Effect of a transverse magnetic field on superlattice miniband transport

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A semiclassical theory for the superlattice miniband transport in crossed magnetic and electric fields is developed here using the nonlinear balance-equation dynamics. Numerical calculations are carried out for the case pertinent to that of the measurement reported recently [Sibille *et al.*, Europhys. Lett. **13**, 279 (1990)]. The present theoretical predictions are in good agreement with the experimental observations.

Interest in superlattice miniband transport has recently been intensified due to the successful experimental observations[1,2] of the negative differential velocity (NDV) in superlattice perpendicular transport and the attribution of this NDV to single-miniband conduction as proposed by Esaki and Tsu.[3] Recent theoretical analysis of Bloch miniband transport by Lei, Horing, and Cui[4] based on a balance-equation approach for a single arbitrary energy band,[5] nicely produced the experimental peak drift velocity v_p and the critical electric field E_c as functions of the miniband width,[1] further supporting the pure miniband-conduction mechanism as the observed NDV origin. The effect of a transverse (parallel to the layers) magnetic field on this nonlinear miniband conduction has also been investigated experimentally by Sibille *et al.*[6] They found (1) a positive magnetoresistance under low electric-field bias; (2) with increasing the magnetic field the occurrence of the peak drift velocity shifts to higher electric field; (3) that under large electric-field biases the current first increases then decreases with increasing magnetic field. The preliminary analysis[6] based on a direct extension of the Esaki-Tsu model, though showing qualitative similarity with the experimental data, exhibited a strong quantitative disagreement. Focusing on the distortion of the free-electron trajectories by the magnetic field, Palmier *et al.*[7] made a theoretical investigation using the Boltzmann equation with constant collision time. The qualitative agreement was improved, but detailed comparison with experiment is still waiting for a more careful calculation.[7] In this paper we present a semiclassical balance-equation analysis of the superlattice miniband transport in crossed electric and magnetic fields.

We assume that the conducting carriers of the superlattice are subject to a strong periodic potential in the z direction and thus form one-dimensional minibands. Considering electrons moving within the lowest miniband, we can use a longitudinal wave vector k_z ($-\pi/d < k_z \leq \pi/d$, d being the period of the superlattice) together with the two-dimensional transverse wave vector $\mathbf{k}_{\parallel}$ to describe the electron state with the energy dispersion, under the tight-binding approximation for the z direction, given by

$$\varepsilon(\mathbf{k}) = k_{\parallel}^2/2m + \varepsilon_1(k_z), \tag{1}$$

where

$$\varepsilon_1(k_z) = \frac{\Delta}{2}(1 - \cos k_z d), \tag{2}$$

Δ being the miniband width. The system under consideration involves N interacting electrons in this single miniband subject to impurity and phonon scatterings and under the influence of a uniform electric field along the z direction $\mathbf{E} = E\mathbf{z}$, a uniform magnetic field along the y direction $\mathbf{B} = B\mathbf{y}$ ($\mathbf{x}$, $\mathbf{y}$, and $\mathbf{z}$ stand for the unit vectors in x, y, and z directions). The total Hamiltonian H of the system is the sum of a phonon part H_{ph}, electron-phonon and electron-impurity interactions H_{ep} and H_{ei}, a uniform electric field potential $H_E = -e\mathbf{E}\cdot\sum_j \mathbf{r}_j$, and a band-related effective electron part H_e with the magnetic field and the electron-electron coupling H_{ee} included:

$$H_e = \sum_j h_j + H_{ee}, \tag{3}$$

$$h_j = \frac{1}{2m}(p_{xj} - eBz_j)^2 + \frac{1}{2m}p_{yj}^2 + \varepsilon_1(p_{zj}). \tag{4}$$

Here $\mathbf{r}_j \equiv (x_j, y_j, z_j)$ and $\mathbf{p}_j \equiv (p_{xj}, p_{yj}, p_{zj}) = -i(\partial/\partial x_j, \partial/\partial y_j, \partial/\partial z_j)$ are the position and momentum operators of the jth electron, and we have chosen the vector potential as $\mathbf{A} = Bz\mathbf{x}$. It is convenient to introduce the center-of-mass position and momentum operators $\mathbf{R} = N^{-1}\sum_j \mathbf{r}_j$ and $\hat{\mathbf{P}} = \sum_j \hat{\mathbf{p}}_j$, and the relative electron position and momentum operators $\mathbf{r}'_j = \mathbf{r}_j - \mathbf{R}$ and $\hat{\mathbf{p}}'_j = \hat{\mathbf{p}}_j - \hat{\mathbf{P}}/N$ in developing a balance-equation theory.

