Stochastic Coulomb blockade in a double-dot system

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Coulomb blockade in a system of two dots connected in series is qualitatively different from that of a single dot. We show that, although the conductance G of a double-dot system reveals oscillations with the gate-induced potential V_g, a typical period of these oscillations changes with the temperature. If the capacitance ratio C_1/C_2 for the dots is an irrational number, the system of peaks in $G(V_g)$ becomes increasingly sparse as the temperature decreases. Both the peak-to-peak distance and the activation energies of the conductance at the peaks that persist are random. However, the distribution function of activation energies calculated for a large interval of V_g has a universal shape and may be considered as a characteristic pattern of a double-dot system. If the ratio C_1/C_2 is small, there is a substantial range of intermediate temperatures in which the ordinary periodic Coulomb oscillations are restored. Numerical simulations show that for observation of both stochastic and regular Coulomb blockade for the same sample at different temperatures it is enough to have the ratio $C_1/C_2 \leq 0.5$. The existence of a small interdot capacitance $C \ll C_1, C_2$ is shown to cause, at the lowest temperatures, a splitting of the conductance peaks that persist into doublets with a constant spacing e^2C/C_1C_2.

I. INTRODUCTION

Electron transport through a small-size conducting dot can be affected significantly by the phenomenon of charging (Coulomb blockade).[1] The simplest type of a structure consisting of a single dot separated from two leads by tunnel barriers (Fig. 1) has been studied extensively both experimentally and theoretically (see, e.g., recent papers, Refs. 2–7). Two different manifestations of Coulomb blockade were under consideration. The first one is a staircase structure observed in the I-V characteristic.[4] The second one is oscillations of the linear conductance as a function of the gate voltage which controls the equilibrium charge of the dot. The conductance peaks are periodic in the gate voltage[2,5,7] with the period

$$\Delta V_g = e/C_0 , \tag{1}$$

where C_0 is the capacitance between the gate and the dot [Fig. 1(b)]. Oscillations become well resolved at low temperatures,

$$k_B T \ll k_B T_B \equiv e^2/2(C_0+2C) , \tag{2}$$

where C is the lead-to-dot capacitance. The explicit form of the linear conductance G vs V_g dependence at a given temperature was derived in Ref. 5. The conductance has a finite activation energy at all values of V_g except the points of charge degeneracy,

$$V_g = \frac{e}{C_0}(n+\tfrac{1}{2}), \quad n = 0, \pm 1, \pm 2, \ldots . \tag{3}$$

At these points the low-temperature value of G approaches a constant,

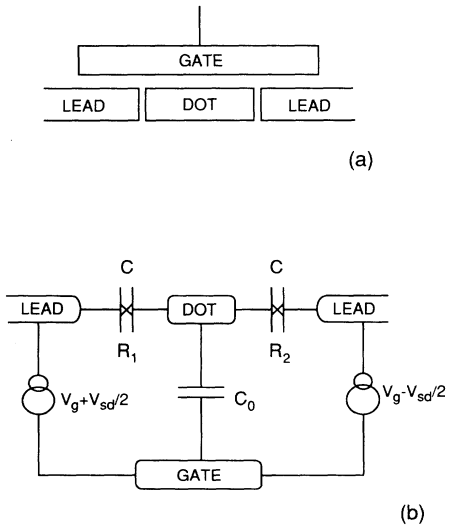


FIG. 1. (a) Single-dot device and (b) the equivalent circuit. R_1 and R_2 are the tunnel resistances of corresponding junctions.

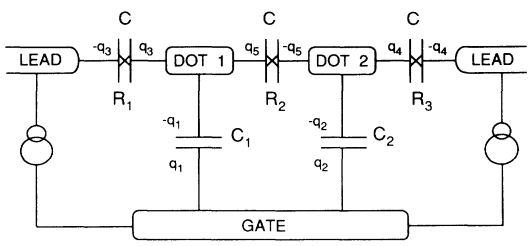


FIG. 2. Equivalent circuit of a double-dot device. Polarization charges $q_1,\ldots,q_5$ are continuous ones; charges en_1, en_2 of the dots (not shown in the figure) are discrete.

$$G(T\to 0)=\tfrac{1}{2}G(T\gg T_B)\ . \tag{4}$$

The conductance (4) remains finite at $T\to 0$ because, under condition (3), the Coulomb energy of a dot having n and $n+1$ electrons is the same and hence the passage of a charge through a dot does not require extra energy.

The situation becomes very different for a sequence of dots connected in series. The simplest case of two dots is shown in Fig. 2. If the dots are not identical, the condition of charge degeneracy cannot be satisfied simultaneously for both dots. In this case the conductance behavior becomes much more complicated. The linear conductance G as a function of temperature T and gate voltage V_g is studied in this paper. We find that, under sufficiently mild requirements on the values of capacitances forming the equivalent circuit shown in Fig. 2, a crossover from a periodic to a quasirandom dependence $G(V_g)$ occurs at low temperatures. This evolution of the $G(V_g)$ function is strongly reminiscent of the oscillatory pattern observed in experiments[8] on In_2O_{3-x} wires as well as some features of oscillations investigated in a GaAs field-effect structure near the depletion threshold.[9] This allows us to make a conjecture that transport properties of mesoscopic wires near the crossover from the metallic to the insulating regime are controlled by the Coulomb blockade in a system formed by dots with random parameters.

In Sec. II, we analyze the electrostatic energy of the two-dot system and derive the conditions favoring conductance through the system. The structure of a single conductance peak at low temperatures is studied in Secs. III and IV, the effect of small interdot capacitance being explicitly discussed in Sec. IV. Numerical results for the crossover from a periodic to a random peak structure are presented in Sec. V. In Sec. VI, we calculate the distribution function of activation energies of the conductance. If the ratio C_1/C_2 for the dots (see Fig. 2) is irrational, the function has a universal shape. Possible relations to experiments are discussed in Sec. VII.

