Transverse magnetoresistance in quantum wells with multiple subband occupancy

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The low-field magnetotransport properties of a two-dimensional electron gas are calculated for a situation in which more than one subband is occupied. The theory is formulated in terms of a Boltzmann transport equation in which only elastic impurity scattering is taken into account. The possibility of intersubband scattering is included and its effect on the transverse magnetoresistance and Hall coefficient is determined. We find that intersubband scattering plays an important role and its neglect can lead to erroneous conclusions regarding the values of the subband mobilities extracted from fits to experimental data.

I. INTRODUCTION

Semiconductor heterostructures are ideal systems to study electronic transport in situations that are not directly accessible in metals. The fact that their fabrication, and hence physical characteristics, can be controlled, facilitates the systematic investigation of the effects of various parameters on the transport properties. $GaAs/Al_xGa_{1-x}As$ heterojunctions are particularly attractive because of their near-perfect interfaces and simple electronic band structure. In addition, the electronic density in these systems can be varied continuously by different means with the interesting possibility of occupying several electronic subbands. Carriers in each of the subbands comprise distinct, but coupled, two-dimensional electron gases (2DEG's) which contribute to the total conductivity.

The dominant scattering mechanism in such systems at low temperatures is elastic scattering from remote ionized impurities. The impurity potential induces transitions within subbands (intrasubband) and between different subbands (intersubband), and it is of interest to determine the rates with which these processes occur. This information can be obtained from conventional transport measurements that yield transport lifetimes, or alternatively, measurements of Shubnikov–de Haas (SdH) oscillation amplitudes, which provide quantum lifetimes. These two lifetimes represent different averages of microscopic transition rates and therefore provide complementary information which is useful in confirming the nature of the scattering mechanisms and the electronic subband structure of the junction.

Recently, measurements of the low-field transverse magnetoresistance in a $GaAs/Al_xGa_{1-x}As$ heterojunction were made in an attempt to determine transport lifetimes.[1] The sample in question had two occupied subbands and a positive magnetoresistance was observed. For a single subband with an isotropic scattering rate, classical theory[2] predicts no magnetoresistance since the Hall field exactly compensates the Lorentz force and the carriers drift in the direction of the applied field. In actual fact, single subband systems typically exhibit a negative magnetoresistance which is attributable to quantum corrections arising from the effects of weak localization or electron-electron interactions.[3,4] However, if the carrier density in such a sample is increased to the point where a second subband is occupied, a *positive* magnetoresistance always seems to be observed.[1,5–7] This change in behavior is clearly related to the occupancy of more than one subband, and it is natural to invoke the classical theory[2] in which the carriers are assumed to comprise distinct groups with different physical attributes. The essential physical idea is that the different subbands constitute parallel conducting channels which are independent apart from the fact that the currents are driven by common macroscopic fields. The situation can be realized when the channels are separated spatially as in a heterostructure[8] or energetically as in a bulk semiconductor. The conventional classical theory for this situation indeed predicts a positive magnetoresistance which varies quadratically with magnetic field B at low fields and then saturates at higher fields.

A fit of the observed magnetoresistance with the classical formula in principle allows one to extract the lifetimes or mobilities for each subband. This procedure was followed by van Houten *et al.*,[1] who came to the conclusion that the mobility of the second subband in their sample was a factor of 4 smaller than that of the first. The quality of their fits also led them to conclude that it was unnecessary to invoke intersubband scattering in the interpretation of their data. However it is not obvious *a priori* what changes intersubband scattering will make to the classical formula, which is derived with the assumption of two independent parallel conducting channels. That intersubband scattering should not play a role is a bit surprising since there is a clear signature of its effect in the observed decrease in mobility associated with second subband occupancy.[9–11] It is therefore clear that the effect of intersubband scattering must be analyzed before any definitive conclusions regarding its relevance can be made.

In this paper we present a Boltzmann transport-theory formulation of the transverse magnetoresistance, which explicitly takes into account the possibility of intersubband scattering. This theory should be valid in the low-field regime below the onset of SdH oscillations that arise from the Landau-level quantization of the electronic states. Formulas are derived for both the magnetoresistance and the Hall coefficient, and the effects of intersubband scattering are explicitly isolated. We find that including these effects can have important implications for the interpretation of the experimental data. In particular, the relative magnitude of the mobilities in the two subbands can be reversed when intersubband scattering is included as compared to when it is not.

