Kinetics of a superlattice array of quasi-one-dimensional electron gases

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Quantum-transport equations and relaxation frequencies are derived for a superlattice array of quantum wires, taking into account dynamical screening and the tunneling between the wires in the tight-binding approximation. The momentum relaxation frequency for scattering by volume and sheet impurities is evaluated and shown to depend on the superlattice period l in a nontrivial manner. Competition between tunneling and screening effects results in a particular value of ℓ for which the relaxation frequency is minimum corresponding to a maximum conduction.

I. INTRODUCTION

Recently arrays of quantum wires have been the subject of several experimental[1] and theoretical[2] studies. In a previous paper[3] we considered transport properties of arrays of quantum wires assuming that the interwire separation was such that tunneling between the wires was negligible. We treated in detail the influence of the interwire Coulomb interaction, which affects the screening, on the transport properties and showed that it can affect, e.g., the mobility drastically. In this paper we generalize the previous theory by including in the calculations tunneling between the wires. The formation of minibands can change the elastic scattering rate significantly. For discrete levels the phase space for scattering of electrons that move along the wire is strongly limited.[3] When minibands appear, scattering within the same band is possible; as a result the scattering rate increases.

The paper is organized as follows. In the next section we present the results without electron screening of the potentials. We follow an approach different from that of Ref. 3. At the end of the section we show how screening can be taken into account. The actual superlattice aspects are presented in Sec. III. Analytical and numerical results for impurity scattering are given in Sec. IV. A summary follows in the last section and the Appendix details the evaluation of some results pertinent to Sec. IV.

II. QUANTUM TRANSPORT EQUATIONS

A. Quantum kinetic equation

We consider an array of identical quantum wires, of length $L_x \equiv L$, arranged periodically along the y axis, with period ℓ, and directed along the x axis as shown in Fig. 1. The confinement along the z direction gives a discrete spectrum while tunneling between the wires gives a set of minibands. The state of a particle in this superlattice is labeled by $A = (k_\alpha, a_y, a_z) \equiv (k_\alpha, a)$. k_α is the continuous wave vector index along the wires, a_z labels the discrete levels along the z direction, and a_y denotes the band index and the continuous wave vector k_y associated with motion, in the band, along the y direction. The corresponding one-particle eigenfunction (normalized) and eigenvalue of the unperturbed Hamiltonian are given, respectively, by

$$\Psi(\mathbf{r}) = e^{ik_\alpha x}\phi_{a_y}(y)\psi_{a_z}(z)/\sqrt{L} \tag{1}$$

and

$$E_A \equiv \hbar\omega_A = \hbar^2 k_\alpha^2/2m^* + \hbar\omega_a, \tag{2}$$

where $\omega_a = \omega_{a_y} + \omega_{a_z}$. We assume that an electric field $\mathbf{E} = E\hat{x}$ is applied along the wires. Then the diagonal component of the density matrix f_A, which is the only one necessary for calculating the current along the wire, satisfies[4] the quantum kinetic equation

$$\frac{\partial f_A}{\partial t} + \frac{eE}{\hbar}\frac{\partial f_A}{\partial k_\alpha} = \mathrm{St} f_A; \tag{3}$$

the collision integral $\mathrm{St}f_A$ is determined by the correlators of the fluctuating parts of the density matrix $\delta\rho$ and of the potential $\delta\varphi$ and is given[3,4] by

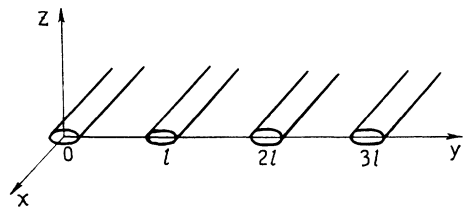

FIG. 1. A superlattice array, with period ℓ, of quantum wires.

$$\mathrm{St}f_A = -\frac{e}{2\pi^2\hbar}\mathrm{Im}\sum_B \int d\omega \int d\omega' e^{-i(\omega+\omega')t} < [\delta\rho_{AB}(\omega), \delta\varphi_{BA}(\omega')]_+ > . \quad (4)$$

Here $[\ ,\ ]_+$ denotes half the anticommutator and the fluctuating parts satisfy the equations

$$\delta\rho_{AB}(\omega) = \delta\rho^0_{AB}(\omega) + eM_{AB}(\omega)\delta\varphi(\omega), \quad (5)$$

$$\delta\varphi_{AB}(\omega) = \delta\varphi^0_{AB} + \frac{1}{e}\sum_{A'B'} V^s_{ABB'A'}(\omega)\delta\rho_{A'B'}(\omega). \quad (6)$$