II. WHEN DOES THE CONDUCTANCE OF A TWO-DOT CHAIN PEAK?

To answer this question, we should find first the dependence of the electrostatic energy U on the discrete charges n_1 and n_2 of the dots. For the sake of simplicity we assume the interdot and the lead-to-dot capacitances to be equal (Fig. 2), though it is not essential to our conclusions. Five continuous polarization charges $q_1,\ldots,q_5$ can be introduced for the circuit shown in Fig. 2, U as a function of these charges having the form

$$U(q_1,\ldots,q_5)=\frac{q_1^2}{2C_1}+\frac{q_2^2}{2C_2}+\frac{q_3^2+q_4^2+q_5^2}{2C}+V_g(q_1+q_2)\ . \tag{5}$$

Here C_1 and C_2 are capacitances between the dots and the gate, C is the value of intercapacitances (lead-to-dot and dot-dot). The first three terms represent the classical charging energy of individual capacitors. The last term arises from the gate voltage V_g applied to the capacitors C_1 and C_2. The discreteness of dot charges implies two constraints on $q_1,\ldots,q_5$:

$$\begin{aligned} q_3+q_5-q_1&=-n_1e\ ,\\ q_4+q_2-q_5&=-n_2e\ . \end{aligned} \tag{6}$$

The equilibrium charges q_i at given n_1,n_2 are found from (6) and from the condition that the energy U is a minimum

$$\partial U/\partial q_i=0,\quad i=3,4,5\ . \tag{7}$$

The resulting energy as a function of the discrete variables n_1 and n_2 has the form

$$U(n_1,n_2)=U_{11}n_1^2+U_{22}n_2^2+U_{12}n_1n_2-eV_g(a_1n_1+a_2n_2)\ , \tag{8}$$

with coefficients

$$U_{ij}=\frac{e^2(2C+C_i\delta_{ij})}{2(3C^2+2CC_1+2CC_2+C_1C_2)}\ , \tag{9}$$

$$a_i=\frac{CC_1+CC_2+C_1C_2+CC_i}{3C^2+2CC_1+2CC_2+C_1C_2}\ . \tag{10}$$

In Eq. (9), summation over repeated subscripts i and j is not assumed. Use of the reduced form (8) of the potential energy that depends only on two charges n_1,n_2 is justified in the study of transport phenomena only if the relaxation time to equilibrium in each dot is much smaller than the dwelling time of the tunneling electron in the dots.

The simplest case corresponds to relatively small cross capacitances between dots and leads,

$$C\ll C_1,C_2\ . \tag{11}$$

These conditions are also easily met in experiments with semiconductor microstructures. We will carry out the analysis of Coulomb oscillations assuming condition (11) to be valid. In zero-order approximation in C, Eqs. (8)–(10) are then reduced to the form

$$U(n_1,n_2)=\frac{\Delta_1n_1^2}{2}+\frac{\Delta_2n_2^2}{2}-eV_g(n_1+n_2)\ . \tag{12}$$

Here

$$\Delta_1 \equiv \frac{e^2}{C_1}, \quad \Delta_2 \equiv \frac{e^2}{C_2} . \tag{13}$$

At low temperatures, a system with potential energy (12) favors certain values of n_1, n_2. Passing of a charge requires a subsequent change of these values by one. Hence, the linear conductance is comparable to its high-temperature value only if these changes of n_1, n_2 cause sufficiently small variations in the potential energy:

$$\begin{aligned} &|U(n_1,n_2)-U(n_1+1,n_2)| \le T , \\ &|U(n_1,n_2+1)-U(n_1,n_2)| \le T . \end{aligned} \tag{14}$$

Taking into account Eq. (12), conditions (14) can be rewritten:

$$|eV_g-(n_1+\tfrac{1}{2})\Delta_1| \le T , \tag{15a}$$

$$|eV_g-(n_2+\tfrac{1}{2})\Delta_2| \le T . \tag{15b}$$

Conditions (15) can be easily satisfied at high temperatures for *any* value of V_g. The Coulomb blockade starts to be important at low temperatures, $T < \Delta_1, \Delta_2$. A significant difference of the two-dot system from a single-dot one, is that *two* conditions (15) on a *single* parameter V_g should be met simultaneously. This becomes increasingly difficult when T is low enough and C_1/C_2 is an irrational number. For significantly different capacitances, $C_1 \ll C_2$, conditions (15) for two different temperature regimes are demonstrated in Figs. 3(a) and 3(b). At intermediate temperatures,

$$\Delta_2/2 \le T \ll \Delta_1/2 , \tag{16}$$

the energy spectrum for the larger dot 2 can be regarded as continuous [Fig. 3(a)]. That is why the total resistance R is small each time V_g is in the vicinity of *any* step of a sparsed "ladder" corresponding to the smaller dot 1 [Fig. 3(a)]. In other words, when condition (16) is met, the discreteness of charge on dot 2 is not important, and dot 2 is equivalent to a massive lead. As a result, the total resistance is given by the sum of R_3 (Fig. 2) and the resistance of a single-dot system like that shown in Fig. 1(b), with the capacitance C_0 replaced by C_1. (The values R_1, R_2, and R_3 shown in Fig. 2 denote the tunnel resistances of corresponding junctions.) The expression for the resistance of a single-dot system was derived in Ref. 5. After adding the constant term R_3, this result takes the form

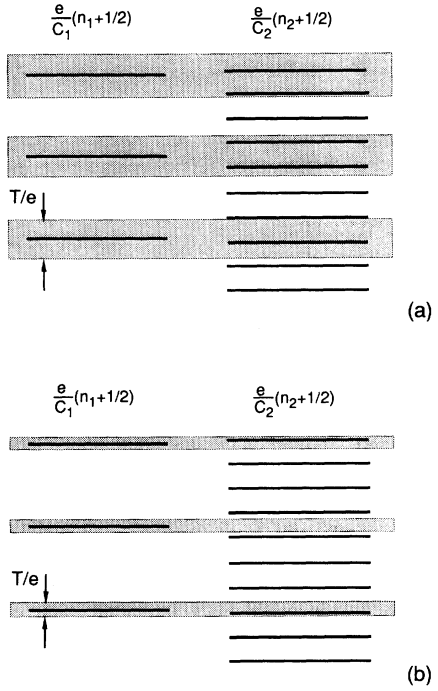


FIG. 3. The ladder of V_g values favoring the tunneling into dots, $C_1 \ll C_2$. If two rungs of different ladders belong to the same strip, charge easily passes through the system. (a) $T \ge e^2/C_2$, (b) $T \ll e^2/C_2$, the system of favorable V_g becomes sparsened.

$$R(V_g,T)=2(R_1+R_2)\sum_n \left[\frac{k_B T}{eV_g-(n+1/2)\Delta_1} \sinh\left[\frac{eV_g-(n+1/2)\Delta_2}{k_B T} \right] \right] + R_3 . \tag{17}$$

Hence, the total resistance $R(V_g)$ experiences sharp periodic oscillations with the period e/C_1.

However, in the low-temperature limit given by inequality

$$T \ll e^2/2C_2 , \tag{18}$$

only some of the levels of dot 1 are available because of restrictions imposed by condition (15b), see Fig. 3(b). Hence, in the temperature regime (18), some of the peaks that are present in (17) still show up, but the number of nonsuppressed peaks is reduced at low temperatures, the system of peaks becoming increasingly sparsed.

