Temperature dependences of the E_0 transitions in bulk Ge and a Ge-rich $(Si)_m/(Ge)_n$ superlattice

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Using photoreflectance (PR) spectroscopy, the temperature dependences of the E_0 transition in bulk Ge and a $(Si)_m/(Ge)_n$ strained-layer superlattice (SLS) were measured at temperatures from 87 to 295 K. A rapid decrease in the PR amplitude with temperature was observed for the SLS as compared with the bulk Ge E_0 transition. This decrease is explained by using a model relating the PR amplitude to minority-carrier lifetime. The overall temperature dependences of the measured PR amplitudes were found to be consistent with the assumptions of this model. The model implies that there may be a different trapping mechanism in the superlattice than in bulk Ge. Fits to the Varshni relation also indicate a different temperature dependence of the E_0 transition energies in the SLS as compared with bulk Ge. The measured bulk germanium Varshni coefficients are found to be in reasonable agreement with the results of an earlier determination.

I. INTRODUCTION

Si-Ge strained-layer superlattices (SLS's) and quantum wells (QW's) have been under increased attention because of their capability to have novel electronic and optical properties while being compatible with existing Si processing technology.[1–4] In order to grow Si-Ge SLS structures consisting of an arbitrary number of periods, strain-symmetrized alloy buffer layers have been employed, thereby permitting structures to be grown thick enough for device applications.[4] Such structures are of great interest and have been under recent investigation owing to the possibility of artificially synthesizing a group-IV quasi-direct-band-gap material using indirect-band-gap hosts (Si and Ge).[5–7]

As shown by Kangarlu *et al.*,[8] photoreflectance (PR) can be used as a nondestructive means for measuring the temperature dependence of superlattice transition energies. Recently, PR has been shown to be useful in the characterization of electronic transitions in $(Si)_m/(Ge)_n$ SLS's.[9] However, the full temperature dependence of E_0 transitions in Si-Ge SLS's has not yet been investigated. In this study we report the temperature dependences of the strain- and confinement-lifted E_0 transitions in a Ge-rich SLS's and compare them with the corresponding dependence of the bulk Ge E_0 transition.[10–12] The dependence of both the measured transition energies and PR amplitudes will be discussed.

II. EXPERIMENT

The structure under investigation was a 60-period $(Si)_8/(Ge)_{32}$ SLS grown on an 8000-Å 80% Ge alloy buffer layer on a ⟨100⟩ Si substrate. The SLS region of the sample therefore consisted of a 45-Å Ge layer followed by a 11-Å Si region, repeated 60 times. Details of the sample growth and structural characterization have been presented elsewhere.[9,13] A p^--type Ge sample was used for comparison with the $(Si)_m/(Ge)_n$ sample grown on a p^--type wafer.

During PR investigations, monochromator slits were set to 0.5 mm for measurements of the Ge spectra, while a slit width of 3 mm was needed to provide a sufficient signal-to-noise ratio (SNR) for the $(Si)_m/(Ge)_n$ structure. The larger slit width was justified by the observation that the measured $E_0^1(1)$ transition[9] linewidth was the narrowest and did not change for both 0.25- and 3-mm slit widths. The corresponding energy dispersions of the 3- and 0.5-mm slit widths are given by $\delta E/E^2 = 6.05\times10^{-3}$ and 2.02×10^{-3} eV^{-1}, respectively.

Spectral measurements of the SLS sample were taken at temperatures from 87 to 295 K. The spectra of bulk Ge sample were taken between 83 and 335 K. An MMR Joule-Thompson refrigerator and temperature controller were used to maintain stable temperatures to within 2 K. PR measurements were taken using a 632.8-nm 10-mW He-Ne laser as a modulation source, while a Judson thermoelectrically cooled photovoltaic germanium detector was used to measure the light reflected from each sample. Reflective optics were used to collect the light after the monochromator, and a 100-W tungsten lamp was used as a probe during all of these measurements. Additional apparatus used in PR spectroscopy, both above and below the band gap, is discussed elsewhere.[14–16]

