Inelastic Coulomb scattering in a diffusive two-band system

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A calculation of the inelastic scattering rate resulting from Coulomb scattering of conduction ("*s*") electrons coupled to a narrow "*d* band" is given for a disordered system. The general analysis is carried out by extending the Keldysh formalism for nonequilibrium processes to the situation of two overlapping bands, including both intraband and interband elastic scattering, and interband Coulomb coupling. The general result is applied to the case of two overlapping bands, the *s* band by itself being in the ballistic regime. Then, when the interband *elastic* scattering vertex u_{sd} is negligible, the inelastic rate of the conduction electrons, at two dimensions, changes from the well-known $T^2\ln T$ behavior at very low temperatures into a $T^{3/2}$ law at higher ones. In three dimensions it behaves like T^2, the ususal result for a ballistic band. When u_{sd} is sufficiently strong to render the motion of the *s* electrons diffusive, the inelastic rate is proportional to $T^{d/2}$ at low temperatures, for $d=2$ and 3, as in the case of a single band. The implications of the results to the temperature-dependent resistivity of a two-band system and to experiments involving two quantum wells in heterostructures are discussed.

I. INTRODUCTION

The picture of two electronic bands coupled together by interband Coulomb interactions has been used quite extensively in condensed-matter physics. The model usually describes a system of two types of electrons with widely different effective masses, i.e., a wide conduction band coupled to a much narrower one.[1] It has been recently applied also to two, spatially separated electron systems (e.g., two quantum wells in a heterostructure[2]). The two species of electrons will be denoted here, for convenience, the "*s*" and "*d*" band, respectively.

The two-band picture is very intriguing in the context of high-transition-temperature superconductivity, as it leads to nonphononic mechanisms for attraction between electrons. In this respect one should distinguish among different types of interband Coulomb couplings.[1] In particular, there is the *pair-transfer* Coulomb vertex (a transfer of two electrons from the *s* to the *d* band, and vice versa), which was proposed[3] to explain the superconducting properties of some transition metals and the *A* 15 compounds. It may be relevant to the high-T_c oxides as well.[4,5] A different interband Coulomb coupling is described by the *s-d vertex,* in which each of the electrons is scattered within its respective band. It leads to a collective motion of the two types of electrons that may, under suitable conditions, result in a well-defined low-frequency mode[6] ("acoustic plasmons"). The *s-d* coupling applies also to two spatially separated electron gases.

Interband Coulomb coupling may be manifested also in the inelastic scattering rate of the conduction electrons. When two coupled bands are considered, there is in addition to the *intraband* scattering the inelastic rate due to *interband* scattering. This additional scattering is of particular interest when the temperature dependence of the resistivity (due to the *s* electrons) is examined.[7] The reason is that when umklapp scattering is unimportant, single-band electron-electron scattering does not contribute to the resistivity. The additional interband inelastic scattering has recently received considerable attention because of the linear temperature dependence of the in-plane resistivity (above the superconducting transition temperature) exhibited by several copper oxides.[8] Among the various attempts to explain this behavior within the Fermi-liquid theory, several concentrate upon interband processes. Thus it was found that inelastic Coulomb scattering of extended electrons by localized ones[9] gives rise to an unusual temperature dependence of the electron-electron scattering rate [$T^2(\ln T)^3$ at two dimensions] that appears as linear over a significant temperature region. Another explanation invokes the *s-d* vertex discussed above, claiming that it leads to an inelastic rate linear in T, at two dimensions.[10] However, it was shown[7] that this is not the case: when the two bands are ballistic, the $T^2\ln T$ behavior characterizes the temperature dependence of the inelastic electron-electron scattering rate for both the *s-d* and the pair-transfer coupling mechanisms, at two dimensions (for the case of two overlapping bands). There are indeed experimental indications[11] for a $T^2\ln T$ dependence in the resistivity of electron-doped Cu oxides. We believe[7] that this finding provides a strong support for a two-band model for these materials, since in a single band without umklapp scattering the $T^2\ln T$ behavior would not show up in the resistivity.

However, the question still remains whether a different temperature dependence can be obtained when *elastic* scattering by defects is added, as is known for the single-band case.[12–14] This paper is devoted to exploring consequences of the two-band picture in situations where the narrow *d* band is much more sensitive to disorder than the wide *s* band. We have in mind systems in which the elastic scattering of the conduction electrons *by itself* is weak, but that of the narrow-band electrons is strong

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enough to make the two-band inelastic scattering diffusive. The extreme limit of this idea is that of a ballistic band coupled to a localized one.[9] This question may also relate to experiments carried out on two spatially separated electron gases in the layered $GaAs/Al_xGa_{1-x}As$ structure.[2] The setup allows the injection of current into one of the electron systems and the detection of an induced current flow or a small drag voltage in the other. The effect is explained by momentum transfer between the two electron systems due to mutual Coulomb scattering.[15] The explicit comparison of the data (in particular, the temperature dependence) invokes "interband" electron-electron interaction of the *s-d* vertex type.

We present in the next section and in the Appendixes a general derivation of the electron-electron inelastic rate in a system of two overlapping bands, having a common Fermi level. We use the Keldysh formalism[16] for nonequilibrium processes, and allow for elastic scattering within each band, as well as *interband elastic scattering.* The *Coulomb* coupling of the two bands is assumed to be of the form of the *s-d* vertex, as the pair-transfer vertex does not develop a diffusion pole due to repeated elastic scattering events. (The inelastic rate due to that vertex in the case of ballistic bands is discussed in Ref. 7.) In Sec. III we use the general expression to consider the situation in which the elastic scattering within the *s* band is weak. We distinguish there between two cases: (i) a weak interband elastic scattering; (ii) a strong interband elastic scattering. In the first case we have the situation of a ballistic wideband coupled to a narrow diffusive one by the Coulomb interaction alone. We find that under such conditions, the temperature dependence of the electron-electron scattering rate, at *two dimensions,* shows a crossover from the $T^2\ln T$ behavior into a $T^{3/2}$ dependence, depending on the relative magnitudes of the unscreened Coulomb vertices. In the second case the *elastic* coupling between the bands renders the conduction band to be diffusive. Then the temperature dependence of the inelastic rate behaves like $T^{d/2}$ ($d=2,3$), as is the case for a single band.[12,13,17] Section IV includes some conclusions.

