Theory of optical phase conjugation in disordered media: Coherent properties of the backscattered field

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Recently it was shown that in disordered media, nonlinear backscattering of phase-conjugated signal light is possible in the presence of only a single pump wave, in contrast to the case of an ordered medium where two counterpropagating pump waves are necessary. Formally, this disorder-induced nonlinear process corresponds to the cooperon-channel (back)scattering of the signal wave in the presence of diffusion propagation of the pump wave. (The cooperon is the Cooper particle-particle diffusion propagator.) In this contribution, the question is considered whether the disorder-induced backscattering of phase-conjugated light can restore the disturbed signal wave front as it does in the ordered case. Examining correlations of the outgoing waves corresponding to different incoming signal waves, it is shown that the backscattered waves possess a "phase memory" even for a large difference of the angles of incidence of incoming waves. This is in contrast to the case of linear backscattering, where such correlations take place only at a very small difference of the angles of incidence (of the order of backscattering peak width). Thus the phase conjugation in disordered media *does* possess the peculiar coherent properties of that in ordered media.

I. INTRODUCTION

Coherent wave phenomena originating in multiple scattering of light in disordered media have attracted considerable attention. Besides investigations of linear optical effects, the problems of nonlinear optics in disordered media are at present of considerable interest. Recently the theory of nondegenerate phase conjugation of light in disordered nonlinear media has been developed[1] for the case of multiple linear scattering of probe (frequency ω_i), pump (ω_p), and signal ($\omega_s=2\omega_p-\omega_i$) waves.[2] It has been shown that even for $l \ll L$ (l is a mean free path of elastic scattering of light and L is a sample thickness), a peak of conjugated light may occur in the direction opposite to the direction of incidence of the probe wave. The peak intensity of this backscattered signal wave may be some orders of magnitude larger than the diffuse background at the same frequency ω_s. It is in contrast to the well-known case of linear backscattering[3-5] where only at most a factor of 2 is realized for the signal-to-background ratio. The angular width $\Delta\theta_{\rm nl}$ of the nonlinearly backscattered peak is considerably smaller ($\Delta\theta_{\rm nl}\sim\lambda/L$, λ is the wavelength) than that ($\Delta\theta_l\sim\lambda/l$) for the linear backscattering. Another feature of the considered optical mixing in disordered media is that it may occur in presence of a single pump wave, while in the case of ordered (transparent) media, two counterpropagating pump waves are required. Formally the disorder-induced phase-conjugation process corresponds to the Cooper-channel backscattering of the probe wave in the presence of diffusion propagation of the pump wave.

It should be stressed that in contrast to the case of transparent media, in disordered media the amplitude E_s of the signal (conjugated) wave averaged over the realizations of the disorder is zero. In this situation it is not obvious if the "phase conjugation" in disordered media possesses the striking coherence properties of that in transparent media, e.g., the restoration of a disturbed incident wave front. To answer this question it is not sufficient to consider the disorder-averaged intensity of a single signal wave, as it was done in Ref. 1, but it is necessary to examine the correlations of the amplitudes of several outgoing (signal) waves corresponding to different incoming (probe) waves. Such an analysis is just the subject of the present work.

II. FORMULATION AND GENERAL ANALYSIS OF THE PROBLEM

First, we recall the basic properties of the optical-phase-conjugation phenomenon (see, e.g., Ref. 6). Figure 1 shows the process of image formation due to nonlinear (conjugating) reflection. Incident (probe) waves with amplitudes $E'_{i\alpha}(\alpha=1,2,\ldots)$ and wave vectors $\mathbf{k}'_{i\alpha}$ starting from point A of a source are disturbed by some linear scatterer T, and are transformed into waves with wave vectors $\mathbf{k}_{i\alpha}$ and amplitudes $E_{i\alpha}=t_\alpha E'_{i\alpha}$. Phase-conjugating reflection of these waves by the nonlinear medium M results in the transformation

$$E_{i\alpha}\to E_{s\alpha}=r_\alpha E^*_{i\alpha}\ , \tag{1}$$

$$\mathbf{k}_{i\alpha}\to\mathbf{k}_{s\alpha}\approx-\mathbf{k}_{i\alpha}\ . \tag{2}$$