Here the superscript 0 indicates the so-called "sources" and

$$M_{AB}(\omega) = \frac{f_B - f_A}{\hbar(\omega - \omega_{AB} + i0)}, \quad (7)$$

with $\omega_{AB} = \omega_A - \omega_B$. For an external system with a uniform polarizability, characterized by the dielectric function $\epsilon_s(\omega, \mathbf{q})$, we have

$$V^s_{ABB'A'}(\omega) = \frac{e^2}{2\pi^2}\int \frac{d^3q}{q^2\epsilon_s(\omega,\mathbf{q})}\, b^{\mathbf{q}}_{AB}\, b^{-\mathbf{q}}_{B'A'}, \quad (8)$$

where

$$b^{\mathbf{q}}_{AB} = \int d^3r\ \Psi^*_A(\mathbf{r})\, e^{i\mathbf{q}\cdot\mathbf{r}}\, \Psi_B(\mathbf{r}). \quad (9)$$

In previous papers[3,4] we have developed a self-consistent procedure for solving Eqs. (5) and (6) which takes into account the screening of the scattering potentials by electrons. However, the presence of tunneling prevents this simple procedure of obtaining the screening function. We, therefore, follow a different method neglecting, to begin with, the electron screening. We return to this point later. For the moment we use ordinary perturbation theory to second order.

We substitute Eq. (5) into Eq. (6) and vice versa keeping only terms to second order in the interaction constant. We assume that the sources are uncorrelated, i.e., $< \delta\rho^0\delta\varphi^0 > = < \delta\rho^0 >< \delta\varphi^0 > = 0$. For the evaluation of the average values after the substitution of the solutions of Eqs. (5) and (6) in Eq. (4) we use Eq. (30) of Ref. 4, a similar equation for ρ replaced by φ, and the identities $\mathrm{Im}M_{AB}(\omega) = (\pi/\hbar)(f_A - f_B)\delta(\omega - \omega_{AB})$ and $[V^s_{ABA'B'}(\omega)]^* = V^s_{BAB'A'}(-\omega)$, where

$$\mathrm{Im}V^s_{ABBA}(\omega) = \frac{e^2}{2\pi^2}\int \frac{d^3q}{q^2}|b^{\mathbf{q}}_{AB}|^2 \mathrm{Im}\frac{1}{\epsilon_s(\omega,\mathbf{q})}. \quad (10)$$

We then obtain, as usual, $\mathrm{St}f_A = \mathrm{St}_{es}f_A + \mathrm{St}_{ee}f_A$. The first term represents electron collisions with the external system while the second term describes interelectron collisions. They are given, respectively, by

$$\mathrm{St}_{es}f_A = -\frac{e^2}{\hbar^2}\sum_B \Big((f_A - f_B) < \delta\varphi^2_s >^{AB}_{\omega_{AB}} - \frac{\hbar}{e^2}\eta_{AB}\ \mathrm{Im}V^s_{ABBA}(\omega_{AB})\Big), \quad (11)$$

and

$$\mathrm{St}_{ee}f_A = \frac{2\pi}{\hbar^2}\sum_{B,A',B'} |V^s_{ABB'A'}(\omega_{AB})|^2\delta(\omega_{AB} - \omega_{A'B'}) \times (F_{BAA'B'} - F_{ABB'A'}). \quad (12)$$

Here $\eta_{AB} = f_A(1-f_B) + f_B(1-f_A)$ and $F_{ABCD} = f_A(1-f_B)f_C(1-f_D)$. The correlator $< \cdots >$ is evaluated in the Appendix for some particular cases of impurity scattering. A frequently encountered case is that when the external system, e.g., impurities, phonons, etc., is in equilibrium at temperature T_s. If in this case f^{eq}_A is the Fermi-Dirac function the condition $\mathrm{St}_{es}f^{\mathrm{eq}}_A = 0$ leads to the fluctuation-dissipation relation

$$< \delta\varphi^2_{s,0} >^{AB}_{\omega} = -\frac{\hbar}{e^2}\coth\left(\frac{\hbar\omega}{2k_BT_s}\right)\mathrm{Im}V^{s,0}_{ABBA}(\omega), \quad (13)$$

where the index 0 denotes equilibrium quantities. With this relation Eq. (11) can be simplified considerably. $\mathrm{St}_{es}f_A$ can be expressed in terms either of the potential correlator $< \delta\varphi^2_{s,0} >$ or of the dielectric function if Eq. (13) is used. The first form is suitable for calculations involving scattering by impurities while the second is more suitable for optical-phonon scattering and sometimes for interparticle interactions. Using Eqs. (11), (13), and (A7) we can write $\mathrm{St}_{es}f_A$ in the form

$$\mathrm{St}_{es}f_A = -\frac{e^2}{4\pi^2\hbar^2}\sum_B \left[f_A - f_B + \eta_{AB}\tanh\left(\frac{\hbar\omega_{AB}}{2k_BT_s}\right)\right]\int dq_x \int dq_y |b^{q_x}_{ab}|^2 |b^{q_y}_{ab}|^2 < \delta\varphi^2_{s,0} >^{a_xb_x}_{\omega_{AB},\mathbf{q}_\perp} . \quad (14)$$

