

## Electrical conductivity and Wigner crystallization

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(Received 13 January 1992; revised manuscript received 30 March 1992)

We have measured the temperature dependence of both the dc resistance and the rf conductivity for several high-quality two-dimensional GaAs/Al<sub>x</sub>Ga<sub>1-x</sub>As electron samples in the high-magnetic-field Wigner-crystallization regime. We have found an increase in the activation energy of the dc resistance at the temperature where the dynamical conductivity becomes frequency dependent. We interpret this temperature as the melting temperature of the Wigner crystal and discuss the conduction mechanisms on both sides of the transition.

The physics of a clean, classical two-dimensional electron system (2DES) is dominated by the Coulomb repulsion between the electrons. Only at temperatures well above the nearest-neighbor interaction energy

$$E_I = \frac{e^2 \sqrt{\pi n}}{4\pi\epsilon\epsilon_0}$$

should the free-electron ideal-gas picture provide a sufficient description. At lower temperatures,  $k_B T < E_I$ , the electron gas becomes a strongly correlated liquid which ultimately freezes into a Wigner solid at a critical temperature  $T_c \approx E_I/100$ .<sup>1</sup> In addition to the  $e$ - $e$  correlations, strong fluctuations are important in the low-temperature phases of the 2DES and their presence explains the surprisingly large ratio of  $E_I/k_B T_c$ .

How does the electrical conductivity reflect the above-described gas-liquid-solid phase diagram of the 2DES? In the ideal-gas regime,  $k_B T > E_I$ , the electrons scatter independently from the lattice impurities and the conductivity obeys the well-known Boltzmann formula and is roughly temperature independent. In the other extreme, in the Wigner crystal phase a Coulomb gap  $\approx E_I/2$  opens between the ground state and the conduction band.<sup>2</sup> The solid ground state is presumably pinned by the impurities and the electrical current is carried by thermally activated excitations like interstitials, vacancies, or dislocation pairs which are predicted to have a much smaller activation energy than the conduction-band electrons.<sup>3,4</sup> The resistance of the Wigner crystal will depend both on the mobility and the density of the carriers and is expected to increase roughly exponentially toward zero temperature. Finally, in the intermediate correlated liquid state between  $E_I$  and  $T_c$  the details of the conduction mechanism are not well known. However, one can speculate that both the strong fluctuations typical to a 2D system and the  $e$ - $e$  correlations will modify the conductivity from the free-electron Boltzmann value. An interesting question remains: what will happen to the dc conductivity at the critical temperature  $T_c$ ? So far the Wigner crystallization has been studied in two classical 2DES, namely in electrons on He (Refs. 1 and 5) and in electrons in high mobility GaAs/Al<sub>x</sub>Ga<sub>1-x</sub>As heterostructures in high magnetic fields.<sup>6-13</sup> The dc resistance of the 2D electrons on thin He films<sup>5</sup> is found to be nearly temperature indepen-

dent above  $T_c$  where the electron-ripplon scattering is the dominant scattering mechanism. At  $T_c$  one finds a sharp kink in the resistance when it becomes activated in the crystal phase. In contrast, in all the previous studies of the 2DES in GaAs/Al<sub>x</sub>Ga<sub>1-x</sub>As heterostructures the dc resistivity is found to be activated in both the correlated liquid state and in the putative crystal phase near the expected transition temperature and surprisingly no feature is found in this data specifying the location of  $T_c$ .<sup>6,8,9</sup>

In this context an interesting question arises about the effect of the disorder on the Wigner crystal. A weak disorder potential  $eV_{\text{dis}} < E_I$  will perturb the long-range crystalline order resulting in a glassy electron state with a finite correlation length  $\xi \gg n^{-1/2}$ . Because the shear modulus of a 2D triangular charged lattice is very much smaller than the bulk modulus, the deformations induced by disorder will be almost entirely transverse, and the charge density will fluctuate very little. Of course in the presence of a sufficiently high disorder  $eV_{\text{dis}} > E_I$ ,  $\xi \approx n^{-1/2}$  and single electron localization will take place leading to an activated resistance but with an activation energy much larger than observed. Furthermore, based on the appearance of the  $\frac{1}{5}$  fractional quantized Hall state in the middle of the reentrant localized phase, it is hard to believe that the single electron localization is dominating over the correlated solid phase.<sup>9</sup> A reasonable estimate of  $eV_{\text{dis}} \approx 1$  K is obtained in these samples by comparing the theoretical and measured values of the energy gap of the  $\frac{1}{5}$  fractional quantum Hall effect (FQHE) state.<sup>9</sup> This gives an  $E_I/eV_{\text{dis}}$  ratio larger than 10, which according to the recent computer simulations by Aoki<sup>14</sup> should result in a highly ordered crystal state with a correlation length  $\xi \gg n^{-1/2}$ . These ideas are confirmed with the recent estimates of  $\xi$  based on both nonlinear  $IV$  curve measurements<sup>8,10,12</sup> and rf conductivity measurements.<sup>7,11</sup> Even though these estimates are model dependent and vary over a wide range they all indicate that the correlation length is over 10 lattice constants in samples where the mobility  $\mu > 10^6$  cm<sup>2</sup>/Vs.

