Energy relaxation of hot two-dimensional excitons in a GaAs quantum well by exciton-phonon interaction

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The power loss of hot two-dimensional excitons in $GaAs/Al_xGa_{1-x}As$ multiple quantum wells and the corresponding energy relaxation times are calculated in the present work. A simplified variational envelope function is assumed for the 1*s* excitonic level in an infintely deep well and the exciton distribution is described by an effective exciton temperature. For deformation-potential acoustic-phonon scattering, a simple expression for the power loss derived by Takagahara is used, while for polar-optic-phonon scattering the expression is developed in the present paper. The calculated values indicate that for lattice and exciton temperatures below 150 K, polar-optic-phonon scattering contributes little to the power loss. The energy relaxation time due to the acoustic-phonon scattering is found to be about 3–4 times higher than the experimental estimate and hence the higher-lying states have to be considered to obtain enhanced energy relaxation rate.

I. INTRODUCTION

Excitons play an important role in determining the optical properties of semiconductor quantum wells (QW's).[1] A number of potentially useful photonic devices made of QW's have been reported,[2–4] the operation of which depends on the properties of two-dimensional (2D) excitons. Excitonic phenomena in QW's and multiple QW's (MQW's) are, therefore, being studied extensively for both the fundamental and applied reasons. Very recently Damen *et al.*[5] made a thorough study of the dynamics of formation and relaxation of hot excitons in $GaAs/Al_xGa_{1-x}As$ MQW's. They concluded that energetic electron-hole pairs (EHP's) are formed immediately after the photoexcitation; these EHP's then give up some energy and form excitons in different levels (1*s*, 2*s*, 2*p*, etc.) with large wave vector K. The hot excitons then relax to zero momentum state ($K=0$) of the singlet (1*s*) state by phonon emission.

Damen *et al.*[5] studied the formation time of 2D excitons as well as the time needed for the hot 2D excitons to relax to the $K=0$ state. This energy relaxation time was estimated by them for different lattice temperatures and excitation intensities. They estimated that the effective relaxation time due to exciton-acoustic phonon interaction is about 30 ps at a lattice temperature of 10 K. No theoretical estimate for the relaxation time of excitons within the exciton branch was available then. The calculation of the energy-loss rate of hot excitons in a QW has since then been performed by Takagahara[6] by considering the interaction with acoustic phonons.

In the present work we make an estimate of the energy relaxation time by using the expression given by Takagahara. At low temperatures excitons can relax via the emission of acoustic phonons only; however, at higher temperatures the relaxation mechanisms may take place through the emission of longitudinal polar-optic (LO) phonons.[7] In this paper we derive an expression for the energy loss rate due to the exciton–bulklike-LO-phonon interaction. In the preliminary work reported here we consider only the 1*s* excitonic level and the relaxation in that particular exciton branch only. We also employ a simple variational wave function[8] for the excitons in the well material assuming an infinite barrier. An essentially similar model has previously been employed by us[9] to calculate the momentum relaxation rate and the mobility of 2D excitons in a MQW.

We have developed the theory of the energy loss rate in Sec. II of this paper. The theory is then applied to calculate the values of the energy relaxation times in $GaAs/Al_xGa_{1-x}As$ MQW's for different values of lattice and exciton temperatures. The calculated values are presented in Sec. III where the relative importance of the two scattering processes is assessed. A comparison is also made between the present calculated values and the values given by Damen *et al.*[5] Section IV gives the conclusion of the paper.

II. THEORY

A. Exciton wave function and matrix element

The theory of scattering of 2D excitons is given in a number of papers.[6–12] The QW is of width L along the z direction and the dispersion relation for excitons is assumed to be parabolic. The wave function of excitons in a QW may be expressed in the Bloch representation as

$$|n,\mathbf{K}\rangle = \sum_{\mathbf{k},\mathbf{k}'} F_n(\mathbf{k},\mathbf{k}',\mathbf{K})\delta_{\mathbf{k}-\mathbf{k}',\mathbf{K}} a^{\dagger}_{c\mathbf{k}} a_{v\mathbf{k}'}|0\rangle \ , \tag{1}$$

where $\mathbf{k}$ ($\mathbf{k}'$) is the wave vector of electron (hole) and

$$F_n(\mathbf{k},\mathbf{k}',\mathbf{K}) = S^{-1}\int d^2r \int dz_e \int dz_h f_n(r,z_e,z_h) \times \exp[i(\alpha_e\mathbf{K}-\mathbf{k})\cdot\mathbf{r} - ik_z z_e + ik_z z_h] \ . \tag{2}$$