So far we have not considered the screening of the scattering potentials by electrons. We do it now adapting the formalism of Ref. 3 which was applied when tunneling was absent. In the presence of the latter we assume that the minibands are narrow and that the screening factor can be treated in the tight-binding approximation. Technically it means that the above procedure is repeated using the approximate form (28), see below, for the y part of Eq. (9). The result for $\mathrm{St}_{es}f_A$ is given by Eq. (14) with a factor $|S(\omega_{AB}, q_x, \ell q_y)|^{-2}$ appearing under the integration signs where

$$S(\omega, q_x, \ell q_y) \approx 1 - \frac{4e^2}{L_xL_y}\int \frac{dq_z}{q^2\epsilon_s(\omega,\mathbf{q})} \times \sum_{A,B} M_{AB}(\omega)|b^{\mathbf{q}}_{AB}|^2. \quad (15)$$

Here we have assumed that the width of the wires is much smaller than the period ℓ of the array. In most cases we can safely ignore the tunneling effect in the screening factor. For instance, when only the lowest miniband is occupied we can use for $S(\cdots)$ the form[3]

$$S(\omega, q_x, \ell q_y) = 1 + \frac{\Delta\epsilon_{00}(\omega, q_x)}{\epsilon_s(\omega, q_x, \ell q_y)}. \tag{16}$$

In this case the function $1/\epsilon_s(\omega, q_x)$ is given by

$$\frac{1}{\epsilon_s(\omega, q_x, \ell q_y)} = \frac{2}{\ell}\sum_{p=-\infty}^{\infty}\int \frac{dq_z}{q_x^2+q_z^2+q_{yp}^2} \times \frac{|b_{00}^{q_z}\, b_{00}^{q_{yp}}|^2}{\epsilon_s(\omega, q_x, q_{yp}, q_z)}. \tag{17}$$

Here $q_{yp} = q_y - 2\pi p/\ell$ and the matrix elements involve the wave functions of separate wells when only the lowest subband is occupied (indicated by 00).

B. Balance equations and relaxation frequencies

From the QKE we derive the momentum and energy balance equations by applying to it the operators $(2e/L)\sum_A \hbar k_\alpha/m^*$ and $(2/L)\sum_A \hbar^2 k_\alpha^2/2m^*$, respectively. Denoting the total carrier number, current, and energy (linear) densities, respectively, by n', j, and ϵ we have

$$\frac{\partial j}{\partial t} = \frac{en'}{m^*}(eE - R) \tag{18}$$

and

$$\frac{\partial \epsilon}{\partial t} = jE - Pn', \tag{19}$$

where $-R = -R_{es} - R_{ee}$ and $P + P_{es} + P_{ee}$ are the friction force in the x direction against the external system and the power transferred, per particle, respectively. Now using Eq. (12) it is easy to see that the terms R_{ee} and P_{ee} vanish identically. Hence, using Eq. (11) we obtain, in matrix form,

$$\begin{pmatrix} R \\ P \end{pmatrix} = \frac{e^2}{4\pi^2\hbar n'}\sum_{AB}\int dq_x \int dk_\alpha \int dk_\beta \int d\omega\, \delta(\omega - \omega_{AB})\delta(k_\alpha - k_\beta - q_x) \times \begin{pmatrix} q_x \\ \omega \end{pmatrix} < \delta\varphi_{s,0}^2 >_{\omega,q_x}^{ab} \left[f_A - f_B + \eta_{AB}\tanh\left(\frac{\hbar\omega}{2k_BT_s}\right)\right]. \tag{20}$$

For simplicity we model the distribution function f_A by a displaced Fermi-Dirac function at temperature T. That is, we replace its argument k_α by $k_\alpha - m^*u/\hbar$, where u is the drift velocity. Then the previous expression takes the simpler form

$$\begin{pmatrix} R \\ P \end{pmatrix} = \frac{1}{4\pi^2}\sum_{a,b}\int dq_x \int d\omega \begin{pmatrix} q_x \\ \omega \end{pmatrix} \mathrm{Im}\Delta\epsilon_{ab}^{eq}(\omega - uq_x, q_x) < \delta\varphi_{s,0}^2 >_{\omega,q_x}^{ab} \left[\tanh\left(\frac{\hbar\omega}{2k_BT_s}\right)\coth\left(\frac{\hbar(\omega - uq_x)}{2k_BT}\right) - 1\right], \tag{21}$$

where the electron dielectric function $\Delta\epsilon_{ab}(\cdots)$ is given by Eq. (23) of Ref. 4. Another form of this expression involving the dielectric function can be written using Eq. (13).

If the electric field E is weak we may linearize R over u and P over $T - T_s$ and define the momentum ν^m and energy ν^T, relaxation frequencies through $R = -m^*\nu^m u$ and $P = \nu^T(T - T_s)_B$. Using Eq. (21) we obtain

$$\begin{pmatrix} \nu^m \\ \nu^T \end{pmatrix} = \frac{\hbar}{4\pi^2 n' k_B^2 T_s^2}\sum_{a,b}\int dq_x \int d\omega \begin{pmatrix} k_BT_sq_x^2/m^* \\ \omega^2 \end{pmatrix} < \delta\varphi_{s,0}^2 >_{\omega,q_x}^{ab} \frac{\mathrm{Im}\Delta\epsilon_{ab}^{eq}(\omega, q_x)}{\sinh(\hbar\omega/k_BT_s)}. \tag{22}$$

These expressions neglect the screening of the scattering potentials by electrons. Again this can be incorporated by inserting the screening factor $S(\cdots)$ given by Eq. (15) under the integral signs.

