Intermodulation in the oscillatory magnetoresistance of a two-dimensional electron gas

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The oscillatory magnetoresistance wave form of a two-dimensional electron gas shows multiple structures when two subbands are populated. In addition to high-field oscillations at a frequency equal to the sum of the two frequencies corresponding to the concentrations of the subbands, and to a superposition at intermediate fields, we observed oscillations at the difference frequency at low fields and higher temperatures. The field range at which the frequency difference is observed increases with increasing temperature. The crossover from superposition to frequency difference is accompanied by a beat. Similar beats, whose field location shows identical temperature dependence, can be observed in data obtained by other groups on different structures. The various components of the wave form can be attributed to different phase relations between the diagonal and off-diagonal elements of the conductivity tensor. It is shown how the intermodulation term, when inserted into the extended oscillatory equation, can give rise to all three structures.

INTRODUCTION

The oscillatory pattern of magnetoresistance, the Shubnikov–de Haas (SdH) effect, is extremely useful in the characterization of a two-dimensional electron gas (2DEG). As a function of the inverse magnetic field, the frequency of oscillation is linearly proportional to the concentration of the carriers generating the given oscillations. In addition, the decay of the oscillation amplitude with increasing temperature at a given magnetic field or with decreasing field at a given temperature allows for the determination of the effective mass or the quantum relaxation time within the subband. The wave form becomes complex when either more than one subband is populated, or more than one carrier type takes part in the conduction process. This complex SdH pattern is generated by the presence of two Landau-level ladders, one for each carrier type or subband. At large enough magnetic fields, when all the levels are sharp, a single oscillatory wave form is obtained, the frequency of which corresponds to the sum of the concentrations of the various carriers.[1] At lower fields, a superposition of the two frequencies is usually observed, whereby the oscillations at the higher frequency, corresponding to the ground subband concentration, f_1, are modulated by the lower oscillation frequency associated with the excited subband f_2.[2,3]

An abrupt change in the frequency of magnetoresistance oscillations as a function of the inverse magnetic field is encountered under various conditions. Most common is a doubling of the frequency when the spin degeneracy is removed, usually at fields of several teslas.[4,5] For some particular structures, a frequency shift was observed due to the different types of carriers present in the 2D gas. When an asymmetric quantum-well structure was investigated in $Ga_{1-x}Al_xAs/GaAs$ heterojunction,[4] the frequency of oscillation increased by a factor of 3.4 from low to high fields. The oscillations at low fields were attributed to the light holes only, for which the lower effective mass m^* results in a larger cyclotron-resonance frequency $\omega_c = qB/m^*$, where B is the magnetic field. Since the separation between the Landau levels is $\hbar\omega_c$, only the ladder produced by the light holes generates oscillations detectable at low fields. In a second paper,[5] a similar asymmetric well was investigated in a GaSb/AlSb system. Here, too, the oscillation frequency of the SdH pattern recorded for this structure increases rather abruptly. However, the change in frequency is much smaller than in the previous sample, only a factor of 1.27. The authors attributed the low-field oscillations to the heavy holes.

In this paper, we report the observation of a third SdH frequency recorded on a modulation-doped field-effect transistor (MODFET) structure when two subbands are populated. At lower fields, a single intermediate frequency f_I is present, while at higher fields a higher frequency f_H prevails, modulated by a third, much lower frequency, f_L. This feature is temperature dependent, i.e., the crossover magnetic field between f_I and f_H decreases with decreasing temperature such that at very low temperatures the intermediate frequency is absent. The theory behind these phenomena is discussed. Various possible mechanisms are investigated, and it is shown that the presence of three frequencies and their temperature dependence is closely related to the intermodulation between the subbands, recently introduced by Coleridge.[3]