After the linear transformation at the scatterer T, $E_{s\alpha}\to E'_{s\alpha}=t_\alpha E_{s\alpha}$, the signal field at point A has an amplitude

$$E'_s=\sum_\alpha E'_{s\alpha}\approx|t|^2\sum_\alpha r_\alpha E'^*_{i\alpha}\ . \tag{3}$$

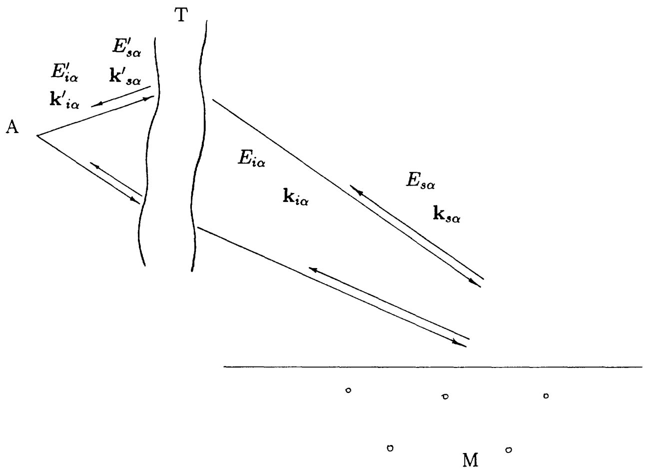

FIG. 1. The process of the restoration of the initial field. The waves from the source A pass the scatterer T and are reflected (with phase conjugation) from the nonlinear medium M.

Obviously, there is compensation for the random phase caused by the scatterer T [we neglect in (3) a dependence of the transmission coefficient $|t_\alpha|^2$ on α, assuming a not too large aperture]. In the usual case of an ordered nonlinear medium the factor r_α is a constant and E'_s is just proportional to the conjugated amplitude of the incident (probe) field at the same point A:

$$E'_s \sim E'^*_i = \sum_\alpha E'^*_{i\alpha} \ . \tag{4}$$

In this case the intensity $I'_s = |E'_s|^2$ of the signal is obviously given by

$$I'_s \sim |E'_i|^2 = \sum_{\alpha,\beta} E'_{i\alpha} E^*_{i\beta} \ . \tag{5}$$

So, the image field reproduces the source field up to some factor and conjugation.

A more complicated situation occurs for conjugation by disordered media. In this case the factor r_α in (3) is a random quantity vanishing under the averaging over disorder (see below). Therefore relation (4) does not hold for the averaged signal amplitude. And what about the intensity of the signal light at a given point of the "image"? (Note that a measurement of intensity rather than of amplitude is a common recording procedure, for which ordinary photography is an example.) Now we are going to check if relation (5) remains valid for the disorder-averaged intensity of the signal (conjugated) field.

The disorder-averaged intensity $\langle I'_s \rangle$ of the signal field can be expressed in the following form:

$$\langle I'_s \rangle = \sum_{\alpha,\beta} \langle E'_{s\alpha} E'^*_{s\beta} \rangle = \sum_{\alpha,\beta} t_\alpha t^*_\beta \langle E_{s\alpha} E^*_{s\beta} \rangle \ . \tag{6}$$

The presence of a random coefficient r_α in the conjugation transformation (1) results in the appearance of some factor $f_{\alpha,\beta}$ in the following expression:

$$\langle E_{s\alpha} E^*_{s\beta} \rangle = f_{\alpha,\beta} E^*_{i\alpha} E_{i\beta} \ . \tag{7}$$

Using $E_{i\alpha} = t_\alpha E'_{i\alpha}$, the averaged intensity (6) takes the form

$$\langle I'_s \rangle \approx |t|^4 \sum_{\alpha,\beta} f_{\alpha,\beta} E'^*_{i\alpha} E'_{i\beta} \ . \tag{8}$$

Thus, the validity of relation (5) for the averaged intensity of the signal field depends crucially on the smoothness of the correlation function $f_{\alpha,\beta}$. Below we shall show that this function is constant in a rather wide region of the parameters α,β.