II. BOLTZMANN TRANSPORT

We take the 2DEG to lie in the x-y plane with a static, uniform magnetic field $\mathbf{B}$ in the z direction and a uniform electric field $\mathbf{E}$ oriented in the x direction. The transport properties of the multiple-subband system are obtained following the theoretical formulation of Siggia and Kwok.[12] The subband states $|n\mathbf{k}\rangle$ are labeled by a subband index n and a two-dimensional wave vector $\mathbf{k}$, and the corresponding subband energies are given by

$$\varepsilon_{n\mathbf{k}}=\varepsilon_n+\frac{\hbar^2k^2}{2m^*}\ , \tag{1}$$

where ε_n is the position of the nth subband edge. The nonequilibrium electron distribution in each subband $f_n(\mathbf{k})$ is assumed to satisfy the Boltzmann transport equation

$$-\frac{e}{\hbar}(\mathbf{E}+\mathbf{v}_{n\mathbf{k}}\times\mathbf{B})\cdot\nabla_{\mathbf{k}}f_n(\mathbf{k})=-\sum_{n',\mathbf{k}'}w_{n\mathbf{k},n'\mathbf{k}'}[f_n(\mathbf{k})-f_{n'}(\mathbf{k}')]\ , \tag{2}$$

where $\mathbf{v}_{n\mathbf{k}}$ is the electron velocity and $w_{n\mathbf{k},n'\mathbf{k}'}$ is the transition rate from state $(n'\mathbf{k}')$ to the state $(n\mathbf{k})$ due to elastic impurity scattering, which is given by the golden-rule formula

$$w_{n\mathbf{k},n'\mathbf{k}'}=\frac{2\pi}{\hbar}\overline{|\langle n\mathbf{k}|\hat{V}|n'\mathbf{k}'\rangle|^2}\delta(\varepsilon_{n\mathbf{k}}-\varepsilon_{n'\mathbf{k}'})\ . \tag{3}$$

Here $V(\mathbf{r})$ is the screened impurity potential due to the ionized donors near the heterojunction interface. The bar denotes an average over impurity configurations. We need not specify the transition rates further at the present time.

Introducing the deviation of the electron distribution function from its equilibrium value

$$f_n(\mathbf{k})=f^0(\varepsilon_{n\mathbf{k}})+g_n(\mathbf{k})\ , \tag{4}$$

where $f^0(\varepsilon_{n\mathbf{k}})$ is the Fermi-Dirac distribution function, the linearized Boltzmann equation takes the form

$$-\frac{e\hbar}{m^*}\mathbf{E}\cdot\mathbf{k}\frac{\partial f^0(\varepsilon_{n\mathbf{k}})}{\partial\varepsilon_{n\mathbf{k}}}+\omega_c\frac{\partial g_n(\mathbf{k})}{\partial\phi}=-\sum_{n'\mathbf{k}'}w_{n\mathbf{k},n'\mathbf{k}'}[g_n(\mathbf{k})-g_{n'}(\mathbf{k}')]\ . \tag{5}$$

$\omega_c=eB/m^*$ is the cyclotron frequency, and the angular variable ϕ is the angle between the wave vector $\mathbf{k}$ and the direction of the electric field. The right-hand side of this equation can be simplified by making use of the energy-conserving δ function in the definition of the transition rate. The wave vectors in the two subbands ($k_n\equiv k$, $k_{n'}\equiv k'$) are then related by

$$k_{n'}^2=k_n^2+\frac{2m^*}{\hbar^2}(\varepsilon_n-\varepsilon_{n'})\ . \tag{6}$$

Performing the sum over $\mathbf{k}'$, Eq. (5) becomes

$$-\frac{e\hbar}{m^*}\mathbf{E}\cdot\mathbf{k}\frac{\partial f^0(\varepsilon_{n\mathbf{k}})}{\partial\varepsilon_{n\mathbf{k}}}+\omega_c\frac{\partial g_n(k_n,\phi)}{\partial\phi}=-\sum_{n'}\int_0^{2\pi}\frac{d\phi'}{2\pi}P_{nn'}(\phi-\phi')\times[g_n(k_n,\phi)-g_{n'}(k_{n'},\phi')]\ , \tag{7}$$

where the angular transition rate is defined as

$$P_{nn'}(\phi-\phi')=\frac{m^*}{\hbar^3}\overline{|\langle n\mathbf{k}|\hat{V}|n'\mathbf{k}'\rangle|^2}\ . \tag{8}$$

