

## Weak-localization effects in a resonant-tunneling junction

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We study the effect of disorder in the metallic contacts on the conductance of a resonant-tunneling junction. The reason for the effect is that the resonant tunneling involves multiple virtual transitions between the impurity state in the barrier and the extended states in the contacts. The diffusive motion of the electron between subsequent visits to the impurity results in a weak-localization correction to the conductance of the junction. The magnetoresistance of the junction, which is caused by the suppression of this correction, is shown to be directly expressed, in the weak-disorder approximation, via the magnetoresistance of the contacts. If the conductance of the junction is dominated by only a few impurities, it exhibits random mesoscopic fluctuations with the magnetic field. A typical amplitude of the fluctuations turns out to be much higher than in the case of the universal conductance fluctuations.

### I. INTRODUCTION

Observation of resonant tunneling through impurity states in the insulating barrier has been reported in several experimental papers.<sup>1-4</sup> Resonant tunneling occurs when the energy position  $\epsilon_0$  of the impurity state is close to the Fermi level  $\epsilon_F$  in the metallic contacts. Under perfect resonance conditions (an impurity is strictly in the middle of the barrier and  $\epsilon_0 = \epsilon_F$ ) the value of conductance provided by the impurity reaches  $e^2/\pi\hbar$ .<sup>5-8</sup> This is a maximal possible value for the conductance of an ideal channel.<sup>9</sup>

Various derivations of the expression for resonant conductance<sup>5-8</sup> were based on the assumption that the contacts are ideal metals. However, the methods of fabrication of the experimental structures described, e.g., in Refs. 1 and 10 suggest that the contacts, either deposited films<sup>1</sup> or heavily doped semiconductors,<sup>10</sup> are partially disordered. A question thus arises about the influence of the disorder in the contacts on the resonant tunneling. It is well known that disordered metallic samples at low temperature are anomalously sensitive to very low magnetic fields due to weak-localization effects.<sup>11</sup> Then the question may be reformulated as whether the weak-localization effects reveal themselves in the resonant transport, and in particular in the magnetoconductivity? Since the voltage applied to the junction drops entirely in the barrier and not in the contacts, the effect might be expected to be absent. However, as we shall demonstrate below, this is not the case. The reason is that the resonant-tunneling process implies the multiple virtual transitions of electrons between the impurity state in the barrier and the extended states in the contacts. In the presence of the elastic scatterers in the contacts, the motion of the electron between subsequent visits to the

impurity is diffusive. This diffusive motion allows the interference between time-reversed closed trajectories, which is responsible for the weak localization.<sup>11</sup> A weak magnetic field suppresses the interference, causing therefore the magnetoresistance of the junction to be of the same nature as the magnetoresistance of the disordered metal.

In the present paper we calculate the conductance of the resonant-tunneling junctions with elastic scatterers in the contacts. We show that the disorder-induced correction to the conductance depends anomalously on the magnetic field and, to the lowest order in the weak-disorder parameter, can be directly expressed in terms of the magnetoconductance of the contacts.

In the case when the area of the junction is so small that the conductance is dominated only by few resonant impurities in the barrier, we show the conductance to exhibit random oscillations with magnetic field. These oscillations have the mesoscopic origin similar to that for the well-known conductance fluctuations in the disordered metals.<sup>12,13</sup>

### II. CALCULATION OF THE WEAK-LOCALIZATION CORRECTION

To study the role of weak-localization effects on the resonant-tunneling process we extend the derivation of the expression for the resonant conductance proposed in Ref. 8 for ideal contacts to the case of the disordered contacts. Let  $\Psi_{l\nu}, \epsilon_{l\nu}$  and  $\Psi_{r\mu}, \epsilon_{r\mu}$  be exact wave functions and energies of the electronic states  $\nu$  and  $\mu$  in the left and in the right contact, respectively, and  $\Psi_0, \epsilon_0$  be the exact wave function and the energy of the impurity state in the barrier. One can express the exact time-dependent wave function of the system as

$$\Psi(\mathbf{r}, t) = \sum_{\nu} A_{l\nu}(t) \Psi_{l\nu}(\mathbf{r}) + \sum_{\mu} A_{r\mu}(t) \Psi_{r\mu}(\mathbf{r}) + A_0(t) \Psi_0(\mathbf{r}), \quad (1)$$