Experimental Results

The sample measured in this paper is a MODFET structure composed of an $Al_xGa_{1-x}As/GaAs$ heterojunction with $x = 0.3$. The $Al_xGa_{1-x}As$ barrier is 500-Å layer doped with Si (3×10^{18} cm^{-3}) and a 60-Å undoped spacer. The 2DEG concentration of the sample at dark was measured by both Hall and SdH techniques, and was found to be 6.10×10^{11} cm^{-2} with a Hall mobility $\mu_H = 295\,000$ cm^2/V s at 4.2 K. After exposing the sample to light, the Hall concentration increased to

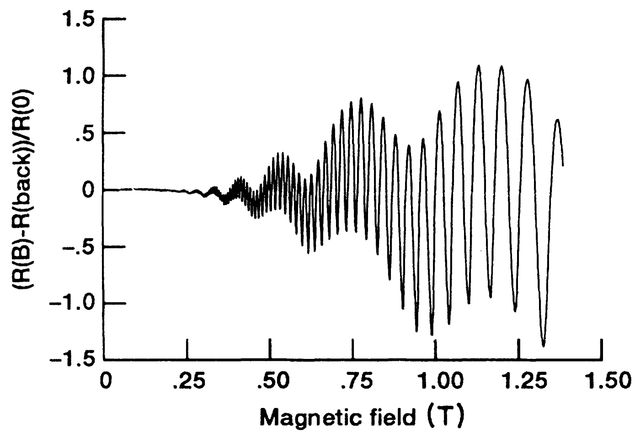

FIG. 1. SdH wave form taken at 1.45 K, showing the superposition of the two frequencies corresponding to two populated subbands.

1.066×10^{12} cm^{-2}, due to persistent photoconductivity, with a mobility of $\mu_H=415\,000$ cm^2/V s. The SdH wave form measured on the exposed sample at 1.45 K is presented in Fig. 1. A superposition of two frequencies can be clearly observed. The high frequency, $f_H=19.9$ T, corresponds to a 2DEG concentration of 9.65×10^{11} cm^{-2}, while the low frequency, $f_L = 1.83$ T, renders a concentration of 8.9×10^{10} cm^{-2}. The sum of these two concentrations equals the result obtained from the Hall experiment. At this temperature, these two frequencies prevail throughout the experimentally available magnetic field, i.e., 1.4 T.

As the temperature is increased, a third frequency of 18.1 T is observed at the lower magnetic fields. This intermediate frequency f_I is equal to the difference between the two previous frequencies, i.e., $f_I=f_H-f_L$. The three frequencies f_L, f_I, and f_H are observed as three peaks in the fast Fourier transform (FFT) of Fig. 2. The presence of a difference frequency was recently reported by Coleridge.[3]

Figure 3 shows an expanded view of the SdH pattern as a function of the inverse magnetic field. The oscillations at f_H are observed at the left part of the wave form (low $1/B$), while f_I is at the right side. A beat appears at the crossover field. The indexed extrema are present in Fig. 4. Although the slopes are very close to one another, they are clearly distinct, and the two frequencies are properly resolved by the computer to render the above concentrations. Such intersects were employed in order to determine the crossover field at the various temperatures of measurement.

The increase in the crossover field with increasing temperature is shown in Fig. 5. The SdH spectrum measured at 1.6 K shows only one high frequency, namely f_H. (There is also a FFT peak at f_L, but this frequency is not relevant to the crossover estimate.) The reason is that the crossover field at this temperature should be about 0.3 T,

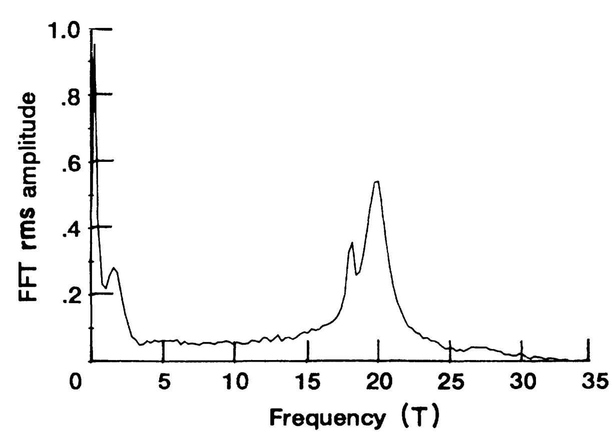

FIG. 2. FFT of the SdH pattern measured at 3.9 K, showing the peaks at f_H (19.9 T), f_L (1.8 T), and f_I (18.1 T).