III. STRUCTURE OF A SINGLE PEAK AT LOW TEMPERATURES

Here we discuss the structure of a particular peak in the conductance that "survives" under condition (18). As is obvious from the previous discussion, this requires anomalous closeness between two rungs belonging to two "ladders," as shown in Fig. 3(b). Hence, we will calculate the conductance assuming that, for a particular pair (n_1, n_2) under consideration, the condition

$$|(n_1+\tfrac{1}{2})\Delta_1-(n_2+\tfrac{1}{2})\Delta_2| \ll \Delta_2 \tag{19}$$

is satisfied and hence conductance may not be suppressed even at low temperatures (18). Because we are studying

the peak in $G(V_g)$, both deviations

$$v_1 = -eV_g + (n_1 + \tfrac{1}{2})\Delta_1 ,$$
$$v_2 = -eV_g + (n_2 + \tfrac{1}{2})\Delta_2 , \tag{20}$$

which show the difference between the current value of V_g and two closest rungs of two ladders [Fig. 3(b)], are also assumed to be small,

$$|v_1|, |v_2| \ll e^2/C_2 . \tag{21}$$

Calculating the resistance of three junctions (Fig. 2) connected in a series, we assume that tunneling does not violate thermal equilibrium in each dot. In other words, electrons are thermalized after each act of tunneling, so that subsequent acts are completely incoherent. This means that the resistances of three junctions add to each other. To determine each resistance, one has to calculate the current I caused by a small bias V applied to a junction. This calculation is a straightforward one at low temperatures (21) because the current is due to transitions between states with only two different charges for each dot. For example, transitions $n_1+1 \rightarrow n_1$ and $n_1 \rightarrow n_1+1$ determine the current between the lead and dot 1:

$$I = -\frac{eV}{R_1}\int d\varepsilon_1 \int d\varepsilon_2 [w_1 + (1-2w_1) f(\varepsilon_2)] \times \frac{\partial f}{\partial \varepsilon_1}\delta(\varepsilon_1 - \varepsilon_2 - v_1) . \tag{22}$$

Here R_1 is a tunnel resistance for the junction between the lead and dot 1, $\varepsilon_{1(2)}$ are kinetic energies of an electron in the lead and dot 1, $f(\varepsilon)$ is the Fermi distribution function,

$$w_1 = \frac{1}{1+\exp(-v_1/k_BT)} \tag{23}$$

is the probability for dot 1 to have a charge en_1. Expression (22) follows directly from the "golden rule" if one assumes that dot 1 can be only in two charge states. The use of the Fermi distribution $f(\varepsilon)$ for a given number of electrons in the dot (i.e., for a fixed charge of dot) is justified when this number is large. We have also allowed the spacing of the kinetic energy in dot 1 to be much less than the Coulomb spacing e^2/C_1. Integration in (22) brings us the following result for the resistance of this junction, affected by the Coulomb blockade:

$$\mathcal{R}_1(T, v_1) = R_1 \frac{2k_BT}{v_1}\sinh(v_1/k_BT) . \tag{24}$$

The resistance of a junction between dot 2 and the lead (i.e., junction 3) can be obtained by substitution $R_1, v_1 \rightarrow R_3, v_2$ in (24). Calculation of the resistance between the dots is also similar; the only difference is that, instead of (23), a probability of having simultaneously given charges $e(n_1+1)$ and en_2 on the dots should be used. The final answer for the sum of resistances of three junctions has the form

$$\mathcal{R}(T, v_1, v_2) = R_1 \frac{2k_BT}{v_1}\sinh(v_1/k_BT) + R_2 \frac{2k_BT}{v_2 - v_1}\{\sinh[(v_2-v_1)/k_BT] + \sinh(v_2/k_BT) - \sinh(v_1/k_BT)\} + R_3 \frac{2k_BT}{v_2}\sinh(v_2/k_BT) . \tag{25}$$

The resulting resistance (25) has, at low temperatures and apart from a prefactor, an activated form, $\mathcal{R} \propto \exp(-\varepsilon/k_BT)$, with the activation energy ε given by

$$\varepsilon = \max(\varepsilon_1, \varepsilon_2, \varepsilon_3) , \tag{26}$$

where activation energies $\varepsilon_1, \varepsilon_2, \varepsilon_3$ correspond to the resistances $\mathcal{R}_1, \mathcal{R}_2, \mathcal{R}_3$ defined by three terms in Eq. (25), respectively. For the first and third term in Eq. (25), these energies are given by $\varepsilon_1 = |v_1|$ and $\varepsilon_2 = |v_2|$. For the second term we have

$$\varepsilon_2 = \max(|v_1|, |v_1 - v_2|, |v_2|) . \tag{27}$$

As a result, for ε from Eq. (26) we get $\varepsilon = \varepsilon_2$, i.e., the transport is always dominated by the dot-dot resistance $\mathcal{R}_2$, and the net activation energy is given by Eq. (27).

Hence, as it follows from Eqs. (20) and (27), the dependence $\ln G \equiv -\ln \mathcal{R}$ vs V_g should have a flat top in the interval

$$\min[(n_1+\tfrac{1}{2})\Delta_1, (n_2+\tfrac{1}{2})\Delta_2] < eV_g < \max[(n_1+\tfrac{1}{2})\Delta_1, (n_2+\tfrac{1}{2})\Delta_2] , \tag{28}$$

where the activation energy ε is constant and equals to

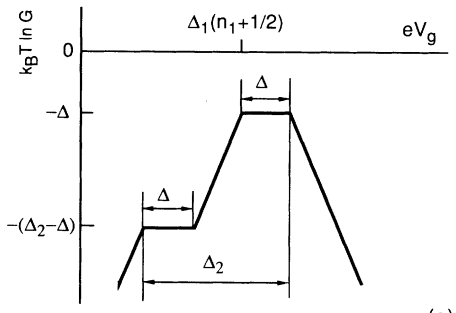
$$|v_1 - v_2| = |(n_1+\tfrac{1}{2})\Delta_1 - (n_2+\tfrac{1}{2})\Delta_2| \equiv \Delta . \tag{29}$$

The conductance drops exponentially in V_g on both sides of this plateau, as is shown in Fig. 4(a) for the case $(n_1+\tfrac{1}{2})\Delta_1 < (n_2+\tfrac{1}{2})\Delta_2$.

