RAPID COMMUNICATIONS

PHYSICAL REVIEW B VOLUME 45, NUMBER 22 1 JUNE 1992-II

Superfluid fraction of ^{4}He very close to T_λ

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(Received 23 December 1991)

Measurements of the superfluid fraction ρ_s/ρ of ^{4}He are presented for the reduced-temperature range $3\times10^{-7} \lesssim t \equiv 1 - T/T_\lambda \lesssim 10^{-2}$. The data were obtained from the second-sound velocity u_{20}, which in turn was determined from nonlinear-pulse transit-time measurements. The exponent ζ, which describes the leading singularity of ρ_s/ρ, is found to be 0.6705 ± 0.0006. This result is in good agreement with existing theories and previous data much further away from T_λ.

Measurements of the superfluid fraction ρ_s/ρ in ^{4}He near the superfluid-transition temperature T_λ by a variety of techniques have a long history,[1] and since about two decades ago ρ_s/ρ has been perhaps the most accurately and extensively known parameter with a singularity near a critical point. The most precise of these early measurements[2] demonstrated the need for the inclusion of confluent singular terms in the dependence upon the reduced temperature $t \equiv (T_\lambda - T)/T_\lambda$ in experimentally accessible ranges of t, and suggested the functional form

$$\rho_s/\rho = k(1 + D_\rho t^\Delta) t^\zeta . \tag{1}$$

A recent analysis[3] of the data at saturated vapor pressure[2] over the wide range $3\times10^{-5} \lesssim t \lesssim 0.05$ and with the temperature-dependent leading amplitude

$$k = k_0(1 + k_1 t) \tag{2}$$

yielded the results[4]

$$\zeta = 0.6717 \pm 0.0004 , \tag{3}$$

$k_0 = 2.403 \pm 0.008$, $k_1 = -1.46 \pm 0.05$, and $D_\rho = 0.33 \pm 0.02$.

At this time it is widely believed that there exists an exact theory of critical phenomena, namely, the renormalization-group theory.[5] Implied in this belief is the expectation that a knowledge of the behavior of a singular property over one range of reasonably small values of t, say $10^{-5} \lesssim t \lesssim 10^{-2}$, enables us to extrapolate *via* a power-law like equation (1) to any other range of even smaller values of t that previously had not been accessible to measurement. Developments in thermometry,[6–14] which have extended the range of t accessible to measurement by about two decades, have provided us with the opportunity to put this belief to a crucial test.

In one very recent test, Marek, Lipa, and Philips[15] (MLP) determined second-sound resonant frequencies, and thereby the exponent ζ, over the range $4\times10^{-7} \lesssim t \lesssim 10^{-3}$. Their data yielded an exponent $\zeta = 0.6740 \pm 0.0005$, in disagreement with previous measurements. Perhaps of greater concern was that these data also gave significant deviations from the functional form Eq. (1) for $t \lesssim 5\times10^{-6}$ which increased dramatically with decreasing t and became as large as 8% (Ref. 16) of ρ_s/ρ for $t \simeq 4\times10^{-7}$. A more recent analysis of this data[17] seems to suggest that these difficulties can be resolved if T_λ is allowed to shift by 149 nK (Ref. 17) relative to its original experimentally determined value.[15]

In this paper we present experimental results for ρ_s/ρ, obtained using a substantially different method than has been used previously, which also extend very close to T_λ. The superfluid fraction was obtained by a time-of-flight method for nonlinearly evolving second-sound pulses,[18] and cover the range $3\times10^{-7} \lesssim t \lesssim 0.01$. The nonlinearities in the equations of hydrodynamics of the superfluid become progressively more important as T_λ is approached very closely. Therefore the shapes of the arriving pulses had to be analyzed in terms of a model which properly took the nonlinearities into account[18] in order to obtain the linear transit time and the corresponding second-sound velocity u_{20}. Thermodynamic data and u_{20} were then used in the usual manner[3] to give ρ_s/ρ. We made an independent determination of T_λ with systematic errors no larger than 20 nK which was based on the determination of the onset of thermal resistance in the fluid.[13] Our results yielded $\zeta = 0.6705 \pm 0.0006$, in satisfactory agreement[19] with the result Eq. (3).