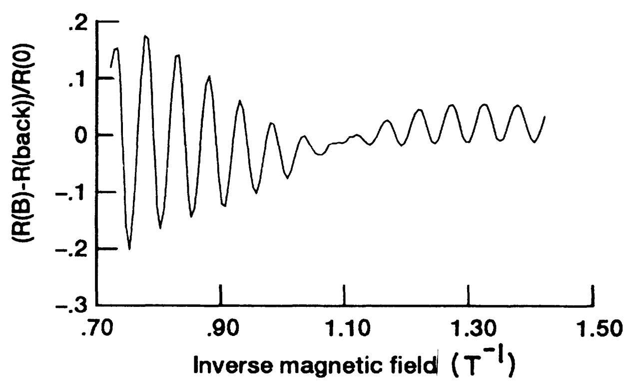

FIG. 3. SdH pattern vs the inverse magnetic field measured at 5.28 K. Two frequencies (f_H and f_I) are observed. A beat is clearly seen at the crossover point.

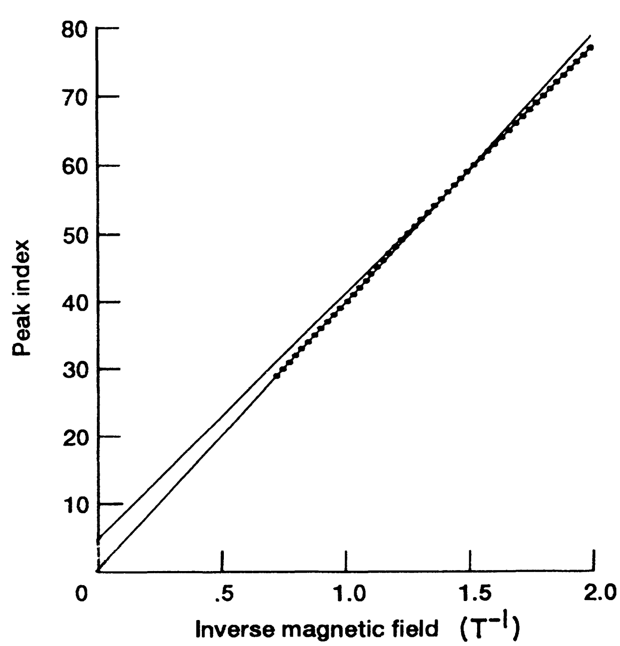

FIG. 4. Indices of extrema points in the experimental SdH pattern vs the inverse magnetic field at 4.6 K. Two slopes and a crossover are clearly observed.

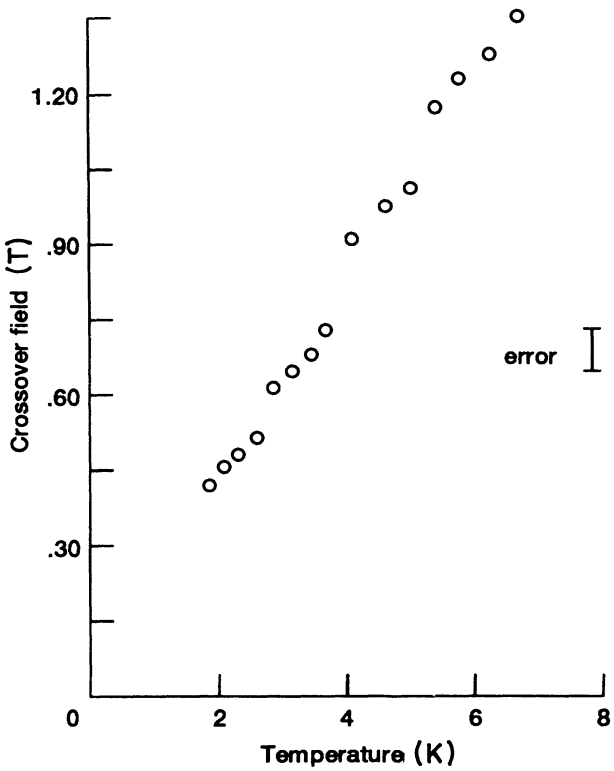

FIG. 5. Magnetic field at which a transition between f_H and f_I takes place vs temperature.