We note that this quantity is only a function of the magnitude of the momentum transfer $\mathbf{q}=\mathbf{k}'-\mathbf{k}$ which is given by

$$q^2=k_n^2+k_{n'}^2-2k_nk_{n'}\cos(\phi-\phi')\ . \tag{9}$$

The angular dependence of $P_{nn'}(\phi-\phi')$ arises through its dependence on q.

To solve Eq. (7) it is convenient to introduce a Fourier expansion in the angular variable. In particular we write

$$g_n(k,\phi)=\sum_m g_n^{(m)}(k)e^{im\phi} \tag{10}$$

with

$$g_n^{(m)}(k)=\frac{1}{2\pi}\int_0^{2\pi}d\phi\, e^{-im\phi}g_n(k,\phi)\ . \tag{11}$$

The scattering rate can be expanded in a similar way. Since $P_{nn'}(\phi-\phi')$ is an even function of its argument, the expansion coefficients satisfy $P_{nn'}^{(-m)}=P_{nn'}^{(m)}$. The Fourier transform of Eq. (7) then yields

$$-\frac{e\hbar}{2m^*}Ek_n\frac{\partial f^0}{\partial\varepsilon_{n\mathbf{k}}}(\delta_{m,1}+\delta_{m,-1})+im\omega_c g_n^{(m)}=-\sum_{n'}(P_{nn'}^{(0)}g_n^{(m)}-P_{nn'}^{(m)}g_{n'}^{(m)})\ . \tag{12}$$

It is clear that only the $m=\pm1$ components of $g_n^{(m)}$ are finite and that these components are independent. The final form of the transport equation is obtained by expressing the nonequilibrium distribution in the form

$$g_n^{(\pm1)}\equiv\frac{e\hbar}{2m^*}Ek_n\frac{\partial f^0}{\partial\varepsilon_{n\mathbf{k}}}\tau_n^{(\pm)}\ , \tag{13}$$

where $\tau_n^{(\pm)}$ is the subband transport lifetime. Substituting this expression into Eq. (12) then gives

$$\sum_{n'}\left[\left(\sum_m P_{nm}^{(0)}\pm i\omega_c\right)\delta_{nn'}-P_{nn'}^{(1)}\right]k_{n'}\tau_{n'}^{(\pm)}=k_n\ . \tag{14}$$

The magnetic field appears explicitly only in the term involving ω_c. Setting this term equal to zero recovers the form of the multiband transport equation derived by Siggia and Kwok.[12] In this limit the two transport lifetimes $\tau_n^{(\pm)}$ are equal.

Once the distribution functions are known the transport properties follow immediately. The current density is given by

$$\mathbf{J} = -2e \int \frac{d^2k}{(2\pi)^2} \sum_n \mathbf{v}_{n\mathbf{k}} f_n(\mathbf{k}) \; . \tag{15}$$

Using Eqs. (4) and (13) and defining the complex current

$$J_\pm = J_x \pm i J_y \; , \tag{16}$$

we find

$$J_\pm = -E \left[\frac{e\hbar}{2m^*} \right]^2 \sum_n \frac{2}{\pi} \int_0^\infty dk\, k^3 \frac{\partial f^0}{\partial \varepsilon_{n\mathbf{k}}} \tau_n^{(\mp)} \; . \tag{17}$$