III. RESULTS AND DISCUSSION

A. Line-shape analysis

The PR spectra from homogeneously (or inhomogeneously) broadened quantum wells are best fit by the first derivative of a Lorentzian (or Gaussian) line-shape function (i.e., first derivative of a complex dielectric function with a Lorentzian or Gaussian absorption profile).[14,17,18] The nonexcitonic spectra from thin-barrier superlattices and bulk materials, however, are best fit using a third-derivative Lorentzian functional form[14,17,18] derived by Aspnes.[19] The PR spectra from the narrow-barrier-width (11 Å) $(Si)_8/(Ge)_{32}$ superlattice may contain both first- and third-derivative components, depending on the degree of quantum-mechanical confinement. As a result, the measured spectra were fit to both the first derivative of a Gaussian and the third derivative of a Lorentzian complex dielectric function. The measured temperature dependences of the transition energies and PR amplitudes were found, within experimental error, to be independent of which derivative line shapes were used. The data, however, showed a better fit in the shoulders of the line shape by the first-derivative Gaussian expression. This may indicate that the spectra measured from the SLS are a result of a first-derivative modulation of the excitonic interaction in the SLS.[18] One should be cautioned, however, that the onset of high-field effects may cause nonlinearities in the dielectric function[19] that result in a line shape that appears to be better fit by a first-derivative Gaussian expression when, in fact, neither of the above line shapes is rigorously appropriate.

B. Bulk and SLS spectra

The spectral features observed from the superlattice are believed to originate from strain- and confinement-induced splittings[20,9] of the E_0 transition in Ge (E_0^{Ge}).[11] Figure 1 illustrates measured PR spectra (dotted curve) of the E_0^{Ge} transition in the bulk Ge sample at 87, 200, and 295 K. The solid curve is a nonlinear least-squares fit to a third-derivative Lorentzian line-shape function.[19,14] These spectra were also fit to a first-derivative Gaussian expression, but as mentioned before, the values of the transition energies and relative amplitudes were found to be independent of which derivative line shape was used. Figure 2 shows similar measured spectra (dotted curve) due to transitions from the strain- and confinement-split heavy- $[E_0^1(1)]$ and light- $[E_0^1(2)]$ hole band to the conduction band (at $\Gamma_{2'}$) in a Ge-rich $(Si)_8/(Ge)_{32}$ SLS at 87, 200, and 295 K. Again, these spectra were also fit to be the first-derivative Gaussian expression, discussed in Sec. III A. As illustrated by these spectral fits, the bulk germanium E_0 line shape is much more poorly represented by the third-derivative Lorentzian line shape then the SLS spectra. This was also found to be the case when comparing the corresponding fits to a first-derivative Gaussian expression. The poor fit of Ge spectra to either the first- or third-derivative expressions may be due to the Franz-Keldysh (FK) effect, given as a possible explanation for a similar problem in the fitting of PR data from a $Ga_{0.82}Al_{0.18}As$ alloy sample.[21] The present Ge line shape closely resembles the features that we have observed from FK oscillations (i.e., the first few extrema) measured from other bulk Ge samples.

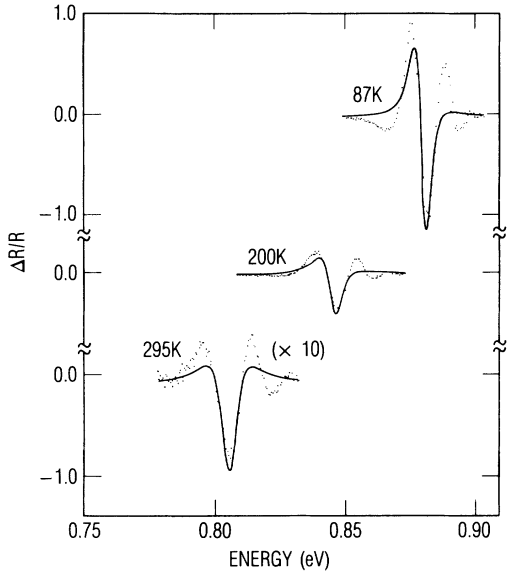

FIG. 1. Measured photoreflectance spectra (dotted curve) from a p-type germanium wafer at 87, 200, and 295 K. Each spectrum was normalized to the measured value of $|\Delta R/R|_{max}$ at 87 K. The solid curve is a nonlinear least-squares fit to a third-derivative Lorentzian line shape. These spectra were also fit to a first-derivative Gaussian expression, but the values of the transition energies and relative amplitudes were found to be independent of the derivative line-shape functions used.