and at these low-field values the oscillatory magnetoresistance was not detectable. At the other extreme, the SdH spectrum measured at 9 K also shows only one high frequency; this time it is f_I. The reason this time is the crossover field is above 1.4 T, which is the maximum value attainable in our experimental setup. In between these temperatures, the amplitude of the FFT peak corresponding to f_H diminishes, while that due to f_I increases with temperature. It is possible to isolate f_H and f_I frequencies completely by obtaining the transforms of the lower fields and higher fields separately. The analysis of the SdH pattern was extended to temperatures as high as

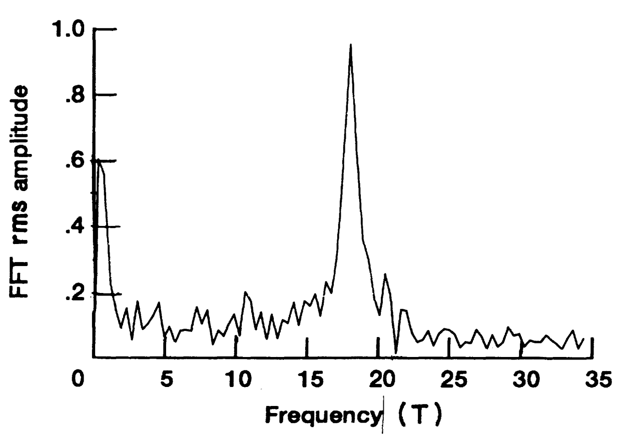

FIG. 6. FFT of the SdH wave form measured at 13 K, showing a single peak at 18.1 T.

15 K. A FFT of a wave form recorded at 13 K is shown in Fig. 6. The only peak observed is, as expected, at the intermediate frequency f_I.

Even though the temperature dependence of this effect has not been reported previously, a close examination of two recent publications which present SdH patterns recorded at various temperatures[2,3] shows beats at magnetic fields which are very close to the crossover fields shown in Fig. 5. Thus, it seems that the presence of an intermediate frequency and its temperature dependence are general phenomena, appearing in samples of different structures and composition.

DISCUSSION

The presence of oscillations at difference frequencies is not apparent from previous analyses. The fact that the two peaks of the FFT corresponding to f_H and f_I can be isolated by choosing appropriate field regions indicates that we are not dealing with a "conventional" wave form in which a superposition of two frequencies is observed.[6]

The interpretation of the data depends first and foremost on the way the three frequencies are associated with the carriers in the various subbands. While f_L definitely corresponds to the concentration of carriers in the excited subband f_2, there are two possible approaches to associate the two high frequencies with f_1. For the sake of the clarity we will call these interpretations model 1 and model 2. In model 1, f_H is associated with the 2DEG in the ground subband, i.e., $f_H = f_1$. Hence, for this model, f_I corresponds to the difference between the two concentrations, i.e., $f_I = f_1 - f_2$. Alternatively, in model 2, f_I is assumed to be generated only by the ground subband carriers, $f_I = f_1$, so that f_H corresponds to the sum of the concentrations $f_H = f_1 + f_2$. While for model 1 the sum of the concentrations derived from f_H and f_L agrees perfectly with the Hall data, it necessitates the introduction of a difference frequency into the system. The only previous report of such a frequency is that of Coleridge,[3] who attributes this frequency to a result of the intermodulation between the two subbands generating frequency modulation (FM). FM produces two side frequencies of equal amplitudes, one at the difference and the other at the sum frequency. However, no sum and difference frequencies are observed simultaneously at the *same* magnetic-field range.

