Elementary excitations in one-dimensional quantum wires: Exact equivalence between the random-phase approximation and the Tomonaga-Luttinger model

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We show—contrary to the viewpoint that the random-phase approximation (RPA) is progressively worse in lower dimensions—that the simple random-phase approximation provides an *exact* description for intrasubband plasmon dispersion in one-dimensional quantum wires, by establishing a general equivalence between the RPA and the strongly correlated Tomonaga-Luttinger model for the elementary-excitation spectra in one-dimensional Fermi systems. We also discuss the formal analogy between *intrasubband* and *intersubband* collective modes in quantum wires by showing that the one-dimensional *intrasubband* collective excitations can also be thought of as depolarization-shifted single-particle excitations. Our results explain why recent experimentally observed one-dimensional-plasmon dispersion in GaAs quantum wires can be quantitatively described by the RPA.

Very recently, Goni *et al.* have observed[1] one-dimensional (1D) plasmons in semiconductor (GaAs) quantum wire structures using inelastic-light-scattering spectroscopy at low ($\sim$4 K) temperatures. There have been earlier[2] experimental studies of plasmons in GaAs quantum wires using infrared-absorption and inelastic-light-scattering techniques; however, Ref. 1 reports an observation of 1D plasmons in the extreme-quantum-limit situation where essentially only one 1D electronic subband is occupied by the carriers. Earlier experiments probed much wider wires (and higher electron densities) where many (typically more than ten) confined subbands were populated by electrons, and, therefore, the system was more accurately represented as a narrow two-dimensional[3] electron gas than a pure 1D electron gas. The quantum wire structure of Ref. 1 has a width of roughly 300 Å, and an electron density of 6.5×10^5 cm^{-1}, with the Fermi level just around the bottom of the first excited subband.

Interesting and important as the observation of 1D plasmons in semiconductor quantum wires most certainly is, what is even more remarkable (at least from a theoretical perspective) is that the observed 1D plasmon dispersion and the line shape of Ref. 1 agree *quantitatively* with a simple theory[4] based on the random-phase approximation (RPA). While RPA theories for elementary excitations in quasi-two-dimensional[3] semiconductor quantum wells, heterojunctions, inversion layers, and multilayer superlattices have been quantitatively highly successful in explaining an abundance of experimental data, it is generally believed[5] that a simple Fermi-gas model for 1D electron systems is simplistic and inadequate because a one-dimensional Fermi system is a singular, strongly correlated many-body system where *any* interaction between the electrons, no matter how weak, leads to essential singularities in the electronic spectral function, and the usual diagrammatic perturbation theory, certainly an approximation as simple as the RPA, is doomed to failure unless special care is taken to incorporate the various 1D singularities into the calculation. In fact, effects of interaction are so strong in one dimension that the Fermi surface disappears[5] in the Tomonaga-Luttinger model, leading to the nomenclature of the Luttinger liquid (for 1D Fermi systems) in contrast to the Landau Fermi liquid where the Fermi surface exists. On the other hand, experimental results of Ref. 1 are quantitatively well described by the RPA theory of Ref. 4, which treats the 1D system simply as a one-dimensional Landau Fermi-liquid-type "metal" ignoring all the sophistication and subtleties[5] of the Tomonaga-Luttinger model,[6] which has been the theoretical paradigm for 1D Fermi systems. This is particularly surprising because the RPA is supposed to be progressively worse in lower dimensions. We clearly have a puzzle here: Why does the simple RPA theory provide an essentially quantitatively exact description of the experimentally observed 1D elementary-electronic-excitation spectra?

In this paper, we solve this puzzle by demonstrating that the standard Bohm-Pines-Lindhard RPA theory for elementary excitations is exactly the same as the known exact solution for the low-lying excitations of the Tomonaga-Luttinger model obtained earlier[5] by Bethe-ansatz, bosonization, and renormalization-group techniques. This apparently surprising fact, that the standard RPA provides an exact solution for the strongly correlated Tomonaga-Luttinger 1D model, seems not to have been appreciated in the existing theoretical literature[5] (spanning four decades) on the subject. We believe that our work is significant not only in the context of understanding collective modes in quantum wires,[1,2] but also in view of the currently renewed interest[7,8] in strongly correlated (non-Fermi-liquid-like) low-dimensional systems as models for high-temperature superconductors. In fact, the Tomonaga-Luttinger model has recently been claimed to be relevant to topics ranging from high-temperature superconductors[7] to edge excitation in the fractional quantum Hall effect.[8] Thus, our demonstration that the RPA elementary excitations are exactly the same as those in the strongly correlated Tomonaga-Luttinger model may have some general consequences well beyond understanding the experimental data of Ref. 1.