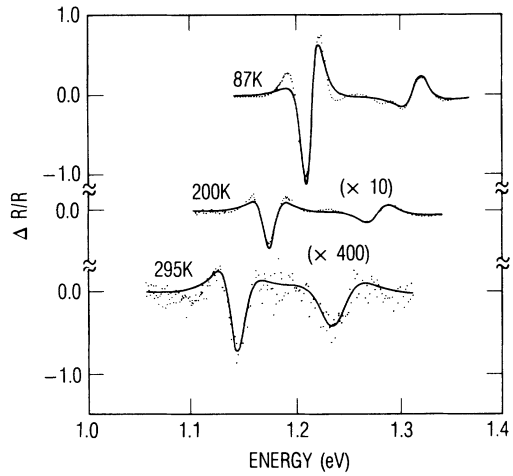

FIG. 2. Measured photoreflectance spectra (dotted curve) from a strain-symmetrized $(Si)_8/(Ge)_{32}$ superlattice at 87, 200, and 295 K. Each spectrum was normalized to the measured value of $|\Delta R/R|_{max}$ at 87 K. The solid curve is a nonlinear least-squares fit to a third-derivative Lorentzian line-shape function. These spectra were also fit to a first-derivative Gaussian expression, but the values of the transition energies and relative amplitudes were found to be independent of the derivative line-shape functions used.

C. E_0 transition energy temperature dependence

Figure 3 gives the measured temperature dependence of the bulk Ge E_0^{Ge} sample transition energy, as well as the $E_0^1(1)$ and $E_0^1(2)$ heavy- and light-hole to conduction-band transition energies (solid circles) in the $(Si)_m/(Ge)_n$ SLS. These spectra are shown fit (solid curve) to the Varshni relation.[22] The results of these fits are summarized in Table I. We find that a linear approximation between 87 and 295 K yields a temperature coefficient of $\Delta E/\Delta T=-3.56\times10^{-4}$ eV/K for E_0^{Ge} and values of $\Delta E/\Delta T=-3.85\times10^{-4}$ and -3.37×10^{-4} eV/K for the $E_0^1(2)$ and $E_0^1(1)$ transitions, respectively. These values are in reasonable agreement with prior interpolations of the electroreflectance data from the E_0 transitions in $Si\text{-}Si_{1-x}Ge_x$ multiple-quantum-well samples measured at 77 and 295 K.[20]

For the measured temperature dependence of the E_0^{Ge} transition, we find that good agreement between prior results[22] is obtained for the values α, β, and E_0 of the Varshni relation. The experimental values obtained from a prior fit to the Varshni relation for the germanium E_0 transition are 0.889 eV, 6.84×10^{-4} eV/K, and 398 K for E_0, α, and β, respectively.[22] These values are compared to the data for the E_0^{Ge} transition given in the Table I. As illustrated by these results, there is a noticeable difference between the temperature dependence of E_0^{Ge}, $E_0^1(1)$, and $E_0^1(2)$ transition energies. These differences are believed to be attributed to both strain and confinement effects in the SLS, not present in bulk Ge. As a result, the Debye-Waller (DW) and self-energy (SE) terms[23,24] in the electron-phonon interaction, which dominate this transition energy dependence, could be correspondingly different.

It might also be true that the shift in the transition energy due to lattice dilation with increasing temperature[25,23] is larger for the strained-layer $(Si)_m/(Ge)_n$ superlattice than for the bulk case. To examine this possibility,

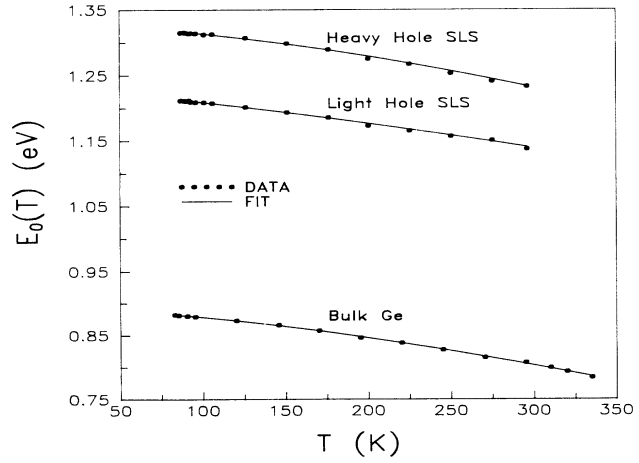

FIG. 3. Measured temperature dependences of the bulk Ge E_0^{Ge} transition energy, as well as the $E_0^1(1)$ and $E_0^1(2)$ heavy- and light-hole to conduction-band $n=1$ SLS transition energies (solid circles). The solid line is a fit to the Varshni relation given in Table I.

TABLE I. Results of fits to the Varshni relation $E=E_0-\alpha T^2/(T+\beta)$.