At zero temperature the k integration can be performed with the result

$$J_\pm = \frac{e^2}{m^*} \sum_i n_i \tau_i^{(\mp)} E \; , \tag{18}$$

where the lifetimes are now evaluated at the Fermi energy and $n_i = k_{Fi}^2/2\pi$ are the subband densities. Equation (18) defines the complex conductivities

$$\sigma_\pm = \frac{e^2}{m^*} \sum_i n_i \tau_i^{(\mp)} \; , \tag{19}$$

which are related to the Cartesian components by

$$\sigma_\pm = \sigma_{xx} \pm i \sigma_{yx} \; . \tag{20}$$

The solution to Eq. (14) is given by

$$k_i \tau_i^{(\pm)} = \sum_j (\mathbf{K} \pm i\omega_c \mathbf{I})^{-1}{}_{ij} k_j \; , \tag{21}$$

where we have defined the scattering matrix

$$K_{ij} = \sum_k P_{ik}^{(0)} \delta_{ij} - P_{ij}^{(1)} \; . \tag{22}$$

From Eq. (19), the conductivity tensor is

$$\begin{aligned} \sigma_\pm &= \frac{e^2}{2\pi m^*} \sum_{ij} k_i k_j (\mathbf{K} \mp i\omega_c \mathbf{I})^{-1}{}_{ij} \\ &= \frac{e^2}{m^*} \mathrm{Tr} \mathbf{N} (\mathbf{K} \mp i\omega_c \mathbf{I})^{-1} \; , \end{aligned} \tag{23}$$

where Tr stands for the trace of the matrix product and the matrix $\mathbf{N}$ is defined as

$$N_{ij} = \frac{k_i k_j}{2\pi} = \sqrt{n_i n_j} \; . \tag{24}$$

We now define the complex resistivity by

$$\rho_+ = \rho_{xx} + i\rho_{yx} = \sigma_+^{-1} \tag{25}$$

from which we obtain the resistivity components

$$\rho_{xx} = \frac{m^*}{e^2} \mathrm{Re} \left[\frac{1}{\mathrm{Tr}\mathbf{N}(\mathbf{K} - i\omega_c \mathbf{I})^{-1}} \right] , \tag{26}$$

$$\rho_{yx} = \frac{m^*}{e^2} \mathrm{Im} \left[\frac{1}{\mathrm{Tr}\mathbf{N}(\mathbf{K} - i\omega_c \mathbf{I})^{-1}} \right] . \tag{27}$$

These expressions simplify for a single occupied subband. In this limit we find

$$\rho_{xx} = \frac{m^*}{ne^2\tau} \tag{28}$$

and

$$\rho_{yx} = -\frac{1}{ne} B \; , \tag{29}$$

where n is the electron gas density and τ is the transport lifetime

$$\frac{1}{\tau} = \frac{m^*}{2\pi\hbar^3} \int_0^{2\pi} d\phi (1 - \cos\phi) \overline{|\langle 1,\mathbf{k}|\hat{V}|1,\mathbf{k}+\mathbf{q}\rangle|^2} \; . \tag{30}$$

These are the expected results.[2] In particular, Eq. (28) shows that classically there is no transverse magnetoresistance for a single band.

III. MAGNETORESISTANCE

The transverse magnetoresistance is conventionally defined as $(\rho_{xx} - \rho_0)/\rho_0$, where ρ_0 is the zero-field resistivity. In terms of the quantities defined previously, we have

$$\frac{\Delta\rho_{xx}}{\rho_0} \equiv \frac{\rho_{xx} - \rho_0}{\rho_0} = \mathrm{Re} \left[\frac{\mathrm{Tr}\mathbf{N}\mathbf{K}^{-1}}{\mathrm{Tr}\mathbf{N}(\mathbf{K} - i\omega_c \mathbf{I})^{-1}} \right] - 1 \tag{31}$$

with

$$\rho_0 = \frac{m^*}{e^2} \left[\frac{1}{\mathrm{Tr}\mathbf{N}\mathbf{K}^{-1}} \right] . \tag{32}$$

Although Eq. (31) can be used for any number of subbands, we now consider the special case of only two occupied subbands ($i = 1,2$). The K matrix then has the form

$$\begin{aligned} \mathbf{K} &= \begin{bmatrix} K_1 & K_3 \\ K_3 & K_2 \end{bmatrix} \\ &= \begin{bmatrix} P_{11}^{(0)} - P_{11}^{(1)} + P_{12}^{(0)} & -P_{12}^{(1)} \\ -P_{12}^{(1)} & P_{22}^{(0)} - P_{22}^{(1)} + P_{12}^{(0)} \end{bmatrix} . \end{aligned} \tag{33}$$

We note that intersubband scattering terms appear both in the diagonal and off-diagonal matrix elements. The diagonal elements contain $P_{12}^{(0)}$, which is the average intersubband scattering rate, while the off-diagonal elements contain the average scattering rate weighted by the momentum transfer.