The center-of-mass velocity, i.e., the average velocity of all the electrons, is given by

$$\hat{\mathbf{v}} = -i[\mathbf{R}, H] = \frac{1}{N}\sum_j (\hat{v}_{xj}\mathbf{x} + \hat{v}_{yj}\mathbf{y} + \hat{v}_{zj}\mathbf{z}) \tag{5}$$

with $\hat{v}_{xj} = (p_{xj} - eBz_j)/m$, $\hat{v}_{yj} = p_{yj}/m$, and $\hat{v}_{zj} = v(p_{zj})$, where $v(k_z) = d\varepsilon_1(k_z)/dk_z$ is the velocity function of the superlattice miniband. The drift velocity $\mathbf{v}$, i.e., the statistical average of the operator $\hat{\mathbf{v}}$, has nonzero components in the x and z directions: $\mathbf{v} = (v_x, 0, v_z)$. The derivation of the force and the energy balance equations involves the determination of the statistical average of the acceleration of the center of mass: $\dot{\mathbf{v}} = -i[\mathbf{R}, H]$. The statistical averages are taken with respect to the density matrix evolving from an initial state in which the center of mass moves at a constant velocity v_x in the x direction and a constant momentum $P_d \equiv N p_d$ in the z direction, and the relative electrons are subject to a thermoequilibrium distribution at temperature T_e, which is generally not equal to the lattice temperature T.

The nice thing about the separation of the center of mass from the relative electrons is that the electric field acts only on the center of mass. If we neglect the intracollisional field effect the relative electron system is free from the electric field. The magnetic field acts on the center of mass. This is an effect of the classical Lorentz force, which plays an important role in transport and we are going to consider it carefully. The magnetic field also appears in the relative part of the Hamiltonian.[8] In the case of a superlattice, however, the nondispersive Landau levels due to a transverse magnetic field exist only within the miniband energy range.[9–11] These Landau levels are smeared by impurity and phonon scatterings at the temperature which is of the order of, or larger than, the miniband width. In this case the quantization due to a transverse magnetic field is not important in contributing to the transport properties. Therefore, once we have separated the effect of the magnetic field on the center of mass we can neglect the Landau quantization in taking the statistical average over the relative electron ensemble and treat the relative electrons as if they have the same energy dispersion as Eq. (1) without the magnetic field.

With these considerations and with the identification of the statistical average of $\dot{\mathbf{v}}$ as the center-of-mass acceleration $d\mathbf{v}/dt = (dv_x/dt, 0, dv_z/dt)$, we obtain

$$\frac{dv_z}{dt} = \frac{eB}{m_z^*} v_x + \frac{eE}{m_z^*} + A_{iz} + A_{pz}, \tag{6}$$

$$\frac{dv_x}{dt} = -\frac{eB}{m} v_z + A_{ix} + A_{px}. \tag{7}$$

Here the electrons in a superlattice miniband are described by an inverse-effective-mass tensor $\mathcal{K}_{ij}$ $(i,j = x,y,z)$, with $\mathcal{K}_{xx} = \mathcal{K}_{yy} = 1/m$, $\mathcal{K}_{i\neq j} = 0$, and

$$\frac{1}{m_z^*} = \mathcal{K}_{zz} = \frac{2}{N}\sum_{\mathbf{k}} \frac{d^2\varepsilon_1(k_z)}{dk_z^2} f(\bar{\varepsilon}(\mathbf{k}), T_e). \tag{8}$$