Transition	E_0 (eV)	α (eV/K)	β (K)
$E_0^1(1)$	1.23	4.35×10^{-4}	147
$E_0^1(2)$	1.33	9.56×10^{-4}	598
E_0^{Ge}	0.892	7.25×10^{-4}	433

we will consider the linear shift in the $E_0^1(1)$ and $E_0^1(2)$ interband transition energies due to lattice-dilation effects. For $T\gg\beta$ the sum of this shift and the linear shift due to the electron-phonon interaction should yield α [i.e., $\alpha\sim(\partial E_0/\partial T)_{\text{lattice dia.}}+(\partial E_0/\partial T)_{\text{electron-phonon}}$]. As discussed in Ref. 24, the lattice-dilation effects of the E_0 interband transitions in bulk materials may be described by

$$\left[\frac{\partial E_0}{\partial T}\right]_{LD}=-3B\alpha_{TE}(T)\left[\frac{\partial E_0}{\partial P}\right], \tag{1}$$

where B is the bulk modulus, related to the elastic constants in cubic materials by $B=(c_{11}+2c_{12})/3$, $\alpha_{TE}=(1/L)(\partial L/\partial T)$ is the coefficient of linear thermal expansion, and P is the pressure. For the $E_0^1(1)$ and $E_0^1(2)$ transitions under study, the large conduction-band offset between the $\Gamma_{2'}$ points in the band structure of Si and Ge,[16] and the small $\Gamma_{2'}$ effective mass[26] result in the following approximation to the lattice-dilation effect in the SLS:

$$\left[\frac{\partial E_0}{\partial T}\right]_{LD}\approx-\alpha_{TE}(T)\left[2E_\infty+3B\left[\frac{\partial E_0}{\partial P}\right]\right], \tag{2}$$

where $E_\infty=\hbar^2\pi^2/2\mu L^2$, L is the width of the Ge well region (45 Å), and μ is the effective mass at $\Gamma_{2'}$. Note that Eq. (2) is only valid for $(Si)_m/(Ge)_n$ SLS's with lightly strained Ge layers, such as the $(Si)_8/(Ge)_{32}$ SLS under study.

Using $(\partial E_0/\partial P)_T=13$ meV,[24] $\alpha_{TE}=6\times10^{-6}$ K^{-1},[27] and the elastic constants given by Ref. 28, the single-band model described by Eq. (2) yields $(\partial E_0/\partial T)_{LD}\approx-1.77\times10^{-4}$. As seen by Table I, this contribution alone does not account for the measured discrepancies in α. Furthermore, since $2E_\infty\ll3B(\partial E_0/\partial P)$, one would not expect to measure a significant difference between values of α for heavy and light holes, as observed in Table I. Therefore, the observed differences in Varshni parameters between bulk and SLS transitions must be due to strain- and confinement-induced changes in the DW and SE contributions of the electron-phonon interaction in the $(Si)_8/(Ge)_{32}$ sample.

From the theory of Allen and Cardona (AC),[23,24,29] the DW and SE contributions to the electron-phonon interaction are given in terms of the one-electron states $|\mathbf{k},n\rangle$ with wave vector $\mathbf{k}$ and band index n as

$$\Delta E_0^{\mathrm{DW}} = \langle \mathbf{k},n|(H_1+H_2)|\mathbf{k},n\rangle \ , \tag{3}$$

$$\Delta E_{\mathbf{k},n}^{\mathrm{SE}} = \sum_{\mathbf{k}',n'\neq\mathbf{k},n} \frac{|\langle \mathbf{k}',n'|H_1|\mathbf{k},n\rangle|^2}{\varepsilon_{\mathbf{k},n}-\varepsilon_{\mathbf{k}',n'}+i\eta} \ . \tag{4}$$

H_1 and H_2 are the leading-order terms of the Taylor expansion of the crystal potential $V(r-\mathbf{R}(l,\kappa)-\mathbf{u}(l,\kappa))$; $\mathbf{R}(l,\kappa)$ are the positions of atoms of species κ having displacements $\mathbf{u}(l,\kappa)$, and l labels the unit cells. Expressions for H_1 and H_2 are given in Ref. 29.

In order evaluate the shifts given by Eqs. (3) and (4), they must first be thermally averaged as follows:

$$\Delta E_{\mathbf{k},n}(T) = \sum_{\mathbf{q},j} \left[\frac{\partial E_{\mathbf{k},n}}{\partial n_{\mathbf{q},j}}\right]\left[n_{\mathbf{q},j}(T)+\frac{1}{2}\right] , \tag{5}$$

where $n_{\mathbf{q},j}=1/(e^{\hbar\omega_{\mathbf{q},j}/k_BT}-1)$ is the Bose-Einstein occupation factor for phonon mode $\mathbf{q},j$ having energy $\hbar\omega$. As demonstrated elsewhere,[23,24] evaluation of Eqs. (3)–(5) requires the ability to determine both the lattice dynamics and a reasonably simple model of the band structure. As a detailed calculation, such as the one performed by AC,[23,24] is beyond the scope of this work, we will only qualitatively investigate effects of strain and confinement on the SE contribution using an effective-mass (deformation-potential) model of this interaction.[30,31] As discussed by AC, this contribution is important for the full treatment of shift due to the electron-phonon interaction. Although we will not discuss effects on the DW term, the SE term alone illustrates how strain and confinement can influence the measured temperature dependence of $(\mathrm{Si})_m/(\mathrm{Ge})_n$ superlattices.