As we shall show, the transitions between the same charge states (n_1, n_1+1 and n_2, n_2+1) determine the conductance for the interval of gate voltages corresponding to $|v_2| < \Delta_2 - \Delta$. Beyond this interval, another charge state of dot 2, n_2+2 or n_2-1, becomes important de-

pending on which energy is larger, $(n_1+\frac{1}{2})\Delta_1$ or $(n_2+\frac{1}{2})\Delta_2$, so that plot G vs V_g has another plateau with a larger activation energy $\varepsilon=\Delta_2-\Delta$ [Fig. 4(b)]. In order to demonstrate the existence of this auxiliary plateau, one can, of course, repeat the calculations cited above taking into account six instead of four charge states of the two-dot system. This method, however, becomes increasingly tedious for more complex configurations of dots. Here we demonstrate another approach which allows one to account easily for all relevant (at low temperature) states of the system and to operate directly with activation energies instead of derivation of the resistance *ab initio*.

At first, we note that Eq. (27) coincides with the well-known expression for the activation energy of the equivalent resistance between two sites in a hopping system of noninteracting electrons.[10] In order to make this analogy complete, we consider the system of one-electron levels shown in Fig. 5. Here the filled and empty states of the level 1 (or level 2) correspond, at definition, to the charges n_1+1 and n_1 (or n_2+1 and n_2), respectively.



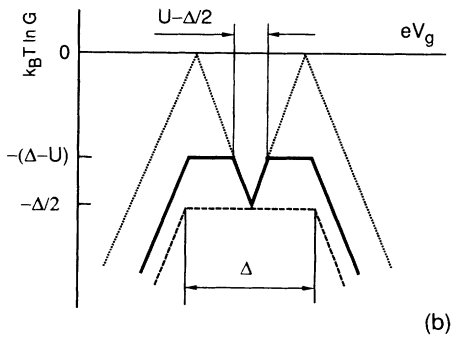
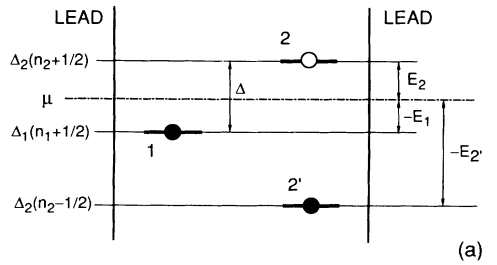


FIG. 4. Structure of a single conductance "peak" G vs V_g at low temperatures for a double-dot device. Here $\Delta_1=e^2/C_1$ is supposed to be much larger than $\Delta_2=e^2/C_2$. The conductance G is normalized to its high-temperature value. (a) "Peak" shape in the absence of interdot and dot-to-lead capacitances ($C=0$, Fig. 2), Δ is given by Eq. (29). (b) Transformation of the upper plateau in (a) due to the dot-dot Coulomb interaction ($C\neq 0$), cf., Sec. IV. Here Δ is the spacing between two levels shown in Fig. 6(a). The effective interdot repulsion energy $U\equiv e^2C/C_1C_2$ is inside the interval $\Delta/2<U<\Delta$. The dashed and dotted lines are plotted for the critical values $U=\Delta/2$ and Δ, respectively.



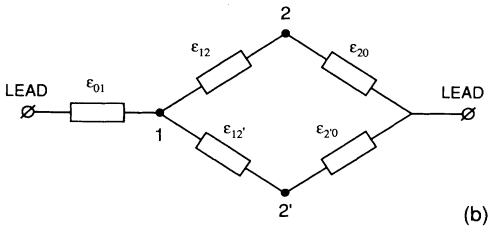


FIG. 5. (a) A diagram of the equivalent one-electron levels illustrating the structure of conductance "peak" in the two-body system plotted in Fig. 4. Absolute values of level energies shown in the left correspond to the degeneracy points in V_g of charge states of dots (Fig. 3). Open and solid circles display the levels filling in the ground state for the position of Fermi level $\mu\equiv eV_g$ shown by dash-dotted line in the figure. (b) Equivalent circuit of the hopping system. Activation energies of the resistances are given by (32).

The additional level 2′ (ascribed to the same site as level 2) is introduced to account for the charge state n_2-1 which corresponds to empty level 2′. The Fermi level μ is equal to eV_g, the absolute values of energies of all the levels being determined by corresponding points of charge degeneracy in V_g. Each of these energies counted from the Fermi level is equal to the work required to bring the next charge from a lead to the dot and is given by

$$\begin{aligned} E_1&=U(n_1+1,n_2)-U(n_1,n_2)\,,\\ E_2&=U(n_1,n_2+1)-U(n_1,n_2)\,,\\ E_{2'}&=U(n_1,n_2)-U(n_1,n_2-1)\,, \end{aligned} \tag{30}$$

that yields

$$\begin{aligned} &E_1=v_1,\quad E_2=v_2\,,\\ &E_{2'}=v_2-\Delta_2\,, \end{aligned} \tag{31}$$

for levels 1, 2, and 2′, respectively. To be consistent, we should prohibit level 2 being filled when 2′ is empty. However, lifting this restriction does not change the low-temperature results in which we are interested.

Obviously, the transport in the system occurs either via levels 1 and 2 (path I) or via 1 and 2′ (path II). Now, with the use of the diagram shown in Fig. 5(a), and taking into account that the levels below the Fermi level $\mu=eV_g$ are almost always occupied and the levels above it are almost

always empty, the origin of the plateaus and slopes in Fig. 4(a) becomes obvious. When μ is situated between levels 1 and 2 (Fig. 5), the activated hop $1\rightarrow 2$ controls the transport (which, for certainty, is considered from the left lead to the right one). The activation energy ε is then equal to the difference E_2-E_1 which, of course, does not depend on the position of the Fermi level μ [the upper plateau in Fig. 4(a)]. Then, when μ is shifted below level 1, an electron has to pass through the system by two successive hops upward in energy, LEAD$\rightarrow 1\rightarrow 2$, followed by the final hop downward in energy from level 2 to the right lead. The highest energy is acquired by the electron on site 2. Hence, the value of ε is equal to $\varepsilon=E_2=v_2$ increasing with decreasing V_g. This manifests itself as the sloped segment between plateaus in the $\ln G$ vs V_g plot in Fig. 4(a). The lower plateau appears in the plot when μ is still between levels 1 and 2′ but already far enough from level 2 to make path II dominate. In this case, ε is determined by two successive activated hops LEAD$\rightarrow 1$ and $2'\rightarrow$LEAD, so that $\varepsilon=E_1+(-E_{2'})=\Delta_2-\Delta$.