III. CORRELATION FUNCTION OF CONJUGATED LIGHT

Consider the same geometry (Fig. 2) of conjugation as in Ref. 1. A nonlinear disordered medium M occupies the half-space $z>0$. Weak probe waves (of frequency ω_i) with amplitudes $E_{i\alpha}(\alpha=1,2\ldots)$ and wave vectors $\mathbf{k}_{i\alpha}$ and an intense pump wave (of frequency ω_p) with an amplitude E_p and wave vector $\mathbf{k}_p$ are incident from outside the surface with incidence angles $\theta_{i\alpha}$ and $\theta_p=0$, respectively. The conjugated signal waves (of frequency ω_s) with amplitudes $E_{s\alpha}$ are generated with wave vectors $\mathbf{k}_{s\alpha}$ very close to $-\mathbf{k}_{i\alpha}$. The amplitude of the generated signal field may be written in the following form:

$$E_{s\alpha}(\mathbf{r}) = \eta \frac{\omega_s^2}{c^2} \int d\mathbf{r}' G_s(\mathbf{r},\mathbf{r}') E_p^2(\mathbf{r}') E^*_{i\alpha}(\mathbf{r}') \ , \tag{9}$$

with η being the nonlinear susceptibility and integration over $\mathbf{r}'$ is carried out within the medium $z>0$. Here $G_s(\mathbf{r},\mathbf{r}')$ is the nonaveraged retarded Green's function of the Maxwell equation for the field $E_{s\alpha}(\mathbf{r})$. The disorder is described by a small random part $\delta\epsilon(\mathbf{r})$ of the dielectric function, with the correlation function given by

$$\frac{\omega^4}{c^4} \langle \delta\epsilon(\mathbf{r}) \delta\epsilon(\mathbf{r}') \rangle = \frac{4\pi}{l} \delta(\mathbf{r}-\mathbf{r}') \ . \tag{10}$$

In (9) the fields $E_p(\mathbf{r}'), E^*_{i\alpha}(\mathbf{r}')$ inside the medium can be expressed by means of the Green's functions G_p, G^*_i

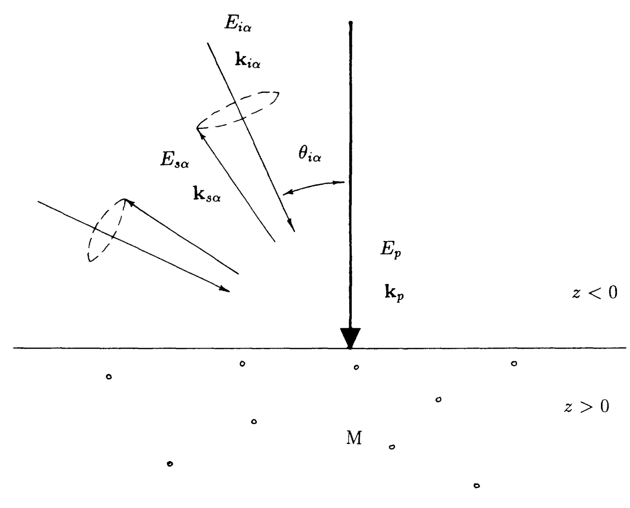

FIG. 2. Experimental geometry of phase conjugation.

through their amplitudes outside the medium $E_p, E_{i\alpha}^*$. Thus the amplitude (9) contains four Green's functions and is illustrated in Fig. 3(a), in which the nonaveraged Green's functions and the nonlinear-interaction vertex are represented by heavy lines and a circle, respectively. Therefore, the correlation function $\langle E_{s\alpha} E_{s\beta}^* \rangle$ (7) involves eight Green's functions. The averaging over disorder is carried out in a conventional way.[7] The disorder-averaged amplitude (9) vanishes because of the randomness of its phase caused by disorder. Figure 3(b) describes the main contribution to the correlation function (7) of conjugated light to the lowest order in the parameter $\lambda/l \ll 1$. Solid lines in the diagram correspond to the averaged Green's functions. In the bulk region they are given by the following expression[7] (where $a = i,p,s$):

$$\bar{G}_a(\mathbf{r},\mathbf{r}') = \frac{\exp[(ik_a - 1/2l)|\mathbf{r}-\mathbf{r}'|]}{4\pi|\mathbf{r}-\mathbf{r}'|} . \tag{11}$$