According to model 2, f_H is due to the sum of the subband concentrations. The presence of oscillations at the sum frequency is documented and the theory is established.[1] However, the sum frequency usually appears as a single frequency at fields of several teslas, following a region of superposition of the two constituent frequencies. On our data, f_H is a part of a superposition wave form. More important, since this model assumes that $f_H = f_1 + f_2$, one concludes that at fields below the crossover point the oscillations are generated by the ground subband only, i.e., $f_I = f_1$. Thus, for model 2 to be valid it is essential to find the physical justification for the absence of oscillations due to the excited subband at these fields. This can be the case only if the Landau ladder produced by the excited subband is not sharp enough. One condition for clearly resolved SdH oscillations is that the separation between these levels, $\hbar\omega_c$, will be larger than the scattering-induced energy broadening of a single level. The width of each level, Γ, is related to the quantum scattering time τ_q by $\Gamma=\hbar/2\tau_q$. A second condition is that $\hbar\omega_c$ should be larger that the thermal energy broadening kT. Translated to magnetic fields, these conditions can be expressed as

$$\begin{aligned} &B > 8.64\times 10^3 (m^*/m_0)\Gamma\ , \\ &B > 0.744(m^*/m_0)T\ , \end{aligned} \tag{1}$$

where B is in teslas, Γ in eV, and T in kelvin. For GaAs, with $m^*/m_0=0.068$ one obtains

$$\begin{aligned} &B > 587\Gamma = 1.93\times 10^{-13}/\tau_q\ , \\ &B > 5.06\times 10^{-2} T\ . \end{aligned} \tag{2}$$

If both broadening mechanisms are significant and if they are not correlated, they can be combined to give the condition $\hbar\omega_c > [(kT)^2+\Gamma^2]^{1/2}$. The value of τ_q is derived from the decay of the SdH amplitude with decreasing magnetic field. Unfortunately, the calculated value of τ_q depends on the mathematical formalism assumed.[6,7] Typically τ_q is about 10^{-12} s for both the first and the excited subbands. Thus, the threshold of resolution is within the range of fields discussed in this work and it increases with temperature.

According to Eq. (1), it is possible that at lower magnetic fields the magnetoresistance oscillations due to the excited subband are not observed, if either the effective mass of electrons in the excited subband is substantially larger than that in the ground level, or the relaxation time τ_q is significantly shorter in the former, or both. Practically neither of these statements seems correct. All reported data indicate that the quantum relaxation time of electrons in the excited subband is three to five times larger than that in the ground level.[3,8] The difference between the effective masses of carriers in the two subbands is minimal and again seems to be in the opposite direction, i.e., it is larger for the ground level.[9] Moreover, even though Eq. (1) in the combined form $\hbar\omega_c > [(kT)^2+\Gamma^2]^{1/2}$ indicates that the crossover field should increase with temperature, the predicted increase is much less steep than that measured, as shown in Fig. 5. In addition, according to the present model, there is a discrepancy (of about 10%) between the SdH and the Hall data. Thus we rule out model 2, associating f_H with the sum of the concentrations.

Let us return to model 1, in which f_H corresponds to the concentration of the ground subband. To validate this model, it is essential to provide for a mechanism through which oscillations at the difference frequency $f_I=f_1-f_2$ can be generated. We intend to show that this is the result of the intermodulation which prevails when more than one subband is occupied.

The theory of Isihira and Smrcka[10] for the oscillatory behavior of conductivity under magnetic fields was extended by Coleridge to the case of two subbands.[3] Based on the introduction of intrasubband and intersubband scattering in the presence of carriers in two subbands, the oscillatory part ΔR_{xx} of the resistance R_{xx} (with a zero-field value of R_0) was expressed by[3]

$$\frac{\Delta R_{xx}}{R_0}=A_1\frac{\Delta g_1}{g_0}+A_2\frac{\Delta g_2}{g_0}+B_{12}\frac{\Delta g_1\,\Delta g_2}{g_0^2}\ . \tag{3}$$

Here g_0 is the zero-field density of states and Δg is its oscillatory part. A_1 and A_2 are the amplitudes of single-subband oscillations.

