Nonlinear excitation in an $S=1/2$ Heisenberg ferromagnetic chain

Zhu-Pei Shi
Department of Physics, New York University, 4 Washington Place, New York, New York 10003
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Nonlinear excitations in an $S=\frac{1}{2}$ Heisenberg ferromagnetic chain are investigated by using the squeezed fermion coherent states and the Jordan-Wigner fermionic transformation of spin-$\frac{1}{2}$ operators. The equation of motion for a fermion operator is a modified Schrödinger equation. Solitonlike excitations are obtained, and their spectrum has a linear dispersion at low energies.

The nonlinear excitations such as solitary waves and solitons in the Heisenberg spin chain have generated a great deal of experimental[1-3] and theoretical[4-14] interest. Theoretically, there are several methods to study these nonlinear excitations in one-dimensional magnets. In the classical method,[4,5] general soliton solutions are obtained for a continuum version of the classical Heisenberg chain. Zakharov and Takhtajan[6] showed that there is a gauge equivalence between the Heisenberg ferromagnet and nonlinear Schrödinger system. It is well known that there are several boson representations of spin operators for the quantum spin system. In Schwinger's boson representation,[15] two sets of boson operators are introduced. By using this representation, Cieplak and Turski[7] investigated soliton excitations in a homogeneous ferromagnetic chain in the continuum limit. They obtained an effective Hamiltonian after some quartic terms of boson operators were neglected (this results in only one set of bosons being retained). By using the coherent spin state,[16] Balakrishnan and Bishop[8] studied nonlinear excitations in an isotropic quantum ferromagnetic chain in the continuum approximation (the Hamiltonian is expanded to a^2 order, where a is a lattice constant). This is a useful method to study the nonlinear dynamics in a magnetic chain, but it seems that there are some difficulties in the anisotropic case. The other coherent-state treatments[9-12] use a severely truncated Holstein-Primakoff expansion[17] for $S_i^{\pm}$. Working with Glauber's coherent-state representation,[18] and by making the small-amplitude and lone-wave approximations, one finds solitary wave profiles identical to classical solitons. When this representation is used to investigate the nonlinear excitations in the spin system, attention should be paid to the relative ratio of ϵ to η for determining the modified terms of the equation of motion;[13,14] here $\epsilon=1/\sqrt{S}$ and $\eta=a/\lambda$, where S is spin length, a is lattice constant, and λ is characteristic wavelength.

The above methods are not suitable for studying the nonlinear excitation in an $S=\frac{1}{2}$ Heisenberg chain. Generally, the $S=\frac{1}{2}$ operators can be written in terms of exact fermion operators through the Jordan-Wigner transformation.[19] To solve the equation of motion it is necessary to introduce a coherent-state representation. Recently Svozil[20] derived the squeezed fermion coherent states in analogy to squeezed light. This allows us to study the nonlinear excitation of an $S=\frac{1}{2}$ Heisenberg ferromagnetic chain. I obtain the spectrum of solitonlike excitations at low energies which agrees with Haldane's recent result on the exact spectrum of spinon excitations in an $S=\frac{1}{2}$ Heisenberg chain.[21]

The Hamiltonian for an $S=\frac{1}{2}$ Heisenberg ferromagnetic chain is

$$H=J\sum_j (\mathbf{S}_j\cdot\mathbf{S}_{j+1})_\Delta \ . \tag{1}$$

Exchange anisotropy is controlled by a parameter Δ and defined as follows:

$$(\mathbf{A}\cdot\mathbf{B})_\Delta = A^xB^x + A^yB^y + (1-\Delta)A^zB^z \ . \tag{2}$$

The spin-$\frac{1}{2}$ operators in one dimension can be written in terms of exact fermion operators through the Jordan-Wigner transformation:

$$C_l = e^{i\pi\sum_{m=1}^{l-1}C_m^\dagger C_m}S_l^- \ ,$$
$$C_l^\dagger = e^{-i\pi\sum_{m=1}^{l-1}C_m^\dagger C_m}S_l^+ \ . \tag{3}$$