We now introduce the eigenvectors and eigenvalues of the K matrix:

$$\mathbf{K}\mathbf{u}_i = \lambda_i \mathbf{u}_i \; , \tag{34}$$

where the eigenvalues are given explicitly by

$$\bar{\tau}_{1,2}^{-1} \equiv \lambda_{1,2} = \tfrac{1}{2}(K_1+K_2) \pm \tfrac{1}{2}\sqrt{(K_1-K_2)^2+4K_3^2} \ . \tag{35}$$

We have used the eigenvalues to define the characteristic lifetimes $\bar{\tau}_{1,2}$, which will be used later. Defining the two-component column vectors

$$\mathbf{v}^{(\pm)} = \begin{bmatrix} k_1\tau_1^{(\pm)} \\ k_2\tau_2^{(\pm)} \end{bmatrix} , \quad \mathbf{k} = \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} \tag{36}$$

the solution to Eq. (14) is

$$\mathbf{v}^{(\pm)} = (\mathbf{K} \pm i\omega_c \mathbf{I})^{-1}\mathbf{k} \ . \tag{37}$$

If the vector $\mathbf{k}$ is now expanded in terms of the eigenvectors of K,

$$\mathbf{k} = \alpha\mathbf{u}_1 + \beta\mathbf{u}_2 \ , \tag{38}$$

we find

$$\mathbf{v}^{(\pm)} = \alpha(\lambda_1 \pm i\omega_c)^{-1}\mathbf{u}_1 + \beta(\lambda_2 \pm i\omega_c)^{-1}\mathbf{u}_2 \ . \tag{39}$$

The complex conductivity is now given by

$$\begin{aligned} \sigma_\pm &= \frac{e^2}{2\pi m^*}\mathbf{k}\cdot\mathbf{v}^{(\mp)} \\ &= \frac{e^2}{2\pi m^*}\left[\frac{\alpha^2}{\lambda_1 \mp i\omega_c} + \frac{\beta^2}{\lambda_2 \mp i\omega_c}\right] \end{aligned} \tag{40}$$

and the zero-field conductivity is obtained simply by setting ω_c equal to zero. Thus

$$\begin{aligned} \sigma_0 &= \frac{e^2}{2\pi m^*}\left[\frac{\alpha^2}{\lambda_1} + \frac{\beta^2}{\lambda_2}\right] \\ &\equiv \tilde{\sigma}_1 + \tilde{\sigma}_2 \ . \end{aligned} \tag{41}$$

The partial conductivities defined by this equation are not the conductivities of each subband, however they are useful for algebraic purposes. The actual subband conductivities are given by

$$\sigma_i = \frac{n_i e^2 \tau_i}{m^*} \ , \tag{42}$$

where τ_i is the zero-field limit of $\tau_i^{(\pm)}$. In terms of $\tilde{\sigma}_i$ we have

$$\sigma_\pm = \frac{\tilde{\sigma}_1}{1 \mp i\gamma_1} + \frac{\tilde{\sigma}_2}{1 \mp i\gamma_2} \ , \tag{43}$$

where

$$\gamma_i = \frac{\omega_c}{\lambda_i} = \bar{\tau}_i\omega_c \ . \tag{44}$$

The expression for the magnetoresistance can now be written in the form

$$\frac{\Delta\rho_{xx}}{\rho_0} = \frac{\tilde{\sigma}_1\tilde{\sigma}_2(\gamma_1-\gamma_2)^2}{\sigma_0^2 + (\tilde{\sigma}_1\gamma_2 + \tilde{\sigma}_2\gamma_1)^2} \ . \tag{45}$$