II. DERIVATION OF THE KINETIC EQUATION IN THE TWO-BAND PICTURE

In the case of a dirty system, it is convenient to use the Keldysh diagram technique for nonequilibrium processes.[16] The derivation of the kinetic equation in this formalism, for the single-band case, has been carried out by Al'tshuler and Aronov.[18] Here we extend their treatment for two bands.

The elastic scattering in the system is characterized by elastic scattering within each band, of matrix element squared u_s and u_d, respectively, as well as interband elastic scattering with matrix element squared u_{sd}. The retarded and advanced Green's function in each of the bands, averaged over impurity scattering, has the usual form[17]

$$G^R_{s,d}(\mathbf{p},E)=1/[E-E_{s,d}(\mathbf{p})+i/2\tau_{s,d}]\ ,$$
$$G^A_{s,d}(\mathbf{p},E)=[G^R_{s,d}(\mathbf{p},E)]^*\ , \tag{2.1}$$

but the mean free (elastic) times of each band, τ_s and τ_d, include the effect of interband elastic scattering

$$\frac{1}{\tau_s}=2\pi N_s u_s+2\pi N_d u_{sd}\ ,$$
$$\frac{1}{\tau_d}=2\pi N_d u_d+2\pi N_s u_{sd}\ . \tag{2.2}$$

(Units in which $\hbar=1$ are used.) In Eqs. (2.1) and (2.2), $E_{s,d}(\mathbf{p})$ denotes the dispersion relation in each band (energies are measured from the Fermi energy, common to both bands) and N_s and N_d are the densities of states at the Fermi energy of the *s* and *d* band, respectively.

We shall calculate the energy relaxation of the electrons in the *s* band. Denoting their nonequilibrium distribution function by $f_s(E)$, the Keldysh formalism yields for the time derivative the equation[16]

$$\frac{\partial f_s(E)}{\partial t}=\frac{1}{4\pi N_s}\sum_{\mathbf{p}}[F_s(\Sigma_s^A-\Sigma_s^R)-\Omega_s(G_s^A-G_s^R)]\ , \tag{2.3}$$

where the functions in the square brackets depend upon $\mathbf{p}$ and E. Here

$$F_s(\mathbf{p},E)=[2f_s(E)-1][G_s^A(\mathbf{p},E)-G_s^R(\mathbf{p},E)]\ . \tag{2.4}$$

Since in the Keldysh technique it is convenient to work with a Green's-function matrix,

$$\hat{G}_s=\begin{bmatrix}0 & G_s^A\\ G_s^R & F_s\end{bmatrix} \tag{2.5}$$

(and similarly for the *d*-band Green's function), the self-energy part is a matrix as well

$$\hat{\Sigma}_s=\begin{bmatrix}\Omega_s & \Sigma_s^R\\ \Sigma_s^A & 0\end{bmatrix}\ , \tag{2.6}$$

which defines Ω_s in Eq. (2.3). The self-energy part of the *s* electrons is presented in Fig. 1. It is given by

$$\hat{\Sigma}_s(\mathbf{p},E)=\int\frac{d\omega}{2\pi i}\sum_{\mathbf{q}}\hat{\Gamma}_s^i(-\mathbf{q},\omega,E-\omega)\hat{G}_s(\mathbf{p}-\mathbf{q},E-\omega)\times\hat{\Gamma}_s^k(\mathbf{q},\omega,E)V_s^{ki}(\mathbf{q},\omega)\ , \tag{2.7}$$

where repeated indices (i,k) are summed over. In this equation $\hat{V}_s$ is the screened Coulomb interaction within the *s* band. In the Keldysh formalism it is again a 2×2

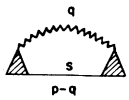

FIG. 1. The self-energy part of the *s* electrons. The hatched triangles represent the vertices, the curly line is the screened Coulomb interaction within the *s* band, and the solid line is the *s*-electron Green's function, averaged over impurity scattering.

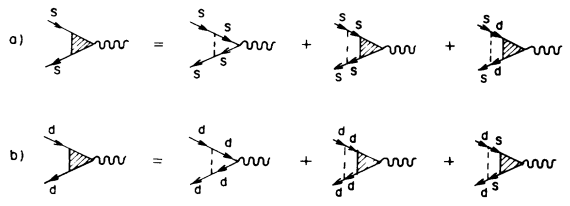

FIG. 2. The vertex corrections. (a) The equation for Γ_s; (b) the equation for Γ_d. The dashed lines represent impurity scattering, within and in between the two bands.

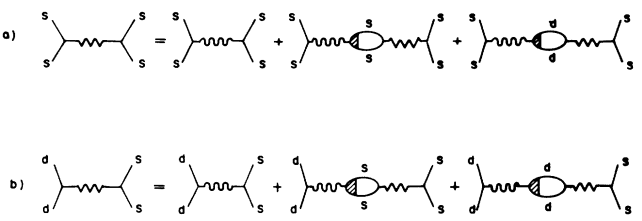

FIG. 3. The screened Coulomb interaction. (a) The intraband interaction within the s band; (b) the interband interaction between the two bands. The "bubbles" are the polarization parts.

matrix (see below). Hence $\hat{V}_s^{ki}$ is its ki matrix element $(k,i=1,2)$. The vertex corrections due to multiple impurity scattering are denoted by $\hat{\Gamma}_s^i$, $i=1,2$. In the following we calculate the various terms that appear on the right-hand side (rhs) of Eq. (2.3), and use that equation to obtain the electron-electron rate. The final result (in the low-temperature limit) is given in Eqs. (2.24) and (2.25) below.