If we assume that te dominant contribution to Eq. (4) is due to absorption and emission of acoustic phonons, the second-order shift in the first-order electron-phonon interaction may be described by the hydrostatic deformation potential constant E_1, defined by

$$H_1 = E_1 \mathrm{Tr}(\epsilon_{i,i}) \ , \tag{6}$$

where $\mathrm{Tr}(\epsilon_{i,i})=\epsilon_x+\epsilon_y+\epsilon_z$ for cubic crystals and ϵ is the strain. Under this approximation and assuming $n=n'$ in Eq. (4), we obtain

$$\Delta E_{\mathbf{k},n}^{\mathrm{SE}} = \sum_{q} \frac{|E_1|^2\hbar q}{2\rho c_s}\left[\frac{n_q}{\varepsilon_{0,n}-\varepsilon_{0-q,n}-\hbar\omega_q} + \frac{n_q+1}{\varepsilon_{0,n}-\varepsilon_{0+q,n}+\hbar\omega_q}\right] , \tag{7}$$

where ρ is the crystal density and c_s is the longitudinal velocity of sound. The leading-order electron-phonon interaction is zero to first order in perturbation theory because of the selection rule $\Delta\mathbf{k}\neq 0$, required by momentum conservation. Now, under the effective-mass approximation at the Γ point, we may rewrite Eq. (7) as

$$\Delta E_{\mathbf{k},n}^{\mathrm{SE}} = \sum_{q} \frac{|E_1|^2\hbar q}{2\rho c_s} \frac{(2n_q+1)\hbar^2q^2/2m^*-\hbar\omega_q}{(\hbar^2q^2/2m^*)^2-(\hbar\omega_q)^2} \ , \tag{8}$$

where m^* is the average parabolic effective mass (over all directions) of the band n under consideration.

It is well known that strain and confinement effects can cause the band structure[32,5,33,34] and phonon dispersion[34–37] of $(\mathrm{Si})_m/(\mathrm{Ge})_n$ superlattices to be very different from that of Si, Ge, or the corresponding random alloy (in our case $\mathrm{Si}_{0.2}\mathrm{Ge}_{0.8}$). Although Eq. (8) is by no means exact (no DW term, acoustic phonons, and parabolic bands), it does serve to illustrate how the temperature-dependent electron-phonon interaction can depend upon phonon dispersion $[\omega_q=\omega(q)]$ and band structure $[m^*=\hbar^2/(\partial^2E/\partial k^2)]$. Thus, without a detailed calculation, we have illustrated how strain and confinement can cause the Varshni coefficients, used to empirically model $\Delta E_{\mathbf{k},n}$, to differ between SLS and bulk E_0 transitions.

From the results of Kangarlu *et al.*[8] in the measurement of Varshni coefficients in the GaAs-$\mathrm{Al}_x\mathrm{Ga}_{1-x}$As system, no appreciable difference in the coefficients was observed between confined (unstrained) and unconfined GaAs. This indicates that strain must be present in a superlattice before there is an appreciable difference between bulk and SLS relations for $\Delta E_{\mathbf{k},n}$. Whether this difference can be attributed to strain alone or is the result of strain-induced mixing of the various *s*- and *p*-like wave functions involved in the E_0 transitions[5] is under current study.

D. Amplitude temperature dependence

In addition to the observed deviations in α and β, there is a difference in the temperature dependence of the PR amplitude of bulk and SLS E_0 transitions. This difference is the most striking characteristic of the data shown in Figs. 1 and 2. Figure 4 shows the measured temperature dependence of the PR amplitude, C_{E_0}, for the E_0 transitions in bulk Ge (C_{E_0}) as well as $E_0^1(1)$ and $E_0^1(2)$ (denoted by $C_{E_0(1)}$ and $C_{E_0(2)}$, respectively) in the $(\mathrm{Si})_m/(\mathrm{Ge})_n$ SLS. These dependences exhibit two regions with unique activation energies that may correspond to different recombination mechanisms within the SLS and Ge samples. Let us denote the transition temperature (corresponding to the "knee of" the curve) by T_{tr}. As noted earlier, the normalized amplitude dependence was found to be independent of the type of derivative line shape used to fit the data. This indicates that the observed dependence was not due to fitting with an inappropriate line shape. Since, in PR, the dominant form of electric-field modulation is modulation of the built-in electric field through photoinjection of electron-hole pairs,[38] the photoreflectance amplitude is expected to be directly related to carrier lifetime. The regimes above and below T_{tr} may therefore be explained using a steady-state model of the dependence of $C=C(T)$ on minority-carrier lifetime.