In Eq. (2) $\alpha_{e\,(h)} = m_{e\,(h)}/M$, $\mathbf{K}$ is the total wave vector of the exciton, and S is the normalization area. The matrix element for transition from a state $|1s,\mathbf{K}\rangle$ to another state $|1s,\mathbf{K}'\rangle$ due to the interaction with the jth-mode phonon is[6,9]

$$|M_j^{\pm}(\mathbf{K},\mathbf{K}')|^2=\sum_{q_z}[E_c^j(\mathbf{K}-\mathbf{K}',q_z)H_{1s}(-\alpha_h(\mathbf{K}-\mathbf{K}'),q_z)-E_v^j(\mathbf{K}'-\mathbf{K},q_z)H_{1s}(\alpha_e(\mathbf{K}-\mathbf{K}'),_q)]^2(N_Q+\tfrac{1}{2}\pm\tfrac{1}{2})\ , \tag{3}$$

where

$$H_{1s}(\alpha\mathbf{q},q_z)=\int d^2r\int dz_e\int dz_h|f_{1s}(\mathbf{r},z_e,z_h)|^2\exp(iq_zz_e+i\alpha\mathbf{q}\cdot\mathbf{r})\ . \tag{4}$$

In Eqs. (2) and (4), $\mathbf{r}=\mathbf{r}_e-\mathbf{r}_h$, $M=m_e+m_h$, $E^j_{c\,(v)}$ is a factor that measures the strength of interaction between an electron (hole) an bulklike jth-mode phonon of 3D wave vector Q, $m_{e\,(h)}$ is the electron (hole) mass, q_z is the component of Q along the z direction, and N_Q is the equilibrium-phonon occupation number with $+$ $(-)$ corresponding to the emission (absorption) process. The transition probability is then expressed as

$$W_j^{\pm}(\mathbf{K},\mathbf{K}')=(2\pi/\hbar)|M_j^{\pm}(\mathbf{K},\mathbf{K}')|^2\delta(E_{\mathbf{K}'}-E_{\mathbf{K}}\pm\hbar\omega_Q)\ . \tag{5}$$

In the present work we employ the following variational envelope function for the 1s excitonic level in an infinite QW:[8]

$$f_{1s}(\mathbf{r},z_e,z_h)=A\exp(-\beta r/2)\cos(\pi z_e/L)\times\cos(\pi z_h/L)\ . \tag{6}$$

B. Energy loss rate

1. Deformation-potential scattering

The power loss due to an interaction between excitons and deformation potential (DP) acoustic phonons has been expressed by Takagahara[6] as follows:

$$\langle dE/dt\rangle_{\mathrm{DP}}=-(1/N_{\mathrm{ex}})\sum_Q\hbar\omega_Q(dN_Q/dt)\ , \tag{7}$$

where dN_Q/dt is the rate of growth of the phonon occupation number and N_{ex} is the exciton density defined as $N_{\mathrm{ex}}=\sum_{\mathbf{K}}f_{\mathrm{ex}}(\mathbf{K})$, f_{ex} being the exciton distribution function which we have assumed to follow the Maxwell Boltzman statistics. The final expression for the power loss is as follows:

$$\langle dE/dt\rangle_{\mathrm{DP}}=-(2\pi M/k_BT_{\mathrm{ex}})^{1/2}\frac{(D_c-D_v)^2}{4\pi^2\rho}\int dQ\,Q^3\exp(-E_Q/k_BT_{\mathrm{ex}})[(1+N_Q)\exp(-\hbar\omega_Q/k_BT_{\mathrm{ex}})-N_Q]\ , \tag{8}$$

where $E_Q=\hbar^2Q^2/8M$. The energy relaxation time τ_E is then expressed as[13]

$$\tau_E^{\mathrm{DP}}=k_B(T_{\mathrm{ex}}-T_L)/\langle dE/dt\rangle_{\mathrm{DP}}\ . \tag{9}$$

2. LO phonon scattering

The expression for the power loss can be written as[14,15]

$$\langle dE/dt\rangle_{\mathrm{LO}}=-(\hbar\omega_{\mathrm{LO}}/N_{\mathrm{ex}})\times\sum_{\mathbf{K}}f_{\mathrm{ex}}(K)\sum_{\mathbf{K}'}[W_{\mathrm{LO}}^{+}(\mathbf{K},\mathbf{K}')-W_{\mathrm{LO}}^{-}(\mathbf{K},\mathbf{K}')]\times[1-f_{\mathrm{ex}}(\mathbf{K}')]\ , \tag{10}$$

where $\hbar\omega_{\mathrm{LO}}$ is the energy of the LO phonon. Since the exciton density is small and f_{ex} is the Maxwellian distribution function, one may put $f_{\mathrm{ex}}\ll 1$, and may rewrite Eq. (10) as

$$\langle dE/dt\rangle_{\mathrm{LO}}=-(\hbar\omega_{\mathrm{LO}}/k_BT_{\mathrm{ex}})\times\int_0^{\infty}dE[W_{\mathrm{LO}}^{+}(E)-W_{\mathrm{LO}}^{-}(E)]e^{-E/k_BT_{\mathrm{ex}}}, \tag{11}$$

where

$$W_{\mathrm{LO}}^{\pm}(E)=\sum_{\mathbf{K}'}(W_{\mathrm{LO}}^{\pm}\mathbf{K},\mathbf{K}')\ .$$