where the coefficients  $A(t)$  satisfy the following system of equations:

$$\begin{aligned} i\hbar \frac{dA_{l\nu}}{dt} &= \varepsilon_{l\nu} A_{l\nu} + T_{l\nu} A_0, \\ i\hbar \frac{dA_{r\mu}}{dt} &= \varepsilon_{r\mu} A_{r\mu} + T_{r\mu} A_0, \\ i\hbar \frac{dA_0}{dt} &= \varepsilon_0 A_0 + \sum_{\nu} T_{l\nu}^* A_{l\nu} + \sum_{\mu} T_{r\mu}^* A_{r\mu}. \end{aligned} \quad (2)$$

Here  $T_{l\nu}$  is the matrix element connecting the state  $\nu$  in the left contact and the localized state in the barrier

$$\begin{aligned} T_{l\nu} &= \int d^d r \Psi_{l\nu}^*(\mathbf{r}) V(\mathbf{r}) \Psi_0(\mathbf{r}) \\ &= \Psi_{l\nu}^*(z_0, 0) \int d^d r V(\mathbf{r}) \Psi_0(\mathbf{r}), \end{aligned} \quad (3)$$

where  $V(\mathbf{r})$  is the short-range impurity potential and  $z_0$  is the displacement of the impurity from the midplane in the barrier (Fig. 1). Since the wave function  $\Psi_{l\nu}$  decays exponentially under the barrier, one can write

$$\Psi_{l\nu}(z_0, 0) = \Psi_{l\nu}(-a/2, 0) \exp[-k(a/2 + z_0)], \quad (4)$$

where  $\Psi_{l\nu}(-a/2, 0)$  is the wave function at the edge of the left contact,  $a$  is the width of the barrier, and  $k$  is the inverse decay length. Combining Eqs. (3) and (4) one has

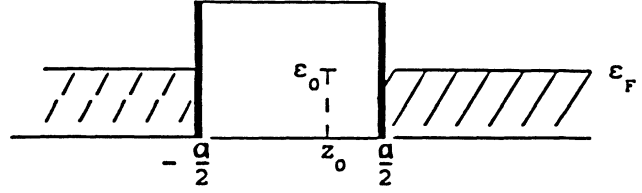


FIG. 1. Schematic of the barrier described here.

$$\begin{aligned} T_{l\nu} &= \beta_l \Psi_{l\nu}(-a/2, 0), \\ \beta_l &\equiv \exp[-k(a/2 + z_0)] \int d^d r V(\mathbf{r}) \Psi_0(\mathbf{r}). \end{aligned} \quad (5)$$

Similarly one defines

$$\begin{aligned} T_{r\mu} &= \beta_r \Psi_{r\mu}(a/2, 0), \\ \beta_r &\equiv \exp[-k(a/2 - z_0)] \int d^d r V(\mathbf{r}) \Psi_0(\mathbf{r}). \end{aligned} \quad (6)$$

The rate of the electron transfer from some state  $\nu_0$  in the left contact to some state  $\mu_0$  in the right contact can be expressed as

$$W(l\nu_0|r\mu_0) = \lim_{t \rightarrow \infty} \left[ \frac{d}{dt} |A_{r\mu_0}(t)|^2 \right], \quad (7)$$

where  $A_{r\mu_0}(t)$  is the solution of the system of equations (2) with the initial conditions

$$A_{l\nu}(0) = \delta_{\nu\nu_0}, \quad A_{r\mu} = 0, \quad A_0(0) = 0. \quad (8)$$

The solution is easily found and leads to

$$W(l\nu_0|r\mu_0) = \frac{2\pi}{\hbar} \frac{|T_{l\nu_0}|^2 |T_{r\mu_0}|^2 \delta(\varepsilon_{l\nu_0} - \varepsilon_{r\mu_0})}{\left| \varepsilon_{l\nu_0} - \varepsilon_0 - \sum_{\nu} \frac{|T_{l\nu}|^2}{\varepsilon_{l\nu_0} - \varepsilon_{l\nu} + i\delta} - \sum_{\mu} \frac{|T_{r\mu}|^2}{\varepsilon_{r\mu_0} - \varepsilon_{r\mu} + i\delta} \right|^2}. \quad (9)$$