The value of the activation energy ε for our system of levels at any μ can be readily evaluated in a formal way following the standard hopping recipe: to each pair of levels on adjacent sites a resistance with the activation energy

$$\varepsilon_{ij}=\max(|E_i|,|E_j|,|E_i-E_j|) \tag{32}$$

should be ascribed. Here E_i,E_j are the energies of levels i,j counted from the Fermi level. In our case, the energies of levels are given by Eqs. (31) (for leads, $E=0$ by a definition). The total resistance is determined by the equivalent circuit shown in Fig. 5(b). Because of the exponential difference between resistances, the net activation energy ε is equal to one of the energies ε_{ij} indicated in Fig. 5(b). As a result, we get

$$\varepsilon=\min(\varepsilon_{12},\varepsilon_{12'}), \tag{33}$$

where ε_{12} is given by the right-hand side of Eq. (27), and

$$\varepsilon_{12'}=\max(|v_1|,|v_1-v_2+\Delta_2|,|v_2-\Delta_2|)\ . \tag{34}$$

The final dependence $\ln G=-\varepsilon/k_BT$ vs V_g is shown in Fig. 4(a) for the case when level 1 is situated below and close to level 2 [as it is assumed in our level diagram in Fig. 5(a)]. In this case, as one can check by means of the same approach, other levels of dot 2 do not contribute to the low-temperature transport. (We recall that, in our consideration, the condition $C_1\ll C_2$ is supposed to be met.) The hopping approach suggested above turns out to be especially convenient when studying effects of interdot capacitance, which are the subject of the next section.

IV. EFFECTS OF COULOMB INTERACTION BETWEEN THE DOTS

The previous consideration was based on the Coulomb Hamiltonian (12) which was written in the zero-order approximation for small values of the interdot and interlead capacitances C (Fig. 2). Let us now return to the initial form of the Hamiltonian (8). Suppose the capacitance C is small but finite: $C\ll C_1,C_2$. Taking into account in Eq. (8) the first-order corrections in C leads to three kinds of changes with respect to the zero-order form (12).

First, the diagonal coefficients U_{11} and U_{22} become slightly different from $\Delta_1/2=e^2/2C_1$ and $\Delta_2/2=e^2/2C_2$, respectively. This causes only a minor renormalization of the spacing of both Coulomb "ladders" (Fig. 3) and cannot be important in the general case of incommensurate capacitances.

Second, the coefficients a_i in Eq. (8) are no longer equal to unity nor to each other: $a_i=1-C/C_i$ (in our case $C_2>C_1$ and $a_2>a_1$). The main effect of this is that the energy $U(n_1+1,n_2)-U(n_1,n_2+1)$ required to activate an electron from dot 1 to dot 2 becomes V_g dependent and changes as $eV_g(C/C_2-C/C_1)$. Because, as we have seen in Sec. III, this activation energy dominates the total conductance in the region of a "plateau" (Fig. 4), the latter acquires a small finite slope

$$\frac{\partial \ln G}{\partial V_g}=\frac{eC}{k_BT}\left[\frac{1}{C_1}-\frac{1}{C_2}\right]\ . \tag{35}$$

Below we shall neglect both of these effects and dwell on the most interesting features arising from a finite off-diagonal coefficient U_{12} in (8), which is equal to

$$U_{12}=U\equiv\frac{e^2C}{C_1C_2}\ , \tag{36}$$

the Coulomb Hamiltonian (8) taking the form

$$U(n_1,n_2)=\Delta_1\frac{n_1^2}{2}+\Delta_2\frac{n_2^2}{2}+Un_1n_2-eV_g(n_1+n_2)\ . \tag{37}$$

Consider again the equivalent system of one-electron levels in Fig. 5(a). Because we shall be interested in the fate of the upper plateau only [Fig. 4(a)], we restrict ourselves to levels 1 and 2. The important difference from the situation studied before is that the energy, e.g., of level 1 defined by the first equality in (30) and by Eq. (37), depends now on the charge state of dot 2: whether it is n_2 or n_2+1 (or, in "one-electron" language, whether level 2 is empty or filled). This is because the third term in the Hamiltonian (37) is equivalent to the Coulomb repulsion between electrons placed on these two levels. For definiteness, we denote now by E_1 the energy of level 1 for the state when level 2 is empty (i.e., charge n_2 on dot 2), and by E_2 the energy of level 2 for the state when level 1 is filled (charge n_1+1 on dot 1). This filling corresponds to the ground state of the system at the Fermi-level position shown in Fig. 6(a). As follows from Eqs. (30) and (37), E_1 and E_2 are equal to

$$\begin{aligned}E_1&=\Delta_1(n_1+\tfrac{1}{2})-eV_g+n_2U\ ,\\ E_2&=\Delta_2(n_2+\tfrac{1}{2})-eV_g+(n_1+1)U\ ,\end{aligned} \tag{38}$$

respectively.

As before, we shall consider an anomalously close pair of levels with the spacing $\Delta\equiv E_2-E_1\ll\Delta_2$. This condition is automatically met for "peaks" of the conductance surviving at sufficiently low temperatures. Before dis-

cussing transport in this system, we note that standard formalism of the hopping theory based on Eq. (32) does not help any more since (32) does not take into account the Coulomb correlations between electrons on different sites. However, the picture of one-electron states shown in Fig. 6(a) is still very useful because it permits one to monitor explicitly a charge passing through the system and, in this way, to evaluate the activation energy.

Suppose, at first, the repulsion energy U is very small, $U \ll \Delta$. Then, as in the previous discussion, the hop of an electron from level 1 to level 2 entirely controls the net activation energy ε provided the Fermi level μ is situated between the levels, i.e., $E_1 < 0$, $E_2 > 0$. After this hop, due to the change of the level occupations, the one-electron level 1 shifts up and the level 2 shifts down by U, as shown in Fig. 6(a). Then the hops 2→LEAD and LEAD→1 return the system to the initial state. Hence, the value of ε is equal to $\varepsilon = E_2 - E_1 - U$, where the last term can be understood as a negative energy of the exciton created in the act of the dominating hop 1→2. As a result, the $\ln G$ vs V_g plot reveals a plateau with the activation energy $\Delta - U$. Thus, a small interdot interaction does not result in a qualitative change of the "peak" shape.