The drift velocity in the z direction is given by

$$v_z = \frac{2}{N}\sum_{\mathbf{k}} v(k_z) f(\bar{\varepsilon}(\mathbf{k}), T_e), \tag{9}$$

where

$$f(\bar{\varepsilon}(\mathbf{k}), T_e) = (\exp\{[\bar{\varepsilon}(\mathbf{k}) - \mu]/T_e\} + 1)^{-1} \tag{10}$$

is the Fermi distribution function at the electron temperature T_e, μ is the chemical potential determined by the condition $N = 2\sum_{\mathbf{k}} f(\bar{\varepsilon}(\mathbf{k}), T_e)$, and

$$\bar{\varepsilon}(\mathbf{k}) = \varepsilon_{\mathbf{k}_\parallel} + \varepsilon_1(k_z - p_d) \tag{11}$$

is the energy of the relative electron. Note that there are three parameters, i.e., p_d, v_x, and T_e, in our theoretical formulation. The frictional accelerations in the z and x directions due to randomly distributed impurities and due to phonons are given separately by

$$A_{iz} = \frac{2\pi n_I}{N}\sum_{\mathbf{k},\mathbf{q}} |u(\mathbf{q})|^2 |g(q_z)|^2 [v(k_z+q_z) - v(k_z)]\,\delta(\varepsilon(\mathbf{k}+\mathbf{q}) - \varepsilon(\mathbf{k}) + q_x v_x) \times \frac{f(\bar{\varepsilon}(\mathbf{k}), T_e) - f(\bar{\varepsilon}(\mathbf{k}+\mathbf{q}), T_e)}{|\epsilon(\mathbf{q}, \bar{\varepsilon}(\mathbf{k}) - \bar{\varepsilon}(\mathbf{k}+\mathbf{q}))|^2}, \tag{12}$$

$$A_{pz} = \frac{4\pi}{N}\sum_{\mathbf{k},\mathbf{q},\lambda} |M(\mathbf{q},\lambda)|^2 |g(q_z)|^2 [v(k_z+q_z) - v(k_z)]\,\delta(\varepsilon(\mathbf{k}+\mathbf{q}) - \varepsilon(\mathbf{k}) + q_x v_x + \Omega_{\mathbf{q},\lambda}) \times \frac{f(\bar{\varepsilon}(\mathbf{k}), T_e) - f(\bar{\varepsilon}(\mathbf{k}+\mathbf{q}), T_e)}{|\epsilon(\mathbf{q}, \bar{\varepsilon}(\mathbf{k}) - \bar{\varepsilon}(\mathbf{k}+\mathbf{q}))|^2} \left[n\left(\frac{\Omega_{\mathbf{q},\lambda}}{T}\right) - n\left(\frac{\bar{\varepsilon}(\mathbf{k}) - \bar{\varepsilon}(\mathbf{k}+\mathbf{q})}{T_e}\right)\right], \tag{13}$$

$$A_{ix} = \frac{2\pi n_I}{N}\sum_{\mathbf{k},\mathbf{q}} |u(\mathbf{q})|^2 |g(q_z)|^2 \frac{q_x}{m}\delta(\varepsilon(\mathbf{k}+\mathbf{q}) - \varepsilon(\mathbf{k}) + q_x v_x) \frac{f(\bar{\varepsilon}(\mathbf{k}), T_e) - f(\bar{\varepsilon}(\mathbf{k}+\mathbf{q}), T_e)}{|\epsilon(\mathbf{q}, \bar{\varepsilon}(\mathbf{k}) - \bar{\varepsilon}(\mathbf{k}+\mathbf{q}))|^2}, \tag{14}$$

$$A_{px} = \frac{4\pi}{N}\sum_{\mathbf{k},\mathbf{q},\lambda} |M(\mathbf{q},\lambda)|^2 |g(q_z)|^2 \frac{q_x}{m}\delta(\varepsilon(\mathbf{k}+\mathbf{q}) - \varepsilon(\mathbf{k}) + q_x v_x + \Omega_{\mathbf{q},\lambda}) \times \frac{f(\bar{\varepsilon}(\mathbf{k}), T_e) - f(\bar{\varepsilon}(\mathbf{k}+\mathbf{q}), T_e)}{|\epsilon(\mathbf{q}, \bar{\varepsilon}(\mathbf{k}) - \bar{\varepsilon}(\mathbf{k}+\mathbf{q}))|^2} \left[n\left(\frac{\Omega_{\mathbf{q},\lambda}}{T}\right) - n\left(\frac{\bar{\varepsilon}(\mathbf{k}) - \bar{\varepsilon}(\mathbf{k}+\mathbf{q})}{T_e}\right)\right]. \tag{15}$$