The expression for the resonant conductance  $G$  provided by the impurity in the barrier is obtained by summing Eq. (9) over all the initial and all the final states at the Fermi level<sup>8</sup>

$$G(\varepsilon_0, z_0) = 2e^2 \sum_{\nu, \mu} W(l\nu|r\mu) \delta(\varepsilon_F - \varepsilon_{l\nu}). \quad (10)$$

On substituting Eq. (9) into Eq. (10) and making use of Eqs. (5) and (6) for the matrix elements  $T_{l\nu}, T_{r\mu}$  one can rewrite Eq. (10) as

$$G(\varepsilon_0, z_0) = -\frac{e^2}{\pi\hbar} \frac{\beta_l^2 \beta_r^2 (\mathcal{G}_l^+ - \mathcal{G}_l^-)(\mathcal{G}_r^+ - \mathcal{G}_r^-)}{|\varepsilon_F - \varepsilon_0 - \beta_l^2 \mathcal{G}_l^+ - \beta_r^2 \mathcal{G}_r^+|^2}, \quad (11)$$

where the exact retarded (advanced) Green's function for the disordered left contact is introduced:

$$\mathcal{G}_l^{\pm}(\mathbf{r}, \mathbf{r}'; \omega) = \sum_{\nu} \frac{\Psi_{l\nu}^*(\mathbf{r}) \Psi_{l\nu}(\mathbf{r}')}{\varepsilon_F + \omega - \varepsilon_{l\nu} \pm i\delta}, \quad (12)$$

$$\mathcal{G}_l^{\pm} \equiv \mathcal{G}_l^{\pm}(-a/2, -a/2; 0). \quad (13)$$

A similar definition for  $\mathcal{G}_r^{\pm}$  is implied.

Expression (11) could also be derived from the Kubo formula for the conductance, if we express the amplitude of the transition from the point  $\mathbf{r}$  in the left contact to the point  $\mathbf{r}'$  in the right contact directly in terms of the electron Green's functions of the corresponding contacts.

In the absence of the disorder in the contacts, Eq. (11) reduces to the standard Landauer-like expression for the conductance caused by the resonant impurity<sup>8</sup>

$$G_0(\varepsilon_0, z_0) = \frac{e^2}{\pi\hbar} \frac{\Gamma_l \Gamma_r}{(\varepsilon_F - \varepsilon_0)^2 + \frac{1}{4}(\Gamma_l + \Gamma_r)^2}, \quad (14)$$

where

$$\Gamma_{l(r)} = 2\beta_{l(r)}^2 \text{Im} \mathcal{G}_{l(r)}^+ = 2\pi \nu_{l(r)} \beta_{l(r)}^2 \quad (15)$$

is the coupling of the impurity to the left (right) contact and  $\nu_{l(r)}$  is the density of the electron states in the appropriate contact.

If there are several impurities in the barrier and they are sufficiently far apart, the total conductance of the

junction  $G_t$  is calculated by summing up all the contributions (14),

$$G_t = \sum_{\varepsilon_0, z_0} G_0(\varepsilon_0, z_0). \quad (16)$$

This quantity is self-averaging, if the number of impurities is large. On averaging over the positions  $z_0$  and energies  $\varepsilon_0$  of the impurities, one finds

$$G_t = \frac{e^2}{\pi\hbar} M, \quad M = \frac{\pi^2 \rho S \Gamma_0}{2k}. \quad (17)$$

Here the effective number of the resonant-tunneling channels  $M$  is defined<sup>8</sup> where  $\rho$  is the density of the impurity states in the barrier,  $S$  is the area of the junction,  $k$  is the inverse decay length entering Eqs. (5) and (6), and  $\Gamma_0$  is the coupling constant (15) for the impurity placed in the middle of the barrier.