The intermodulation, which prevails when two subbands are occupied, is represented by the B_{12} term. We intend to show that this component incorporates several mixed conductivity terms leading to various oscillatory wave patterns. We start from the equations describing a material with a single carrier type. The conductivity tensor elements as obtained in Ref. 10 for this structure are

$$\begin{aligned} \sigma_{xx}&=\frac{\sigma_0}{1+\omega_c^2\tau_0^2}\left[1+\frac{2\omega_c^2\tau_0^2}{1+\omega_c^2\tau_0^2}\frac{\Delta g}{g_0}\right]\ , \\ \sigma_{xy}&=-\frac{\sigma_0\omega_c\tau_0}{1+\omega_c^2\tau_0^2}\left[1-\frac{3\omega_c^2\tau_0^2+1}{\omega_c^2\tau_0^2(1+\omega_c^2\tau_0^2)}\frac{\Delta g}{g_0}\right]\ , \end{aligned} \tag{4}$$

where σ_0 is the zero-field longitudinal conductivity and τ_0 is the mobility scattering time.[11] When the magnetoresistance in a sample with two subbands is analyzed, the two components of the conductivity tensor must be combined and the matrix inverted. This results in

$$\rho_{xx}=\frac{\sigma_{xx_1}+\sigma_{xx_2}}{(\sigma_{xx_1}+\sigma_{xx_2})^2+(\sigma_{xy_1}+\sigma_{xy_2})^2}\ . \tag{5}$$

The intermodulation component is obtained from the second derivative term in the expansion of the resistivity as series in $\Delta g/g_0$. To reduce the complexity of the following expressions, we will denote the nonoscillatory part of σ_{xx} as σ_x, that of σ_{xy} as σ_y $[\sigma_y=-\sigma_0\omega_c\tau_0/(1-\omega_c^2\tau_0^2)]$, and the denominator of Eq. (5) by D.

The mixed derivative provides the following term:

$$\begin{aligned} \frac{\partial^2\rho_{xx}}{\partial x_1\partial x_2}=\frac{1}{D^2}\Bigg[&-6(\sigma_{xx_1}+\sigma_{xx_2})(\sigma_{x_1}\sigma_{x_2}\alpha_1\alpha_2+\sigma_{y_1}\sigma_{y_2}\beta_1\beta_2)-2(\sigma_{xy_1}+\sigma_{xy_2})(\sigma_{x_1}\sigma_{y_2}\alpha_1\beta_2+\sigma_{x_2}\sigma_{y_1}\alpha_2\beta_1) \\ &+\frac{8(\sigma_{xx_1}+\sigma_{xx_2})}{D}[(\sigma_{xx_1}+\sigma_{xx_2})^2\sigma_{x_1}\sigma_{x_2}\alpha_1\alpha_2+(\sigma_{xy_1}+\sigma_{xy_2})^2\sigma_{y_1}\sigma_{y_2}\beta_1\beta_2 \\ &+(\sigma_{xx_1}+\sigma_{xx_2})(\sigma_{xy_1}+\sigma_{xy_2})(\sigma_{x_1}\sigma_{y_2}\alpha_1\beta_2+\sigma_{x_2}\sigma_{y_1}\alpha_2\beta_1)]\Bigg]\ , \end{aligned} \tag{6}$$

where i represents the subband $(i=1,2)$ and $\sigma_{xx_i}=\sigma_{x_i}(1+\alpha_i x_i)$; $\sigma_{xy_i}=\sigma_{y_i}(1+\beta_i x_i)$ with $x_i=|\Delta g_i/g_0|$; $\alpha_i=\alpha_{0_i}\cos(2\pi f_i/B+\phi_i)$; $\beta_i=\beta_{0_i}\cos(2\pi f_i/B+\theta_i)$. The temperature reduction factor and the exponential decay due to the quantum relaxation time τ_q are included in the oscillatory part of the density of states Δg_i, i.e., in x_i. One should note that both ω_c and τ_0 depend on the carrier type and therefore are different for the two subbands. In our case this prevents the very simple representation that was shown in the single-subband case.[10] No *a priori* assumption is made regarding the relative phase between the two oscillation phases ϕ_i and θ_i.