The second-sound cell was located in the vacuum can of a cryostat described previously.[20] The cell was the same as the one used further away from T_λ.[18] The working volume was 0.1203 cm high by 3.18×3.18 cm^2. Its top and bottom were bounded by Pyrex plates, 3.81 cm on a side and 0.318 cm thick. The side walls were formed by a copper spacer. The inner surface of the bottom plate contained a square chromium thin-film heater that completely spanned the lateral dimensions of the cell. A small (0.254 cm on a side) thin-film (PbAu) superconducting bolometer[18] was deposited on the inside surface of the top Pyrex plate. Without signal averaging the device could typically resolve 0.1-μK changes in temperature in a 300-kHz bandwidth. With signal averaging, and in a smaller bandwidth, the rms noise in the second-sound signal was as small as 2 nK in some of the measurements. The cell temperature could be controlled with a stability of 1 nK through the use of a ^{4}He melting-pressure thermometer,[13,14] and T_λ could be determined with a precision of 5 nK with a fixed-point device.[13,21] The accuracy of T_λ was ± 20 nK. For $t > 5.5\times10^{-4}$, germanium thermometry was used and the error in the temperature measurement was ± 1 μK. Systematic errors in ρ_s/ρ due to uncertainties in the cell length are 0.2%.

A voltage pulse consisting of a single cycle of a haversine was applied at the heater to launch a second-sound pulse into the helium. This pulse propagated across the cell and was detected each time it reflected off the surface containing the bolometer. The geometry of the cell was such that the pulse remained planar for the first six echoes.[18] Due to the reflection at the surface of the bolometer, the temperature amplitudes recorded at the bolometer were roughly a factor of 2 larger than the amplitude of the second sound as it propagated across the cell. Signal averaging of up to 2×10^4 pulse sequences were used to increase the signal-to-noise ratio. The model used to extract u_{20} from the data was the same as that described in Ref. 18. The first arrival of the pulse at the detector was used as an initial condition for the numerical integration of Burgers' equation with damping in one dimension.[22] These numerical solutions were fit to a later arrival of the pulse at the detector. The only two adjustable parameters were the second-sound velocity u_{20} and damping D_2. They are not strongly correlated, and both could be determined with meaningful accuracy.

The use of Burgers' equation to describe the nonlinear propagation of second sound had been examined in detail[18] for $t \gtrsim 10^{-3}$. Its validity closer to T_λ was tested in this work by data taken for a great variety of initial pulse widths and amplitudes. For initial pulses significantly shorter than the width of the cell (not larger than 0.34 mm), and for amplitudes $\delta t/t$ between 8×10^{-4} and 0.07, no systematic trends of the fitting parameters with pulse amplitude were noted. Changing the bolometer power from 0.043 to 0.52 μW did not affect the results for u_{20} noticeably. The effect of the interaction of the pulse with itself as it reflected off the bolometer and heater substrates was tested for with the use of double-pulse sequences as described in Ref. 18, also with a null result. The second arrival of a pulse obtained very close to T_λ, for $t=3.07\times 10^{-7}$, is shown in Fig. 1 as open circles, along with the fit (solid line) and the residuals (solid circles). In this case the rms deviation is only about 2 nK.

Due to the effect of gravity,[23] T_λ is shifted vertically by 1.273 μK/cm. For this reason, our cell was quite short (0.1203 cm). With the reduced temperature measured with respect to T_λ at the *middle* of the cell, it can be shown that the effect of gravity on the data is negligible even at our smallest reduced temperatures. Thus no gravity corrections were required.

The results for u_{20} were converted to ρ_s/ρ using the relation

$$u_{20}^2 = \sigma^2 \rho_s T/\rho_n C_p \, , \tag{4}$$

where σ and C_p are the entropy and heat capacity per unit mass. The entropy is taken from Ref. 24 and C_p from Ref. 25.

