Mutual drag of two- and three-dimensional electron gases: A collective-collisions approach

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In the recent paper by Laikhtman and Solomon [Phys. Rev. B **41**, 9921 (1990)] a theoretical description of the mutual-drag effect of two- and three-dimensional electron gases in heterostructures was given. In this paper a different approach based upon the concept of collective collisions between electrons is suggested. The mutual drag of two two-dimensional gases is considered briefly.

In a recent experiment,[1] the phenomenon of mutual drag of two-dimensional (2D) and three-dimensional (3D) electron gases (EG) has been discovered. The effect manifests itself via the occurrence of induced electric current in one of the gases when a driving voltage is applied to the other. The phenomenon of mutual drag was predicted by Price[2] in 1983 for the case of two layers of nondegenerate 2DEG. In Price's consideration,[2] the effect is a consequence of momentum transfer caused by the scattering of an electron in one layer on the electrons of the other layer. The scattering of nondegenerate 2D electrons on 3D ones was considered by Boiko and Sirenko.[3] However, the experiment[1] showed that the momentum transfer dominated only at sufficiently high temperatures while at lower temperatures the energy transfer played a crucial role. Recently, Laikhtman and Solomon[4] (LS) suggested an explanation for the mutual drag under conditions similar to those in experiment.[1]

In the present Brief Report, we suggest another approach to the mutual-drag effect. We assume that the energy and momentum transfer is caused by the interaction of a given electron with the fluctuating electric field formed by all electrons in the system (collective collisions) rather than binary collisions as assumed by LS. Although it can be shown (and it will be at the end of this paper) that these approaches are identical under the assumptions about the Thomas-Fermi character of the screening, our result differs from that of LS. Since the calculations in the collective-collisions approach are much easier than in the binary-collisions approach (because of the kinematics of only one type of electron—2D or 3D—should be considered), we may attribute this difference to the additional approximations adopted by LS to obtain results in analytical form. For the sake of clarity we shall confine ourselves to consideration of momentum transfer only, which is a much simpler process than that of energy transfer.

As in Ref. 4, we consider a 2D channel separated from the half-space filled with 3D electrons by an insulating layer of thickness a. Let the driving voltage be applied to the 2D channel. We first calculate the momentum relaxation time of 2D electrons determined by the scattering of the 3D electrons. Let $\phi(\mathbf{r},t)$ be the random electrostatic potential formed by all the electrons in the system. We can describe the effect of this potential on the 2D electrons by including the corresponding term into the Hamiltonian

$$H_{\rm int}=e\phi \ , \tag{1}$$

where e is the electron charge. As shown in Ref. 5 (cf. also Ref. 6), the probability of transition from state $\mathbf{p}$ to state $\mathbf{p}'$ caused by the interaction with a random potential can be expressed in terms of the correlation function of this potential. Applying the method described in Ref. 5 we have

$$W_{\mathbf{p}\to\mathbf{p}'}=\frac{e^2}{\hbar S}\int dz_1\int dz_2\psi_n^2(z_1)\psi_n^2(z_2)K(\omega,\mathbf{q},z_1,z_2) \times(1-f_{\mathbf{p}'})(1-f_{\mathbf{p}})^{-1} \ , \tag{2}$$

where K is the Fourier transform of the correlation function

$$K(\omega,\mathbf{q},z_1,z_2)=\int d^2\rho\int dt\, e^{-i(\mathbf{q}\cdot\rho-\omega t)} \times\langle\phi(0,z_1,0)\phi(\rho,z_2,t)\rangle \ . \tag{3}$$

ρ and $\mathbf{q}$ are directed parallel to the channel, the z axis is directed perpendicular to the channel, ψ_n is the size quantization wave function, $f_{\mathbf{p}}$ the Fermi distribution function, and S the normalization area. The zero of the z axis is chosen at the 3DEG/insulator interface. Due to the conservation of momentum and energy, $\mathbf{q}=\mathbf{p}-\mathbf{p}'$ and $\omega=(\varepsilon_{\mathbf{p}}-\varepsilon_{\mathbf{p}'})/\hbar$. With the probability defined by Eq. (2) the momentum relaxation time is

$$\frac{1}{\tau}=\sum W_{\mathbf{p}\to\mathbf{p}'}(1-\cos\theta) \ , \tag{4}$$

where θ is the angle between $\mathbf{p}$ and $\mathbf{p}'$. According to the fluctuation-dissipation theorem, the correlation function is connected with the imaginary part of the electrostatic retarded Green's function

$$K(\omega,\mathbf{q},z_1,z_2)=-2\hbar[N(\omega)+1]\,\mathrm{Im}\mathcal{D}(\omega,\mathbf{q},z_1,z_2) \ , \tag{5}$$

where $N(\omega)$ is the Bose factor, and $\mathcal{D}$ is the Fourier transform of the electrostatic potential at point z_1 induced by a point charge at point z_2. We shall consider the case of an infinitely thin 2D channel and shall replace

ψ_n in Eq. (2) by $\delta(z-a)$. Thus, to calculate $W_{p\to p'}$ we have to calculate $\mathrm{Im}\mathcal{D}$ and insert $z_1=z_2=a$.