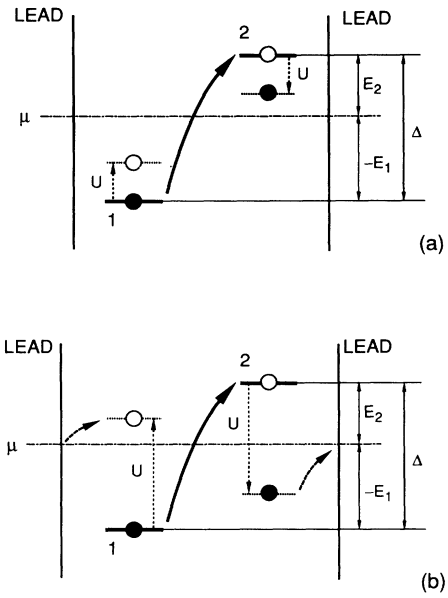


FIG. 6. Illustration of transport through a two-dot system with a finite cross capacitance C (Fig. 2). Two closest levels 1,2 belonging to the different dots and responsible for a conductance maximum $G(V_g)$ are shown [analogous to levels 1,2 in Fig. 5(a)]. Effective interdot repulsion energy $U \equiv C/C_1C_2$, where $C \ll C_1, C_2$. Open and solid circles denote empty and filled levels, respectively. The Fermi level $\mu = eV_g$ is shown by a dash-dotted line. (a) $U < \Delta/2$. The only activated hop 1→2 controls the conductance. Coulomb shifts of levels after this hop are shown by dotted lines and arrows. (b) $U > \Delta/2$ and μ is close to a middle point between levels. As a result of Coulomb shifts, both levels cross μ. An additional activated hop is needed to complete the charge transfer (one of those that are shown by dashed arrows).

Suppose now that $U > \Delta/2$ and μ is approximately equidistant in energy from both levels, i.e., $-E_1 \approx E_2$. It is easy to see that, in this case, after the first hop 1→2 takes place, level 2 "sinks" under the Fermi level, and level 1 ascends above it [Fig. 6(b)]. Now, in order to continue the charge transfer, another *activated* hop is necessary: Whether the same electron should hop from "2" to the lead with the energy increase $U - E_2$, or another electron hops from the left lead to "1" with the energy increase $U + E_1$ [two dashed arrows in Fig. 6(b)]. The final hop (LEAD→1 or 2→LEAD, respectively) returns the system to the initial state and proceeds with energy decreasing. One of these two processes, which is the cheapest in terms of energy, is responsible for the charge transfer in the system. The net activation energy is given by the sum over two hops

$$\begin{aligned}\varepsilon &= E_2 - E_1 - U + \min(E_1 + U, U - E_2) \\ &= \min(-E_1, E_2) \, . \end{aligned} \tag{39}$$

Thus, the value ε depends on each energy E_1, E_2, counted from the Fermi level, separately rather than on their difference and hence is μ dependent. Expression (39) is obviously valid until both arguments in the first minimum are positive, i.e., until the following condition holds:

$$|\mu - \mu_0| < U - \frac{\Delta}{2} \, , \tag{40}$$

where μ_0 is the middle point between levels 1 and 2. Outside the interval (40) for μ, the only activated hop is again 1→2, and ε is equal to $\Delta - U$.

The resulting G vs V_g dependence is shown in Fig. 4(b). The flat top of the conductance maximum is now split consisting of two identical plateaus separated by a valley with the depth $(U - \Delta/2)/k_BT$. The valley appears for the first time at the critical value of the repulsion energy $U = \Delta/2$, at the center of the plateau. With U increasing, it expands until, in the limit $U = \Delta$, it occupies the whole width of the incipient plateau. In this limit, the valley is surrounded by two sharp peaks with $\varepsilon = 0$ at their maxima. [Obviously enough, the case when $U > \Delta$ is impossible, in principle, because it would mean a negative activation energy ε. In other words, the state shown in Fig. 6(b) would not be a ground state of the two-level system: bringing the electron from "1" to "2," one would get a gain in the energy and would obtain the true ground state with the new spacing $2U - \Delta < U$ between the levels.]

As we already mentioned in previous sections, the temperature decrease selects the conductance peaks with smaller and smaller activation energy. As becomes clear from the present discussion, at very low temperatures, $T \ll U/k_B$, only those sparse pairs of levels will reveal themselves in the G vs V_g plot for which the condition $\Delta - U \leq k_BT$ is met. It means that all survived peaks will look like that shown in Fig. 4(b) by a dotted line: They will be split in V_g by a constant quantity $\delta V_g = U$ which neither varies from one peak to another nor depends on V_g: as is clear from its definition (36), U is determined by the geometry of the conducting channel only. Such a behavior was experimentally observed in Ref. 11.

V. CROSSOVER FROM THE STOCHASTIC COULOMB BLOCKADE TO PERIODIC OSCILLATIONS

In Secs. II–IV we studied analytically two limiting temperature regimes of oscillations $G(V_g)$ for the case of strongly different capacitances, $C_1 \ll C_2$. Two questions arise in this context: (1) How robust are these results regarding the increase of the ratio C_1/C_2? (2) How does the crossover in temperature between stochastic and periodic oscillations in $G(V_g)$ occur?