Equation (6) can be simplified assuming that $\omega_c\tau_0 \gg 1$ since τ_0 is the mobility scattering time not the quantum relaxation time[11] and is related to the electron mobility μ_e through $\omega_c\tau_0=\mu_e B$. In all high-mobility structures this condition is fulfilled at fields as low as a few tenths of a tesla. The relative magnitude of the various conductivity components should be examined. The ratio between the longitudinal and transverse nonoscillatory conductivities is well known, $\sigma_{y_i}=-\omega_c\tau_0\sigma_{x_i}$. It is important to notice that this is not the case for the oscillatory part of σ_{xx_i}, $\Delta\sigma_{xx_i}=\sigma_{xx_i}-\sigma_{xx_i}(\Delta g_i=0)$, according to the derivation of Isihara and Smrcka.[10] Rather, from Eq. (4) one obtains the inverse relationship, i.e., $\Delta\sigma_{xx_i}\approx\frac{2}{3}\omega_c\tau_0\,\Delta\sigma_{xy_i}$ for $\omega_c\tau_0\gg1$. Thus, for this approximation $\alpha\approx\frac{2}{3}\omega_c^2\tau_0^2\beta$. Hence, $\sigma_{x_i}^2\sigma_{x_j}\alpha_1\alpha_2$ is of the same order of magnitude as $\sigma_{x_i}\sigma_{y_1}\sigma_{y_2}\alpha_i\beta_j$, while $\sigma_{x_i}\sigma_{y_1}\sigma_{y_2}\beta_1\beta_2$ and $\sigma_{x_i}^4\sigma_{x_j}\alpha_1\alpha_2/D$ are smaller by a factor of $\omega_c^{-2}\tau_0^{-2}$ (j equals either 1 or 2 but is different from i). Since all the terms multiplied by $8(\sigma_{x_1}+\sigma_{x_2})/D$ are of the same order of magnitude, this component is negligible as well as the second term in the first set of parentheses. Therefore, Eq. (6) can be reduced to

$$\frac{\partial^2\rho_{xx}}{\partial x_1\,\partial x_2}\simeq\frac{1}{(\sigma_{xy_1}+\sigma_{xy_2})^4}\times[-6(\sigma_{xx_1}+\sigma_{xx_2})\sigma_{x_1}\sigma_{x_2}\alpha_1\alpha_2-2(\sigma_{xy_1}+\sigma_{xy_2})(\sigma_{x_1}\sigma_{y_2}\alpha_1\beta_2+\sigma_{x_2}\sigma_{y_1}\alpha_2\beta_1)]\ . \tag{7}$$

Thus, the mixed term in the expansion of the resistivity can be written as

$$\left.\frac{\partial^2\rho_{xx}}{\partial x_1\,\partial x_2}\right|_0 x_1x_2\simeq\frac{1}{(\sigma_{y_1}+\sigma_{y_2})^4}[-6(\sigma_{x_1}+\sigma_{x_2})\Delta\sigma_{xx_1}\Delta\sigma_{xx_2}-2(\sigma_{y_1}+\sigma_{y_2})(\Delta\sigma_{xx_1}\Delta\sigma_{xy_2}+\Delta\sigma_{xx_2}\Delta\sigma_{xy_1})]\ . \tag{8}$$