Outside the 3DEG the function $\mathcal{D}$ satisfies the ordinary Poisson equation

$$\left[\frac{\partial^2}{\partial z_1^2}-q^2\right]\mathcal{D}=\frac{4\pi}{\epsilon}\delta(z_1-z_2)\ , \tag{6}$$

where ϵ is the dielectric constant of the insulator. Equation (6) must have boundary conditions. The first condition is imposed at the channel and takes into account the presence of 2D free electrons:

$$\left.\frac{\partial\mathcal{D}}{\partial z_1}\right|_{z_1=a+}-\left.\frac{\partial\mathcal{D}}{\partial z_1}\right|_{z_1=a-}=-\frac{4\pi\sigma}{\epsilon}$$

$$=4\pi q^2\chi(q,\omega)\mathcal{D}|_{z_1=z_2=a}\ , \tag{7}$$

where $\chi(q,\omega)$ is the polarizability of 2DEG (Ref. 7) [in deriving Eq. (7) it was taken into account that the potential is continuous through the channel]. Another boundary condition is imposed at $z_1=0$,

$$\left.\frac{\partial\mathcal{D}}{\partial z_1}\right|_{z_1=0}=\frac{\bar{\epsilon}}{\epsilon}q\mathcal{D}\Big|_{z_1=0}\ , \tag{8}$$

where

$$\frac{1}{\bar{\epsilon}}=\frac{q}{\pi\epsilon}\int_{-\infty}^{\infty}\frac{dk_z}{k^2\epsilon_\infty(k,\omega)}\ , \tag{9}$$

and $k^2=k_z^2+q^2$. $\epsilon_\infty(k,\omega)$ is the permittivity of the infinite 3DEG. This condition was derived for the case of specular reflection of the electrons from the boundary (see Refs. 6, 8, and 9 and also Boiko and Sirenko.[3]) It can be shown that the condition (8) is identical with the condition used by LS under the assumption concerning the Thomas-Fermi character of screening in the 3D electron gas. With all the foregoing boundary conditions imposed, we have for the Green's function at $z_1=z_2=a$

$$\mathcal{D}=\frac{\mathcal{D}_0}{1+q^2\chi(q,\omega)\mathcal{D}_0}\ , \tag{10}$$

where $\mathcal{D}_0$ is the Green's function for the system without the 2DEG

$$\mathcal{D}_0=\frac{2\pi}{q\epsilon}\left[1-\frac{\bar{\epsilon}-\epsilon}{\bar{\epsilon}+\epsilon}e^{-2qa}\right]\ . \tag{11}$$

Interpreting Eq. (10) in terms of an effective dielectric permittivity coincides with that used by Boiko and Sirenko.[3]

To calculate the imaginary part of Eq. (10) we shall make the same assumptions as in the work by LS:[4] the screening is assumed to be static and of the Thomas-Fermi type. In the case of static screening the imaginary part of $\mathcal{D}$ is expressed as a sum of two terms: one is proportional to $\mathrm{Im}\mathcal{D}_0$ and contains only $\mathrm{Re}\chi$, and the reverse is true for the second. The first term corresponds to fluctuations caused by dissipation in 3DEG and screened by the 2DEG, and vice versa. Hence the total $\mathrm{Im}\mathcal{D}$ corresponds to the interaction of 2D electrons with 3D electrons as well as with 2D ones. Since we are interested in the calculation of the drag between 2D and 3D electrons we should omit the second term in $\mathrm{Im}\mathcal{D}$, i.e., put $\mathrm{Im}\chi=0$. Taking into account all the assumptions mentioned above for $\mathrm{Im}\mathcal{D}$, at $z_1=z_2=a$, we obtain

$$-\mathrm{Im}\mathcal{D}=\frac{2\pi\omega}{q_s^2(q_s^{\mathrm{ch}})^2v_F\epsilon}\frac{\ln(2q_s/q)\exp(-2qa)}{[1-\exp(-2qa)]^2}\ , \tag{12}$$

where q_s^{ch} and q_s are the screening parameters of the channel and of the gate, respectively, and v_F is the Fermi velocity in the gate.