Dashed lines in Fig. 3(b) correspond to the correlation function (10). Each of the four sets of the three parallel dashed lines represents an infinite series of ladder or maximally crossed diagrams, referred to as, respectively, diffusion propagator $\mathcal{D}(\mathbf{r},\mathbf{r}')$ or the Cooper particle-particle diffusion propagator[8]

$$\mathcal{C}(\mathbf{r},\mathbf{r}') = \frac{12\pi D}{l^3}\mathcal{D}(\mathbf{r},\mathbf{r}') , \tag{12}$$

where $D = cl/3$ is the diffusion coefficient for light in the disordered medium. The diffusion propagator $\mathcal{D}(\mathbf{r},\mathbf{r}')$ obeys the equation

$$(i\Delta\omega + D\nabla^2)\mathcal{D}(\mathbf{r},\mathbf{r}') = -\delta(\mathbf{r}-\mathbf{r}') \tag{13}$$

and an effective boundary condition[1,5] at $z=0$

$$\mathcal{D}(\mathbf{r},\mathbf{r}') - hl\frac{\partial}{\partial z}\mathcal{D}(\mathbf{r},\mathbf{r}') = 0, \quad z' > 0 . \tag{14}$$

$\Delta\omega$ in (13) is the difference between the frequencies of the Green's functions G and G^* in a ladder series; h in (14) is a dimensionless phenomenological constant[1,5] depending on the reflectivity of the surface.

Note that in the particular case of a single probe wave ($\mathbf{k}_{i\alpha} = \mathbf{k}_{i\beta}$) diagram 3(b) coincides with diagram 2(b) of Ref. 1 describing the intensity of the phase-conjugated light. The presence of Cooper propagators connecting the probe and the signal light lines corresponds to the time-inverted propagation of the nonlinearly generated signal wave along the path of the probe wave. The diffusion propagators, connecting the pump-light lines, describe the diffusion of the pump light inside the medium. Diagram 3(b) contains the minimum number of Cooper and diffusion propagators required to get the bulk contribution to the correlation function (7). On the other hand, extra Cooper (diffusion) propagators would result in an additional small factor λ/l. In the leading order given by diagram 3(b), the expression for the correlation function (7) is evidently a convolution of two factors (corresponding to α and β waves, respectively) with some kernel K (see below) describing the diffusion propagation of the pump field:

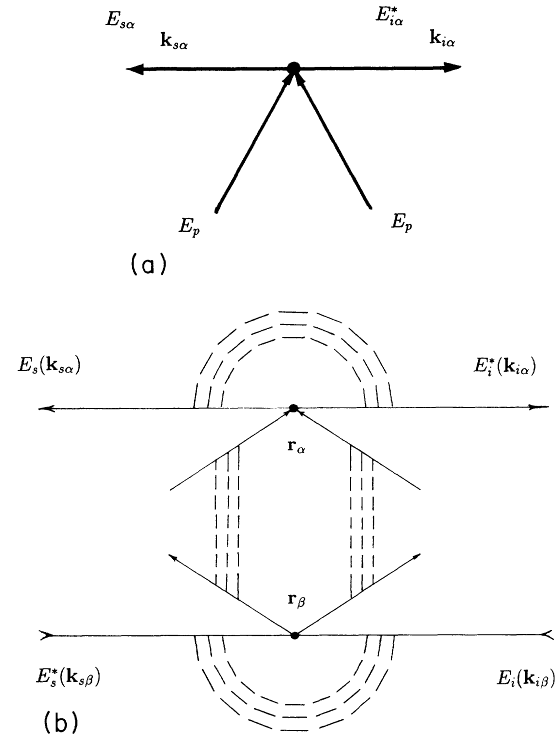

FIG. 3. Diagrams describing phase conjugation of light by a disordered medium: (a) nonaveraged amplitude of the signal wave; (b) correlation function of amplitudes of backscattered conjugated waves.