The elementary excitations of an interacting electron gas are given by[9] the zeros of the dielectric function:

$$\epsilon(q,\omega)=0 \ . \tag{1}$$

The exact dielectric function is given by

$$\epsilon^{-1}(q,\omega)=1+V(q)\tilde{\Pi}(q,\omega) \ , \tag{2}$$

where $V(q)$ is the Fourier transform of the relevant Coulomb interaction, and $\tilde{\Pi}(q,\omega)$ is the reducible[10] density response function or the polarizability function, which obeys Dyson's equation

$$\tilde{\Pi}(q,\omega)=\Pi(q,\omega)+\Pi(q,\omega)V(q)\tilde{\Pi}(q,\omega) \ , \tag{3}$$

where $\Pi(q,\omega)$ is the exact irreducible[10] polarizability function. Combining Eqs. (2) and (3), we get the *exact* relationship,

$$\epsilon(q,\omega)=1-V(q)\Pi(q,\omega) \ . \tag{4}$$

Equation (4) may be considered an identity, defining the relationship between the dielectric function and the irreducible polarizability function. The RPA consists of the simple approximation of replacing the exact $\Pi(q,\omega)$ of the system by $\Pi_0(q,\omega)$, which is the irreducible polarizability (the "Lindhard function") for the corresponding *noninteracting* electron gas, yielding

$$\epsilon_{\mathrm{RPA}}(q,\omega)=1-V(q)\Pi_0(q,\omega) \ . \tag{5}$$

Equation (5) can be derived by a number of alternative equivalent techniques (e.g., collective coordinates, self-consistent field, equation of motion, time-dependent Hartree, summing the most divergent diagrams), all of which were well-established[9,10] in the 1950s. The RPA forms the basis of extensive dynamical linear-response studies in three[9] and two[11,3] -dimensional electron systems. Equation (5) leads to the well-known long-wavelength collective mode dispersion of $\omega(q\to 0)\sim\omega_p$ (a constant independent of q) and $\omega(q\to 0)\sim\sqrt{q}$ in three and two dimensions, respectively.

In a 1D electron gas, it can easily be shown[4] that the noninteracting polarizability or the polarization bubble diagram is given by

$$\Pi_0(q,\omega)=\frac{m}{\pi q}\ln\left[\frac{\omega^2-\omega_-^2}{\omega^2-\omega_+^2}\right] \ , \tag{6}$$

with

$$\omega_\pm=qv_F\pm q^2/2m \ , \tag{7}$$

where v_F is the 1D Fermi velocity ($\hbar=1$ throughout). Solving Eq. (1) with Eqs. (5) and (6), we get the following for the RPA elementary excitation spectra of the 1D system:

$$\omega_p^2=\frac{A(q)\omega_+^2-\omega_-^2}{A(q)-1} \ , \tag{8}$$

with

$$A(q)=\exp\left[\frac{q\pi}{mV(q)}\right] \ . \tag{9}$$

To explicitly see the connection between the RPA and the Tomonaga-Luttinger model, we expand Eq. (8) up to second order in q/k_F (the expansion is motivated by the linear dispersion[5,6] of the known exact elementary excitation spectrum in the Tomonaga-Luttinger model) to get

$$\omega_p^2\simeq\frac{mV(q)q^2}{\pi}\left[\frac{2v_F}{m}+\frac{v_F^2\pi}{mV(q)}\right]+O(q^3)$$
$$=q^2\left[v_F^2+\frac{2v_FV(q)}{\pi}\right] \ , \tag{10}$$

leading to the following RPA dispersion for the long-wavelength collective mode ("plasmon") in 1D systems:

$$\omega=|q|\left[v_F^2+\frac{2}{\pi}v_FV(q)\right]^{1/2} \ . \tag{11}$$

Note that the collective excitation given by Eq. (11) has the distinctive form of a shifted single-particle excitation (remembering that qv_F is the single-particle energy in the Tomonaga-Luttinger model)—this is unique to 1D; in higher dimensions plasmon energy cannot be written as a shifted single-particle energy. Equation (11) is *exactly* the same[5,12] as the eigenenergy of the elementary-excitation spectrum in the Tomonaga-Luttinger model. Since the Tomonaga-Luttinger result is thought to be generically exact for 1D systems (note that the RPA assumes the electron single-particle energy to be parabolic, i.e., $E=q^2/2m$ in contrast to the Tomonaga-Luttinger model, where the single-particle energy has the well-known linear v_Fq dispersion), we assert that the RPA is exact in 1D, in contrast to the viewpoint that the RPA becomes progressively worse in lower dimensions.