Now we substitute Eq. (12) into Eq. (5) and calculate τ according to Eq. (4). For temperatures higher than the typical value for energy transfer, the Bose factor in Eq. (5) reduces to the classical Nyquist factor $k_BT/\hbar\omega$ and the factor in Eq. (2) including the Fermi distribution function reduces to unity. Thus, the dependence $\tau^{-1}\propto T$, which was obtained by LS (Ref. 4), follows immediately. The integration over d^2p' is easily carried out. Once τ is calculated, it is easy to derive the expression for the induced current in the 3DEG via simple considerations of momentum conservation. If the number of 2D electrons in the channel is N_{ch} and their drift velocity caused by the applied electric field F is u, then the total momentum transferred to 3D electrons per unit time is

$$\frac{dP}{dt}=\frac{N_{\mathrm{ch}}mu}{\tau}\ . \tag{13}$$

If τ_g is the elastic relaxation time in the 3DEG determined by some scattering mechanism which is much stronger than the mutual drag, then the total momentum gained by the 3DEG is $\tau_g dP/dt$. The influence of a mutual drag force on the 3DEG leads to the establishment of a drift velocity field $v(z)$. Thus, the total induced current in the 3DEG is $J=L_yen_g\int_0^{L_z}dz\,v(z)$, where L_y is a sample size in the direction perpendicular to the current, L_z is the size in the z direction, and n_g is the concentration of the 3DEG. Using the obvious relation between $\tau_g dP/dt$ and J we have

$$J=n_{\mathrm{ch}}n_g\frac{\tau_g\tau_{\mathrm{ch}}}{\tau}\frac{e^2}{m}L_yF\ , \tag{14}$$

where $n_{\mathrm{ch}}=N_{\mathrm{ch}}/S$, and τ_{ch} has the same meaning for the channel as τ_g for the 3DEG. The derivation demonstrated above is equivalent to the solution of the Boltzmann equation only if all relaxation times do not depend on the electron energy. For the reasons discussed in Ref. 4 this is the case in question. Finally, substituting into Eq. (10) the expression for τ, we have

$$J=\frac{15}{16}\frac{\zeta(5)}{\pi}\tau_g\tau_{\mathrm{ch}}\frac{e^2}{\hbar v_F\epsilon}\frac{k_BT}{\hbar}\frac{\ln(4q_sa)}{(q_s^{\mathrm{ch}})^2q_s^2a^6}\frac{e^2FL_y}{m}\ , \tag{15}$$

where $\zeta(x)$ is the Riemann function. Denoting by J_{LS} the result for the induced current obtained by LS for the ratio J/J_{LS}, we find

$$J/J_{\mathrm{LS}}=\frac{1}{\pi^2}\frac{e^2}{\hbar v_F\epsilon}\ln(4q_s a)(q_s/k_F)^2 . \tag{16}$$

For the given set of the experimental parameters chosen for comparison by LS, $J/J_{\mathrm{LS}}\approx 0.09$. Our results agree better with the observed magnitudes of the induced current than those of LS, which are about an order of magnitude larger than the observed current. The difference between J and J_{LS} can be found experimentally by varying n_g, if the dependence of τ_g on n_g may be discriminated by a special experiment. Then, according to Eq. (15)

$$J/\tau_g \propto \ln(n_g)/n_g^{2/3} , \tag{17}$$

while J_{LS}/τ_g does not depend on n_g. Note that if we substitute $k_B T=\hbar^2 k_F^2/2m$ in the expression for τ derived by Boiko and Sirenko in the nondegenerate case, then their result agrees with ours.

Now we are in a position to prove the identity of the collective-and binary-collisions approaches. The idea of the proof is straightforward. The transition probability in the collective-collisions approach is expressed through the imaginary part of the Green's function or, neglecting its dimensionality, of the electrostatic potential. The transition probability in the binary-collisions approach is expressed through the matrix element of the potential. Requiring that this probability should be equal we obtain some identity for the potential. In the case of simple geometry (3D or 2D electron gases) it can be shown that this identity is satisfied if the permittivity of the electron gas obeys the expression derived in the random-phase approximation (RPA). (The well-known example of the violation of the identity of the collective- and binary-collisions approach beyond the RPA is the electron-electron interaction in impure metals.[10]) In our case, with a complicated geometry, we are able to prove this identity only under stronger assumptions regarding the screening which were considered above.