III. THE SUPERLATTICE MODEL

We consider a superlattice potential along the y direction added to a confining potential along the z direction. The total potential is written as

$$V(y, z) = \sum_{p=-\infty}^{\infty} V(y - p\ell) + V(z). \tag{23}$$

For definiteness we choose for both terms the parabolic forms $V(y) = \hbar^2 q_1^4 y^2/2m^*$ and $V(z) = \hbar^2 q_2^4 z^2/2m^*$. Moreover, we assume that only the lowest miniband is occupied, along y; the same holds for the lowest level in the z direction ($a_z = 0$). First we write the wave function for an electron in this superlattice potential in the tight-binding approximation as ($a_y \to k_y$)

$$\phi_{k_y}(y) = \sum_{p=-\infty}^{\infty} e^{ik_y\ell p}\phi_0(y - p\ell), \tag{24}$$

where $\phi_0(y)$ is the wave function of the isolated well and $-\pi/\ell \le k_y \le \pi/\ell$. Now the tight-binding model is valid

when only the wave functions of neighboring wells have a (small) overlap, i.e., for $\ell q_1 \gg 1$. We then have the superlattice dispersion relation $E_{k_y} = -\Delta_\ell \cos k_y \ell$, the width of the miniband being $2\Delta_\ell$ and given by the overlap integral. All energies are measured from the center of the miniband. With $\phi_0(y) = (q_1/\sqrt{\pi})^{1/2}\exp(-q_1^2 y^2/2)$ we obtain

$$\Delta_\ell = \frac{\hbar^2 q_1^2}{2m^*}(1 + 0.5 q_1^2 \ell^2) e^{-q_1^2\ell^2/4}. \tag{25}$$

For a strongly degenerate electron gas we approximate f_A by a step function. When the Fermi level E_F lies in the miniband, i.e., when $E_F < \Delta_\ell$, the normalization condition gives the density of one wire, $n = n'\ell/L_y$, as

$$n = \frac{4\tilde{k}_F}{\pi^2 E_+}[E_-^2\, \mathcal{K}(E_+) + \mathcal{E}(E_+)], \tag{26}$$

where $E_\pm = \sqrt{(E_F \pm \Delta_\ell)/2\Delta_\ell}$ and $E_F = -\Delta_\ell + \hbar^2\tilde{k}_F^2/2m^* = \hbar^2 k_F^2/2m^*$. Further, $\mathcal{K}(x)$ and $\mathcal{E}(x)$ are the complete elliptic integrals, and L_y is the dimension of the array in the y direction. For $E_F \to -\Delta_\ell$ we have $n \approx \hbar\tilde{k}_F^2/\pi\sqrt{2m^*\Delta_\ell}$. If, on the other hand, the Fermi level lies above the miniband, i.e., for $E_F > \Delta_\ell$, we have

$$n = \frac{4\tilde{k}_F}{\pi^2}\,\mathcal{E}(1/E_+). \tag{27}$$

For $E_F \gg \Delta_\ell$ we obtain the standard relation $n \approx 2\tilde{k}_F/\pi \approx 2k_F/\pi$.

The superlattice aspects enter the results of the previous section through the energy spectrum and through the screening function $S(\cdots)$, cf. Eq. (17), and in particular through the factor $b_{kk'}^{q_y}$. In the tight-binding limit ($q_1\ell \gg 1$) with the help of Eq. (24) we can approximately write

$$|b_{k_y k_y'}^{q_y}|^2 \approx \frac{2\pi}{L_y}\, e^{-q_y^2/2q_1^2} \sum_{p=-\infty}^{\infty} \delta(q_y - k_y + k_y' + q_p), \tag{28}$$

where $q_p = 2\pi p/\ell$.

IV. SCATTERING BY CHARGED IMPURITIES

A. Analytical results

As an application of the formalism we will now evaluate the momentum relaxation frequency for scattering by a random distribution of volume or sheet impurities. We assume that only the lowest level in the z direction is occupied. Then Eq. (22), taking into account the electron screening, becomes

$$\begin{pmatrix}\nu^m\\ \nu^T\end{pmatrix} = \frac{\hbar}{8\pi^3 n' k_B^2 T_s^2} \sum_{k_y k_y'} \int dq_x \int dq_y \int d\omega |b_{k_y k_y'}^{q_y}|^2 < \delta\varphi_{s,0}^2 >_{\omega,\mathbf{q}_\perp}^{00} \begin{pmatrix} k_B T_s q_x^2/m^*\\ \omega^2\end{pmatrix} \frac{\mathrm{Im}\,\Delta\epsilon_{k_y k_y'}^{eq}(\omega, q_x)}{\sinh(\hbar\omega/k_B T_s)|S(\omega, q_x, \ell q_y)|^2}, \tag{29}$$

and $S(\cdots)$ is given by Eq. (16).