$$\langle E_s(\mathbf{k}_{s\alpha})E_s^*(\mathbf{k}_{s\beta})\rangle \propto E_{i\alpha}^* E_{i\beta} \int d\mathbf{r}_\alpha d\mathbf{r}'_\alpha d\mathbf{r}''_\alpha d\mathbf{r}_\beta d\mathbf{r}'_\beta d\mathbf{r}''_\beta [\exp(-i\mathbf{Q}_\alpha\cdot\mathbf{r}'_\alpha)\mathcal{C}(\mathbf{r}'_\alpha,\mathbf{r}''_\alpha)\bar{G}_s(\mathbf{r}''_\alpha,\mathbf{r}_\alpha)\bar{G}_i^*(\mathbf{r}''_\alpha,\mathbf{r}_\alpha)]K(\mathbf{r}_\alpha,\mathbf{r}_\beta)[\alpha\to\beta]^* . \tag{15}$$

In Eq. (15) all the integration variables $\mathbf{r} = (z, \mathbf{r}_\parallel)$ are inside the medium ($z > 0$, $\mathbf{r}_\parallel$ describes components parallel to the surface); the presence of factors like $\exp(-i\mathbf{Q}_\alpha \cdot \mathbf{r}'_\alpha)$ is due to the external lines of s and i photons in diagram 3(b) with $\mathbf{Q}$ being defined by (an index α or β is implied)

$$\mathbf{Q} = \mathbf{k}_s + \mathbf{k}_i^* . \tag{16}$$

The components $\mathbf{k}_{s\parallel}$ and $\mathbf{k}_{i\parallel}$ in (16) coincide with the corresponding ones of the photon wave vectors outside the medium. The components $\mathbf{k}_{sz}$ and $\mathbf{k}_{iz}$ are connected with the corresponding outside values by means of the refraction law having an imaginary part $\mathrm{Im}\mathbf{k} \sim 1/l$, which describes the damping of an averaged field amplitude inside the disordered medium.

Using the Fourier transformation with respect to $(\mathbf{r}_\parallel - \mathbf{r}'_\parallel)$ in (12) to (14) one obtains directly for $\mathcal{C}(\mathbf{Q}_\parallel; z', z'')$ at $z', z'' > 0$

$$\mathcal{C}(\mathbf{Q}_{\parallel};z',z'')\propto\frac{1}{Q}\left\{\exp[-Q|z'-z''|]-\frac{1-hQl}{1+hQl}\exp[-Q(z'+z'')]\right\}, \quad (17)$$

where

$$Q=\sqrt{\mathbf{Q}_{\parallel}^2-iL_{\Delta\omega}^{2}}\,,\quad \mathrm{Re}Q>0\,, \quad (18)$$

with $L_{\Delta\omega}=(D/|\omega_s-\omega_i|)^{1/2}$. Just the functions (17) (with indices α and β) appear in the integrand of (15) after integration over $\mathbf{r}'_{\alpha\parallel}$ and $\mathbf{r}'_{\beta\parallel}$. Because of the fast decay of the factor $\exp(-iQ_z z')$, the variable z' in (15) is restricted to the region $z'\sim 1/l$, while the other variable z'' in (15) takes values deeply in the bulk of the medium. Being interested in the vicinity of the backscattering peak (where $Q\ll 1/l$) one can use instead of (17) the following simplified expression, taking into account the lowest power in Ql and Qz':

$$\mathcal{C}(\mathbf{Q}_{\parallel};z',z'')\sim(z'-2hl)\exp(-Qz'')\,. \quad (19)$$