Equations (1) and (2) were fit to the results for ρ_s/ρ. We fixed[26] Δ at 0.5. A fit was first performed to the data with $t > 10^{-5}$ in order to determine the parameters k_1 and D_ρ. The corresponding terms in Eq. (1) contribute about 0.1% or less for $t < 10^{-5}$ and it was desirable to determine them further away from T_λ because small but systematic deviations of the data at smaller t influence them unduly. The remaining parameters, ζ and k_0, were then determined by a fit over the entire data set, with D_ρ

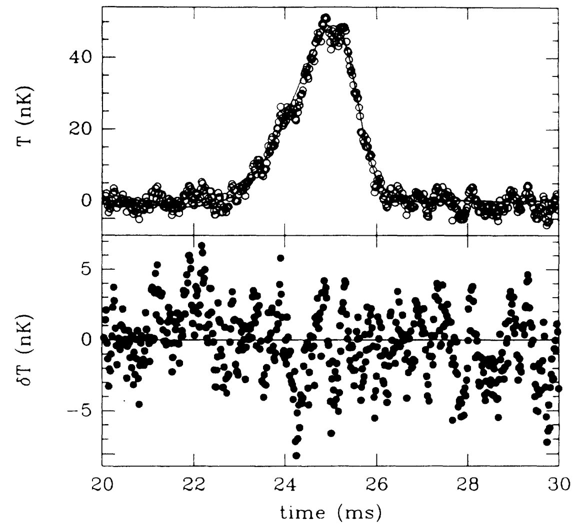

FIG. 1. Second arrival of a pulse at the bolometer, and the corresponding fit, for $t=3.07\times 10^{-7}$. The pulse width at the heater was 0.32 mm and the amplitude of the first echo, used as an initial condition, was $\delta t/t=0.057$. This fit gave $u_{20}=0.1553$ m/s and $D_2=0.0031$ cm^2/s. A pulse was launched at the heater every 1.3 s and pulse sequences at the bolometer were averaged for about 10 h in order to achieve the 2-nK rms deviations shown.

and k_1 held constant. The values obtained for the parameters in Eq. (1) were

$$\zeta = 0.6705 \pm 0.0006 \, , \tag{5a}$$

$$k_0 = 2.380 \pm 0.015 \, , \tag{5b}$$

$$k_1 = -1.74 \pm 0.20 \, , \tag{5c}$$

$$D_\rho = 0.396 \pm 0.035 \, , \tag{5d}$$

in good agreement with the results[3] based on the older data[2] which we quoted above.

The u_{20} data were also analyzed using the heat-capacity measurements of Lipa and Chui (LC).[27,28] The formula for C_p provided in Ref. 25 [given in Eq. (4.3)], although it fits the data well for $t \gtrsim 10^{-5}$, has a logarithmic form which is not correct closer to the transition. The data of LC extend to the smallest values of t used in this work; however, they do not go to larger values than $t=10^{-3}$ and had to be normalized to the results of Ref. 25. When our u_{20} data are analyzed using the LC results inside of 10^{-3}, and the results of Ref. 25 outside, the exponent and leading amplitude are substantially unchanged from those given in Eqs. (5).

The percent deviations of the data from the function are shown in Fig. 2. Note that this differs from the procedure of MLP (Ref. 15) and Swanson, Chui, and Lipa,[17] who show the deviations of the resonant frequency of a second-sound cavity.[16] It is clear from Fig. 2 that there is a region $t \lesssim 10^{-5}$ where Eq. (1) does not fit the data within their random errors. The reasons for this discrepancy are not known. As discussed above, a thorough search for systematic effects was carried out. In addition, the fit shown in Fig. 2 was repeated for only the smallest-

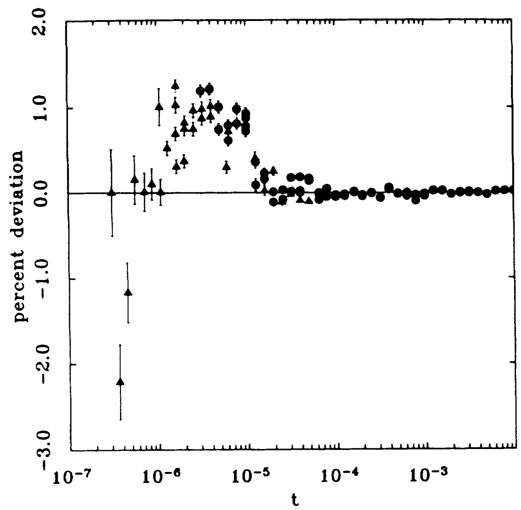

FIG. 2. Percent deviation of the ρ_s/ρ data from Eq. (1) with the parameters of Eqs. (5).

amplitude pulses and again for only the shortest-length pulses. In either case, the results were unchanged. Nonetheless, we expect that the deviations are associated with undetermined experimental problems which arise when working at these extremely small reduced temperatures.