The number operator for each site is $n_l=C_l^\dagger C_l$. By substituting Eq. (3) into Eq. (1) we obtain the Hamiltonian

$$H_s=-J\sum_{l,\delta}C_l^\dagger C_{l+\delta}+J(1-\Delta)\sum_l C_l^\dagger C_l - \frac{J(1-\Delta)}{2}\sum_{l,\delta}C_l^\dagger C_{l+\delta}^\dagger C_{l+\delta}C_l \ , \tag{4}$$

where $H_s=H-H_0$, $H_0=-J(1-\Delta)N/4$, $\delta=\pm1$, and N is the number of lattice sites. The fermion operators satisfy the following equation of motion:

$$i\hbar(\partial/\partial t)C_j=[C_j,H_s]$$
$$=J(1-\Delta)C_j - J\sum_\delta C_{j+\delta} - J(1-\Delta)\sum_\delta C_{j+\delta}^\dagger C_{j+\delta}C_j \ . \tag{5}$$

To solve Eq. (5) we introduce the squeezed fermion coherent state[20]

$$|\beta\rangle = \prod_i |\beta(i)\rangle \;,$$
$$C_i|\beta\rangle = \beta_i|\beta\rangle \;, \qquad (6)$$
$$|\beta\rangle = e^{(C^\dagger\beta - \beta^* C)}|0\rangle \;.$$

When Eq. (5) acts on a squeezed coherent state and goes to the continuum limit $[\beta_i(t) \to \beta(x,t)]$ we find

$$i(\partial/\partial t)\beta = -A_1\beta - A_2\beta_{xx} - A_3|\beta|^2\beta - A_4|\beta|^2_{xx}\beta \;,$$
$$A_1 = J(1+\Delta)/\hbar, \quad A_2 = Ja^2/\hbar \;, \qquad (7)$$
$$A_3 = 2J(1-\Delta)/\hbar, \quad A_4 = Ja^2(1-\Delta)/\hbar \;.$$

where $\beta_{i+\delta}$ is expanded to a^2 order, and a is a lattice constant. To solve Eq. (7) we use the method of multiple scales.[22] For weak nonlinear waves with dispersion, we can introduce the slow variables

$$\xi = \rho(x - V_g t), \quad \tau = \rho^2 t \qquad (8)$$

and the asymptotic expansion

$$\beta = \rho\beta^{(1)} + \rho^2\beta^2 + \rho^3\beta^3 + \cdots \;, \qquad (9)$$

where ρ is a small parameter denoting the relative amplitude of the excitation. $V_g = d\omega/dk$ is the group velocity of the linear wave. Then we obtain

$$iU_\tau + A_2 U_{\xi\xi} + A_3|U|^2 U = 0 \qquad (10)$$

with

$$\beta = \rho\beta^{(1)} + O(\rho^2) \;,$$
$$\beta^{(1)} = U(\xi,\tau)e^{i(kx-\omega t)} \;, \qquad (11)$$
$$\omega = A_2 k^2 - A_1 \;,$$

where k is the wave number. Then the single soliton solution is

$$\beta(x,t) = \left[\frac{2A_2}{A_3}\right]^{1/2} k_0\rho\,\mathrm{sech}[k_0\rho(x - x_0 - V_g' t)]$$
$$\times e^{i[(k+\alpha_0)x - \Omega t - \phi_0]} \;,$$
$$\Omega = \omega + (\alpha_0^2 - k_o^2\rho^2)A_2 \;, \qquad (12)$$
$$V_g' = 2(k+\alpha_0)A_2 \;,$$

where k_0, α_0, x_0 are integral constants and V_g' is the velocity of the soliton. The normalization of $\beta(x,t)$ sets $k_0\rho = aA_3/2A_2$. Then the occupation number $n_l \to n(x,t) = |\beta(x,t)|^2$ is

$$n(x,t) = (1-\Delta)\mathrm{sech}^2[(x - x_0 - V_g')/\lambda] \;, \qquad (13)$$

where the soliton width is $\lambda = a/2(1-\Delta)$ and its velocity is $V_g' = 2Ja^2(k+\alpha_0)/\hbar$. The local magnetization distribution $S^z(x,t)$ is given by

$$S^z(x,t) = n(x,t) - \tfrac{1}{2}$$
$$= (1-\Delta)\mathrm{sech}^2[(x - x_0 - V_g' t)/\lambda] - \tfrac{1}{2} \;. \qquad (14)$$