While in the Tomonaga-Luttinger model the interaction V is purely a parameter, for 1D semiconductor quantum wires $V(q)$ is the matrix element of the $1/r$ Coulomb interaction in the lowest quantized subband and is, therefore, exactly known if the confinement potential is known. It can be shown[4] that for $q\ll a$, where a is the typical confinement width, $V(q)\sim|\ln(qa)|$, leading to (for very small q)

$$\omega_p\sim|q||\ln(qa)|^{1/2} \ , \tag{12}$$

a result obtained earlier[4] by Das Sarma and Lai. In strict 1D (i.e., $a\to 0$), Coulomb interaction is logarithmically singular without a cutoff. This follows simply from the fact that $\int dx\, e^{iqx}/x$ is logarithmically divergent without an infrared cutoff. It may also be worthwhile to point out that for small q ($\ll k_F$) the correct long-wavelength expansion for the irreducible RPA polarizability in 1D is given by

$$\Pi(q,\omega)=\frac{N}{m}\frac{q^2}{\omega^2-q^2v_F^2}+\cdots \tag{13}$$

(where N is the 1D electron density), in contrast to the higher-dimensional result given by $\Pi(q,\omega)=(N/m)(q^2/\omega^2)+\cdots$. Again, Eq. (13) is of the exact Tomonaga-Luttinger form.

In Fig. 1, we show our calculated 1D plasmon dispersion for the full RPA [Eq. (8)], the long-wavelength RPA, or, equivalently, the Tomonaga-Luttinger result [Eq. (11)], and, the Das Sarma–Lai long-wavelength result [Eq. (12)] for the parameters of Ref. 1, assuming a harmonic confinement (which is a reasonable[13] approximation for GaAs quantum wires) with the energy-level separation of 5.2 meV and 1D electron density 6×10^5 cm^{-1}. Changing the confinement to a square-well-type confinement does not affect the result very much. Note that in the wave-vector range of Ref. 1, the first two approximations (i.e., the full RPA and its long-wavelength expansion) are indistinguishable. The experimental results of Ref. 1 are very close to the theoretical results shown here (cf. Ref. 1, in particular Fig. 3, for details).

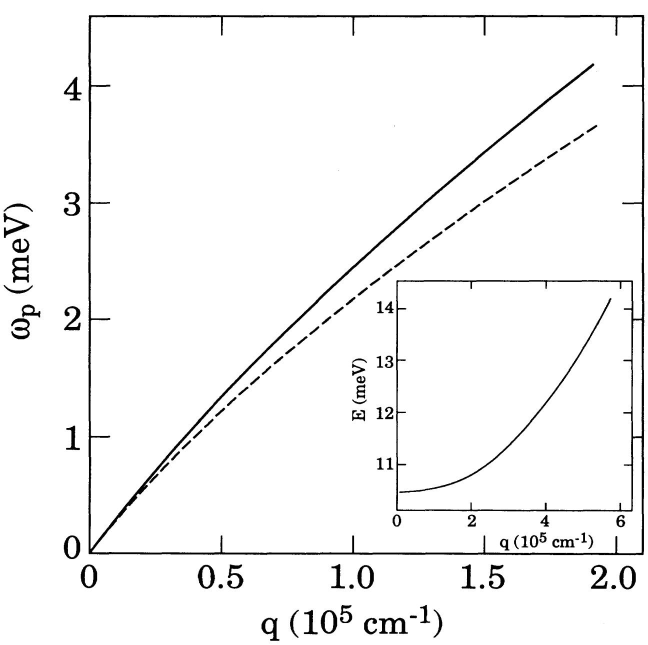

FIG. 1. The calculated intrasubband 1D plasmon dispersion for the parabolic GaAs quantum wire ($N=6\times10^5$ cm^{-1}) of Ref. 1: Full RPA, i.e., Eq. (8) of text (solid line); long-wavelength RPA or, equivalently, the Tomonaga-Luttinger result, i.e., Eq. (11) of text (indistinguishable from the solid line); the logarithmic formula of Eq. (12) (dashed line). The experimental results of Ref. 1 are close to the solid line. Inset: The calculated lowest *intersubband* 1D RPA collective mode dispersion for the same sample.