Let the $\tilde{w}_{\mathbf{p}\to\mathbf{p}'}$ be the probability of the transition of a 2D electron from the state p into the state p' caused by the collision with a 3D electron. Then, following the Fermi "golden rule"

$$\tilde{W}_{\mathbf{p}\to\mathbf{p}'}=\frac{2\pi}{\hbar(\mathrm{LS})^2}\sum_{\mathbf{k},\mathbf{k}'}\delta(\varepsilon_{\mathbf{k}}+\varepsilon_{\mathbf{p}}-\varepsilon_{\mathbf{k}'}-\varepsilon_{\mathbf{p}'})\delta_{\mathbf{p}-\mathbf{p}',\mathbf{k}'_{\parallel}-\mathbf{k}_{\parallel}}\left|\int_{-\infty}^{0}dz_2 V(\omega,\mathbf{q},a,z_2)e^{iz_2(k_z-k'_z)}\right|^2(1-f_{\mathbf{p}'})(1-f_{\mathbf{p}})^{-1}f_{\mathbf{k}}(1-f_{\mathbf{k}'}) , \tag{18}$$

where $\mathbf{k}=(k_z,\mathbf{k}_{\parallel})$ is the momentum of the 3D electron, L is the normalization length in z direction, and $V(\omega,q,a,z_2)$ is the Fourier transform of the potential at $z=a$ induced by the 3D electron located at z_2 inside the gate. Note that $V=e^2\mathcal{D}$. Due to the reciprocity principle $\mathcal{D}(\omega,\mathbf{q},a,z_2)=\mathcal{D}(\omega,\mathbf{q},z_2,a)$. Comparing Eq. (18) with Eq. (2) we obtain the following identity for $\mathcal{D}$:

$$-\mathrm{Im}\mathcal{D}(\omega,\mathbf{q},a,a)=\frac{e^2}{2(2\pi)^3}\int d^2k_{\parallel}\int dk_z\int dk_{z'}\delta(\epsilon_{\mathbf{k}_{\parallel},k_z}+\epsilon_{\mathbf{p}}-\epsilon_{\mathbf{k}_{\parallel}+\mathbf{q},k'_z}-\epsilon_{\mathbf{p}'})$$
$$\times\left|\int_{-\infty}^{0}dz_2\mathcal{D}(\omega,\mathbf{q},z_2,a)e^{iz_2(k_z-k_{z'})}\right|^2(1-f_{\mathbf{k}_{\parallel}+\mathbf{q},k'_z})f_{\mathbf{k}_{\parallel},k_z}/[N(\omega)+1] . \tag{19}$$

The integration over z is accomplished by using the fact that in the Thomas-Fermi approximation $\mathcal{D}$ satisfies the Poisson equation inside the gate: $\Delta\mathcal{D}=q_s^2\mathcal{D}$. Then, integrating by parts, we obtain for this integral

$$\frac{\left|\left(\frac{\partial\mathcal{D}}{\partial z_2}\right|_{z_2=0}\right|^2+(k_z-k_{z'})^2(\mathcal{D}|_{z_2=0})^2}{(k_z-k'_z)^2+q_s^2} . \tag{20}$$

Integrating further over $\mathbf{k}_{\parallel}$, k_z, and k'_z, and making use of the boundary conditions (7) and (8), we obtain that the left part of Eq. (19) coincides with Eq. (12). Thus, the identity of the two approaches is proven.

Finally, we shall discuss briefly the mutual drag of two 2D electron gases. This situation was considered previously in the pioneer work by Price[2] for the case of nondegenerate gases. The binary-collision approach was applied, and screening was completely neglected. Here we consider the case of degenerate gases, take into account the screening in both gases, and apply the collective-collision approach in a way similar to that discussed above. The gases are assumed to be identical. The final result for the induced current is

$$J=\frac{\pi^3}{60}\frac{e^2(\tau_{\mathrm{ch}})^2}{\hbar v_F\epsilon}\frac{k_B T}{\hbar}\frac{1}{(q_s^{\mathrm{ch}})^2 a^5}\frac{e^2}{m}FL_y , \tag{21}$$

where now a stands for the distance between 2D gases. Equation (21) exhibits a slower decrease of current with distance a ($\propto a^{-5}$) as compared to the 3D/2D case ($\propto \ln a/a^6$). This is quite natural because of the weak character of screening in the two-dimensional systems. If τ_{ch} is determined by the acoustic phonon scattering, as was assumed in Ref. 4, then the total dependence of the induced current is T^{-1}, contrary to the 2D/3D case where induced current does not depend on temperature.

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