According to electromodulation (EM) theories of interband and excitonic spectra, the change in the real and imaginary parts of the dielectric function, $\Delta\epsilon_1$ and $\Delta\epsilon_2$, are proportional to some power of the modulating electric field $\mathcal{E}$.[39,40] Therefore, we may relate the normalized change in reflectivity to $\mathcal{E}$, using the Seraphin

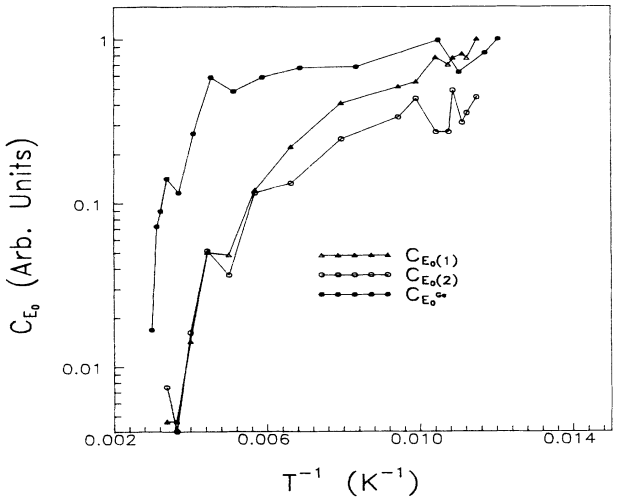

FIG. 4. Measured temperature dependence of the PR amplitude C_{E_0} for the bulk Ge E_0 transitions (C_{E_0}) as well as the $E_0^{\downarrow}(1)$ and $E_0^{\downarrow}(2)$ heavy- and light-hole to conduction-band transitions ($C_{E_0(1)}$ and $C_{E_0(2)}$) in the $(Si)_m/(Ge)_n$ SLS. The normalized amplitude dependence was found to be independent of the form of the derivative line-shape functions used to fit the data. These dependences exhibit two regions with unique activation energies that correspond to different recombination mechanisms within the SLS and Ge samples.

coefficients,[41] as follows:

$$\frac{\Delta R}{R}=\alpha\,\Delta\epsilon_1+\beta\,\Delta\epsilon_2\propto\mathcal{E}^r\ , \tag{9}$$

where R is the reflectivity, α and β are the Seraphin coefficients, and $r=2$ for the low-field limit of interband transitions when the excitonic interaction may be neglected.[42,19] For excitonic interactions and electromodulation involving high-field or Franz-Keldysh effects, r may take on noninteger values.[39,11]

As mentioned above, PR is a type of EM where the field modulation takes place through periodically perturbing the built-in field near a semiconductor surface by photoinjection of carriers.[38] We may then determine a relationship between the amplitude of the PR signal and the carrier lifetime for the conditions of this experiment. Consider the case for a p-type semiconductor sample whose built-in $\mathcal{E}$ field is modulated by a pump beam of intensity I_0 that decreases as $I=I_0e^{-\alpha z}$ at a distance z in the sample, where α is the absorption coefficient. We assume that the beam enters the sample at normal incidence. Let us further assume that the probe beam is chopped at a modulation frequency $\nu_m\ll 1/\tau$, where τ is the carrier lifetime and that a fraction γ of the photogenerated electrons not recombining with holes in the bulk can drift to the surface and neutralize positively charged trap levels at the surface, thereby establishing a reduced field. The injected carrier density at z may be taken at its steady-state value given by the product of τ and the generation rate.[43] The photoinjected excess electronic charge density at this point is then given by

$$\rho_v(z)=\frac{d\rho_s(z)}{dz}=\frac{-q\tau\eta\gamma[dI(z)/dz]}{\hbar\omega}=\frac{q\tau\alpha\eta\gamma I(z)}{\hbar\omega}\ , \tag{10}$$

