experiment,[6–8] and leaves room and need for the quantum-mechanical investigation.[8,9]

The solid line in Fig. 5(b) shows, for $\alpha=1$, the rescaled

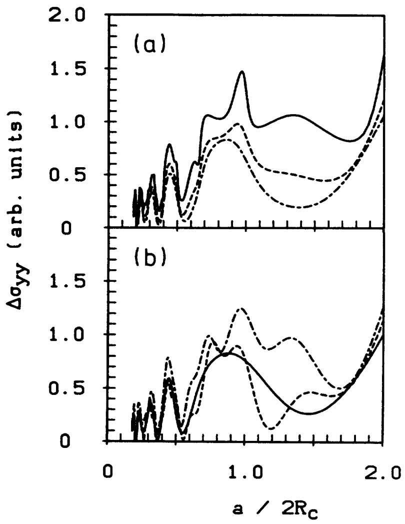

FIG. 5. (a) Rescaled band conductivity created by the step-function potential (15) with square symmetry ($\alpha=1$) and aspect ratio $d/a=0.5$ at a distance z from the 2DEG, for $z/a=0.015$ (solid line), 0.04 (dashed line), and 0.08 (dashed-dotted line). Only the effect of Poisson's equation is taken into account, not that of screening by the 2DEG. (b) Models $(\Delta\sigma_{yy})^{\rm res}=B^{\rm res}\sum_{v=1}^{4}v_v\cos^2[\pi(g_v/B^{\rm res}-\frac{1}{4})]$ vs $B^{\rm res}=a/2R_c$, with $v_1=g_1=1$, $g_2=\sqrt{2}$, $g_3=3$, and $g_4=\sqrt{10}$ for $v_2=0.35$, $v_3=v_4=0$ (solid line), for $v_2=v_3=0, v_4=0.3$ (dashed line), and for $v_2=0.6, v_3=0.15, v_4=0.2$ (dashed-dotted line).

conductivity for the model (13),

$$(\Delta\sigma_{yy})^{\rm res}=B^{\rm res}\{\cos^2[\pi(1/B^{\rm res}-\tfrac{1}{4})] +(\sqrt{2}/4)\cos^2[\pi(\sqrt{2}/B^{\rm res}-\tfrac{1}{4})]\}\ . \quad (14)$$

The second cosine term, resulting from the mixed term ($\propto V_{1,1}$) in Eq. (13), raises the values at the minima and shifts the minimum at $B^{\rm res}=\frac{4}{3}$ [Fig. 1(a)] to a somewhat higher value.

As a multiplicative model for a 2D step-function potential we consider a square lattice with a unit cell $|x|\le a/2$, $|y|\le a/2$, where the potential is

$$V(x,y)=\begin{cases}0 & \text{if } |x|>d/2\\ \tilde{V} & \text{if } |x|<d/2,\ |y|<d/2\\ (1-\alpha)\tilde{V} & \text{if } |x|<d/2,\ |y|>d/2\end{cases} \quad (15)$$

again with $0\le\alpha\le1$. The relevant Fourier components are

$$V_{n_x,0}=[1-\alpha(1-d/a)]\sin(\pi n_x d/a)\tilde{V}/(\pi n_x)\ ,$$
$$V_{0,n_y}=\alpha(d/a)\sin(\pi n_y d/a)\tilde{V}/(\pi n_y)\ , \quad (16)$$
$$V_{n_x,n_y}=\alpha\sin(\pi n_x d/a)\sin(\pi n_y d/a)\tilde{V}/(\pi^2 n_x n_y)\ .$$

For an aspect ratio $d/a=0.5$ the modulation towards square symmetry ($\alpha=1$) again reduces the amplitude $V_{1,0}$ of the fundamental period to one-half of its value for the 1D lattice ($\alpha=0$). Again for $d/a=0.5$, Fig. 5(a) shows the rescaled band conductivity for three values of the distance z between the plane of the potential (15) and the 2DEG, where the exponential decrease of the Fourier components according to Poisson's equations is taken into account, but not the screening due to the 2DEG. The origin of the rich structure shown by the solid curve in Fig. 5(a) ($z/a=0.015$) is analyzed in Fig. 5(b). It is qualitatively well understood, if one adds to the 1D result, shown in Fig. 2(b) for $z/a=0.01$, the contributions due to the most important mixed terms given by the Fourier components $V_{1,1}$ and $V_{3,1}$ in (16) [Eq. (8)].