1. Volume impurities

For simplicity we assume that the dielectric function of the system, $\epsilon(\omega, \mathbf{q})$ can be approximated by the constant ϵ_L and denote the impurity concentration by $n_I^{(3)}$. Further, we use Eq. (A9) with $b_{00}^{q_z} = \exp(-q_z^2/4q_2^2)$ and the expression for the correlator $< \delta\varphi_s^2 >_{\omega,\mathbf{q}_\perp}^{q_z q_z'}$. We then obtain[6]

$$< \delta\varphi_s^2 >_{\omega,\mathbf{q}_\perp}^{00} = \frac{8\pi^3 e^2 n_I^{(3)}}{\epsilon_L^2 q_\perp^3}\, G(q_\perp/\sqrt{2}q_2)\, \delta(\omega), \tag{30}$$

where

$$G(x) = 2x/\sqrt{\pi} + (1 - 2x^2)e^{x^2}\mathrm{erfc}(x); \tag{31}$$

here $\mathrm{erfc}(x) = (2/\sqrt{\pi})\int_x^\infty \exp(-t^2)dt$, $G(x \ll 1) \approx 1 - x^2$ and $G(x \gg 1) \approx 2/x\sqrt{\pi}$.

To proceed further we consider a strongly degenerate electron gas, i.e., we replace f_A^{eq} by a step function. Using the standard expression for $\Delta\epsilon^{eq}$ together with the miniband spectrum we obtain from Eqs. (29) and (30) the result

$$\nu^m = \frac{2e^4 m^* n_I^{(3)}\ell}{\pi n \epsilon_L^2 \hbar^3} \int_0^{\pi/\ell} dk_y \int_{-\infty}^{\infty} dq_y\, e^{-q_y^2/2q_1^2}\, \Theta(Q_{k_y}^2)\, \Theta(Q_{k_y - q_y}^2)\, \frac{1}{Q_{k_y} Q_{k_y - q_y}} \times \left[\frac{Q_+^2}{\bar{Q}_+^3}\, G\left(\frac{\bar{Q}_+}{2q_2}\right) S^{-2}(0, Q_+, \ell q_y) + \frac{Q_-^2}{\bar{Q}_-^3}\, G\left(\frac{\bar{Q}_-}{2q_2}\right) S^{-2}(0, Q_-, \ell q_y)\right]. \tag{32}$$

Here $Q_\pm \equiv Q_\pm(k_y, q_y) = |Q_{k_y} \pm Q_{k_y - q_y}|$, $Q_{k_y} = \sqrt{2m^*(E_F - E_{k_y})}/\hbar$, and $\bar{Q}_\pm = \sqrt{q_y^2 + Q_\pm^2}$. To arrive at Eq. (32) we have used the property $Q_{k_y - q_y - q_p} = Q_{k_y - q_y}$ which results from the periodicity. As for the screening factor $S(\cdots)$ using Eqs. (16), (17), and (28) we obtain

$$S(0,q_x,\ell q_y)=1+\frac{2m^*e^2}{\ell\hbar^2\epsilon_L q_x}e^{q_x^2/2q_2^2+(q_y^2/2)(1/q_2^2-1/q_1^2)}\ln|K_+/K_-|\sum_{p=-\infty}^{\infty}\mathrm{erfc}\left(\frac{Q_p}{\sqrt{2}q_2}\right)\Big/Q_p, \tag{33}$$

where $K_\pm=(k_F\pm q_x/2)^2-(m^*k_BT_s/\hbar^2q_x)^2$ and $Q_p=\sqrt{q_x^2+(q_y-q_p)^2}$.

In the limit of isolated wires $\ell\to\infty$ we have $\Delta_\ell\to 0, Q_{k_y}\to k_F, Q_+\to 2k_F$, and $Q_-\to 0$. Then Eq. (32) goes over to Eq. (69) of Ref. 5. If we neglect the elctron screening and assume $q_1\gg n, q_2\gg n$ we obtain

$$\nu^m(\ell\to\infty)\approx\frac{4e^4m^*n_I^{(3)}}{\epsilon_L^2n\hbar^3q_1^2}e^{Q_{1n}}\{K_1(Q_{1n})-K_0(Q_{1n})\}\approx\frac{16e^4m^*n_I^{(3)}}{\pi^2\epsilon_L^2\hbar^3n^3}, \tag{34}$$

where $Q_{1n}=(\pi n/2q_1)^2$ and $K_n(x)$ is the modified Bessel function.