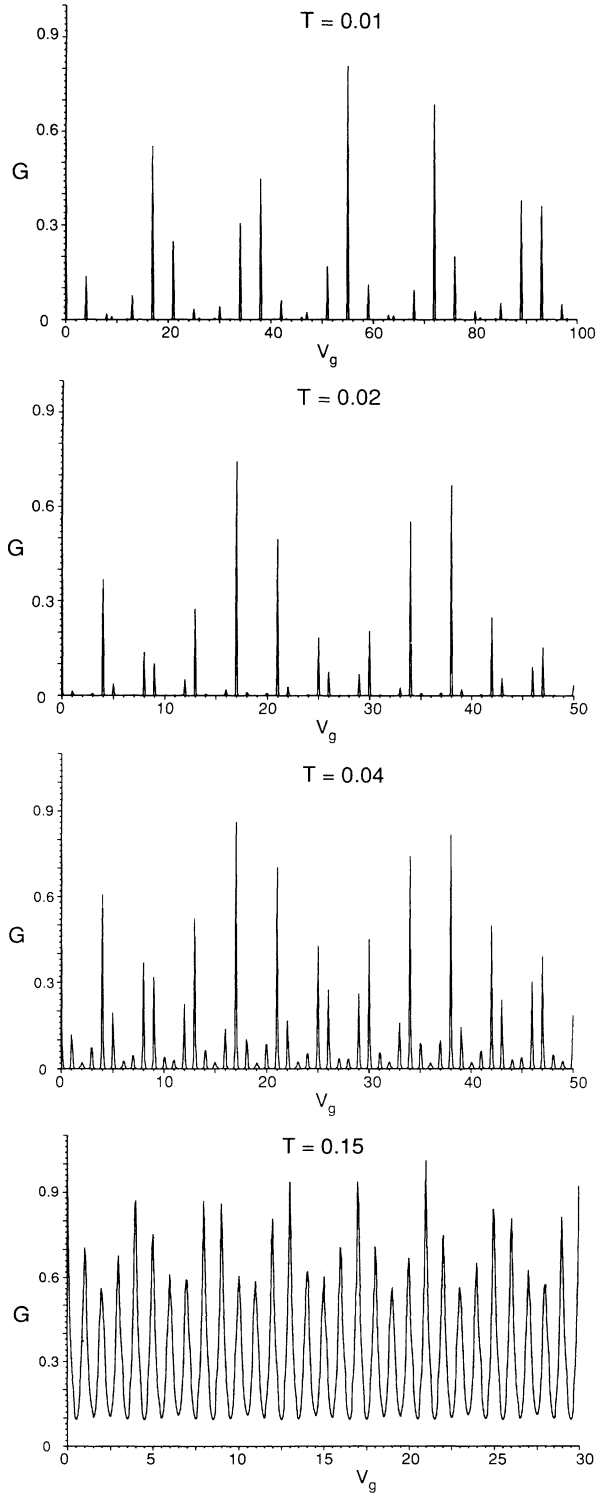


FIG. 7. Peak structure G vs V_g at four different temperatures T. Conductance G is plotted in arbitrary units which are the same for all four plots; V_g and T are measured in units of e/C_1 and e^2/C_2k_B, respectively.

As follows from the discussion in Sec. IV, a small cross capacitance C is important only in the limit of very low temperatures when it causes splitting of the conductance peaks. That is why, in answering on the above questions, we neglect with C. We have made numerical simulations of $G(V_g)$ for a number of ratios $C_1/C_2=\Delta_2/\Delta_1$ at different values of normalized temperature, T/Δ_2. We find that the above-presented classification of conductance oscillations holds for $C_1/C_2 \leq 0.4$, i.e., in a surprisingly large interval of values C_1/C_2. As an example for an "irrational" value $C_1/C_2=0.36\ldots$ is given in Fig. 7 for four different temperatures. The scale for the conductance G is the same on all four plots, and it is easy to see that a few peaks are almost temperature independent. The density of high peaks (with $G \geq 0.5$ at their maxima) grows with the temperature (linearly at small T) and saturates at $T \simeq 0.4\Delta_1/k_B$ reaching the maximal value: one peak per interval $\Delta V_g = e^2/C_1$. Peaks at the mentioned temperatures are still very well pronounced: the peak-to-valley ratio for the conductance is equal to 10.

The oscillation patterns shown in Fig. 7 are obtained in a simple approximation which accounted for the hops via two rungs of the sparse ladder and four rungs of the dense one, see Fig. 3. The equivalent resistance for each pair of levels was calculated with the use of formula (25).

In addition, direct numerical simulations using a Monte Carlo method were also performed. For a small interdot capacitance C, results are shown in Fig. 8(a); the traces obtained are similar to those in Fig. 7 which are found by simplified calculations.

We have also simulated the case of a large capacitance $C \gg C_1, C_2$, which is relevant to experiments.[8] The temperature region for a pronounced Coulomb blockade effect is determined only by C. At the same time, there are two more scales e/C_1, e/C_2 which govern the dependence of the activation energy on V_g. At intermediate

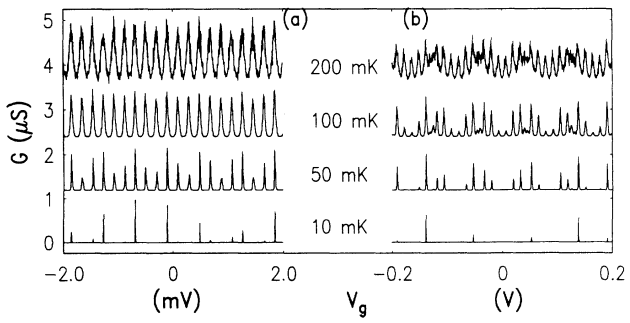


FIG. 8. (a) Numerical simulation of the conductance vs V_g of a double-dot device with $C \ll C_1, C_2$ at four temperatures. The parameters used in simulation are $C=11.2$ aF, $C_1=0.82$ fF, $C_2=4.36$ fF. Each trace is offset from the previous one for clarity. (b) Numerical simulation with $C \gg C_1, C_2$. The parameters are $C=0.55$ fF, $C_1=11.2$ aF, $C_2=1.92$ aF.

temperatures, $T \leq 0.6e^2/Ck_B$, both scales show up in the oscillatory pattern. Unlike the case of a small C, here the basic distance between the conductance peaks is determined by the *smaller* of these two scales. If $C_1 \ll C_2$, the larger scale appears as a modulation of peak heights, see Fig. 8(b).

VI. THERMOFINGERPRINTS OF THE STOCHASTIC COULOMB BLOCKADE

At low temperatures, conductance has an activated temperature dependence under the conditions of Coulomb blockade. For a single-dot device, the activation energy depends on the gate voltage linearly within the period, changing from zero to $e^2/2C$. So the distribution function $\rho(\varepsilon)$ of the activation energies ε determined from the *overall* dependence $G(V_g,T)$ (not only from peaks and valleys) is a constant in the interval $0<\varepsilon<e^2/2C$ and zero for $\varepsilon \geq e^2/2C$. For a double-dot device, the situation is quite different. We can again introduce a "period" in V_g as an interval $(e/C_1)n < V_g < (e/C_1)(n+1)$. The dependences of activation energy $\varepsilon(V_g)$ are similar in different "periods," but this energy does not reach zero value on each period, the minimal value being determined by Δ, Eq. (29). According to the definition, the distribution function is

$$\rho(\varepsilon)=\frac{1}{W}\int_0^W d(eV_g)\delta(\varepsilon-\varepsilon(V_g)), \quad W\to\infty \tag{41}$$

or

$$\rho(\varepsilon)=\left\langle \left| \frac{d\varepsilon}{d(eV_g)} \right|^{-1} \right\rangle , \tag{42}$$

where the angular brackets mean averaging over a large number of "periods." As it follows from Sec. III and Fig. 4(a), for each period

$$\left| \frac{d\varepsilon}{d(eV_g)} \right|^{-1} = \Theta(\varepsilon-\min(\Delta,\Delta_1-\Delta)) + (\delta(\varepsilon-\Delta)+\delta(\varepsilon+\Delta-\Delta_2))\Delta , \tag{43}$$

where Θ is a unit-step function. The distribution function of Δ over different "periods" depends on the algebraic properties of the ratio $C_1/C_2=\Delta_2/\Delta_1$. For an irrational C_1/C_2, the value of Δ is equally distributed in the interval $0<\Delta<\Delta_2/2$. In this case, the averaging of (43) is straightforward and gives

$$\rho(\varepsilon)=\frac{2}{\Delta_1+\Delta_2/2}\begin{cases} 4\varepsilon/\Delta_2 & \text{if } \varepsilon<\Delta_2/2 \\ 1+2(1-\varepsilon/\Delta_2) & \text{if } \Delta_2/2<\varepsilon<\Delta_2 \\ 1 & \text{if } \Delta_2<\varepsilon<\Delta_1/2 \\ 0 & \text{if } \varepsilon>\Delta_1/2 . \end{cases} \tag{44}$$