IV. SUMMARY

We have calculated the contribution of the band conductivity to the magnetoresistance oscillations of a 2DEG subject to a general, nonharmonic two-dimensional superlattice potential in the quasiclassical limit. In general, the effect of higher harmonics contained in a superlattice potential, which is created near a gate at some distance z from the plane of the 2DEG, decreases exponentially with increasing z as a consequence of Poisson's equation. The higher harmonics contained in a step-function potential of period a become absolutely unimportant for $z/a\ge0.2$, provided there are no space charges between the gate and the 2DEG. A very effective mechanism which enhances the importance of higher harmonics is the wave-number-dependent (Thomas-Fermi) screening of the potential by the 2DEG itself. For a step-function potential with aspect ratio $d/a=0.5$, this screening mechanism leads to well-visible effects of the third harmonic up to distances with $z/a\approx0.1$. The calculated magnetic-field dependence of the band conductivity for this situation shows a rich structure and reproduces very closely an experimental result of Winkler, Kotthaus, and Ploog.[3] If the additional structure due to the higher harmonics is not resolved, within the classical calculation the only effect of these harmonics can be slight shifts of the minima and the maxima of the band conductivity. The drastic changes observed recently by Weiss,[6–8] such as an interchange of the positions of resistance maxima and minima and a considerable suppression of the band conductivity by more than an order of magnitude, which occur when the lateral superlattice potential is changed from a one-dimensional to a two-dimensional one, cannot be explained by the classical calculation and point to a quantum-mechanical origin.[8,9,14] Within the classical calculation and the multiplicative models discussed, one obtains at most a reduction of the band conductivity by a factor 0.25.

ACKNOWLEDGMENTS

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APPENDIX

To first-order perturbation theory, the correction to the Landau energies $E_n=\hbar\omega_c(n+\frac{1}{2})$ due to the weak modulation potential $V(x)=\sum_g V_g\exp(igx)$ is given by the expectation value $\langle nx_0|V(x)|nx_0\rangle$ taken with Landau functions $\langle x,y|nx_0\rangle=L_y^{-1/2}\exp(ik_y y)\phi_n(x-x_0)$, where $x_0=-l^2k_y$, and ϕ_n is a normalized oscillator function. The resulting energy eigenvalues of the Landau bands are[15,14]

$$E_n(x_0)=\hbar\omega_c(n+\tfrac{1}{2})+\sum_g V_g e^{igx_0}\mathcal{L}_n(\tfrac{1}{2}g^2l^2)\ , \tag{A1}$$

with $\mathcal{L}_n(G)=\exp(-G/2)L_n(G)$ and L_n a Laguerre polynomial. The Kubo formula yields for the band conductivity $\Delta\sigma_{yy}=-\int dE\, f'(E)\Delta\sigma_{yy}(E)$, with $f'(E)$ the derivative of the Fermi function and[15]

$$\Delta\sigma_{yy}(E)=\frac{\hbar e^2}{l^2}\int_0^a\frac{dx_0}{a}\sum_n|\langle nx_0|v_y|nx_0\rangle A_{n,x_0}(E)|^2\ . \tag{A2}$$

If we neglect the internal band structure of the Landau levels and omit the x_0 dependence of the spectral function $A_{n,x_0}(E)$ and insert the group velocity $\langle nx_0|v_y|nx_0\rangle=\hbar^{-1}dE_n(x_0)/dk_y$ calculated from Eq. (A1), we obtain

$$\Delta\sigma_{yy}(E)=\frac{e^2l^2}{\hbar}\sum_n[A_n(E)]^2\sum_g g^2|V_g|^2[L_n(\tfrac{1}{2}g^2l^2)]^2\ . \tag{A3}$$

Assuming that the shape of the spectral function $A_n(E)$ is independent of n and that its width ($\propto\gamma$) is much smaller than k_BT, we may write $\int dE\, f'(E)[A_n(E)]^2=f'(E_n)/\gamma\pi$. To derive Eq. (9), we exploit $\hbar\omega_c\ll k_BT$, transform the n sum into an integral over E_n, and assume that L_n as a function of n near $n_F=E_F/\hbar\omega_c$ ($\gg 1$) varies slowly on the scale $k_BT/\hbar\omega_c$. Then we obtain

$$\Delta\sigma_{yy}=\frac{e^2}{2\pi\hbar}\frac{2l^2}{\gamma\hbar\omega_c}\sum_g g^2|V_g|^2[L_{n_F}(\tfrac{1}{2}g^2l^2)]^2\ . \tag{A4}$$

For fixed argument $\frac{1}{2}g^2l^2$ and large index n_F ($\gg 1$), $L_{n_F}(\frac{1}{2}g^2l^2)\approx J_0(gR_c)$ holds with $R_c=l\sqrt{2n_F+1}$. With this approximation, with $\gamma=\hbar/\tau$, with $g=2\pi n/a$ for ($n=\pm1,\pm2,\ldots$), and with the asymptotic formula (6), Eq. (A4) reduces to Eq. (9). For a superlattice potential with only conditionally convergent Fourier series, $|V_g|\propto 1/g$, this approximation is, however, not feasible, since $J_0(gR_c)$ ($\propto g^{-1/2}$) decreases only slowly with increasing g, so that the sum in Eq. (9) diverges, whereas $L_{n_F}(\frac{1}{2}g^2l^2)$ decreases exponentially with increasing g, and the g sum in Eq. (A4) converges rapidly. Thus, the possible divergence of Eq. (9) for a steplike superlattice potential is an artifact of the classical approximation using Eq. (4).

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