Although the data closest to T_λ had a reduced temperature determined to a precision of ± 5 nK, the accuracy was ± 20 nK. The effect of shifting T_λ by this amount is shown in Fig. 3. The fitting parameters are not significantly changed. However, the apparent decrease in the residuals for $t \lesssim 10^{-6}$ is influenced strongly.

The result equation (5a) is in good agreement with the result equation (3) which was based on data further away from T_λ. It also agrees with the most recent analysis[17] of the data[15] of Marek, Lipa, and Philips; but as mentioned above, this analysis involved a shift in T_λ which is in apparent disagreement with the measured value. The value for ζ also agrees well with renormalization-group calculations. From theory it is expected[29] that ζ is equal to the correlation-length exponent ν. Values of ν have been obtained by several approximations, and range[30-32] from 0.669 ± 0.002 to 0.672 ± 0.002.

In summary, we have presented results for precise and complete measurements of ρ_s/ρ, over a large range of the reduced temperature t. We have used a technique that accounts for the nonlinearities in the propagation of second sound near T_λ. Agreement with previous data further away from T_λ is excellent where there is overlap, and the exponent values derived from the data agree well with

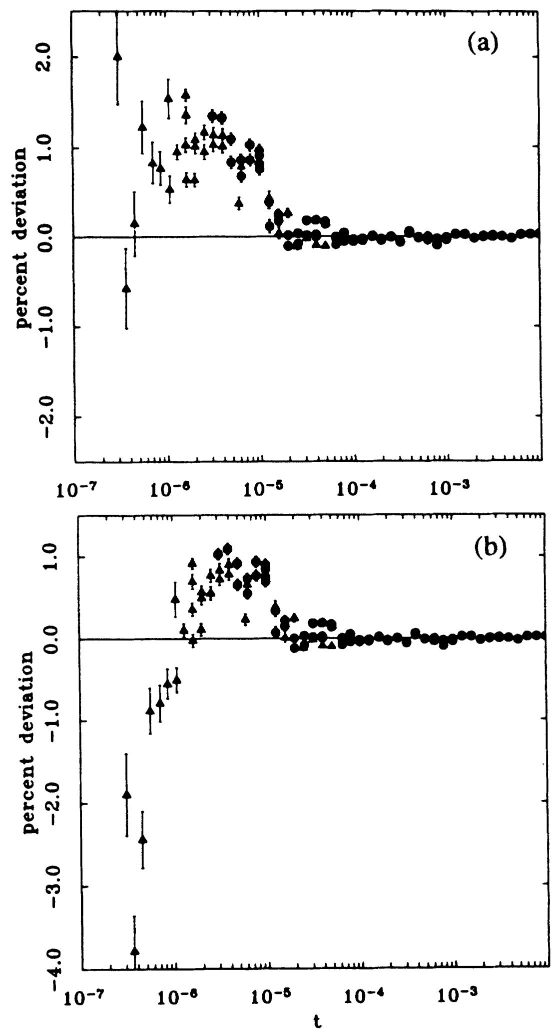

FIG. 3. Percent deviation as in Fig. 2, but with T_λ shifted. (a) $\delta T_\lambda = -20$ nK. This fit gave $\zeta = 0.6704$, $k_0 = 2.378$, $k_1 = -1.77$, and $D_\rho = 0.403$. (b) $\delta T_\lambda = +20$ nK. This fit gave $\zeta = 0.6706$, $k_0 = 2.382$, $k_1 = -1.71$, and $D_\rho = 0.388$.

those obtained much further away from the transition. This agreement supports the validity of the renormalization-group theory of critical phenomena, and lends credence to the belief that the theory gives the behavior of the system arbitrarily close to the transition when measurements are available further away.

This work was supported by National Science Foundation Grant No. DMR 89 18393.

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[1]For a review, see for instance, G. Ahlers, in *The Physics of Liquid and Solid Helium*, edited by K. H. Bennemann and J. B. Ketterson (Wiley, New York, 1976), Pt. I, Chap. 2.