This equation simplifies further using

$$\begin{aligned} \tilde{\sigma}_1\gamma_2 + \tilde{\sigma}_2\gamma_1 &= \frac{e^2}{2\pi m^*}\frac{\omega_c}{\lambda_1\lambda_2}(\alpha^2+\beta^2) \\ &= \frac{ne^2}{m^*}\frac{\omega_c}{\lambda_1\lambda_2} \end{aligned} \tag{46}$$

and

$$\begin{aligned} \tilde{\sigma}_1\tilde{\sigma}_2(\gamma_1-\gamma_2)^2 &= \left[\frac{e^2}{2\pi m^*}\right]^2 \frac{\omega_c^2}{\lambda_1\lambda_2}\left[\frac{\alpha\beta}{\lambda_1} - \frac{\alpha\beta}{\lambda_2}\right]^2 \\ &= \left[\frac{e^2}{2\pi m^*}\right]^2 \frac{\omega_c^2}{\lambda_1\lambda_2} \\ &\quad \times [(\beta\mathbf{u}_1 - \alpha\mathbf{u}_2)\cdot\mathbf{K}^{-1}\mathbf{k}]^2 \\ &= \left[\frac{e^2}{m^*}\right]^2 \frac{n_1 n_2}{\lambda_1\lambda_2}(\mu_1-\mu_2)^2 B^2 \ . \end{aligned} \tag{47}$$

Here we have introduced the subband mobilities μ_i

$$\mu_i = \frac{e\tau_i}{m^*} \tag{48}$$

which are related to the conductivities by $\sigma_i = n_i e \mu_i$. We finally obtain

$$\frac{\Delta\rho_{xx}}{\rho_0} = \frac{r\sigma_1\sigma_2(\mu_1-\mu_2)^2 B^2}{\sigma_0^2 + (ren\mu_1\mu_2 B)^2} \tag{49}$$

with

$$r = (\lambda_1\tau_1\lambda_2\tau_2)^{-1} = \frac{\bar{\tau}_1\bar{\tau}_2}{\tau_1\tau_2} \ . \tag{50}$$

The magnetoresistance has been expressed in a form that resembles as closely as possible the classical expression[2] derived for two independent conducting channels. This limit is recovered by setting the intersubband scattering matrix elements in Eq. (33) to zero, in which case the K matrix is diagonal and its eigenvalues are in fact the reciprocal of the zero-field transport lifetimes. The factor r in Eq. (50) is then 1. It should be emphasized, however, that the effect of intersubband scattering appears not only through the factor r but is also contained implicitly in the transport lifetimes that determine the subband conductivities σ_i and mobilities μ_i.

The magnetoresistance in Eq. (49) has the characteristic form $\Delta\rho_{xx}/\rho_0 = aB^2/(1+bB^2)$ and is therefore determined by the two constants a and b. The curvature at low fields is

$$a = r(\mu_1-\mu_2)^2\frac{\sigma_1\sigma_2}{\sigma_0^2} \ , \tag{51}$$

while the saturation value at high fields is

$$\frac{a}{b} = \frac{n_1 n_2}{rn^2}\frac{(\mu_1-\mu_2)^2}{\mu_1\mu_2} \ . \tag{52}$$

It is seen that the low-field behavior is scaled by the factor r relative to the classical expression, while the saturation value is scaled by $1/r$. As a result, a fit of the magnetoresistance to experimental data will in general lead to different values of the subband mobilities depending on whether intersubband scattering is or is not taken into account.

From Eq. (33), the magnetoresistance can be seen to depend on three independent constants K_1, K_2, and K_3. If K_3 is set equal to zero, intersubband scattering is ignored, and the two remaining constants define the subband lifetimes τ_1 and τ_2. These could be used to fit the experimental a and b parameters, but the lifetimes must also determine the zero-field resistivity ρ_0. Taking this constraint into account does not leave a sufficient number of free parameters to independently fit a and b unless other parameters such as the subband densities are also allowed to vary. Of course this freedom is not available if the densities are fixed by the SdH oscillation frequencies. On the other hand, allowing for intersubband scattering provides a third parameter that will obviously permit a superior fit to a, b, and ρ_0 to be achieved. It should be emphasized, however, that the additional parameter is physically based. An example of this fitting procedure will be given shortly.