where η is the quantum efficiency, q is the charge on the electron, $\hbar\omega$ is the photon energy of the pump beam, and $\rho_v(z)$ is the electronic volume charge density at z. $\rho_s(z)$ represents the excess electronic sheet charge per unit area in a layer of width dz. The new sheet charge density at $z=0$ may then be determined from the integral of Eq. (10) from 0 to l, where l is given by the smaller of either the electron mean free path, the probe beam's penetration depth ($1/\alpha$), or the sample depletion width W. Thus the change in initial sheet charge density at $z=0$ is given by

$$\Delta\rho_s^0=\frac{Aq\tau I_0}{\hbar\omega}\ , \tag{11}$$

$$A=(1-e^{-\alpha l})\eta\gamma\ . \tag{12}$$

The maximum value of the built-in electric field is readily obtained from Gauss's law as

$$\mathcal{E}^{\max}=\frac{q}{\epsilon}\left[N_AW-\frac{A\tau I_0}{\hbar\omega}\right]\ , \tag{13}$$

where ϵ is the low-frequency dielectric constant and N_A is the ionized acceptor concentration. Solving Poisson's equation under the depletion approximation[43] yields

$$\mathcal{E}(z)=\frac{q}{\epsilon}\left[N_A(W-z)-\frac{A\tau I_0}{\hbar\omega}\right]\ . \tag{14}$$

We may now determine the relationship between the PR amplitude and the minority-carrier lifetime using Eq. (9) and assuming, for simplicity, a sinusoidal modulation given by

$$I_0=I_{\rm dc}+\frac{I_{\rm dc}}{2}\sin(2\pi\nu_m t)\ . \tag{15}$$

Inserting Eq. (14) into Eq. (9) and expanding the result to two terms, we have

$$\mathcal{E}^r(z)\simeq\frac{q}{\epsilon}\left[\frac{qN_A}{\epsilon}(W-z)\right]^{r-1}\left[N_A(W-z)-\frac{rA\tau I_0}{\hbar\omega}\right]\ , \tag{16}$$

where we have assumed, in the case of low level injection, that $rA\tau I_0/\hbar\omega\ll N_A(W-z)$ for $0\le z\le l^{\rm probe}$; $l^{\rm probe}$ is given by the smaller of W or $1/\alpha_p$, the penetration depth of the probe beam. Taking the ac component of the previous expression after inserting Eq. (15) into Eq. (16), the resultant PR amplitude, given by Eq. (9), is related to τ by

$$C(T) \propto \frac{\tau I_{\rm dc} N_A^{r-1} r A}{2\hbar\omega} . \quad (17)$$

Furthermore, using the Shockley-Read-Hall (SRH) theory of recombination,[44] we find

$$\tau \propto 1+2\exp\left[\frac{E_F-E_{Fi}}{k_BT}\right]\cosh\left[\frac{E_{Fi}-E_t}{k_BT}\right], \quad (18)$$

where E_F is the Fermi level, E_{Fi} is the intrinsic Fermi level, and E_t is the energy of the trap level. As a detailed knowledge of the trapping levels in the SLS is not available, the above results will be used primarily for qualitative discussion.

Before proceeding to discuss how Eq. (17) and (18) can be used to explain the measured amplitude behavior, we will first verify the assumptions used to derive these equations and review how the parameters used in the above model of $C(T)$ are meaningful in terms of the samples under investigation. The steady-state assumption of our model (i.e., that $\nu_m \ll 1/\tau$) was easily verified experimentally by varying the chopping speed of the pump beam to lower frequencies and noting that the amplitude remained invariant. This is consistent with the modulation frequency of $\nu_m = 281$ Hz used in these experimental and the typical lifetimes $\leq 10^{-3}$ sec for bulk Ge.[43] The lifetime in the SLS should be even less than this. Also, for this small lifetime a probe beam intensity of ~ 100 mW/cm^2 and $A \ll 1$, expected for reasonable values of γ, η, α, and electron mean free path in Ge, we should have $rA\tau I_0/\hbar\omega \ll N_A(W-z)$. These values are largely unknown for the SLS under investigation, but should be such that $rA\tau I_0/\hbar\omega \ll N_A(W-z)$ is also satisfied for the $(Si)_8/(Ge)_{32}$ material (i.e., τ should be smaller, etc.).