The expression for $W_{\mathrm{LO}}^{\pm}(E)$ is

$$W_{\mathrm{LO}}^{\pm}(E)=\frac{e^2\omega_{\mathrm{LO}}\beta^6M}{\pi^2L^2\hbar^2\epsilon'}(N_Q+\tfrac{1}{2}\pm\tfrac{1}{2})\times\int_0^{2\pi}d\vartheta\,I(q,\eta)(\gamma_e-\gamma_h)^2\ , \tag{12}$$

where

$$\gamma_{e\,(h)}=(\beta^2+\alpha^2_{h\,(e)}q^2)^{-3/2}\ ,\quad \eta=2\pi/L\ ,$$

$$I(x,y)=\frac{\pi}{2x^2(x^2+y^2)}\left[\frac{(3x^2+2y^2)\pi}{y}-\frac{y^4(1-e^{-2\pi x/y})}{x(x^2+y^2)}\right]$$

and $1/\epsilon'=1/\epsilon_\infty-1/\epsilon_s$, $\epsilon_{\infty\,(s)}$ being the optical (static) permittivity of the material. The energy relaxation time τ_E^{LO} is expressed exactly as in Eq. (10) with $\langle dE/dt\rangle_{\mathrm{LO}}$ in the denominator.

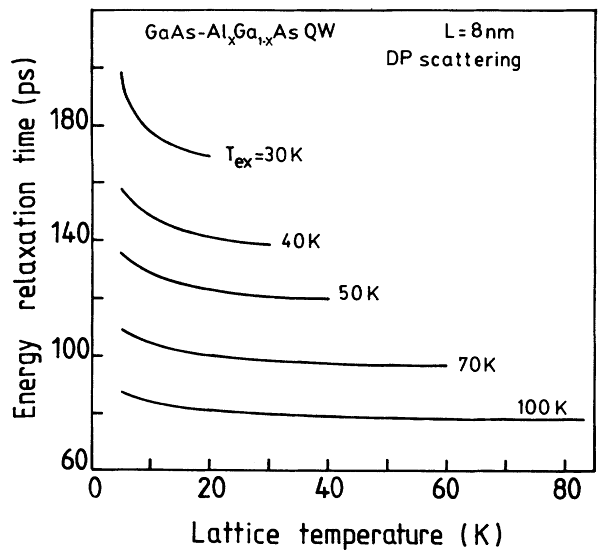

FIG. 1. Energy relaxation time due to scattering with deformation-potential (DP) acoustic phonons, as a function of lattice temperature for an 8-nm-wide well, and for different exciton temperatures (T_{ex}).

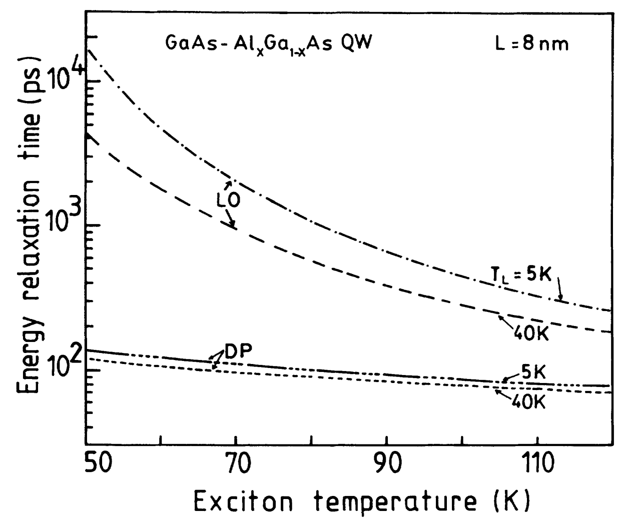

FIG. 3. Energy relaxation time due to the scattering with longitudinal-optic (LO) and deformation-potential (DP) acoustic phonons, as functions of exciton temperature for an 8-nm-wide well, and for two different values of lattice temperature (T_L).