IV. HALL COEFFICIENT

A similar analysis can be given for the Hall resistivity ρ_{yx}. From Eq. (27) we obtain the intermediate expression

$$\rho_{yx}=-\frac{\gamma_1\tilde{\sigma}_1+\gamma_2\tilde{\sigma}_2+\gamma_1\gamma_2(\gamma_1\tilde{\sigma}_2+\gamma_2\tilde{\sigma}_1)}{(\tilde{\sigma}_1+\tilde{\sigma}_2)^2+(\gamma_1\tilde{\sigma}_2+\gamma_2\tilde{\sigma}_1)^2} . \tag{53}$$

Using

$$\begin{aligned}\gamma_1\tilde{\sigma}_1+\gamma_2\tilde{\sigma}_2&=\frac{e^3B}{2\pi m^{*2}}\left[\frac{\alpha^2}{\lambda_1^2}+\frac{\beta^2}{\lambda_2^2}\right]\\&=\frac{e^3B}{2\pi m^{*2}}(\mathbf{K}^{-1}\mathbf{k})\cdot(\mathbf{K}^{-1}\mathbf{k})\\&=eB(n_1\mu_1^2+n_2\mu_2^2)\end{aligned} \tag{54}$$

and Eq. (46), we find

$$\rho_{xy}=-\frac{\langle\mu^2\rangle+(r\mu_1\mu_2B)^2}{\langle\mu\rangle^2+(r\mu_1\mu_2B)^2}\frac{B}{ne} , \tag{55}$$

where the average mobility is defined as

$$\langle\mu\rangle=\frac{n_1\mu_1+n_2\mu_2}{n_1+n_2} , \tag{56}$$

with a similar definition of $\langle\mu^2\rangle$. The Hall coefficient $R_H(B)=\rho_{yx}/B$ has the limiting values

$$R_H(0)=-\frac{\langle\mu^2\rangle}{\langle\mu\rangle^2}\frac{1}{ne} \tag{57}$$

and

$$R_H(\infty)=-\frac{1}{ne} . \tag{58}$$

Since $\langle\mu^2\rangle>\langle\mu\rangle^2$, $|R_H(0)|>|R_H(\infty)|$. Neither of these limits involve the factor r, but the effect of intersubband scattering is implicitly contained in the zero-field mobilities. The factor r, however, does play a role in determining the field at which the Hall coefficient makes the transition from the low- to high-field values.

V. DISCUSSION

We have used our results for the magnetoresistance to reanalyze the data of van Houten *et al.*[1] Their sample was a GaAs/Al_xGa_{1-x}As heterojunction in which two subbands were occupied and in which the density was varied by means of persistent photoconductivity. The original analysis was based on Eq. (49) with $r=1$. A fit to the data is obtained by varying the subband mobilities subject to the constraint that the zero-field conductivity $\sigma_0=en_1\mu_1+en_2\mu_2$ has the observed value. We have repeated this fitting procedure ($K_3=0$), and our results for both the dark and illuminated samples essentially reproduce those obtained previously. We find $\mu_1=39.7$ m^2/V s and $\mu_2=10.2$ m^2/V s for the dark sample and $\mu_1=60.0$ m^2/V s and $\mu_2=15.9$ m^2/V s for the illuminated sample. The main conclusion one comes to if intersubband scattering is neglected is that the second subband mobility is considerably smaller than that of the first subband. The fits to the magnetoresistance data obtained with these values are indicated by the dashed lines in Fig. 1 and are seen to be quite good. The usual explanation[1] for the relative magnitude of the subband mobilities is based on the observation that the Fermi wave vector of the second subband is a factor of 5 smaller than that of the first. The impurity scattering rate is a strongly decreasing function of the momentum transfer so that scattering at $2k_{F2}$ is expected to be stronger than at $2k_{F1}$. However this argument is incomplete since the scattering matrix elements also depend on the subband wave functions. Since the higher subbands extend further away from the interface, they are further removed from the ionized impurities, and as a result, the intrasubband scattering matrix elements decrease with increasing subband index. Thus the relative magnitude of the Fermi wave vectors and the distance from the interface are com-

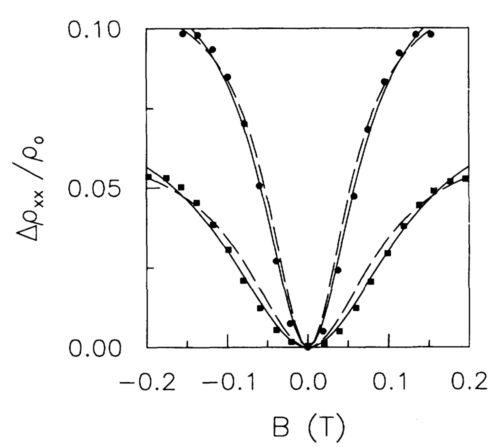

FIG. 1. Magnetoresistance as a function of the magnetic field. The solid points are the experimental data from Ref. 1: squares (dark) and circles (light). The dashed curves are the theoretical fits assuming no intersubband scattering, and the solid lines assume intersubband scattering.