In the disordered case we define the coupling constants as in Eq. (15) with

$$\bar{v}_{l(r)} = \frac{1}{\pi} \langle \text{Im} \mathcal{G}_{l(r)}^+ \rangle, \quad (18)$$

where  $\langle \dots \rangle$  stands for averaging over the realizations of the disorder in the left and right contacts. The mean electron density of states  $\bar{v}$  is well known to be insensitive to weak-localization effects. Thus the value of the coupling constants remains unchanged as compared to the case of pure contacts (15). However, the conductance (11) depends on the Green's functions nonlinearly and thus should depend on the disorder. Assuming the disorder to be weak we define

$$\delta \mathcal{G}_{l(r)}^\pm \equiv \mathcal{G}_{l(r)}^\pm - \langle \mathcal{G}_{l(r)}^\pm \rangle \quad (19)$$

and expand the conductance (11) in series over  $\delta \mathcal{G}$ ,

$$G(\varepsilon_0, z_0) = G_0(\varepsilon_0, z_0) [1 + P_l \delta \mathcal{G}_l^- + P_r \delta \mathcal{G}_r^- + P_l^* \delta \mathcal{G}_l^+ + P_r^* \delta \mathcal{G}_r^+ + Q_l \delta \mathcal{G}_l^+ \delta \mathcal{G}_l^- + Q_r \delta \mathcal{G}_r^+ \delta \mathcal{G}_r^- + \dots] \quad (20)$$

with coefficients

$$P_{l(r)} = \frac{1}{2\pi i \bar{v}_{l(r)}} \frac{\varepsilon_F - \varepsilon_0 + \frac{i}{2}(\Gamma_{l(r)} - \Gamma_{r(l)})}{\varepsilon_F - \varepsilon_0 - \frac{i}{2}(\Gamma_l + \Gamma_r)}, \quad (21)$$

$$Q_{l(r)} = - \left[ \frac{1}{2\pi \bar{v}_{l(r)}} \right]^2 \frac{\Gamma_l \Gamma_r}{(\varepsilon_F - \varepsilon_0)^2 + \frac{1}{4}(\Gamma_l + \Gamma_r)^2}.$$

To find the disorder-induced correction to the conductance (11) of the junction one should average Eq. (20) over positions and energies of the impurities in the barrier and, independently, over the realization of the disorder in the contacts. Up to the first nonvanishing after the averaging terms we get  $\langle G \rangle = G_t + \delta G$ , where  $G_t$  is given by Eq. (16), and the disorder-induced correction reads as

$$\delta G = -G_t \left[ \frac{\langle \delta \mathcal{G}_l^+ \delta \mathcal{G}_l^- \rangle}{(4\pi \bar{v}_l)^2} + \frac{\langle \delta \mathcal{G}_r^+ \delta \mathcal{G}_r^- \rangle}{(4\pi \bar{v}_r)^2} \right]. \quad (22)$$

It is seen that the correction is negative, i.e., disorder in the contacts diminishes conductance of the junction. The quantity  $\langle \delta \mathcal{G}_{l(r)}^+ \delta \mathcal{G}_{l(r)}^- \rangle$  which is the characteristics of the left (right) contact is similar to that appearing in the calculation of the weak-localization correction to conductivity of disordered metal.<sup>11</sup> However, it is expressed (in the first order in the small disorder parameter) as a sum of diffusion and cooperon contribution<sup>14</sup>

$$\langle \delta \mathcal{G}_l^+ \delta \mathcal{G}_l^- \rangle = \frac{2\pi \bar{v}_{l(r)}}{\hbar} (\mathcal{C}_{l(r)} + \mathcal{D}_{l(r)}) \quad (23)$$

while the weak-localization correction to the conductivity of the contact is expressed via the cooperon contribution only:<sup>11</sup>

$$\frac{\delta \sigma_{l(r)}}{\sigma_{l(r)}} = - \frac{1}{\pi \hbar \bar{v}_{l(r)}} \mathcal{C}_{l(r)}. \quad (24)$$

Here the cooperon contribution  $\mathcal{C}_{l(r)}$  is defined as<sup>11</sup>

$$\mathcal{C}_{l(r)} = \int \frac{d^d q}{(2\pi)^d} \frac{1}{D_{l(r)} q^2 + \tau_\phi^{-1}}, \quad (25)$$

where  $D_{l(r)}$  is the diffusion coefficient of the electron in the left (right) contact and  $\tau_\phi$  is a dephasing scattering time. In absence of magnetic field the diffusion contribution  $\mathcal{D}_{l(r)}$  in Eq. (23) is defined in the same way as the cooperon one.<sup>15</sup> The dimensionality of the integral (25) depends on the geometry of the contacts. It is  $d=2$  if the contact is a thin film [with a thickness less than the coherence length  $L_\phi = (D\tau_\phi)^{1/2}$ ] and  $d=3$  if the contact is a bulk metal.