In the two-band picture there are s- and d-vertex corrections. They are shown in Fig. 2. The equations for them have the form[19]

$$\hat{\Gamma}_s^k(\mathbf{q},\omega,E)=\hat{\gamma}^k+u_s\sum_{\mathbf{p}}\hat{\sigma}_x\hat{G}_s(\mathbf{p}-\mathbf{q},E-\omega)\hat{\Gamma}_s^k(\mathbf{q},\omega,E)\hat{G}_s(\mathbf{p},E)\hat{\sigma}_x+u_{sd}\sum_{\mathbf{p}}\hat{\sigma}_x\hat{G}_d(\mathbf{p}-\mathbf{q},E-\omega)\hat{\Gamma}_d^k(\mathbf{q},\omega,E)\hat{G}_d(\mathbf{p},E)\hat{\sigma}_x\ , \tag{2.8}$$

and a similar equation for $\hat{\Gamma}_d$, obtained by interchanging the roles of s and d. In this equation k takes the values 1 and 2, where

$$\gamma_{ij}^1=(\tfrac{1}{2})^{1/2}\delta_{ij},\quad \gamma_{ij}^2=(\tfrac{1}{2})^{1/2}(\hat{\sigma}_x)_{ij}\ , \tag{2.9}$$

are the bare vertices and $\hat{\sigma}_x$ is a Pauli matrix. We list the explicit expressions for the vertex corrections in Appendix A.

Next we consider the screened Coulomb potential. Its equations are plotted in Fig. 3. In the two-band picture, the screening mixes the intraband Coulomb vertices, $v_s(q)$ and $v_d(q)$, with the interband Coulomb interaction, $v_{sd}(q)$ $[v_{sd}(q)=v_{ds}(q)]$. Explicitly, the equations presented in Fig. 3 read

$$\begin{aligned}\hat{V}_s(\mathbf{q},\omega)&=v_s(\mathbf{q})\hat{\sigma}_x+v_s(\mathbf{q})\hat{\sigma}_s\hat{\Pi}_s(\mathbf{q},\omega)\hat{V}_s(\mathbf{q},\omega)\\&\quad+v_{sd}(\mathbf{q})\hat{\sigma}_x\hat{\Pi}_d(\mathbf{q},\omega)\hat{V}_{ds}(\mathbf{q},\omega)\ ,\\ \hat{V}_{ds}(\mathbf{q},\omega)&=v_{ds}(\mathbf{q})\hat{\sigma}_x+v_{ds}(\mathbf{q})\hat{\sigma}_s\hat{\Pi}_s(\mathbf{q},\omega)\hat{V}_s(\mathbf{q},\omega)\\&\quad+v_d(\mathbf{q})\hat{\sigma}_x\hat{\Pi}_d(\mathbf{q},\omega)\hat{V}_{ds}(\mathbf{q},\omega)\ .\end{aligned} \tag{2.10}$$

Here $\hat{V}_s$ and $\hat{V}_{ds}$ are the s-band intraband and the interband screened Coulomb interactions, respectively. $\hat{\Pi}_s$ and $\hat{\Pi}_d$ denote the polarization parts of the s and d bands, respectively (a 2×2 matrix in the Keldysh formalism). They are represented by the vertex-renormalized bubbles in Fig. 3, and their determination is discussed below. Introducing the notations

$$\hat{V}_s=\begin{bmatrix}0 & V_s^A\\ V_s^R & V_s\end{bmatrix},\quad \hat{V}_{ds}=\begin{bmatrix}0 & V_{ds}^A\\ V_{ds}^R & V_{ds}\end{bmatrix}, \tag{2.11}$$

and

$$\hat{\Pi}_{s,d}=\begin{bmatrix}\Pi_{s,d} & \Pi_{s,d}^R\\ \Pi_{s,d}^A & 0\end{bmatrix} \tag{2.12}$$

[in accordance with the Green's function matrix, Eq. (2.5) and the self-energy matrix, Eq. (2.6), respectively], we find for the screened Coulomb interactions

$$\begin{aligned}V_s^{R,A}&=\frac{1}{\epsilon^{R,A}}[v_s(1-v_d\Pi_d^{R,A})+v_{sd}^2\Pi_d^{R,A}]\ ,\\ V_s&=\frac{1}{\epsilon^R\epsilon^A}[v_{sd}^2\Pi_d+|v_s-(v_sv_d-v_{sd}^2)\Pi_d^A|^2\Pi_s]\ ,\end{aligned} \tag{2.13}$$

and

$$\begin{aligned}V_{ds}^{R,A}&=\frac{v_{sd}}{\epsilon^{R,A}}\ ,\\ V_{ds}&=\frac{v_{sd}}{\epsilon^R\epsilon^A}\{[v_s-(v_sv_d-v_{sd}^2)\Pi_d^A]\Pi_s\\&\quad+[v_d-(v_sv_d-v_{sd}^2)\Pi_d^R]\Pi_d\}\ .\end{aligned} \tag{2.14}$$

Here $\epsilon^{R,A}(\mathbf{q},\omega)$ denotes the dielectric function

$$\epsilon^{R,A}(\mathbf{q},\omega)=(1-v_s\Pi_s^{R,A})(1-v_d\Pi_d^{R,A})-v_{sd}^2\Pi_s^{R,A}\Pi_d^{R,A}\ . \tag{2.15}$$

We note that in the absence of the interband Coulomb coupling, $(v_{sd}=0)$, the results reduce to those of Ref. 18.

The last ingredients in the calculation are the polarization parts, $\hat{\Pi}_s$ and $\hat{\Pi}_d$. These are determined by the equation (see Fig. 3)

$$\Pi^{jk}_{s,d}(\mathbf{q},\omega)=\mathrm{Tr}\int\frac{dE}{2\pi i}\sum_{\mathbf{p}}\hat{\Gamma}^{j}_{s,d}(-\mathbf{q},E-\omega)\hat{G}_{s,d}(\mathbf{p}-\mathbf{q},E-\omega)\hat{\gamma}^{k}\hat{G}_{s,d}(\mathbf{p},E)\ , \tag{2.16}$$

where $j,k=1,2$. The renormalized vertices $\hat{\Gamma}_{s,d}$ are given by Eq. (2.8) and the bare vertices by Eqs. (2.9). We present in Appendix B the explicit expressions for the polarization parts.