TABLE I. Transport parameters obtained by fitting Eq. (49) to the magnetoresistance data of Ref. 1. Dark and Light refer, respectively, to before and after illumination of the sample.

	n_1	n_2	K_1	K_2	K_3	$\bar{\tau}_1$	$\bar{\tau}_2$	μ_1	μ_2
	10^{15} m^{-2}			10^{11} s^{-1}		10^{-11} s		m^2/V s	
Dark	4.9	0.16	0.96	3.1	−0.85	0.29	1.5	38	66
Light	7.1	0.45	0.75	1.6	−0.60	0.52	2.3	55	97

peting effects and it is not *a priori* obvious which effect will be dominant.

As discussed at the end of Sec. III, the fit obtained without intersubband scattering is not the optimal one of the form $\Delta\rho_{xx}/\rho_0 = aB^2/(1+bB^2)$ since the a and b parameters are prescribed functions of the mobilities and cannot be varied independently. The optimal fit can be obtained by allowing for intersubband scattering and is shown as the solid lines in Fig. 1. These were obtained by treating the K-matrix elements K_1, K_2, and K_3 as free parameters constrained by the zero-field conductivity. The fit obtained is seen to be an improvement over the fits neglecting intersubband scattering. The subband mobilities for the dark sample are now found to be $\mu_1 = 37.9$ m^2/V s and $\mu_2 = 66.0$ m^2/V s. The first subband mobility is only slightly changed from the earlier value. This is related to the fact that the first subband electrons dominate the conductivity because of their higher density. The second subband mobility, however, is increased from 10.2 m^2/V s to 66.0 m^2/V s and is now roughly twice the first subband mobility. Thus the conclusion one comes to regarding the relative magnitudes of the subband mobilities is completely different depending on whether or not intersubband scattering is taken into account.

The quality of the fits to the data is not the only criterion to use in judging the reasonableness of the results. In Table I we also show the values of the K-matrix elements which according to Eq. (33) are defined in terms of microscopic scattering matrix elements. Considering the dark sample again, we find $K_1 = 9.6\times10^{10}$ s^{-1}, $K_2 = 3.1\times10^{11}$ s^{-1}, and $K_3 = -8.5\times10^{10}$ s^{-1}. Since the intersubband term $P_{12}^{(0)}$ is a common contribution to K_1 and K_2, it is clear that the *intrasubband* scattering rate for the second subband $(P_{22}^{(0)} - P_{22}^{(1)})$ is in fact considerably larger than that of the first subband as might be expected on the basis of the relative magnitude of the subband Fermi wave vectors. Nevertheless, the second subband mobility turns out to be larger than the first. The reason why this is possible is that the transport lifetimes are determined by the full complement of the K-matrix elements as a result of the inversion in Eq. (21). The matrix element K_3 according to Eq. (33) is given by $-P_{12}^{(1)}$. The fit, therefore, indicates that $P_{12}^{(1)}$ is a positive quantity, which is expected in view of the dependence of the scattering matrix elements on the momentum transfer given in Eq. (9).

The values of the various K-matrix elements obtained from the fits are not unreasonable when compared with those calculated for similar samples under similar conditions. In particular we previously found[11] that the transport lifetime for the second subband can be larger than that of the first for a sample that is illuminated with red radiation. Apparently, the sample being considered in the present paper is behaving similarly.

Finally we note that despite the improvement, the fit to the data in Fig. 1 is not perfect. The experimental data exhibit a saturation with increasing field which is more rapid than can be achieved with the derived field dependence of the magnetoresistance. Thus we cannot claim the results for the subband mobilities obtained in this paper to be conclusive. However the main point we are trying to make is that inferences drawn from the analysis of magnetoresistance data are sensitively dependent on the assumptions made about the presence or absence of intersubband scattering. We have shown how the classical expression for the magnetoresistance can be generalized to take intersubband scattering into account. At the very least, this effect should be allowed for when analyzing data in those situations where intersubband scattering might be expected to occur.[1,6]

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