In hindsight, it is easy to see why the RPA is an exact description for 1D plasmon dispersion. In the Tomonaga-Luttinger model, all diagrams containing closed fermion loops consisting of more than two fermion lines vanish.[14] Vertex corrections are, therefore, absent in the Tomonaga-Luttinger model. Thus, the only diagrams which survive[14] are the bubble (or ring) diagrams (without any internal interaction lines) connected by Coulomb interaction lines. This, of course, is the RPA, which is nothing but a geometric series of bubble diagrams, with a single bubble being the noninteracting irreducible polarizability Π_0 of Eq. (6). This geometric series, as is well known,[9] leads to the RPA given by Eq. (5). Compared to 1D electron gas models with nonlinear parabolic dispersions, such as were used in Ref. 4, the Tomonaga-Luttinger model distorts the electron energy spectrum only far away from the Fermi level in making its linear dispersion approximation. Hence, for those physical quantities which depend only on the electrons near the Fermi surface, the Tomonaga-Luttinger model is a very good model, and, therefore, the RPA should be accurate in the calculation of those physical quantities. That is the reason why, in the long-wavelength limit, the calculated 1D plasmon dispersion using the RPA and a quadratic electron dispersion is exactly the same as the low-energy collective spectrum of the Tomonaga-Luttinger model.

Before concluding, we point out that the elementary excitations discussed above are longitudinal charge-density excitations along the length of the wire, which are the only allowed excitations in the Tomonaga-Luttinger model. But, in real quantum wires, even in the extreme 1D limit, there can be collective charge-density excitations associated with the transverse electronic motion representing quantum intersubband transitions. The same RPA theory can be used to obtain the collective excitation spectra for these intersubband transverse excitations (which have no analog in the Tomonaga-Luttinger model, where all higher subbands are strictly neglected), and, assuming occupation only of the lowest subband, one obtains

$$\omega_{12}^2(q)=\frac{B(q)\Omega_+^2(q)-\Omega_-^2(q)}{B(q)-1}, \tag{14}$$

where ω_{12} is the lowest intersubband collective mode, $\Omega_\pm(q)=E_{21}\pm qv_F+q^2/2m$, with E_{21} as the single-particle energy separation between the ground and the first excited subbands associated with confinement, and, the function $B(q)$ is given by

$$B(q)=\exp\left[\frac{q\pi}{mV_{1212}(q)}\right], \tag{15}$$

where $V_{1212}(q)$ is the off-diagonal matrix-element of Coulomb interaction between subbands 1 and 2 [in this notation, $V(q)$ of Eqs. (5)–(11) is $V_{1111}(q)$]. Note that Eqs. (14) and (15) are formally equivalent to Eqs. (8) and (9), except that now we are considering intersubband collective modes, whereas Eqs. (8) and (9) refer to intrasubband collective modes. The long-wavelength expansion of Eqs. (14) and (15) lead to

$$\omega_{12}(q) \simeq [E_{21}^2 + 2E_{21}NV_{1212}(q \to 0)]^{1/2} + O(q^2) , \quad (16)$$

which is formally the same as Eq. (11). Note that in the semiconductor microstructure literature,[3,4,11] the second term of Eq. (16) is called a depolarization shift[15] and therefore we can consider the second term in Eq. (11) *a depolarization shift for 1D intrasubband excitation.* The analogy is formally exact because the first term in Eq. (11), $v_F q$, is the single-particle energy in the Tomonaga-Luttinger model just as the first term in Eq. (16), E_{21}, is the single-particle intersubband excitation energy. As an inset of Fig. 1, we show our calculated *intersubband* collective excitation dispersion for the parameters of Ref. 1. This theoretical result has also been experimentally verified quantitatively in Ref. 1. (Note that the intersubband excitations are massive in the field-theoretic sense, i.e., they have a gap, E_{21}, at $q=0$. The vertex corrections are important for the intersubband excitations and represent an excitonic effect physically.)