Having determined the vertex corrections and the screened Coulomb interactions, it is straightforward to obtain the self-energy $\hat{\Sigma}_s$ [Eq. (2.7)] and the kinetic equation (2.3). After a considerable amount of algebraic manipulations one finds

$$\frac{\partial f_s(E)}{\partial t}=\frac{1}{8\pi N_s}\int\frac{d\omega}{2\pi i}\sum_{\mathbf{q}}\left[\frac{\alpha_d\alpha_d^{A}(\zeta_s^{A}-\zeta_s)}{(1-u_s\zeta_s)(1-u_s\zeta_s^{A})}+\frac{\alpha_d^{*}\alpha_d^{R}(\zeta_s^{R}-\zeta_s^{*})}{(1-u_s\zeta_s^{*})(1-u_s\zeta_s^{R})}\right]$$
$$\times\{[2f_s(E)-2f_s(E-\omega)]V_s+[4f_s(E)f_s(E-\omega)-2f_s(E)-2f_s(E-\omega)](V_s^{R}-V_s^{A})\}\ . \tag{2.17}$$

In this equation the quantities V_s, $V_s^{R,A}$, ζ_s, $\zeta_s^{R,A}$, α_d, and $\alpha_d^{R,A}$ are functions of $\mathbf{q}$ and ω. Here (see Appendix A)

$$\begin{aligned}
&\zeta_s(\mathbf{q},\omega)=\sum_{\mathbf{p}}G_s^{A}(\mathbf{p}-\mathbf{q},E-\omega)G_s^{R}(\mathbf{p},E)\ ,\\
&\zeta_s^{R,A}(\mathbf{q},\omega)=\sum_{\mathbf{p}}G_s^{R,A}(\mathbf{p}-\mathbf{q},E-\omega)G_s^{R,A}(\mathbf{p},E)\ ,\\
&\alpha_d(\mathbf{q},\omega)=(1-u_s\zeta_s)\frac{1-u_d\zeta_d+u_{sd}\zeta_d}{(1-u_d\zeta_d)(1-u_s\zeta_s)-u_{sd}^2\zeta_s\zeta_d}\ ,\\
&\alpha_d^{R,A}(\mathbf{q},\omega)=(1-u_s\zeta_s^{R,A})\frac{1-u_d\zeta_d^{R,A}+u_{sd}\zeta_d^{R,A}}{(1-u_d\zeta_d^{R,A})(1-u_s\zeta_s^{R,A})-u_{sd}^2\zeta_s^{R,A}\zeta_d^{R,A}}\ .
\end{aligned} \tag{2.18}$$

The functions ζ_d and $\zeta_d^{R,A}$ are given by the first two equations of (2.18), with s and d interchanged.

The result (2.17) can be simplified considerably when ω is smaller than E and the elastic rate smaller than the Fermi energy.[17] In this case $\zeta^{R,A}$ are negligible compared with ζ, and $u_{s,d}\zeta_{s,d}^{R,A}\ll 1$. As a result, the large parentheses in Eq. (2.17) yield approximately $-2\,\mathrm{Re}[\alpha_d\zeta_s/(1-u_s\zeta_s)]$. Using the same approximation for the polarization parts [Eqs. (B1)–(B3)] and inserting Eqs. (2.13) for the screened Coulomb interactions into (2.17), we obtain

$$\begin{aligned}
\frac{\partial f_s(E)}{\partial t}=\frac{1}{2\pi N_s}\int\frac{d\omega}{2\pi i}\int\frac{dE_1}{2\pi i}\sum_{\mathbf{q}}&\left[2\,\mathrm{Re}\frac{\alpha_d\zeta_s}{1-u_s\zeta_s}\right]\frac{1}{\epsilon^{R}\epsilon^{A}}\\
&\times\left[v_{sd}^2\left[2\,\mathrm{Re}\frac{\alpha_s\zeta_d}{1-u_d\zeta_d}\right]\{f_s(E-\omega)f_d(E_1)[1-f_s(E)][1-f_d(E_1-\omega)]\right.\\
&\qquad-f_s(E)f_d(E_1-\omega)[1-f_s(E-\omega)][1-f_d(E_1)]\}\\
&+|v_s-(v_sv_d-v_{sd}^2)\Pi_d^{A}|^2\left[2\,\mathrm{Re}\alpha_d\frac{\zeta_s}{1-u_s\zeta_s}\right]\\
&\times\{f_s(E-\omega)f_s(E_1)[1-f_s(E)][1-f_s(E_1-\omega)]\\
&\left.\qquad-f_s(E)f_s(E_1-\omega)[1-f_s(E-\omega)][1-f_s(E_1)]\}\right]\ .
\end{aligned} \tag{2.19}$$

Here $\epsilon^{R}\epsilon^{A}=|\epsilon^{R}|^2$ is the absolute value squared of the dielectric function [Eq. (2.15)]. In the absence of the interband Coulomb interaction ($v_{sd}=0$), Eq. (2.19) reduces to the corresponding single-band expression of Al'tshuler and Aronov[18] (note that α_d [Eqs. (2.18)] is unity in the absence of interband *elastic* scattering). In the present case, the kinetic equation for the s electrons consists of two contributions: The collision term due to the screened *interband* Coulomb interaction [the first term in (2.19)] which involves, as expected, both f_s and f_d, and a collision term due to the screened *intraband* Coulomb interaction [the second term in (2.19)]. Note that the matrix element squared of the inelastic interaction in the latter is indeed the screened intraband Coulomb vertex, Eqs. (2.13).

To obtain an expression for the inelastic rate we write for the occupation numbers in Eq. (2.19) $f_{s,d}(E)=f_{s,d}^{0}(E)+\delta f_{s,d}(E)$, where f^{0} is the equilibrium Fermi distribution. To first order in δf, the term on the rhs proportional to $-\delta f_s(E)$ gives the inelastic rate of the s electrons at energy E. It is convenient to average this rate over the Fermi surface, so that the inelastic rate will be obtained as a function of temperature. This is accomplished by mul-

tiplying with the factor $[-\partial f_s^0(E)/\partial E]$ and integrating over E. The result for the averaged inelastic rate, $1/\tau_{\rm in}$, is then

$$\frac{1}{\tau_{\rm in}}=\frac{1}{(2\pi)^3 N_s}\int d\omega \frac{\beta\omega^2}{(e^{\beta\omega}-1)(1-e^{-\beta\omega})}\sum_{\mathbf{q}}\left[2\,\mathrm{Re}\frac{\alpha_d\zeta_s}{1-u_s\zeta_s}\right]\frac{1}{\epsilon^R\epsilon^A}$$
$$\times\left[v_{sd}^2\left[2\,\mathrm{Re}\frac{\alpha_s\zeta_d}{1-u_d\zeta_d}\right]+|v_s-(v_sv_d-v_{sd}^2)\Pi_d^R|^2\left[2\,\mathrm{Re}\frac{\alpha_d\zeta_s}{1-u_s\zeta_s}\right]\right]. \quad (2.20)$$