Note, that $\rho(\varepsilon)$ has a characteristic peak at $\varepsilon=\Delta_2/2=e^2/2C_2$ (see Fig. 9). This feature in $\rho(\varepsilon)$ exists as long as C_2/C_1 is a noninteger number. The zero value $\rho(0)=0$, however, holds only for irrational C_1/C_2.

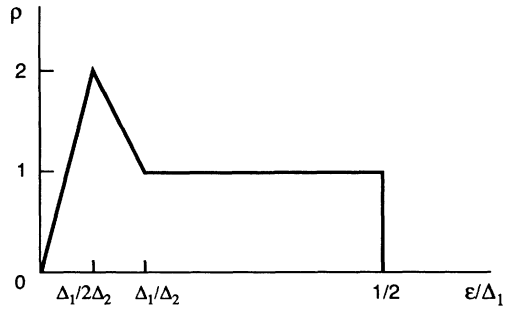


FIG. 9. Distribution density ρ of activation energies for the G vs V_g dependence shown in units of $\Delta_1/2+\Delta_2/4$, see Eq. (44), where $\Delta_1=e^2/C_1$ and $\Delta_2=e^2/C_2$. An irrational value of C_1/C_2 is assumed.

VII. CONCLUSIONS

We have studied Coulomb blockade in two dots connected in a series. It turns out that the behavior of the linear conductance G of this system is entirely different from that of a single dot: At low temperatures, the periodic structure in the G vs gate voltage gives way to a system of random peaks. At first glance, emerging patterns of $G(V_g)$ (see Fig. 7) are similar to those observed in quasi-one-dimensional hopping transport systems:[12] Peaks in $G(V_g)$ become more sparse as the temperature is lowered (the average distance between peaks grows as $1/T$), different peaks decaying with different activation exponents. However, there are strong differences even in this low-temperature regime between these two types of fluctuations. In a double-dot system, each particular peak does not significantly change its position when the temperature is changed. Distances between peaks are proportional to a "basic segment" with a length determined by the capacitance of the smaller dot. In addition, the distribution functions of $\ln G$ are entirely different. For a hopping system, such a function is a smooth one;[13,14] two-dot structure has a distribution function exhibiting a sharp feature. The position of this feature depends on the capacitance of the larger dot. Finally, in the studied system, the absolute derivative $d\ln G/dV_g$ for sloped parts of the peak structure does not change from one peak to another. In a hopping system, this slope is random and fluctuates for different peaks a few times.[12] This effect has been shown to result from Coulomb shifts of energies of localized states responsible for the transport in the course of recharging of other states randomly distributed in the sample.[15]

Above we assumed that the difference between capacitances C_1 and C_2 is large. If these values are, on the contrary, almost equal to each other, another effect occurs: $G(V_g)$ exhibits beatings with the period larger than the period of "filling oscillations;" the ratio of periods is equal to $C_1/|C_1-C_2|$. This kind of pattern was observed experimentally on a system of (almost) equivalent dots in Ref. 16, see upper inset in Fig. 2 therein. Analysis of this figure implies $|C_1-C_2| \simeq 0.1C_1$.

We have studied in detail the case of relatively small

interdot capacitance, $C \ll C_1, C_2$. This is adequate to the experimental realizations.[7,9,11] Though small values of C do not lead to drastic changes in the $G(V_g)$ dependence in the high-temperature part of the stochastic regime, there is one qualitative consequence of nonzero C that can be revealed at low temperatures: The interdot Coulomb interaction generates a splitting of those conductance peaks that survive at low temperatures [see Fig. 4(b)]. This kind of behavior was observed in a number of traces measured on GaAs heterostructures.[11]

The results can be generalized in the case of large $C > C_1, C_2$. Our conclusions about stochastic oscillations at low temperatures based on Eqs. (8)–(10) remain correct and are supported by numerical simulation as well. However, periodic structure in $G(V_g)$ at intermediate temperatures is no longer perfect: The system crosses over from almost V_g-independent conductance at high temperatures directly to oscillations with fluctuating values of its maxima at low T. It is noteworthy also that conductance peaks in the case under discussion do not have flat tops even at $T \to 0$, see Sec. V. This type of $G(V_g, T)$ dependence was observed on some samples in the experiments on In_2O_{3-x} wires.[8]

We have discussed purely incoherent electron transport in the system. Coherent tunneling should modify conductance at the highest peaks that are formed by almost perfect resonance between the electron states of both dots. As a result, in the limit of very low temperatures, the peak conductance increases and may exceed its high-temperature value.

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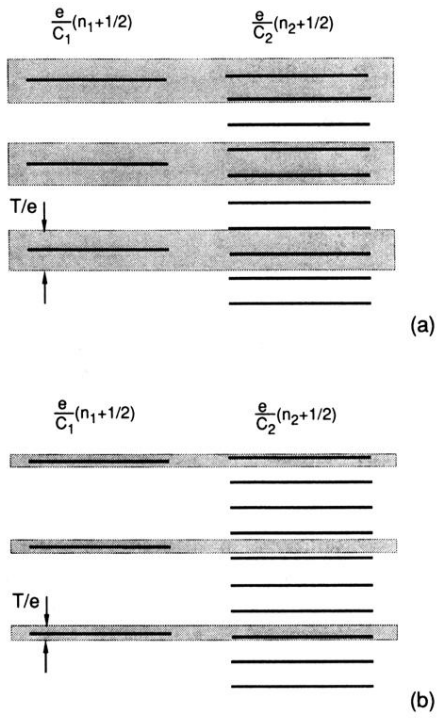

FIG. 3. The ladder of V_g values favoring the tunneling into dots, $C_1 \ll C_2$. If two rungs of different ladders belong to the same strip, charge easily passes through the system. (a) $T \geq e^2/C_2$, (b) $T \ll e^2/C_2$, the system of favorable V_g becomes sparsened.