In summary, we have studied the elementary-excitation spectra of 1D quantum wire structures, obtaining the seemingly surprising result that the RPA provides an *exact* description of the collective mode dispersion in 1D, in contrast to higher dimensions, where it is exact only in the high-density limit.[9,10] The long-wavelength (up to second order in q/k_F) RPA plasmon dispersion is shown to be identical to the exact result in the Tomonaga-Luttinger model of a strongly correlated 1D Fermi system. Our results explain[16] why a recent experimental study[1] of 1D plasmons in GaAs quantum wires can be quantitatively explained by an earlier numerical RPA analysis. In that sense, we have demonstrated that 1D quantum wires, at least in the extreme quantum limit, are Tomonaga-Luttinger systems, which is important in view of the widespread fundamental and technological interest in quantum wires. We also show the curious result that 1D *intrasubband* collective mode dispersion can be thought of as a depolarization-shifted single-particle excitation, whereas in two dimensions[3,11] there is no relationship between collective and single-particle modes. Finally, we obtain the 1D *intersubband* collective mode dispersion and establish the formal analogy between intrasubband and intersubband excitations in 1D.

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[1] A. R. Goni *et al.*, Phys. Rev. Lett. **67**, 3298 (1991).

[2] W. Hansen *et al.*, Phys. Rev. Lett. **58**, 2586 (1987); T. Demel *et al.*, Phys. Rev. B **38**, 12 732 (1988); T. Egeler *et al.*, Phys. Rev. Lett. **65**, 1804 (1990): T. Demel *et al.*, *ibid.* **66**, 2657 (1991).

[3] T. N. Theis, Surf. Sci. **98**, 515 (1981); J. K. Jain and S. Das Sarma, *ibid.* **196**, 466 (1988), and references therein.

[4] Q. P. Li and S. Das Sarma, Phys. Rev. B **40**, 5860 (1989); **41**, 10 268 (1990); **43**, 11 768 (1991); S. Das Sarma and W. Y. Lai, *ibid.* **32**, 1401 (1985).

[5] D. C. Mattis and E. H. Lieb, J. Math. Phys. **6**, 375 (1965); A. Luther and I. Peschel, Phys. Rev. B **9**, 2911 (1974); F. D. M. Haldane, J. Phys. C **14**, 2585 (1981).

[6] S. Tomonaga, Prog. Theor. Phys. **5**, 544 (1950); J. M. Luttinger, J. Math. Phys. **4**, 1154 (1963).

[7] P. W. Anderson, Phys. Rev. Lett. **64**, 1839 (1990); **65**, 2306 (1990); **66**, 2092 (1990); **65**, 3326 (1991); J. R. Engelbrecht and M. Randeria, *ibid.* **66**, 3325 (1991).

[8] F. D. M. Haldane, Bull. Am. Phys. Soc. **35**, 254 (1990); X. G. Wen, Phys. Rev. Lett. **64**, 2206 (1990); M. Stone, Int. J. Mod. Phys. B **5**, 509 (1991).

[9] D. Pines and P. Nozières, *The Theory of Quantum Liquids* (Addison-Wesley, New York, 1989), and references therein.

[10] A. L. Fetter and J. D. Walecka, *Quantum Theory of Many-Particle Systems* (McGraw-Hill, New York, 1971).

[11] T. Ando, A. B. Fowler, and F. Stern, Rev. Mod. Phys. **54**, 437 (1982).

[12] G. D. Mahan, *Many-Particle Physics* (Plenum, New York, 1981), p. 317.

[13] S. E. Laux, D. J. Frank, and F. Stern, Surf. Sci. **196**, 101 (1988).

[14] I. E. Dzyaloshinskii and A. I. Larkin, Zh. Eksp. Teor. Fiz. **65**, 411 (1973) [Sov. Phys. JETP **38**, 202 (1974)].

[15] S. J. Allen, D. C. Tsui, and B. Vinter, Solid State Commun. **20**, 425 (1976).

[16] It just so happens that in the experimental parameter range of Ref. 1, the full wave-vector-dependent RPA plasmon result [Eq. (8)] is quantitatively indistinguishable (cf. Fig. 1) from the long-wavelength RPA result [Eq. (11)] which, in turn, is exactly the same as the Tomonaga-Luttinger result.