[2]D. S. Greywall and G. Ahlers, Phys. Rev. Lett. **28**, 1251 (1972); Phys. Rev. A **7**, 2145 (1973).

[3]A. Singsaas and G. Ahlers, Phys. Rev. B **30**, 5103 (1984).

[4]The statistical errors quoted in this paper are 67% confidence limits.

[5]K. G. Wilson, Phys. Rev. B **4**, 3184 (1971). For a review, see J. Kogut and K. G. Wilson, Phys. Rep. C **12**, 76 (1974); or M. E. Fisher, Rev. Mod. Phys. **46**, 597 (1974).

[6]J. Lipa, B. Leslie, and T. Wallstrom, Physica B **107**, 331 (1981).

[7]V. Steinberg and G. Ahlers, J. Low Temp. Phys. **53**, 255 (1983).

[8]T. C. P. Chui and J. A. Lipa, in *Proceedings of the Seventeenth International Conference on Low Temperatures Physics*, edited by U. Eckern, A. Schmid, W. Weber, and H. Wühl (North-Holland, Amsterdam, 1984), p. 931.

[9]T. Chui and D. Marek, J. Low Temp. Phys. **73**, 161 (1988).

[10]R. V. Duncan, G. Ahlers, and V. Steinberg, Phys. Rev. Lett. **60**, 1522 (1988).

[11]M. Larson and G. Ahlers, Physica B **165**, 37 (1990).

[12]G. K. S. Wong, P. A. Crowell, H. A. Cho, and J. D. Reppy, Phys. Rev. Lett. **65**, 2410 (1990).

[13]L. S. Goldner, Ph.D. thesis, University of California at Santa Barbara, 1991.

[14]L. S. Goldner, N. Mulders, and G. Ahlers, in *Proceedings of the Seventh International Symposium on Temperture, Its Measurement and Control in Science and Technology*, edited by J. Schodey and L. Rubin (American Institute of Physics, New York, in press).

[15]D. Marek, J. Lipa, and D. Philips, Phys. Rev. B **38**, 4465 (1988).

[16]Marek, Lipa, and Philips (Ref. 15) and Swanson, Chui, and Lipa (Ref. 17) show the deviations of a second-sound resonance frequency from a fit. These deviations are smaller by a factor of 2 than the equivalent ones for ρ_s/ρ since ρ_s/ρ is proportional to the square of the frequency.

[17]D. R. Swanson, T. C. P. Chui, and J. A. Lipa (unpublished); and (private communication).

[18]L. S. Goldner, G. Ahlers, and R. Mehrotra, Phys. Rev. B **43**, 12861 (1991).

[19]A fit (see Ref. 3) of the data from Ref. 2 with $t \lesssim 10^{-2}$ gave $\zeta = 0.6710 \pm 0.0010$.

[20]W. Y. Tam and G. Ahlers, Phys. Rev. B **32**, 5932 (1985).

[21]R. V. Duncan and G. Ahlers, in *Proceedings of the Seventh International Symposium on Temperature, Its Measurement and Control in Science and Technology* (Ref. 14).

[22]As described in Ref. 18, heat transport into and out of the bolometer and heater substrates also had to be taken into consideration.

[23]G. Ahlers, J. Low Temp. Phys. **84**, 173 (1991).

[24]A. Singsaas and G. Ahlers, Phys. Rev. B **29**, 4951 (1984).

[25]G. Ahlers, in *The Physics of Liquid and Solid Helium*, edited by K. H. Bennemann and J. B. Ketterson (Wiley, New York, 1976), Pt. I, Chap. 2.

[26]See Ref. 3 for a discussion of the value of Δ.

[27]J. Lipa and T. Chui, Phys. Rev. Lett. **51**, 2291 (1983).

[28]The formula used for C_p and best fit values of the parameters were provided by J. A. Lipa and T. C. P. Chui (private communication).

[29]B. D. Josephson, Phys. Rev. Lett. **21**, 608 (1966).

[30]J. C. LeGuillou and J. Zinn-Justin, Phys. Rev. B **21**, 3976 (1980).

[31]D. Z. Albert, Phys. Rev. B **25**, 4810 (1982).

[32]J. C. LeGuillou and J. Zinn-Justin, J. Phys. (Paris) Lett. **46**, L137 (1985).