2. *Sheet impurities*

We assume that the impurities are distributed in the (x,y) plane with surface concentration $n_I^{(2)}$. For the potential correlator we have

$$<\delta\varphi_s^2>^{00}_{\omega,\mathbf{q}_\perp}=\frac{8\pi^3e^2n_I^{(2)}}{\epsilon_L^2q_\perp^2}[\mathrm{erfc}(q_\perp/2q_2)]^2\,e^{q_\perp/2q_2}\delta(\omega). \tag{35}$$

In this case ν^m takes the form (33) with $n_I^{(3)}G(\bar{Q}_\pm/2q_2)/\bar{Q}_\pm^3$ replaced by $n_I^{(2)}[\mathrm{erfc}(\bar{Q}_\pm/2q_2)]^2\times\exp(\bar{Q}_\pm^2/2q_2^2)/\bar{Q}_\pm^2$. For $\ell\to\infty$ and $q_1\gg n, q_2\gg n$ we obtain upon neglecting the electron screening

$$\nu^m(\ell\to\infty)\approx\frac{8e^4m^*n_I^{(2)}}{\epsilon_L^2\hbar^3n^2}. \tag{36}$$

As for the screening $S(\cdots)$ an approximate result S simpler than that given by Eq. (33) can be obtained as follows. We evaluate the expression (33) for those values of the arguments that give the largest contribution to ν^m as given by Eq. (32). For $q_1\ell\gg 1$ and $q_2\ell\gg 1$ we can evaluate the sum over p in Eq. (33) in the opposite limits $n\ell\gg 1$ and $n\ell\ll 1$. We can then construct the interpolation formula

$$S=1+\frac{2m^*e^2}{\pi^2\epsilon_L\hbar^2n}\ln\left(\frac{\pi^2\hbar^2n^2}{m^*k_BT_s}\right)\left[\ln\left(\frac{\sqrt{8}q_1}{\pi n}\right)+\frac{1}{\sqrt{2}n\ell}\right], \tag{37}$$

which covers all values of $n\ell$ and is nearly exact in the opposite limits $n\ell\gg 1$ and $n\ell\ll 1$.

B. Numerical results

In Figs. 2 and 3 we plot the momentum relaxation frequency ν^m, as evaluated numerically from Eq. (32), as function of the period ℓ, using the dimensionless quantity $\mathcal{L}=q_1\ell$, for $q_1=q_2$ and $E_F>\Delta_\ell$. In these figures $g=\pi n/q_1$, $\nu_i=(e^4m^*/\hbar^3\epsilon_L^2)(n_I^{(i)}/q_2^i)$, $m^*=0.067m_0$, $\epsilon_L=13$, $T_s=4.2$ K, and $i=3$ or 2. Further, the solid, dashed, and dotted curves correspond to $g=0.2, 0.4$, and 1.0, respectively.

The decrease of ν^m with increasing g can be seen from the limiting expressions (34) and (36): for the degenerate electron gas we have approximately $\nu^m\sim n^{-i}$. This is better illustrated in Figs. 4 and 5 on which we comment further below.

The decrease of ν^m for small ℓ can be explained in the following manner. For small ℓ the miniband width becomes large. If the Fermi energy is fixed this broadening of the miniband leads to the appearance of electrons with energy less than E_F (up to $E_F-\Delta_\ell$). For electrons with energy about E_{k_y} the corresponding relaxation frequency behaves as $\sim(E_F-E_{k_y})^{-i/2}$; as a result the frequency increases when the period decreases. Notice that the curves do not start at $\mathcal{L}=0$ since the tight-binding model that we are using is not valid in this case.

The increase of ν^m with ℓ for $\mathcal{L}>10$ is connected with electron screening: for large ℓ screening becomes weak and the scattering rate increases. The same behavior of ν^m with increasing ℓ was reported earlier when tunneling was not considered,[3] i.e., for relatively large periods. As $\ell\to\infty$ the curves saturate as expected for isolated wires.

The importance of screening is most clearly seen in Figs. 4 and 5 where we plot the relaxation frequency as function of the electron density ($g=\pi n/q_1$). In both figures the solid and dashed curves correspond to scattering by volume and sheet impurities, respectively. Notice,

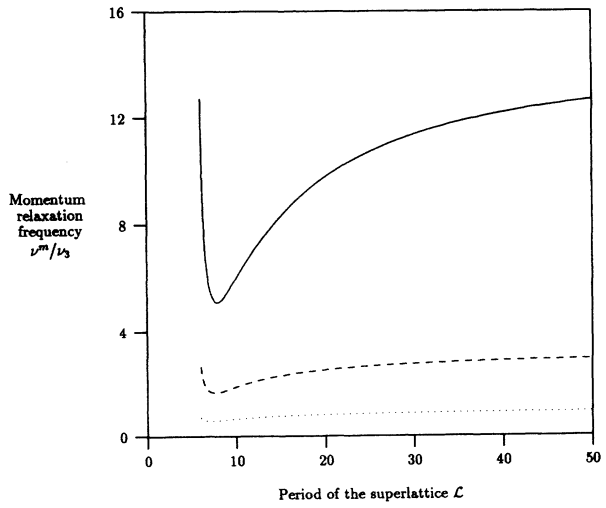

FIG. 2. Momentum relaxation frequency for scattering by volume impurities as function of the period $\ell(\mathcal{L}=q_1\ell)$ for different values of the electron density n ($g=\pi n/q_1$). The solid, dashed, and dotted curves correspond to $g=0.2, 0.4$, and 1.0, respectively.