In the above expressions $u(\mathbf{q})$ and n_I represent the impurity potential and the impurity density, $M(\mathbf{q},\lambda)$ is the electron-phonon matrix element for phonons of wave vector $\mathbf{q}$ in branch λ, having frequency $\Omega_{\mathbf{q},\lambda}$; $n(x) = (e^x - 1)^{-1}$ is the Bose function, $\epsilon(\mathbf{q},\omega)$ is the dielectric function of the carriers in the random-phase approximation, and $g(q_z)$ is a form factor determined by the wave function of the superlattice miniband. In the extreme tight-binding limit for the envelope function it is simply the form factor of a single quantum well: $g(q_z) = d^{-1}\int dz|\phi(z)|^2 \exp(iq_z z)$, $\phi(z)$ being the single well function.

The energy balance equation is obtained by calculating the electron energy-loss rate to the phonon system, i.e., the statistical average of the rate of change of the phonon energy $\dot{H}_{\rm ph} = -i[H_{\rm ph}, H]$. We require that the energy supplied by the electric field equals the sum of the energy increase of the electron system and its energy loss to the phonon system, to give

$$\frac{dh_e}{dt} = eEv_z - W, \tag{16}$$

where h_e is the average relative electron energy (per carrier) and W is the energy-loss rate per carrier from the electron system to the phonon system:

$$W = \frac{4\pi}{N} \sum_{\mathbf{k},\mathbf{q},\lambda} |M(\mathbf{q},\lambda)|^2 |g(q_z)|^2 \Omega_{\mathbf{q},\lambda} \delta(\varepsilon(\mathbf{k}+\mathbf{q}) - \varepsilon(\mathbf{k}) + q_x v_x + \Omega_{\mathbf{q},\lambda}) \times \frac{f(\bar{\varepsilon}(\mathbf{k}), T_e) - f(\bar{\varepsilon}(\mathbf{k}+\mathbf{q}), T_e)}{|\epsilon(\mathbf{q}, \bar{\varepsilon}(\mathbf{k}) - \bar{\varepsilon}(\mathbf{k}+\mathbf{q}))|^2} \left[n\left(\frac{\Omega_{\mathbf{q},\lambda}}{T}\right) - n\left(\frac{\bar{\varepsilon}(\mathbf{k}) - \bar{\varepsilon}(\mathbf{k}+\mathbf{q})}{T_e}\right) \right]. \tag{17}$$

The expressions for A_{iz}, A_{pz}, and W are similar to those of A_i, A_p, and W in the case without a magnetic field.[4] The difference is that now an extra contribution $q_x v_x$ appears in the δ function. Note that we have three parameters, p_d, v_x, and T_e. All physical quantities in the equations are functions of them. Their values and temporal variations can be determined by the three balance equations for the given initial condition.

These equations have been used to explore the effect of a transverse magnetic field on the velocity–electric-field characteristics of the miniband conduction in GaAs/AlAs superlattices having various values of period d, miniband width Δ, and carrier sheet density N_s at lattice temperature $T = 300$ K. Scatterings due to impurities, polar-optic phonons, and acoustic phonons (including deformation potential and piezoelectric couplings) are taken into account, assuming that phonons are three-dimensional bulk modes. All material and electron-phonon coupling constants are taken from the well-known values for pure GaAs. Such a calculation is pertinent to the sample geometry used in the experiment by Sibille *et al.*:[6] the size in the x direction is more than 100 times larger than the total thickness of the superlattice barrier in the z direction, such that the possible establishment of the Hall voltage at the boundary regions has a negligible effect in contrast to the case of a Hall geometry.