III. RESULTS AND DISCUSSION

We have calculated the energy loss rates and the corresponding energy relaxation time for excitons in a GaAs/Al$_x$Ga$_{1-x}$As QW due to DP and LO phonon scatterings. The following parameter values are used in the calculations: $m_e = 0.067m_0$, $m_h = 0.45m_0$, $D_c = 12.0$ eV, $D_V = 6.1$ eV, $\epsilon = 10.9\epsilon_0$, $\epsilon_s = 12.8\epsilon_0$, $\rho = 5.346 \times 10^3$ kg/m^3, $u = 5.2 \times 10^3$ m/sec, $\hbar\omega_{LO} = 36.2$ meV.

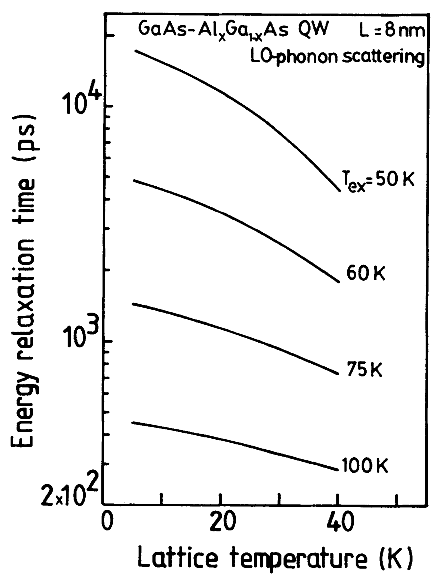

FIG. 2. Energy relation time due to scattering with longitudinal-optic (LO) phonons, as a function of lattice temperature for an 8-nm-wide well, and for different exciton temperatures (T_{ex}).

Figure 1 shows the variation of the energy relaxation time due to DP phonon scattering with lattice temperature using different exciton temperatures as parameters. It is found that for low lattice temperatures in the range 5–10 K and exciton temperature slightly above these values, the relaxation time τ_E^{DP} is about 120 psec and decreases as the exciton temperature increases.

Figure 2 gives a plot of the relaxation time τ_E^{LO} due to LO phonon interaction as a function of lattice temperature for different exciton temperatures as parameters. The relaxation time shows an identical behavior as exhibited by τ_E^{DP} in Fig. 1; however, the time is about one order of magnitude higher, indicating that the LO phonon scattering is quite weak in the range of lattice and exciton temperatures considered here.

To show the difference between the values of the energy relaxation times due to DP and LO phonon scatterings we have plotted in Fig. 3 the times as a function of exciton temperature with the lattice temperature as a parameter. We have chosen higher values of T_{ex} since for low-exciton temperatures τ_E^{LO} is quite high and no useful comparison can be made. The curves indicate that τ_E^{DP} changes very little with lattice temperature, while for τ_E^{LO} the variation is larger. A comparison between these two sets of curves indicates that the LO phonon scattering becomes effective for higher values of the exciton and the lattice temperatures. This conclusion of the relative importance of the DP and LO phonon scatterings is consistent with our findings for the momentum relaxation rates of excitons.[9] Our calculation for the mobility of 2D excitons in GaAs QW's indicates that the DP acoustic-phonon scattering is dominant up to a lattice temperature of 150 K.

We now make a comparison between the present theoretical calculations and the experimental data by Damen *et al.*[5] The authors indicate a value of 30 psec for the energy relaxation time for excitons being scattered by DP acoustic phonons when the lattice temperature is quite low. Our theory involving only the 1*s* excitonic level gives a value about 3–4 times higher than this in the temperature range of interest as indicated in Fig. 1. This value does not alter very much even if we include LO phonon scattering. In our theory we have considered an ideal situation of a QW having an infinite barrier, and also scattering in the 1*s* excitonic branch only. In reality the barrier is finite and the excitons occupy higher-lying (2*s*, 2*p*, etc.) states or even the scattering states in the continuum. The scattering processes within each level and between the levels should therefore be considered to estimate the overall relaxation rates. It is likely that the inclusion of these levels will enhance the scattering rate and the energy relaxation time will be lower accordingly.

IV. CONCLUSION

We have calculated the energy relaxation times of hot 2D excitons scattered by DP acoustic and LO phonons in a GaAs/$Al_xGa_{1-x}As$ MQW taking into consideration the 1*s* excitonic level only. It is found that LO phonon scattering is not important for the lattice and exciton temperatures below about 150 K. The relaxation time due to DP acoustic-phonon scattering is about 3–4 times highr than the value estimated from the experimental data. We suggest that the scattering in the higher excitonic levels should also be included to obtain a lower value of the energy relaxation time.

ACKNOWLEDGMENTS

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