For the isotropic case $\Delta = 0$, the occupation number $n(x,t)$ and localized magnetization $S^z(x,t)$, reduce to

$$n(x,t) = \mathrm{sech}^2[(x - x_0 - V_g' t)/\lambda] \;,$$
$$S^z(x,t) = \mathrm{sech}^2[(x - x_0 - V_g' t)/\lambda] - \tfrac{1}{2} \qquad (15)$$

with

$$n(x_0, t=0) = 1, \quad n(\pm\infty, t) = 0 \;,$$
$$S^z(x_0, t=0) = \tfrac{1}{2}, \quad S^z(\pm\infty, t) = -\tfrac{1}{2} \;. \qquad (16)$$

We see that Eqs. (15) and (16) demonstrate explicitly a nonlinear excitation in the $S = \frac{1}{2}$ spin chain, as shown in Fig. 1. The other interesting result is that there is no soliton excitation for the $S = \frac{1}{2}$ XY Heisenberg spin chain because $n(x,t) = 0$ for $\Delta = 1$.

In the coherent-state representation, the energy of the excitation is given by

$$E = \langle\beta|H_s|\beta\rangle / \langle\beta|\beta\rangle$$
$$= J[(k+\alpha_0)^2 a^2 + 12(1-\Delta)^3 - 12(1-\Delta)^2 - \tfrac{1}{2}(1+\Delta)] \;. \qquad (17)$$

Haldane[23] predicted that integer spins correspond to the standard quantization of the O(3) σ model, therefore the nonlinear excitations have a gap (so-called Haldane gap); the half-integer spins correspond to a nonstandard quantization of the σ model and its nonlinear excitation may not have a soliton gap. Here we use this argument to set $E = 0$ at $k = 0$ and from Eq. (17) we find

$$\alpha_0 = (1/a)[\tfrac{1}{2}(1+\Delta) + 12(1-\Delta)^2 - 12(1-\Delta)^3]^{1/2} \;. \qquad (18)$$

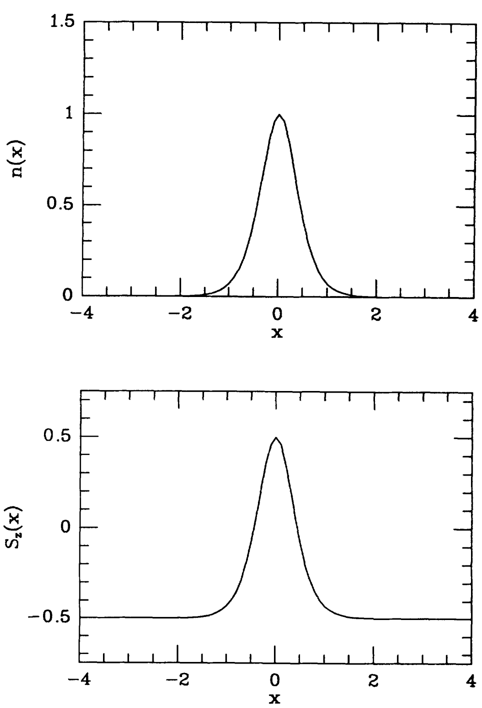

FIG. 1. Occupation number $n(x)$ and local magnetization $S^z(x)$ distributions for a soliton excitation in an $S = \frac{1}{2}$ isotropic Heisenberg ferromagnetic chain at $x_0 = 0$, $t = 0$.

Thus the nonlinear excitation spectrum for the isotropic case ($\Delta = 0$) reads

$$E_{\rm iso} = J(\sqrt{2}ak + k^2a^2) . \quad (19)$$

At low energies ($k \to 0$) this ferromagnet has a linear dispersion which agrees with Haldane's recent result[21] that the $S = \frac{1}{2}$ Heisenberg ferromagnetic chain has a linear magnon dispersion at low energies [Eq. (20) of Ref. 21].

In conclusion, we have obtained the solitonlike excitations for an $S = \frac{1}{2}$ Heisenberg ferromagnetic chain based on squeezed fermion coherent states. The spectrum has the form $ak + bk^2$, which reduces to a linear dispersion at low energies ($k \to 0$), and which agrees with Haldane's recent result on the exact spectrum in terms of fermionic $S = \frac{1}{2}$ spinon excitation. The other interesting result we find is that there may be no solitonlike excitations in an $S = \frac{1}{2}$ XY spin chain.

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