The form (2.20) shows that the inelastic rate of the s electrons is due to Coulomb scattering within the s band (second term in the large square brackets) and into the d band (first term in the large square brackets). However, the interband Coulomb vertex, v_{sd}, appears in both contributions, because of the complicated nature of screening in the two-band picture. In order to separate out the contribution originated from the interband coupling we proceed as follows. Firstly, the first equation in (2.13) is used to obtain

$$\mathrm{Im}V_s^R=\frac{1}{2i}\frac{1}{|\epsilon^R|^2}[v_{sd}^2(\Pi_d^R-\Pi_d^A)+|v_s-(v_sv_d-v_{sd}^2)\Pi_d^R|^2(\Pi_s^R-\Pi_s^A)]\,. \quad (2.21)$$

Secondly, it is noted from Eqs. (B1) and (B2) that in the limit $\omega<E$

$$\Pi_s^R-\Pi_s^A=-\frac{\omega}{2\pi i}2\,\mathrm{Re}\frac{\alpha_d\zeta_s}{1-u_s\zeta_s}\,, \quad (2.22)$$

and an analogous expression for $\Pi_d^R-\Pi_d^A$. Inserting these relations into Eq. (2.20) we find

$$\frac{1}{\tau_{\rm in}}=\frac{1}{\pi^2 N_s}\int d\omega\frac{\beta\omega}{(e^{\beta\omega}-1)(1-e^{-\beta\omega})}$$
$$\times\sum_{\mathbf{q}}\mathrm{Re}\frac{\alpha_d\zeta_s}{1-u_s\zeta_s}\mathrm{Im}V_s^R\,, \quad (2.23)$$

which in the low-temperature limit can be written as

$$\frac{1}{\tau_{\rm in}}=\frac{2}{\pi^2 N_s}\frac{1}{\beta}\int_0^{1/\beta}\frac{d\omega}{\omega}\sum_{\mathbf{q}}\mathrm{Re}\frac{\alpha_d\zeta_s}{1-u_s\zeta_s}\mathrm{Im}V_s^R\,. \quad (2.24)$$

This form shows that the interband Coulomb coupling, v_{sd}, affects the inelastic rate of the s electrons through the *screening* of the intraband Coulomb vertex. Explicitly [cf. Eqs. (2.13)]

$$\mathrm{Im}V_s^R=\mathrm{Im}\left[\frac{1}{v_s}-\Pi_s^R-(v_{sd}/v_s)^2\right.$$
$$\left.\times\frac{\Pi_d^R}{1-(v_sv_d-v_{sd}^2)\Pi_d^R/v_s}\right]^{-1}. \quad (2.25)$$

In the absence of the interband Coulomb coupling, Eqs. (2.24) and (2.25) reproduce the inelastic rate in a single band,[17,18] apart from the *elastic* coupling (u_{sd}) between the two bands, which changes the single-band form $\zeta_s/(1-u_s\zeta_s)$ into $\alpha_d\zeta_s/(1-u_s\zeta_s)$ [see Eq. (2.18)]. The same modification occurs in the polarization parts (Appendix B). Using Eq. (B1) we find

$$\Pi_s^R(\mathbf{q},\omega)\simeq\int\frac{dE}{2\pi i}\left[[f_s(E)-f_s(E-\omega)]\left[\frac{\alpha_d\zeta_s}{1-u_s\zeta_s}\right]^*\right.$$
$$\left.+f_s(E)(\zeta_s^A-\zeta_s^R)\right]\,, \quad (2.26)$$

and an analogous expression for Π_d^R. The last term here is $\sim -N_s$ (where N_s is the density of states at the Fermi energy of the s electrons). Treating the first term in the small-ω limit, we obtain

$$\Pi_s^R(\mathbf{q},\omega)\simeq -N_s-\frac{\omega}{2\pi i}\left[\frac{\alpha_d\zeta_s}{1-u_s\zeta_s}\right]^*. \quad (2.27)$$

Thus, in order to determine the temperature dependence of the inelastic rate, one needs to obtain the ω and $\mathbf{q}$ dependence of the quantities $\alpha_d\zeta_s/(1-u_s\zeta_s)$ and $\alpha_s\zeta_d/(1-u_d\zeta_d)$. In the next section we carry out this analysis for two possible configurations of the static disorder.

III. TEMPERATURE DEPENDENCE OF THE INELASTIC RATE

In a clean one-band system, the Coulomb inelastic rate is proportional to T^2 at three dimensions[20] and to $T^2\ln T$ at two dimensions.[21] The question we address is how this behavior is changed by coupling of the conduction electrons to a narrow band which is diffusive. We shall distinguish between two situations. (1) A *ballistic s* band, coupled to a *diffusive d* band by the interband Coulomb vertex solely. This may be the case when the two electron gases are spatially separated. Here we shall assume that u_s and u_{sd} vanish [cf. Eqs. (2.2)]. (2) When the two electron gases are not separated in space, the interband elastic scattering (of matrix element squared u_{sd}) may render the s band to be diffusive as well. This is because it contributes to the s-electron elastic rate the term $N_d u_{sd}$ [see Eqs. (2.2)], which may be large due to the high density of states of the d band. In this situation both bands are diffusive. In the following we consider the two cases separately.

A. A ballistic s band coupled to a diffusive d band ($u_{sd}=0$)

In the absence of *elastic* coupling between the bands, $\alpha_d=\alpha_s=1$ [Eqs. (2.18)]. Since the d band is in the diffusion regime, we have the usual expression[17]

$$\frac{\zeta_d}{1-u_d\zeta_d}\simeq 2\pi i N_d\frac{1}{\omega+iD_dq^2}\ ,$$
$$\Pi_d^R(\mathbf{q},\omega)\simeq N_d\frac{iD_dq^2}{\omega-iD_dq^2}\ , \tag{3.1}$$

where D_d is the diffusion coefficient of the d band. These expressions hold at two and three dimensions. For ζ_s we have

$$\zeta_s(\mathbf{q},\omega)=\begin{cases}\pi^2N_sm_s/p_Fq, & d=3\\ 2\pi N_sm_s/p_Fq, & d=2\ ,\end{cases} \tag{3.2}$$

where in both dimensions $m_s\omega<p_Fq$, and p_F is the Fermi wave vector of the s electrons. Inserting (3.2) into (2.27),

$$\Pi_s^R(\mathbf{q},\omega)=-N_s+iN_s\omega m_s/p_Fq \tag{3.3}$$

(we omit here the nonimportant difference between two and three dimensions).