In Fig. 1 we plot the perpendicular drift velocity $v_d \equiv v_z$ as a function of the electric field at lattice temperature $T = 300$ K in the presence of a transverse uniform magnetic field of strength $B = 0, 10, 15$, and 20 T for a superlattice of period $d = 57$ Å, miniband width $\Delta = 20$ meV, carrier sheet density $N_s = 2 \times 10^{10}$ cm^{-2}, and weak electric field (zero magnetic field) mobility $\mu_0 = 3.3$ m^2/V s at 4.2 K. In the absence of a magnetic field the drift velocity reaches its peak value $v_p \simeq 1.3 \times 10^6$ cm^2/s at an electric-field $E_c \simeq 8$ kV/cm. The presence of a transverse magnetic field greatly slows down the increase of the drift velocity with increasing E at low electric-field region; thus the drift velocity reaches its peak at much

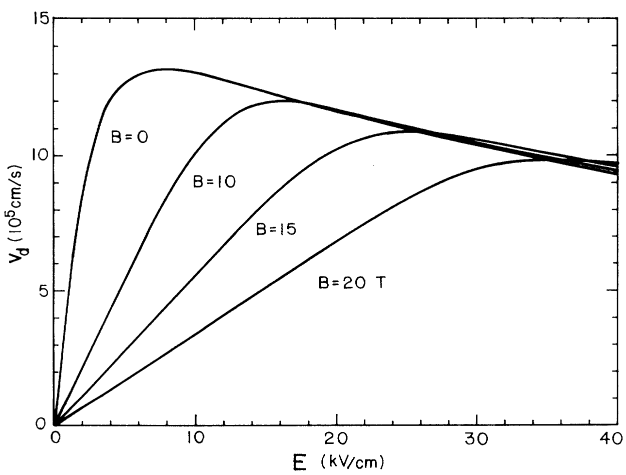

FIG. 1. Calculated drift velocity at lattice temperature $T = 300$ K as a function of electric field for a GaAs/AlAs superlattice in the presence of a transverse magnetic field of various strengths: $B = 0, 10, 15$, and 20 T. The superlattice parameters are carrier sheet density $N_s = 2 \times 10^{10}$ cm^{-2}, period $d = 57$ Å, miniband width $\Delta = 20$ meV, and weak electric field (zero magnetic field) mobility $\mu_0 = 3.3$ m^2/V s at 4.2 K.

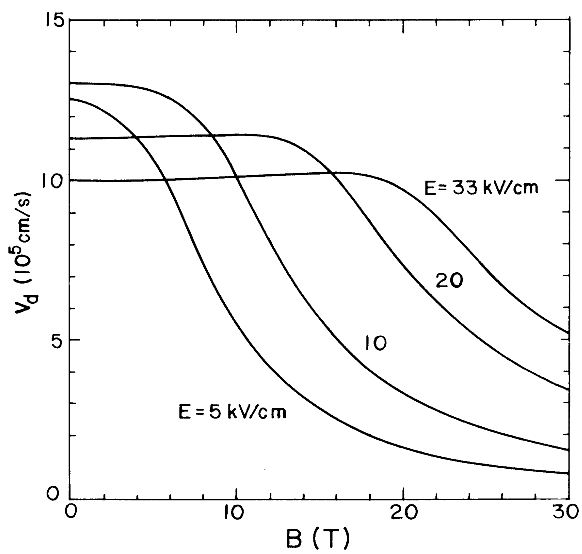

FIG. 2. Drift velocity is plotted as a function of the magnetic field for fixed electric-field strength $E = 5, 10, 20$, and 33 kV/cm at lattice temperature $T = 300$ K for the same superlattice system as described in Fig. 1.

higher electric field than the case without a magnetic field. However, at a fixed electric field of high strength the drift velocity first increases with increasing B then decreases at high magnetic field. These predictions are in good agreement with the experimental observation of Sibille *et al.*[6] To compare our theoretical calculation with their experimental measurement we plot the drift velocity v_d as a function of the magnetic field for several fixed electric fields $E = 5, 10, 20$, and 40 kV/cm, as is shown in Fig. 2. This figure should be compared with Fig. 3-a of Ref. 6. The simple extension of the Esaki-Tsu model in Ref. 6 seems to yield much too strong an effect of the magnetic field (see Fig. 3-b of Ref. 6). The Boltzmann equation analysis by Palmier *et al.*[7] also gives a considerably stronger effect than the present investigation. Both of their theoretical calculations[6,7] predict a significant enhancement of the peak drift velocity by the magnetic field, in contrast to the present theoretical result. The nonlinear balance-equation approach seems to provide a different and adequate solution to the superlattice magnetic miniband transport.

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