Combining Eqs. (22)–(24), we can express the relative value of the weak-localization correction to the conductance of the junction in terms of the relative values of the weak-localization corrections to the conductances of the contacts:

$$\frac{\delta G}{G_t} = \frac{1}{4} \left[ \frac{\delta \sigma_l}{\sigma_l} + \frac{\delta \sigma_r}{\sigma_r} \right]. \quad (26)$$

Note that this relation holds only in the weak disorder approximation (i.e., in the first order of expansion in powers of  $\mathcal{D}$  and  $\mathcal{C}$ ). This approximation is sufficient, however, for the present considerations.

The temperature dependence of  $\delta \sigma_{l(r)}$  results from that of the dephasing scattering time  $\tau_\phi$  in Eq. (25). The latter is given by  $\tau_\phi \propto T^{-p}$ , where  $p$  depends on the mechanism of inelastic scattering,<sup>11</sup> which results in  $\delta \sigma \propto \ln T$  for  $d=2$  and  $\delta \sigma \propto T^{p/2}$  for  $d=3$ . Then, as follows from Eq. (26), the conductance of the junction turns out to be sensitive to very low temperatures in just the same way. Another reason for the temperature dependence of the resonant conductance is the spin correlation between the electron at the impurity and the electrons in the contacts.<sup>16,17</sup> As shown in Refs. 16 and 17, the destruction of this correlation with  $T$  leads to the correction  $\delta \sigma \propto -\ln T$  of the opposite sign to the correction originating from the weak-localization effects. Note that broadening of the distribution function of the electrons in the contacts does not lead<sup>8</sup> to a temperature dependence

of the average conductance  $G_i$ .

The suppression by magnetic field of the cooperon contribution to the disorder-induced correction to the conductance (22) causes the negative (in the absence of the spin-orbit scattering) magnetoresistance of the junction. It follows from Eqs. (22)–(25) that it is expressed directly in terms of the negative magnetoresistance  $\Delta\sigma_{l(r)}$  of the left (right) contact

$$\frac{\Delta G(H)}{G_i} \equiv \frac{\delta G(H) - \delta G(0)}{G_i} = \frac{1}{8} \left[ \frac{\Delta\sigma_l(H)}{\sigma_l} + \frac{\Delta\sigma_r(H)}{\sigma_r} \right], \quad (27)$$

where  $\Delta\sigma_{l(r)} \equiv \delta\sigma_{l(r)}(H) - \delta\sigma_{l(r)}(0)$  can be presented as a function of the ratio  $H/H^*$  with

$$H^* = \frac{\hbar c}{eL_\phi^2}. \quad (28)$$

The exact form of this function is given in Ref. 11. For  $H = H^*$ , the relative magnitude  $\Delta G/G_i$  of magnetoresistance of the junction is of the order of  $\hbar/\varepsilon_F\tau$  for two-dimensional contacts, and of the order of  $\hbar^2/\varepsilon_F^2\tau^3\tau_\phi^{1/2}$  for three-dimensional ones. Here  $\tau$  is the elastic-scattering relaxation time.

### III. MESOSCOPIC FLUCTUATIONS OF CONDUCTANCE

In the case when the area of the junction is so small that the number of channels  $M$  defined by Eq. (17) is of the order of 1, the conductance is dominated by only a few resonant impurities. The conductance provided by a single impurity is sensitive to the interference of the amplitudes of different diffusive trajectories within the region of the size  $L_\phi$  around the impurity. In magnetic field each amplitude acquires some phase factor so that the interference pattern changes. This change will reveal itself in the amplitude of the resonant tunneling. Therefore, the conductance of the junction with a small number of resonant channels will exhibit random fluctuations with magnetic field which are similar to the universal conductance fluctuations (UCF) in small metallic samples.<sup>12,13</sup> We will show, however, a typical relative amplitude of the fluctuations to be considerably greater than that of the UCF. The reason is that the fluctuations of the tunneling conductance reveal the fluctuations in the local densities of states rather than in conductances of the contacts. The local density of states is known<sup>14</sup> to fluctuate much stronger than the conductance or the global density of states characterizing a sample as a whole.<sup>18</sup>