After integration over z'_α,z'_β, Eq. (15) reduces to

$$\langle E_s(\mathbf{k}_{s\alpha})E_s^*(\mathbf{k}_{s\beta})\rangle \propto E_{i\alpha}^*E_{i\beta}\int d\mathbf{r}''_\alpha d\mathbf{r}''_\beta\exp[-Q_\alpha z''_\alpha-i\mathbf{Q}_{\parallel\alpha}\mathbf{r}''_{\parallel\alpha}] \times\mathcal{K}(\mathbf{r}''_\alpha,\mathbf{r}''_\beta)\exp[-Q_\beta^* z''_\beta+i\mathbf{Q}_{\parallel\beta}\mathbf{r}''_{\parallel\beta}]\,, \quad (20)$$

with

$$\mathcal{K}(\mathbf{r}''_\alpha,\mathbf{r}''_\beta)=\int d\mathbf{r}_\alpha d\mathbf{r}_\beta\overline{G}_s(\mathbf{r}''_\alpha,\mathbf{r}_\alpha)\overline{G}_i^*(\mathbf{r}''_\alpha,\mathbf{r}_\alpha)] \times K(\mathbf{r}_\alpha,\mathbf{r}_\beta)\overline{G}_s^*(\mathbf{r}''_\beta,\mathbf{r}_\beta)\overline{G}_i(\mathbf{r}''_\beta,\mathbf{r}_\beta)]\,. \quad (21)$$

In (20) we have omitted factors which are the same for the cases of different ($\alpha\neq\beta$) and coincident ($\alpha=\beta$) wave vectors of the incident light. Also all optical Fresnel factors were omitted in (20): they depend smoothly on the angles of incidence $\theta_{i\alpha}$ (with the characteristic scale $\Delta\theta_{i\alpha}\sim 1$) and therefore are not important in the typical case when the aperture is not too large, when the angle difference is small as compared to, say, $\pi/2$.

The kernel $K(\mathbf{r}_\alpha,\mathbf{r}_\beta)$ in (21) is represented by the part in diagram 3(b) that corresponds only to the pump field. Because of the fast decay of the averaged Green's functions (11) and the short-correlation-length nature of the disorder (10), the typical spacing between points $\mathbf{r}_\alpha$ and $\mathbf{r}_\beta$ does not exceed l [see Fig. 3(b)]. As a consequence, the same holds for points $\mathbf{r}''_\alpha$ and $\mathbf{r}''_\beta$ in the kernel $\mathcal{K}(\mathbf{r}''_\alpha,\mathbf{r}''_\beta)$ in (21). It means that on a characteristic scale of the order $1/Q\gg l$, in the integrand (20) the kernel $\mathcal{K}(\mathbf{r}''_\alpha,\mathbf{r}''_\beta)$ has a short range and can be approximated by $\mathcal{K}\sim\delta(\mathbf{r}''_\alpha-\mathbf{r}''_\beta)$. Integration over $\mathbf{r}''_\alpha,\mathbf{r}''_\beta$ gives for the rhs of (20): $(Q_\alpha+Q_\beta^*)^{-1}\delta^{(2)}(\mathbf{Q}_{\parallel\alpha}-\mathbf{Q}_{\parallel\beta})$. Integrating (20) over $\mathbf{k}_{s\alpha},\mathbf{k}_{s\beta}$ within the small solid angles $\Omega_\alpha,\Omega_\beta$ around the directions $-\mathbf{k}_{i\alpha}$ and $-\mathbf{k}_{i\beta}$ (the size of these solid angles is determined by the small linear angle $\Delta\theta_{\mathrm{nl}}\sim\lambda/L_{\Delta\omega}$ being the angular width of the nonlinear conjugating backscattering peak[1]), we get the final expression for the correlation function $\langle E_{s\alpha}E_{s\beta}^x\rangle$:

$$\langle E_{s\alpha}E_{s\beta}^*\rangle=\int d\Omega_\alpha d\Omega_\beta\langle E_s(\mathbf{k}_{s\alpha})E_s^*(\mathbf{k}_{s\beta})\rangle \propto E_{i\alpha}^*E_{i\beta}\int d\Omega\frac{1}{\mathrm{Re}Q}\,. \quad (22)$$