The inelastic rate is found by inserting Eqs. (3.1)–(3.3) into Eqs. (2.24) and (2.25). This yields

$$\frac{1}{\tau_{\rm in}}\sim\frac{1}{N_s\beta}\int_0^{1/\beta}d\omega\sum_{\mathbf{q}}\left[\left[\frac{m_s}{p_Fq}\right]^2+b\frac{m_s}{p_Fq}\frac{D_dq^2}{\omega^2+\{D_dq^2[1+(v_sv_d-v_{sd}^2)N_d/v_s]\}^2}\right]. \tag{3.4}$$

Here we have introduced the notations

$$b=\frac{N_d}{N_s}\left[\frac{v_{sd}}{v_s}\right]^2. \tag{3.5}$$

The first term in the square brackets of Eq. (3.4) is the inelastic rate of a single ballistic band. The second term gives the inelastic rate due to the interband Coulomb coupling, and is proportional to the coupling parameter b. We now evaluate the latter contribution at two and three dimensions. To this end we define

$$(v_sv_d-v_{sd}^2)\frac{N_d}{v_s}=\begin{cases}(K/q)^2, & d=3\\ K/q, & d=2\ .\end{cases} \tag{3.6}$$

Thus K^{-1} plays the role of an effective "Thomas-Fermi" screening length. When $v_s\sim v_d\sim v_{sd}$ (as estimated in Ref. 1), K is very small, whereas when the coupling parameter $b\ll 1$, K is of the order of the inverse screening length of the narrow d band, and can be rather large.

At two dimensions we find

$$\sum_{\mathbf{q}}\frac{1}{q}\frac{D_dq^2}{\omega^2+D_d^2(q^2+Kq)^2}\sim\begin{cases}1/(D_dK) & \text{for } \omega\ll D_dK^2\\ \sqrt{1/\omega D_d} & \text{for } \omega\gg D_dK^2\ .\end{cases} \tag{3.7}$$

It therefore follows from Eq. (3.4) that

$$\frac{1}{\tau_{\rm in}}\sim\frac{1}{N_s}\frac{m_s}{p_F}(T)^2\left[\frac{m_s}{p_F}\ln\frac{4E_s}{T}+\frac{b}{KD_d}\right],\quad T\ll D_dK^2 \tag{3.8}$$

so that at very low temperatures the inelastic rate is governed by the *intraband* Coulomb scattering. Here E_s is the distance of the Fermi level from the bottom of the s band.[7] At higher temperatures the temperature dependence is changed and is given by

$$\frac{1}{\tau_{\rm in}}\sim\frac{1}{N_s}\frac{m_s}{p_F}\left[T^2\frac{m_s}{p_F}\ln\frac{4E_s}{T}+bT\left[\frac{T}{D_d}\right]^{1/2}\right],$$
$$T\gg D_dK^2\ . \tag{3.9}$$

We note that for $b\sim 1$ [which renders K to be small, see Eqs. (3.5) and (3.6)], the *interband* Coulomb scattering dominates the intraband one for $(V_s/V_d)^2\gg T\tau_d$, where V_s and V_d are the Fermi velocities of the s and d electrons, respectively.

At three dimensions we find

$$\sum_{\mathbf{q}}\frac{1}{q}\frac{D_dq^2}{\omega^2+D_d^2(q^2+K^2)^2}\sim\frac{1}{D_d}\ , \tag{3.10}$$

because in this case the q integration is governed by the upper limit. Therefore both terms in Eq. (3.4) yield T^2, the Baber law.[20]

B. Strong interband elastic scattering ($u_{sd}\neq 0$)

When the interband elastic scattering is strong enough such that $\omega\tau_s, qV_s\tau_s\ll 1$ [where τ_s is given in Eqs. (2.2) and V_s is the s-electron Fermi velocity], then the motion of the s electrons becomes diffusive. In such a situation

$$\zeta_s=\frac{2\pi iN_s}{\omega+i/\tau_s}(1-\tau_sD_sq^2)\ , \tag{3.11}$$

where D_s denotes the diffusion coefficient of the s band. An analogous expression holds for ζ_d. Using these forms in Eqs. (2.18) we obtain

$$\alpha_d\frac{\zeta_s}{1-u_s\zeta s}\simeq 2\pi iN_s\frac{\omega+iD_dq^2+2\pi iu_{sd}(N_s+N_d)}{(\omega+iD_sq^2)(\omega+iD_dq^2)+2\pi iu_{sd}[\omega(N_s+N_d)+iq^2(N_sD_s+N_dD_d)]}\simeq 2\pi iN_s\frac{1}{\omega+iDq^2}\ , \tag{3.12}$$

where

$$D=\frac{D_sN_s+D_dN_d}{N_s+N_d} . \tag{3.13}$$

The last approximate equality in Eq. (3.12) holds under the conditions that the s band is diffusive due to the interband elastic scattering (i.e., $\omega\tau_s, qV_s\tau_s \ll 1$, where $\tau_s^{-1}\simeq 2\pi N_d u_{sd}$) and for $(qV_d\tau_d)^2 \ll u_{sd}(N_s+N_d)/(u_dN_d+u_{sd}N_s)$. Using (3.12) in Eq. (2.27) for Π_s^R (and the analogous equation for Π_d^R) we find

$$\Pi_s^R=N_s\frac{iDq^2}{\omega-iDq^2},\quad \Pi_d^R=N_d\frac{iDq^2}{\omega-iDq^2} . \tag{3.14}$$

It is now straightforward to obtain from Eq. (2.23) the expression

$$\mathrm{Im}V_s^R\simeq\frac{\omega}{N_sDq^2}\frac{1}{1+b}\left[1+b\frac{(Dq^2)^2[(K/q)^{d-1}/(1+b)]^2}{\omega^2+(Dq^2)^2[1+(K/q)^{d-1}/(1+b)]^2}\right],\quad d=2,3 . \tag{3.15}$$

Here b is the dimensionless coupling describing the interband Coulomb interaction [Eq. (3.5)] and K is the effective screening wave vector introduced above [Eq. (3.6)].