To estimate the amplitude of the fluctuations we calculate the variance

$$(\text{var}G)^2 = \langle G^2 \rangle - \langle G \rangle^2. \quad (29)$$

Substituting Eq. (20) into Eq. (29) we get in the first non-vanishing after the averaging order

$$(\text{var}G)^2 = 2G_0^2(\varepsilon_0, z_0) [ |P_l|^2 \langle \delta\mathcal{G}_l^+ \delta\mathcal{G}_l^- \rangle + |P_r|^2 \langle \delta\mathcal{G}_r^+ \delta\mathcal{G}_r^- \rangle ]. \quad (30)$$

Using Eqs. (14) and (20) we can rewrite this expression as

$$(\text{var}G)^2 = 2 \left[ \frac{e^2}{\pi\hbar} \right]^2 \frac{\Gamma_l^2 \Gamma_r^2 [(\varepsilon_F - \varepsilon_0)^2 + \frac{1}{4}(\Gamma_l - \Gamma_r)^2]}{[(\varepsilon_F - \varepsilon_0)^2 + \frac{1}{4}(\Gamma_l + \Gamma_r)^2]^3} \times \left[ \frac{\langle \delta\mathcal{G}_l^+ \delta\mathcal{G}_l^- \rangle}{(2\pi\bar{v}_l)^2} + \frac{\langle \delta\mathcal{G}_r^+ \delta\mathcal{G}_r^- \rangle}{(2\pi\bar{v}_r)^2} \right]. \quad (31)$$

The relative amplitude of the fluctuation may be expressed directly in terms of that for the local densities of states in the contacts. Indeed, both spatial arguments of the Green's functions in Eq. (31) are stuck at the left (or right) edge of the contact, Eq. (13). The variance of the fluctuation  $(\text{var}v)^2 = \langle v^2 \rangle - \bar{v}^2$  of the local density of states is expressed<sup>14</sup> via the same averages (23):

$$\langle (\text{var}v_{l(r)})^2 \rangle = \frac{1}{2\pi^2} \langle \delta\mathcal{G}_{l(r)}^+ \delta\mathcal{G}_{l(r)}^- \rangle. \quad (32)$$

Thus, one finds the relative amplitude of the resonant conductance fluctuations as

$$\frac{(\text{var}G)^2}{G_0^2} = \frac{(\varepsilon_F - \varepsilon_0)^2 + \frac{1}{4}(\Gamma_l - \Gamma_r)^2}{(\varepsilon_F - \varepsilon_0)^2 + \frac{1}{4}(\Gamma_l + \Gamma_r)^2} \times \left[ \frac{(\text{var}v_l)^2}{\bar{v}_l^2} + \frac{(\text{var}v_r)^2}{\bar{v}_r^2} \right]. \quad (33)$$

It is seen from Eq. (33) that the amplitude depends strongly on the location and the energy of the resonant impurity. In particular, under the perfect resonant conditions ( $\Gamma_l = \Gamma_r$  and  $\varepsilon_0 = \varepsilon_F$ ) we obtain  $\text{var}G = 0$  in the leading order. It is quite natural that there are no sample-to-sample fluctuations: all samples reach the maximal possible value of conductance  $e^2/\pi\hbar$ . But, with the same accuracy, there are no random fluctuations with magnetic field in a particular sample. Such a suppression of the fluctuations results from the fact that all the types of the diffusive trajectories making the interference pattern (e.g., those passing through the left contact only, those visiting the right contact once, twice, etc.) have the same relative weight under the perfect resonant conditions. It causes the fluctuations to be insensitive to changing the magnetic field. A difference in weight of trajectories appears in the higher order of perturbation, and the relative amplitude of the fluctuations (33) in such a case does not exceed that of the UCF. However, under typical conditions for the resonant tunneling when either  $|\Gamma_l - \Gamma_r| \sim \Gamma$  or  $|\varepsilon_0 - \varepsilon_F| \sim \Gamma$ , the relative amplitude of the resonant conductance fluctuations is of the same order of magnitude as that of the amplitude of the local density-of-states fluctuations which is much greater than that of the UCF. Let us estimate this amplitude.