The first term represents a product of cosines at the two frequencies which is an amplitude modulation. This result is a superposition as described by Leadley *et al.*[2] and Coleridge.[3] The last two terms of Eq. (8) include the mixed oscillations due to the longitudinal conductivity of one subband, and the transverse conductivity of the other, and therefore are of equal amplitudes, independent of the model assumed, since the ratios between the xx and xy components are crossed. The wave form generated by these terms depends on the relative phase between the longitudinal and transverse oscillatory conductivities, and between the oscillations in the first and the second subbands. The issue of the phases in a single-carrier-type system was addressed by Coleridge, Stoner, and Fletcher,[11] who have measured the phase difference between σ_{xx} and σ_{xy}. According to their findings, ϕ is equal to π, while θ increases from $3\pi/2$ to 2π with decreasing field. Thus, the phase difference increases from $\pi/2$ to π. The presence of the difference frequency at high temperatures and low magnetic fields, as verified experimentally, implies that at the lowest fields there is another change in the relative phases of the four components of conductivity; this time they will be in antiphase to the high-field configuration. Indeed, one can observe in Fig. 3 that the beat is accompanied by a change of phase of π in the wave form. If these terms are dominant, as discussed below, then, taking $\phi_1=\phi_2=\pi$, while $\theta_1=\frac{1}{2}\pi$ and $\theta_2=\frac{3}{2}\pi$, one obtains

$$\left.\frac{\partial^2\rho_{xx}}{\partial x_1\,\partial x_2}\right|_0 x_1x_2\simeq\frac{2\Delta\sigma_{xx_1}\Delta\sigma_{xy_2}}{(\sigma_{y_1}+\sigma_{y_2})^3}\sin\left[\frac{2\pi f_1}{B}-\frac{2\pi f_2}{B}\right]\ . \tag{9}$$

Thus, the difference frequency can be derived from the intermodulation term with the appropriate choice of phases. At higher fields, assuming the phases reported in Ref. 11, $\phi_1=\phi_2=\pi$ and $\theta_1=\theta_2=2\pi$, one gets

$$\left.\frac{\partial^2\rho_{xx}}{\partial x_1\,\partial x_2}\right|_0 x_1x_2\simeq\frac{2\Delta\sigma_{xx_1}\Delta\sigma_{xy_2}}{(\sigma_{y_1}+\sigma_{y_2})^3}\cos\left[\frac{2\pi f_1}{B}\right]\times\cos\left[\frac{2\pi f_2}{B}\right]\ , \tag{10}$$

which is an amplitude modulation as discussed before. Finally, at the highest fields, where $\phi_1=\phi_2=\pi$, while $\theta_1=\theta_2=\frac{3}{2}\pi$, one obtains

$$\left.\frac{\partial^2\rho_{xx}}{\partial x_1\,\partial x_2}\right|_0 x_1x_2\simeq\frac{2\Delta\sigma_{xx_1}\Delta\sigma_{xy_2}}{(\sigma_{y_1}+\sigma_{y_2})^3}\sin\left[\frac{2\pi f_1}{B}+\frac{2\pi f_2}{B}\right]\ . \tag{11}$$

This is the sum of the frequencies observed at large fields (of several teslas, beyond our range), as observed experimentally.

The intermodulation can generate several wave patterns, the relative amplitude of which depends on the effect of the various scattering mechanisms on the relaxation time, as outlined by Coleridge.[3] On the other hand, the intermodulation is one of three components contributing to the oscillatory wave form of a system with two occupied subbands, as presented in Eq. (3). If all three terms were significant, the measured line shape would not manifest a pure oscillation at the difference frequency, as we have recorded. However, the amplitude of oscillation due to the separate subbands decays exponentially with increasing temperature. On the other hand, the intermodulation increases markedly as the temperature is raised.[3] Thus, at higher temperatures, this term becomes more dominant, and oscillations at the difference frequency are observed throughout larger magnetic-field ranges.

CONCLUSIONS

The presence of a third frequency in the SdH wave form was experimentally verified. The analysis shows that this frequency corresponds to the difference between the concentration of the ground and the excited subband. It seems that this feature is present in other structures as well. All three frequencies of the wave form, starting from the sum frequency at high fields, through the superposition at intermediate fields, and down to the difference frequency at the lowest fields, can be attributed to the various relative phases between the longitudinal and transverse conductivities in the intermodulation.

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