Using Eqs. (3.12) and (3.15) in the expression for the inelastic rate [see Eq. (2.24)] it becomes

$$\frac{1}{\tau_{\mathrm{in}}}\sim\frac{1}{N_s\beta}\frac{1}{1+b}\int_0^{1/\beta}d\omega\sum_{\mathbf{q}}\left[\frac{1}{\omega^2+(Dq^2)^2}+b\frac{(Dq^2)^2[(K/q)^{d-1}/(1+b)]^2}{[\omega^2+(Dq^2)^2]\{\omega^2+(Dq^2)^2[1+(K/q)^{d-1}/(1+b)]^2\}}\right] . \tag{3.16}$$

The first term in the large parentheses yields the inelastic rate in a diffusive single-band system,[12,13,16,17] i.e., $\tau_{\mathrm{in}}^{-1}\sim T^{d/2}$, with two modifications: it is renormalized by the factor $(1+b)^{-1}$ and it includes the modified diffusion coefficient D [see Eq. (3.13)]. The second term in the large parentheses in Eq. (3.16) gives the contribution due to the interband Coulomb scattering. In order to study the latter, it is convenient to redefine the effective screening wave vector to be $\bar{K}$

$$\bar{K}=\begin{cases}K^2/(1+b), & d=3\\ K/(1+b), & d=2 .\end{cases} \tag{3.17}$$

At two dimensions we find

$$b\sum_{\mathbf{q}}\frac{(Dq^2)^2(\bar{K}/q)^2}{[\omega^2+(Dq^2)^2]\{\omega^2+(Dq^2)^2[1+(\bar{K}/q)]^2\}}\sim\begin{cases}b/D\omega & \text{for } \omega\ll D\bar{K}^2\\ b\bar{K}^2/\omega^2 & \text{for } \omega\gg D\bar{K}^2 .\end{cases} \tag{3.18}$$

Hence the ω integration is dominated by the lower cutoff, both at low temperatures, $T\ll D\bar{K}^2$, and at higher temperatures. The temperature dependence of the inelastic rate is therefore the same as for a single diffusive band, $\tau_{\mathrm{in}}^{-1}\propto T/DN_s$.

At three dimensions

$$b\sum_{\mathbf{q}}\frac{(Dq^2)^2(\bar{K}/q)^4}{[\omega^2+(Dq^2)^2]\{\omega^2+(Dq^2)^2[1+(\bar{K}/q)^2]^2\}}\sim\begin{cases}b(1/\omega D^3)^{1/2} & \text{for } \omega\ll D\bar{K}^2\\ b\bar{K}^4(D/\omega^5)^{1/2} & \text{for } \omega\gg D\bar{K}^2 .\end{cases} \tag{3.19}$$

It therefore follows that at low temperatures, $T\ll D\bar{K}^2$, the inelastic rate assumes the single-band form[12,16,17]

$$\frac{1}{\tau_{\mathrm{in}}}\propto\frac{1}{N_s}\left[\frac{T}{D}\right]^{3/2},\quad T\ll D\bar{K}^2 . \tag{3.20}$$

At higher temperature, $T\gg D\bar{K}^2$, the inelastic rate acquires a correction term linear in the temperature

$$\frac{1}{\tau_{\mathrm{in}}}\propto\frac{1}{N_s}[(T/D)^{3/2}+bT\bar{K}/D],\quad T\gg D\bar{K}^2 . \tag{3.21}$$

However, to leading order, $\tau_{\mathrm{in}}^{-1}\propto T^{3/2}$ as in the single-band situation.

IV. DISCUSSION

The *interband* inelastic electron-electron rate of conduction electrons coupled to a narrow band is interesting for two reasons: firstly, because of the contribution of this process to the resistivity. When umklapp scattering is unimportant (e.g., in a disordered system), single-band electron-electron scattering does not contribute to ρ. The observation[11] of $T^2\ln T$ behavior in some high-T_c superconductors supplies therefore a strong support for a two-band description [cf. Eqs. (3.8) and (3.9)]. The second reason is the possible relevance for experiments involving two quantum wells in heterostructures.[2,15]

In a previous work,[7] the interband inelastic rate in a clean system was investigated, for the case of two overlapping bands. It was found that the $T^2\ln T$ behavior characterizes its temperature dependence, both for the "s-d" and "pair-transfer" interband inelastic scattering mechanisms, at two dimensions. It was suggested there that by introducing *elastic* scattering into the system this temperature dependence may change, as happens in the

single-band case, even when the s band by itself is ballistic. In the present work we have investigated in detail this possibility. Our main findings are as follows: (i) Multiple elastic scattering events affect the s-d vertex, leading to the appearance of diffusion poles.[17] This does not happen for the pair-transfer vertex. Therefore, in the impure system one can distinguish between the scattering rate due to the s-d vertex and the one due to the pair-transfer vertex, because they have a different temperature dependence. (ii) The temperature dependence is determined by two factors, the interband elastic coupling and the relative strengths of the bare Coulomb vertices. This leads, at two dimensions, to a change from the $T^2 \ln T$ behavior to a $T^{3/2}$ law, when the s band is ballistic. This is a behavior which may be observed, e.g., in experiments on two quantum wells in heterostructures. It relies on the complicated nature of screening in a two-band picture and suggests the possibility of experimental manifestations of this screening in transport measurements.