In this paper we report dc-resistance measurements in the 2DES in high mobility GaAs/Al<sub>x</sub>Ga<sub>1-x</sub>As heterostructures in the Wigner solid regime. We are for the first time able to observe a clear change in the activated temperature dependence of the resistance between the liquid

and solid electron phases. We also determine the transition temperature from the rf-conductivity measurements and find a good agreement between the  $T_c$  values measured by these two methods. Finally we will discuss the origin of the activated resistance behavior on both sides of the transition temperature.

We have used five low-density ( $n \approx 6 \times 10^{10} \text{ cm}^{-2}$ ), high-mobility ( $\mu \approx 2\text{--}4 \times 10^6 \text{ cm}^2/\text{Vs}$ ) samples in these studies. The dc resistance was measured over a rectangular area, approximately  $3 \times 5 \text{ mm}^2$ , where the four electrical contacts were diffused at the corners. This sample area was obtained by etching out a mesa structure on a larger GaAs/ $\text{Al}_x\text{Ga}_{1-x}\text{As}$  chip. The etched areas outside the rectangular sample contained the transducers used for the surface acoustic wave (SAW) measurements reported earlier.<sup>11</sup> The dc-resistance measurements were carried out with a 1-nA ac-current source with 100 M $\Omega$  source resistance using standard lock-in amplification techniques at 2.4-Hz frequency. The rf-conductivity values of the 2DES were obtained from the SAW response at several fixed frequencies between 18 MHz and 1.55 GHz.<sup>11</sup> In these experiments, the samples were cooled down to 30-mK temperatures in a dilution refrigerator inside a 165-kG superconducting magnet.

The temperature dependence of the longitudinal resistance  $\rho_{xx}$  is shown in the Arrhenius plot of Fig. 1 at several filling factors  $\nu = nh/Be$  around the  $\frac{1}{5}$  FQH state. At these filling factors the resistance is found to increase towards lower temperatures but saturate at about 2 M $\Omega$  at the lowest temperatures. In our constant-current measurements, this low-temperature saturation could be due to Ohmic heating or depinning of the Wigner crystal. However, we believe that the saturation is mainly instrumental because it is independent of the filling factor but depends on the measurement frequency, ground capacitance of the current leads, and the increasing contact resistances towards low temperatures. At higher temperatures the resistance cannot be described by a single activation energy. In this temperature regime we see a clear upward bend of the data which takes place at about the expected Wigner crystallization temperature. In the earlier measurements over the same temperature range,  $\rho_{xx}$  was observed to be activated with a single exponent describing

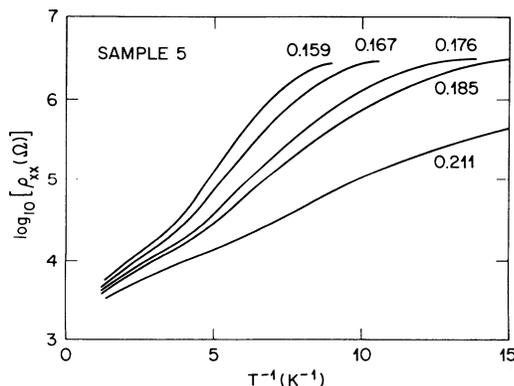


FIG. 1. Arrhenius plot of dc resistance  $\rho_{xx}$  for different filling factors in the Wigner-crystallization regime.