In the derivation (22) it has been taken into account that the presence of $\delta^{(2)}(\mathbf{Q}_{\parallel\alpha}-\mathbf{Q}_{\parallel\beta})$ results in $Q_{\parallel\alpha}=Q_{\parallel\beta}\equiv Q$. In addition some geometrical factor $\propto\cos\theta_{i\beta}$ arises as the Jacobian of the transformation from Ω_β to $\mathbf{Q}_{\parallel\beta}$. Such smooth geometrical factors were again omitted in (22). Thus we have arrived at expression (7) with the correlation factor $f_{\alpha,\beta}$ being independent of α and β. It is essential that the same factor corresponds also to the case of coinciding $\mathbf{k}_{i\alpha},\mathbf{k}_{i\beta}$, which is simply a particular case of the previous derivation. This special case was considered in Ref. 1. Note that the last integral in (22) just gives (up to some proportionality factor) the integral intensity σ_{peak} of the backscattered peak.[9]

We have proved the smoothness of the correlation function $f_{\alpha,\beta}$ in (7) and, as a consequence, the validity of relation (5) for the disorder-averaged intensity of the phase-conjugated field. It means that a measurement of the intensity (e.g., by photography) of the image field will reveal the interference structure of the source field, as it takes place in the usual case of ordered media.

To stress the special role of phase conjugation for the validity of (5), we demonstrate below that this relation breaks down for ordinary linear backscattering[3,4] by a disordered medium.

IV. COMPARISON WITH LINEAR BACKSCATTERING

Consider the same scattering geometry as shown in Fig. 2, but without a pump wave and with the nonlinear medium substituted by a linear one. The correlation function of the two backscattered ($\mathbf{k}_{s\alpha}\approx-\mathbf{k}_{i\alpha},\mathbf{k}_{s\beta}\approx-\mathbf{k}_{i\beta}$) waves is given by the diagram in Fig. 4. For the case of a single probe wave ($\mathbf{k}_{i\alpha}\equiv\mathbf{k}_{i\beta}$) the known singular part of this diagram at $\mathbf{k}_{s\alpha}\approx-\mathbf{k}_{i\alpha}$ just describes the intensity of the backscattered field. In the considered case of different waves, the singular part survives if $\mathbf{k}_{s\alpha}\approx-\mathbf{k}_{i\beta}$, and as a consequence of $\mathbf{k}_{s\alpha}\approx-\mathbf{k}_{i\alpha}$ we have $\mathbf{k}_{i\alpha}\approx\mathbf{k}_{i\beta}$. The wave vector region allowed by these approximate equalities is restricted to the intersection of narrow cones in the directions of the backscattering $-\mathbf{k}_{i\alpha}$ and $-\mathbf{k}_{i\beta}$, with the aperture angle of the cones being just

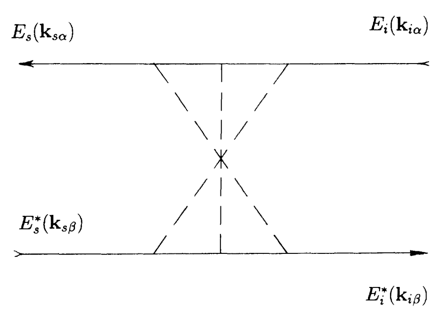

FIG. 4. The diagram describing the correlation function of amplitude of the linearly backscattered waves.

the angular width ($\Delta\theta_l \sim \lambda/l$) of the linearly backscattered peak. Hence, there is no correlation of the scattered wave if the difference of the angles of incidence of the incoming waves exceeds a small value $\Delta\theta_l$. It means that the factor $f^l_{\alpha,\beta}$ in the correlation function of linearly scattered waves

$$\langle E_{s\alpha} E^*_{s\beta} \rangle = f^l_{\alpha,\beta} E_{i\alpha} E^*_{s\beta} \qquad (23)$$

is a sharp function of the angles of incidence (practically $f^l_{\alpha,\beta} \sim \delta_{\alpha,\beta}$). Therefore, in contrast to the case of phase conjugation, for linear backscattering relation (5) is not valid and the intensity of the scattered field does not reproduce the variations of the intensity of the incoming field.