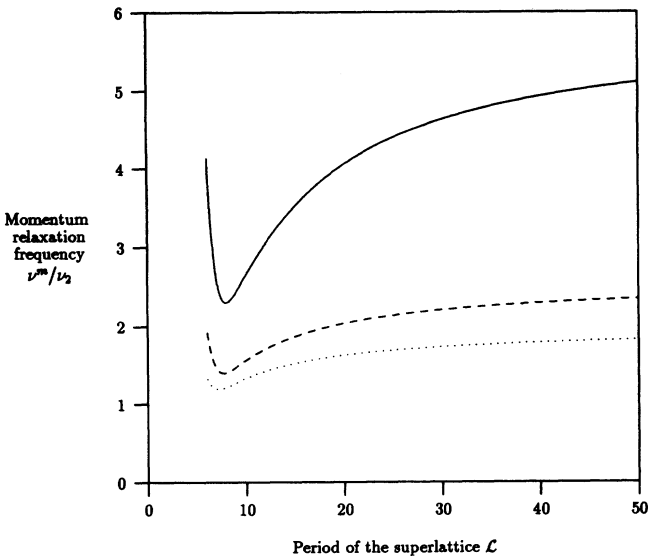

FIG. 3. As in Fig. 2 for scattering by sheet impurities.

however, the vertical scales: in Fig. 4 the screening factor $S(\cdots)$, which enters Eq. (32), or its version for scattering by sheet impurities, is evaluated using Eq. (33) whereas in Fig. 5 we have put $S(\cdots) = 1$, i.e., we have neglected screening. The results in the latter case are about five hundred times larger than those of Fig. 4, i.e., they are severely overestimated. This is due to the large values of S for very low temperatures since the second term in $K_{\pm}$ varies approximately as $\sim k_B T/E_F$ and diverges when $T \to 0$ for $q_x \to k_F$ as Eq. (33) shows. We conclude that it is essential to treat screening as accurately as possible.

At first sight, the large difference in the results with

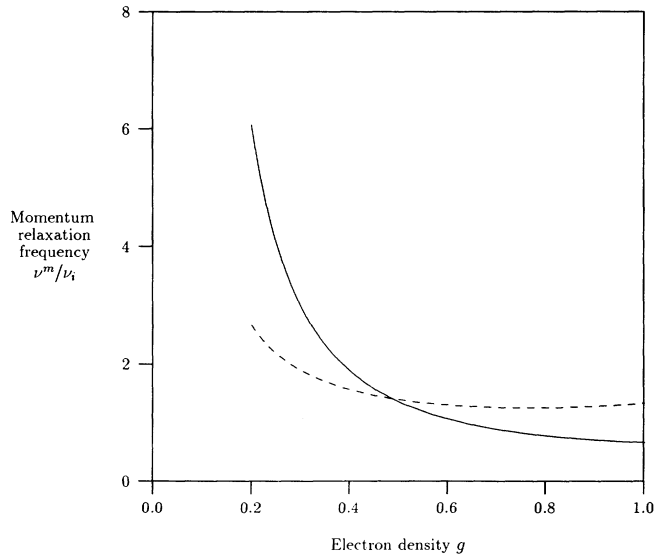

FIG. 4. Momentum relaxation frequency as function of the the electron density. Screening has been taken into account. Curves 1 and 2 are for scattering by volume and sheet impurities, respectively.

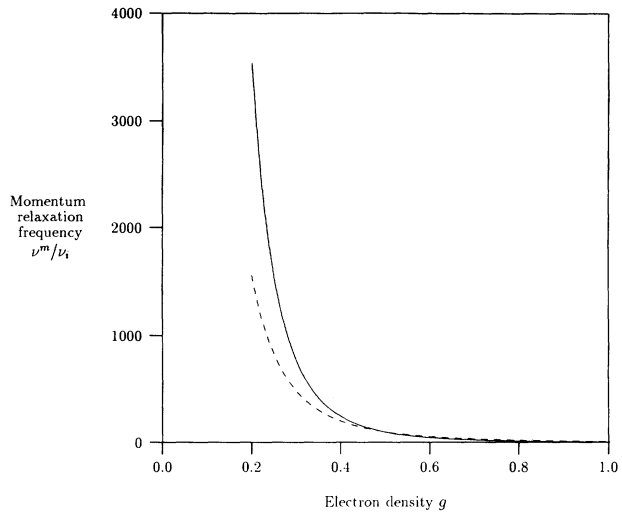

FIG. 5. As in Fig. 4 with neglect of screening.

and without screening appears surprising and needs to be understood. Let us recall that for a three-dimensional electron gas the Coulomb interaction with charged impurities leads, in the absence of electron screening, to a diverging momentum relaxation frequency, i.e., $\nu^m \to \infty$, whereas if electron screening is taken into account we have $\nu^m \sim \ln(k_F/k_s)$, where k_s is the screening wave vector. Formally, the ratio of the two results tends to infinity. Moreover, this finite result for the three-dimensional case is obtained by the summation of only the ring diagrams to all orders[7] which means that it is essentially a nonperturbative result. Something similar holds in the present case. We consider the scattering to second order and improved by including screening. The latter has been taken into account in the self-consistent manner of Ref. 4 upon solving the equations for the fluctuating parts of the density matrix and of the potentials, cf. Eqs. (5) and (6). That is, our results concerning screening have been obtained in essentially a nonperturbative manner. Consequently, they are not so surprising and one could trust them. Notice, however, that our finite result for S diverges when $T \to 0$ for $q_x \to k_F$, cf. Eq. (33). It goes without saying that for the moment we consider this problem as not being completely solved.