the data across the solidification temperature.<sup>6,8,9</sup>

We believe that the high-temperature upward bend in the resistance data is an intrinsic property of the 2DEG. We see this phenomenon in all of our five samples cut from two different wafers. We can also rule out the depinning of the Wigner crystal, because the voltages [ $V = \rho_{xx}(T)I$ ] used at these higher temperatures are well below the reported minimum value of the depinning threshold voltage  $V_{th} \approx 100 \mu\text{V}$ .<sup>8,12</sup> We have also remeasured some of the resistance curves in Fig. 1 at  $I = 0.5\text{-nA}$  current level and find no difference between the 0.5- and 1.0-nA results. Finally, if depinning is affecting the data in the low-temperature Wigner-crystal phase, the intrinsic lower current data should actually have a more dramatic bend than what is seen in Fig. 1.

In Fig. 2 we have studied more closely the  $T$ -dependent resistance at the filling factor  $\nu = 0.176$ . The dashed line in the upper panel shows the activated resistance below the transition temperature  $T_c$ . At temperatures above  $T_c$  the transport also looks activated but with a smaller activation energy than in the solid phase. In the lower panel we have shown a simultaneous measurement of the dynamical conductivity of the same sample between 314 MHz and 1.55 GHz frequencies.<sup>11</sup> Over this frequency range the conductivity is frequency independent in the phase above  $T_c$  but is found to become strongly frequency dependent at lower temperatures. As explained in a previous publication,<sup>11</sup> the frequency dependence results from the collective low-frequency pinning mode in the solid phase. This pinning frequency depends on the sample disorder, has a broad distribution and in our samples we find it to be centered at about 1-GHz frequency in the low filling-factor limit. By comparing the two panels in Fig. 2

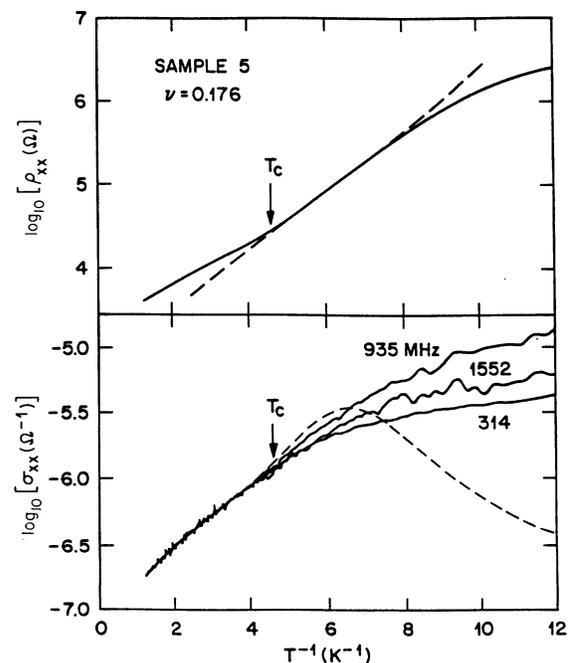


FIG. 2. Upper panel: a semilog plot of dc resistance vs inverse temperature. Lower panel: a similar plot of rf conductivity including dc conductivity as a dashed line.

we find that the kink in the  $\rho_{xx}$  data and the onset of frequency dependence in the conductivity data coincide, confirming our interpretations of both the dc and rf data.

The lower panel also shows the inferred conductivity  $\sigma_{xx}^{\text{dc}}$  (dashed line)  $= \rho_{xx}/(\rho_{xx}^2 + \rho_{xy}^2)$ , where  $\rho_{xy}$  was not directly measured but assumed to be  $B/ne$  and temperature independent. In the regime where crystallization is believed to occur, other measurements have shown that  $\rho_{xy}$  rises above the classical value, so that the dashed line provides an upper bound for  $\sigma_{xx}^{\text{dc}}$ . It is clear that there is a large difference between the dc response and that at 314 MHz, which automatically sets a fairly long length scale for the pinned regions.