It is easy to see from Eqs. (17) and (18) that for $|E_F-E_{Fi}| \ll |E_{Fi}-E_t|$ (corresponding to $T > T_{\rm tr}$), the PR amplitude should have a dominant temperature behavior given by $C(T) \sim \zeta e^{-|E_{Fi}-E_t|/k_BT}$ where ζ is weakly dependent upon temperature. This is valid for the samples studied (very lightly doped) near room temperature. Note that $E_F \rightarrow E_{Fi}$ (i.e., $p \rightarrow n_i$) as we increase T (for example, see p. 19 of Ref. 43). This is consistent with the smaller value of $T_{\rm tr}$ for the SLS than for bulk Ge since the SLS was not intentionally doped. Conversely, as we decrease T, $|E_F-E_{Fi}|$ increases and a different activation energy is expected. This explains the transition from low- to high-temperature regimes observed Fig. 4. Thus, for $T > T_{\rm th}$, an activation energy corresponding to $|E_{Fi}-E_t|$ may be obtained, approximately independent of E_F.

In this study the rapid decrease in PR amplitude with increasing T prevents us from rigorously verifying the exponential threshold $C(T) \sim \zeta e^{-|E_{Fi}-E_t|/k_BT}$ predicted at higher temperature as a result of the number of thermally generated carriers overwhelming those present due to impurity ionization. A more thorough verification of this behavior would require the analysis of the temperature behavior of a narrow-gap material; $n,p \rightarrow n_i$ would be expected to occur at much lower temperatures in that case. To verify the model at $T < T_{\rm th}$ would be nontrivial since one would generally need to solve for $\tau = \tau(\Delta p \sim \Delta n)$, with $\Delta p = \Delta n(\tau)$ obtained through the continuity equation. One would furthermore need to include the temperature dependence of $E_F - E_{Fi}$ and determine the asymptotic behavior of the ac component of the electric field, analogous to Eq. (16). Therefore, although the above asymptotic model of $C(T)$ does predict the two activation energies for the $C(T)$ data measured in these experiments, the actual values of the activation energies for $T < T_{\rm th}$ may not have a concise physical meaning in relation to the parameters used in Eqs. (17) and (18).

For $T > T_{\rm tr}$, however, the parameters in the model have a physical interpretation in terms of activation energies for SRH recombination centers in lightly doped materials. One could model non-SRH-type recombination effects in a similar fashion. Applying this model to the $C(T)$ data, from Fig. 4, we find $|E_t - E_{Fi}| \simeq 0.11$ eV for both $C_{E_0(1)}$ and $C_{E_0(2)}$, while $|E_t - E_{Fi}| \simeq 0.15$ eV for the bulk Ge sample. This indicates that there may be a different trapping mechanism, or trap-level distribution, in the superlattice then in bulk Ge. This difference is responsible for the more rapid decrease in C_{E_0} then $C_{E_0(1)}$ or $C_{E_0(2)}$ with increasing T. Similar PR studies may, therefore, be useful in comparing SRH-like trapping centers in SLS materials. It should be noted, however, that the assumption of a single trapping level responsible for SRH recombination events prevents a detailed knowledge of trap distributions from PR temperature studies alone. Furthermore, the band structure of the $(Si)_8/(Ge)_{32}$ sample is not purely three dimensional and lifetime mechanisms may not be completely accounted for by SRH theory alone. Clearly, more work must be done to fully understand the use of PR amplitude measurements in characterizing carrier lifetime.

IV. SUMMARY AND CONCLUSION

In conclusion, we have compared the measured temperature dependence of PR critical-point energies of the E_0 transition in bulk Ge to the $E_0^1(1)$ and $E_0^1(2)$ transitions in a $(Si)_8/(Ge)_{32}$ SLS. In both cases the transition energies fit well to the nonlinear Varshni relation for bulk semiconductors.[22] The different temperature dependences measured for the $E_0^{\rm Ge}$, $E_0^1(1)$, and $E_0^1(2)$ transitions, summarized in Table I, are believed to be due to differences in the temperature-dependent Debye-Waller and self-energy interactions[23,24] for each of these transitions. It was also found that a two-point linear approximation between 87 and 300 K yields a bulk Ge temperature coefficient $\Delta E_0/\Delta T$ near the value of the coefficients obtained from the $(Si)_8/(Ge)_{32}$ sample. This is in agreement with an earlier work.[20] The values obtained from the Varshni relation fits, α, β, and E_0, are also in good agreement with previous values for the bulk E_0 transition.[22]

Finally, by comparing the measured PR amplitude dependence on temperature, a much more rapid decrease in amplitude with temperature is observed for the E_0 transitions in the SLS than for the corresponding bulk

case. This decrease is believed to be associated with the dependence of the PR amplitude on minority-carrier lifetime. From the results of fitting these dependences to an approximate model, differences in activation energies between SLS and bulk dependences indicate that there may be a different trapping mechanism, or trap-level distribution, in the superlattice than in bulk Ge. The measured temperature dependences, in both cases, were found to be consistent with the assumptions of the model presented above.

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