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APPENDIX A: EXPRESSIONS FOR THE VERTEX CORRECTIONS

Here we list the expressions for the vertex corrections in the two bands, $\hat{\Gamma}_s^k$ and $\hat{\Gamma}_d^k$, $k=1,2$ (see Fig. 2). These are obtained by solving two algebraic equations, Eq. (2.8) and the one obtained from it by interchanging s and d. To this end it is convenient to introduce the following notations:

$$\zeta_s(\mathbf{q},\omega)=\sum_{\mathbf{p}} G_s^A(\mathbf{p}-\mathbf{q},E-\omega)G_s^R(\mathbf{p},E)\ ,$$
$$\zeta_s^{R,A}(\mathbf{q},\omega)=\sum_{\mathbf{p}} G_s^{R,A}(\mathbf{p}-\mathbf{q},E-\omega)G_s^{R,A}(\mathbf{p},E)\ , \tag{A1}$$

and similarly for the d band. We further define

$$\alpha_d(\mathbf{q},\omega)=(1-u_s\zeta_s)\frac{1-u_d\zeta_d+u_{sd}\zeta_d}{(1-u_d\zeta_d)(1-u_s\zeta_s)-u_{sd}^2\zeta_s\zeta_d}\ ,$$
$$\alpha_d^{R,A}(\mathbf{q},\omega)=(1-u_s\zeta_s^{R,A})\frac{1-u_d\zeta_d^{R,A}+u_{sd}\zeta_d^{R,A}}{(1-u_d\zeta_d^{R,A})(1-u_s\zeta_s^{R,A})-u_{sd}^2\zeta_s^{R,A}\zeta_d^{R,A}}\ , \tag{A2}$$

and the analogous quantities obtained by interchanging s and d. It is noted that in the absence of the interband elastic scattering (i.e., for $u_{sd}=0$) the quantities α_d and $\alpha_d^{R,A}$ are equal to 1. In terms of the functions defined in Eqs. (A1) and (A2), the vertex $\hat{\Gamma}_s^1$ to leading order is

$$\Gamma_{s,22}^1=\left[\frac{1}{2}\right]^{1/2}\frac{\alpha_d}{1-u_s\zeta_s}\ ,$$
$$\Gamma_{s,12}^1=\left[\frac{1}{2}\right]^{1/2}[2f_s(E-\omega)-1]\left[\frac{\alpha_d}{1-u_s\zeta_s}-\frac{\alpha_d^R}{1-u_s\zeta_s^R}\right]\ ,$$
$$\Gamma_{s,21}^1=\left[\frac{1}{2}\right]^{1/2}[2f_s(E)-1]\left[\frac{\alpha_d^A}{1-u_s\zeta_s^A}-\frac{\alpha_d}{1-u_s\zeta_s}\right]\ , \tag{A3}$$
$$\Gamma_{s,11}^1=\left[\frac{1}{2}\right]^{1/2}\left[\frac{\alpha_d}{1-u_s\zeta_s}\right]^*-\left[\frac{1}{2}\right]^{1/2}[2f_s(E)-1][2f_s(E-\omega)-1]2\,\mathrm{Re}\left[\frac{\alpha_d}{1-u_s\zeta_s}-\frac{\alpha_d^A}{1-u_s\zeta_s^A}\right]\ .$$

Here we have omitted the E, $\mathbf{q}$, and ω dependences for brevity. The vertex correction $\hat{\Gamma}_d^1$ is obtained by interchanging s and d in Eqs. (A3). The vertex correction $\hat{\Gamma}_s^2$ is given by

$$\Gamma_{s,22}^2=0\ ,$$
$$\Gamma_{s,12}^2=\left[\frac{1}{2}\right]^{1/2}\frac{\alpha_d^R}{1-u_s\zeta_s^R}\ ,\quad \Gamma_{s,21}^2=\left[\frac{1}{2}\right]^{1/2}\frac{\alpha_d^A}{1-u_s\zeta_s^A}\ , \tag{A4}$$
$$\Gamma_{s,11}^2=\left[\frac{1}{2}\right]^{1/2}\left[[2f_s(E)-1]\left[\frac{\alpha_d^*}{1-u_s\zeta_s^*}-\frac{\alpha_d^R}{1-u_s\zeta_s^R}\right]-[2f_s(E-\omega)-1]\left[\frac{\alpha_d^*}{1-u_s\zeta_s^*}-\frac{\alpha_d^A}{1-u_s\zeta_s^A}\right]\right]\ .$$

Again, the vertex correction $\hat{\Gamma}_d^2$ is obtained from (A4) by interchanging s and d. The results (A3) and (A4) reproduce those of Ref. 18 in the absence of interband elastic scattering.

APPENDIX B: EXPRESSIONS FOR THE POLARIZATION PARTS

Here we list the expressions for the polarization parts of the two bands, $\hat{\Pi}_s$ and $\hat{\Pi}_d$ [see Eq. (2.16)]. Using the notations (2.12) and the definitions (A1) and (A2), a straightforward calculation yields

$$\Pi_s^R(\mathbf{q},\omega)\simeq\int\frac{dE}{4\pi i}\left[[2f_s(E)-1]\left[\frac{\alpha_d^*}{1-u_s\xi_s^*}\zeta_s^*-\frac{\alpha_d^R}{1-u_s\xi_s^R}\zeta_s^R\right]+[2f_s(E-\omega)-1]\left[\frac{\alpha_d^A}{1-u_s\xi_s^A}\zeta_s^A-\frac{\alpha_d^*}{1-u_s\xi_s^*}\zeta_s^*\right]\right], \tag{B1}$$

$$\Pi_s^A(\mathbf{q},\omega)\simeq\int\frac{dE}{4\pi i}\left[[2f_s(E-\omega)-1]\left[\frac{\alpha_d}{1-u_s\zeta_s}\zeta_s-\frac{\alpha_d^R}{1-u_s\xi_s^R}\zeta_s^R\right]+[2f_s(E)-1]\left[\frac{\alpha_d^A}{1-u_s\xi_s^A}\zeta_s^A-\frac{\alpha_d}{1-u_s}\zeta_s\right]\right], \tag{B2}$$

$$\Pi_s(\mathbf{q},\omega)\simeq\int\frac{dE}{4\pi i}\{1-[2f_s(E)-1][2f_s(E-\omega)-1]\}\left[\frac{\alpha_d}{1-u_s\zeta_s}\zeta_s+\frac{\alpha_d^*}{1-u_s\xi_s^*}\zeta_s^*\right]. \tag{B3}$$

Here we have made use of the explicit expressions for the vertex corrections (Appendix A). The expressions for the matrix elements of $\hat{\Pi}_d$ are obtained by interchanging s and d in Eqs. (B1)–(B3).

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