We can use the data from Fig. 2 to determine the transition temperature  $T_c$  in two independent ways. As can be seen from Fig. 2, both the dc-resistance and rf-conductivity data are still relatively smooth through the transition temperature and only about 10% accuracy is obtained in the  $T_c$  determination. The results are shown in the lower panel of Fig. 3 as a function of the filling factor  $\nu$  for two of our samples cut from the same wafer. We find in agreement with the previous studies<sup>9</sup> that the Wigner crystal phase is reentrant for  $\nu > 0.2$  and that  $T_c$  approaches zero at the  $\frac{1}{5}$  FQH state. The agreement between the two independent  $T_c$  determinations is better than 15%. However, we notice that the dc-resistance data are giving slightly higher  $T_c$  values than the rf conductivity. They are also about 20% higher than the values reported in the earlier *IV* curve<sup>8,10,12</sup> and rf-conductivity studies.<sup>7</sup> Presumably this is not a significant difference and only demonstrates the difficulties in determining the transition temperature from smooth data such as in Fig. 2.

The activation energy  $E_a$  of the uncondensed carriers can be obtained from the dashed line in the upper level panel of Fig. 2 and it is presented as a function  $\nu$  in the upper panel of Fig. 3. The magnitude of  $E_a$  is of the order of the energy needed to create current carrying defects like interstitials<sup>3</sup> or bound dislocation pairs.<sup>4</sup> The interstitials are found to be quite mobile because only a small energy barrier of the order of  $E_I/1000$  separates the locations of their energy minima at the centers of the neighboring unit cells.<sup>15</sup> For a classical Wigner crystal of point charges, the activation energy of a center interstitial is  $0.13E_I = 4.5$  K.<sup>3</sup> This classical case is only achieved in the infinite magnetic field limit in a 2DES with zero thickness. In realistic samples at finite magnetic field, the finite-size effects are expected to reduce the value of  $E_a$  and this is in a reasonable agreement with the values what we might get by extrapolating the data in Fig. 3 to  $\nu=0$ . Similar estimates have been recently presented about the energy needed to create a bound dislocation pair.<sup>4</sup> The correlation between  $E_a$  and  $T_c$  is exactly as expected for a model of dislocation-mediated melting.

Finally, we would like to discuss the surprising observation that the conductivity is activated also in the correlated electron liquid near the melting temperature  $T_c$  in con-

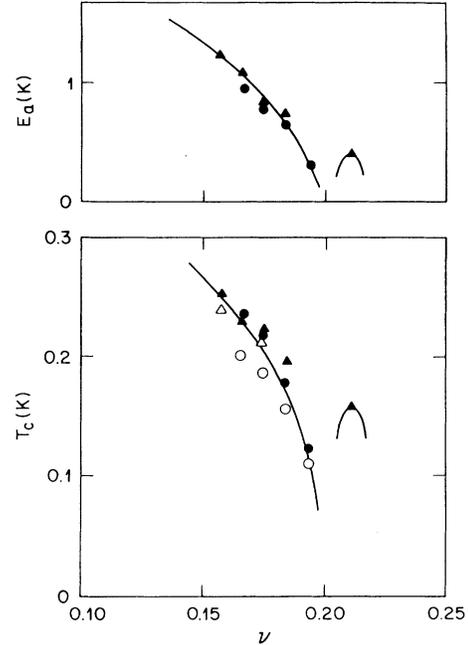


FIG. 3. Activation energy and transition temperature of Wigner crystal vs filling factor measured in samples 2 (circles) and 5 (triangles). The open and solid symbols are determined from rf and dc measurements, respectively.

trast to electrons on He films. We suggest that this is due to the relatively large correlated areas  $\approx (E_I/T)n^{-1}$  typical to two-dimensional systems at low temperatures. These slowly fluctuating correlated areas can be pinned by impurities and the diffusive modes of the system will be damped by fluctuations. Consequently, in analogy with the solid phase, the dominant conduction mechanism remains the defect hopping inside the correlated areas. This can explain the similarity in activation energies across  $T_c$  in GaAs/Al<sub>x</sub>Ga<sub>1-x</sub>As heterostructures. We note that electron localization on He films is very different because the polaronic dimple around each electron forms self-consistently at the solidification transition, producing a large increase in the electron's effective mass. Thus single particle localization (self-trapping) and crystallization occur together.

In conclusion, we have found a feature in the dc-resistivity data of the 2DES in GaAs/Al<sub>x</sub>Ga<sub>1-x</sub>As heterostructures at a temperature  $T_c$ , which coincides with the onset of the frequency dependence in the dynamical conductivity. The activation energy for linear transport scales with  $T_c$ , consistent with a model of dislocation-mediated melting of a Wigner crystal.

We acknowledge the discussions of defect mobilities and activation energies with V. Elser, E. Cockayne, and D. Huse.

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