The sum in the last set of brackets in Eq. (31) has the same form as in Eq. (22), and can also be expressed as the sum of diffusion and cooperon contributions, Eq. (23). These contributions differ, however, from those calculated directly from Eq. (25). The difference is due to the fact that the products of two Green's functions in Eq. (31) correspond, in contrast to Eq. (22), to an averaging of two independent measurements. [On calculating these products in the temperature diagram technique, one finds

that the two Green's functions in the angular brackets have the same Matsubara frequency in Eq. (22), but different ones in Eq. (31).] It is well known from the theory of UCF (see, e.g., Ref. 19) that in such a case diffusion and cooperon contributions are sensitive to the temperature broadening of the electron Fermi distribution functions. The "energy averaging" results in dependence of these contributions on both the coherence length  $L_\phi = (D\tau_\phi)^{1/2}$  and the thermal length  $L_T = (D\hbar/T)^{1/2}$ . Roughly speaking, one can say that the lower cutoff in the singular integral (25) should be taken at  $q^{-1} \sim \mathcal{L} = \min(L_\phi, L_T)$ . On taking this into account and substituting Eq. (23) into Eqs. (32) and (33), one finds the typical relative amplitude of the resonant conductance fluctuations at  $d=2$  as

$$\frac{\text{var}G}{\langle G \rangle} \propto \frac{\text{var}v}{\bar{v}} \sim \left[ \frac{\hbar}{\varepsilon_F \tau} \ln \left[ \frac{\mathcal{L}}{l} \right] \right]^{1/2}, \quad (34)$$

where  $l = v_F \tau$  is the length of the mean free path for elastic scattering. The relative amplitude of the UCF is of the order of  $\hbar/\varepsilon_F \tau$ , which is much smaller than (34) under the weak-disorder condition. In the three-dimensional case, Eqs. (33) and (25) predict the relative amplitude of the fluctuations being of the order of  $\hbar/(\varepsilon_F \tau)$  while the relative amplitude of the UCF is proportional to the second power of this small parameter.

Note finally that Eq. (33) describes the amplitude of the fluctuations for a single resonant-tunneling channel. With the increase of the number of channels  $M$ , Eq. (17), this amplitude falls off as  $M^{-1/2}$ .

#### IV. CONCLUSION

The main result of the present paper is that the conductance of a resonant-tunneling junction with disordered contacts reveals the anomalous sensitivity to very low temperatures and low magnetic fields. The reason for the effect of the disorder on the transport properties of the junction is that the electron during a single act of the

resonant tunneling visits both contacts many times and diffuses there over long distances. Another consequence of coupling of the impurity state to the disordered continuum is the slow, nonexponential decay of the impurity population with time which was studied in Ref. 20.

We have restricted our considerations to the simplest case of resonant tunneling through a single impurity state. With the increase of the barrier thickness  $a$  the resonant tunneling through the pairs of impurities becomes important.<sup>8</sup> By extending considerations of Ref. 8 to the case of disordered contacts it can be shown that the weak-localization corrections to the two-impurity resonant conductance have the same sign and the same order of magnitude as in (22). Thus, our prediction about the increase of  $\delta G$  with temperature and with magnetic field is also valid for this case.

We have studied the conductance at applied voltage  $V \rightarrow 0$ . At finite  $V$  the differential conductance can be presented as

$$G(V) = \frac{1}{2} [G_0(\varepsilon_F + eV/2) + G_0(\varepsilon_F - eV/2)]. \quad (35)$$

For  $eV \gg T$  we have  $\tau_\phi \propto V^p$ . This leads to the singular voltage dependence of the disorder-induced correction  $\delta G(V) \propto \ln V$  for  $d=2$  and  $\delta G(V) \propto V^{d/2}$  for  $d=3$ . Note that a similar correction originates from the electron-electron interaction in the disordered contacts.<sup>21</sup> The latter correction, however, is not sensitive to a weak magnetic field.

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