V. SUMMARY

In this paper we have derived quantum transport equations and relaxation frequencies for an array of quantum wires taking into account screening (dynamically) and tunneling. For impurity scattering the results for the momentum relaxation frequency, and correspondingly those for the conductivity, depend on the period ℓ in a nontrivial manner: at some particular value ℓ_c for ℓ we have maximum conduction reflecting the opposing effects of tunneling and screening. This ℓ_c depends on the material parameters. This is a very interesting result. Its validity, of course, is related to that of the tight-binding approximation. We estimate that the corresponding peak behavior of the conductivity will be more pronounced in

wide miniband samples. Further, our calculations show that it is very important for the conductivity to take into account screening since the results can be about five hundred times larger if one does not. We are not aware of any pertinent experimental data; we hope that our findings will stimulate the relevant experiments.

ACKNOWLEDGMENT

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APPENDIX

The correlator which enters Eq. (11) is related to the correlator $C(\omega)$, resulting from the substitution of Eqs. (5) and (6) in Eq. (4), by the expression

$$C(\omega) = < [\delta\varphi^0_{AB}(\omega), \delta\varphi^0_{BA}(\omega')]_+ > = 2\pi\delta(\omega+\omega') < \delta\varphi^2_s >^{AB}_{\omega} . \quad \text{(A1)}$$

Now denoting by $\delta\varphi^0(\omega, \mathbf{q})$ the Fourier transform of $\delta\varphi^0(\mathbf{r}, t)$ we can write

$$\delta\varphi^0_{AB}(\omega) = \frac{1}{8\pi^3}\int d^3q\, b^{\mathbf{q}}_{AB}\, \delta\varphi^0(\omega, \mathbf{q}). \quad \text{(A2)}$$

If the scattering system is uniform we have

$$C(\omega, \mathbf{q}) = < [\delta\varphi^0(\omega, \mathbf{q}), \delta\varphi^0(\omega', \mathbf{q}')]_+ > = 16\pi^4\delta(\omega+\omega')\delta(\mathbf{q}+\mathbf{q}') < \delta\varphi^2_s >_{\omega,\mathbf{q}} . \quad \text{(A3)}$$

Combining the last two equations with Eq. (A1) we obtain

$$< \delta\varphi^2_s >^{AB}_{\omega} = \frac{1}{8\pi^3}\int |b^{\mathbf{q}}_{AB}|^2 \ < \delta\varphi^2_s >_{\omega,\mathbf{q}} . \quad \text{(A4)}$$

If the scattering system is uniform in the (x, y) palne and nonuniform in the z direction Eq. (A3) becomes

$$C(\omega, \mathbf{q}) = 8\pi^3\delta(\omega+\omega')\delta(\mathbf{q}_\perp + \mathbf{q}_\perp) < \delta\varphi^2_0 >^{q_z, q'_z}_{\omega, \mathbf{q}_\perp} . \quad \text{(A5)}$$

If the system is uniform only in the x direction we can write

$$< \delta\varphi^2_s >^{AB}_{\omega} = (1/L) < \delta\varphi^2_s >^{ab}_{\omega, k_\alpha - k_\beta} . \quad \text{(A6)}$$

The first two cases can be combined in the form

$$< \delta\varphi^2_s >^{AB}_{\omega} = \frac{1}{4\pi^2}\int dq_x \int dq_y |b^{q_x}_{ab}|^2 \ |b^{q_y}_{a_y b_y}|^2 \times < \delta\varphi^2_s >^{a_z b_z}_{\omega, \mathbf{q}_\perp} . \quad \text{(A7)}$$

For the system that is uniform in all directions we have

$$< \delta\varphi^2_s >^{a_z b_z}_{\omega, \mathbf{q}_\perp} = \frac{1}{2\pi}\int dq_z |b^{q_z}_{a_z b_z}|^2 < \delta\varphi^2_s >_{\omega,\mathbf{q}}, \quad \text{(A8)}$$

whereas for that which is uniform only in the (x, y) plane

$$< \delta\varphi^2_s >^{a_z b_z}_{\omega, \mathbf{q}_\perp} = \frac{1}{4\pi^2}\int dq_z \int dq'_z b^{q_z}_{a_z b_z} b^{q'_z}_{b_z a_z} \times < \delta\varphi^2_s >^{q_z q'_z}_{\omega, \mathbf{q}_\perp} . \quad \text{(A9)}$$

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