V. AMPLITUDE OF THE BACKSCATTERED CONJUGATED FIELD

Consider another interesting feature of the phenomenon. As was mentioned above, the amplitude (1) of a conjugated backscattered wave $E_{s\alpha} = r_\alpha E^*_{i\alpha}$ averaged over disorder is zero because of the random phase of the factor r_α. However, it is not clear if this amplitude is zero (or negligible) in a given disordered sample, or whether it vanishes only after averaging over an ensemble of samples. Independence of the correlation function $f_{\alpha,\beta}$ in (7) on α and β proved above allows us to suggest that the randomness of all the factors r_α is described by a single complex random variable $r\exp(i\phi)$ (with positive r), which does not depend on the angle of incidence of incoming waves (neglecting a smooth variation on the scale $\Delta\theta_i \sim 1$). Thus, the phase ϕ could be considered as a characteristic parameter of a given sample. As a consequence, the amplitude $E_s = \sum_\alpha E_{s\alpha}$ of the conjugated-backscattered field generated by a given disordered nonlinear sample should not be zero. In fact it has a typical magnitude $|E_s| = \sqrt{\langle I_s \rangle}$ and its phase differs in a random manner from sample to sample.

These coherence properties of the amplitude of the conjugated light allow a simple qualitative explanation. The conjugated light is generated by the mixing of two pump waves and the conjugated probe wave with wave vectors $\mathbf{k}_{p1}, \mathbf{k}_{p2}$, and $\mathbf{k}_i$, respectively. Because of multiple scattering, these wave vectors have arbitrary directions in the bulk of the disordered sample. As it was noted in Ref. 1, the processes where $\mathbf{k}_{p1} + \mathbf{k}_{p2} \neq 0$ contribute to the incoherent diffuse background at frequency ω_s, while the backscattered signal peak corresponds to the processes with the phase-matching condition $\mathbf{k}_{p1} + \mathbf{k}_{p2} \approx 0$. Consider now expression (9) for the amplitude of the conjugated field, where the quantity $[E_p(\mathbf{r})]^2$ can be represented as

$$[E_p(\mathbf{r})]^2 = \sum_{\mathbf{k}_{p1}, \mathbf{k}_{p2}} E_{\mathbf{k}_{p1}} E_{\mathbf{k}_{p2}} \exp[i(\mathbf{k}_{p1} + \mathbf{k}_{p2}) \cdot \mathbf{r}] \ . \qquad (24)$$

To find out more about the coherent (backscattered) component of the conjugated field one can extract from (24) the terms corresponding to the phase-matching condition $\mathbf{k}_{p1} + \mathbf{k}_{p2} \approx 0$:

$$[E_p(\mathbf{r})]^2_{\text{coh}} = \sum_{\mathbf{k}_p} E_{\mathbf{k}_p} E_{-\mathbf{k}_p} \equiv C\exp(i\Phi) \ , \qquad (25)$$

where C and Φ are the (positive) modulus and phase of the complex quantity $[E_p]^2_{\text{coh}}$. It is important that C and Φ are characteristic parameters for a given sample and pump geometry and do not depend on the incident probe field E_i. Thus, the quantity (25) enters Eq. (9) simply as a common multiplicative factor resulting in the coherence of the amplitudes of backscattered conjugated signal waves corresponding to different (coherent) incident waves.

VI. CONCLUSIONS

We have shown that the phase-conjugated field generated in a disordered nonlinear medium is strongly correlated even if the angular width of an incident probe beam exceeds the angular width of the peak of the conjugated nonlinear backscattering of a single wave by several orders of magnitude. This is in contrast to the case of linear backscattering where such correlations are absent outside the narrow angular region of the backscattering peak.

As a consequence, the intensity of the backscattered conjugated field in disordered media does reproduce interference variations of the incoming field, as takes place in the ordered case. Moreover, the amplitude of the backscattered conjugated field reproduces the conjugated amplitude of an incident (probe) field up to some constant complex factor, which is a characteristic parameter for a given sample and experimental configuration.

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[9]We use this opportunity to correct expression (18) in Ref. 1 for σ_{peak}, which should read $\sigma_{\text{peak}} \propto (\delta n_{\text{nl}})^2 (2\pi l / L_{\Delta\omega})$. As a consequence, Eq. (21) in Ref. 1 should be substituted by $\sigma_0 \propto (L_0 L_{\Delta\omega} / l\lambda) \sigma_{\text{peak}}$.