Ultrasonic attenuation of a superconductor with a spiral spin-density wave

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We have calculated the longitudinal ultrasonic attenuation of a simple clean superconducting metal with a spiral spin-density wave (SSDW). The superconducting gap $\Delta(\mathbf{k})$ vanishes along a circle on the Fermi surface. The ultrasonic attenuation coefficient $\alpha_\mathbf{q}$ for wave vector $\mathbf{q}$ parallel to the SSDW axis is not directly related to the superconducting density of states, its temperature dependence cannot be fitted by a power law and depends on the ratio between Fermi and sound velocities. For $\mathbf{q}$ perpendicular to the SSDW axis, $\alpha_\mathbf{q}(T)$ is linear in T for low enough temperature T, as for the polar phase of p-wave superconductors.

I. INTRODUCTION

Several years ago there was a considerable research effort devoted to the longitudinal ultrasonic attenuation of heavy-fermion superconductors. The temperature dependence of $\alpha_\mathbf{q}$ in UPt_3,[1,2] UBe_{13},[3] and $(U,Th)Be_{13}$ (Ref. 4) differs from the exponential behavior predicted by the usual BCS theory. Instead, T^2 (Refs. 1 and 3) and T^3 (Ref. 2) temperature dependences have been reported. It has been argued that a T^2 behavior corresponds to a clean triplet superconductor in a polarlike state.[1] However, taking into account the crystal symmetry, Volovik and Gorkov showed that triplet states with $\Delta_\mathbf{k}$ vanishing along a line on the Fermi surface are ruled out.[5] In addition, the experiments are done in the hydrodynamic limit. Several theories valid in this limit have been proposed.[6,7] The experiments[1-4] also show a peak in $\alpha_\mathbf{q}(T)$ slightly below the superconducting critical temperature T_c. There are also several explanations for this peak,[6,8,9] including a Landau-Khalatnikov relaxational model of the order parameter[8] and a change in the coherence factor due to time-reversal (for singlet) or inversion (for triplet pairing) symmetry breaking.[9] For high-T_c systems, $\alpha_\mathbf{q}(T)$ is qualitatively similar, with a peak below T_c and a nonexponential temperature dependence.[10,11]

Overhauser and Daemen solved analytically the superconducting gap equation for a spiral or linear spin-density wave.[12] They also calculated the specific heat and obtained a result that agrees qualitatively with the experimentally observed in UPt_3.[13] For the spiral spin-density wave (SSDW), $\Delta_\mathbf{k}$ vanishes along a line on the Fermi surface. Thus, although the electronic structure of heavy-fermion systems is much more complicated than that of a simple metal with a SSDW, it seems that the study of the simpler system might shed light on the physics of some properties of heavy-fermion and eventually high-T_c systems. We have calculated the ultrasonic attenuation of a superconductor with a SSDW. The system is described in Sec. II. In Sec. III we calculate the longitudinal ultrasonic attenuation for wave vectors parallel and perpendicular to the SSDW axis. Section IV contains a discussion.

II. THE SYSTEM

Following Ref. 12, the effective one-particle Hamiltonian can be written in the form

$$H=H_0+H_{\rm int}\ , \tag{1}$$

$$H_0=\sum_{\mathbf{k},\sigma}\varepsilon_\mathbf{k}c^\dagger_{\mathbf{k}\sigma}c_{\mathbf{k}\sigma}-G\sum_\mathbf{k}\left[c^\dagger_{\mathbf{k}+\mathbf{q}\downarrow}c_{\mathbf{k}\uparrow}+\text{H.c.}\right]\ , \tag{2}$$

$$H_{\rm int}=-V\sum_{\mathbf{k},\mathbf{k}'}c^\dagger_{\mathbf{k}\uparrow}c^\dagger_{-\mathbf{k}\downarrow}c_{-\mathbf{k}'\downarrow}c_{\mathbf{k}'\uparrow}\ . \tag{3}$$

$c^\dagger_{\mathbf{k}\sigma}$ creates an electron in a plane-wave state with momentum $\mathbf{k}$ and spin σ. $\varepsilon_\mathbf{k}=\hbar^2k^2/2m$. $H_{\rm int}$ is the reduced BCS interaction,[14] and the wave vector of the SSDW is given by

$$\mathbf{Q}=2k_F\hat{\mathbf{z}}\ ,\quad k_F=(2m\varepsilon_F)^{1/2}/\hbar\ , \tag{4}$$

where ε_F is the Fermi energy. The operators that create an electron in a one-particle eigenstate of H_0 with predominantly spin σ can be written in the form

$$a^\dagger_{\mathbf{k}\sigma}=\cos\theta_{\mathbf{k}\sigma}c^\dagger_{\mathbf{k}\sigma}+\sin\theta_{\mathbf{k}\sigma}c^\dagger_{\mathbf{k}+\sigma\mathbf{Q},-\sigma}\ . \tag{5}$$

and

$$\cos(2\theta_{k\uparrow})=\frac{\varepsilon_{\mathbf{k}+\mathbf{Q}}-\varepsilon_\mathbf{k}}{2r_\mathbf{k}}\ ,$$

$$r_\mathbf{k}=[(\varepsilon_{\mathbf{k}+\mathbf{Q}}-\varepsilon_\mathbf{k})^2/4+G^2]^{1/2}\ , \tag{6}$$

$$\text{sgn}\theta_{\mathbf{k}\uparrow}=\text{sgn}(\varepsilon_{\mathbf{k}+\mathbf{Q}}-\varepsilon_\mathbf{k})\ ,\theta_{-\mathbf{k}\downarrow}=\theta_{\mathbf{k}\uparrow}\ .$$

The energies of these eigenstates are

$$E_{\mathbf{k}\uparrow}=E_{\mathbf{k}\downarrow}=\frac{\varepsilon_\mathbf{k}+\varepsilon_{\mathbf{k}+\mathbf{Q}}}{2}-\text{sgn}(\varepsilon_{\mathbf{k}+\mathbf{Q}}-\varepsilon_\mathbf{k})r_\mathbf{k}\ . \tag{7}$$

The Fermi surface for states $a^\dagger_{\mathbf{k}\sigma}|0\rangle$ has a hole of radius $(2mG)^{1/2}/\hbar$ centered at the wave vector $(0,0,-\sigma k_F)$ and with its perimeter lying in the plane $k_z=-\sigma k_F$. The density of states obtained using Eq. (7) is shown in Fig. 1. In comparison with the unperturbed case for the same chemical potential, part of the spectral weight is displaced from slightly above to slightly below the Fermi energy stabilizing the SSDW.

In terms of the $a_{\mathbf{k}\sigma}$ and $a^\dagger_{\mathbf{k}\sigma}$, assuming $\Delta_0 \ll G$ [see Eq. (16)], the relevant part of H_{int} takes the form

$$H_{\text{int}}=-\sum_{\mathbf{k},\mathbf{k}'} V_{\mathbf{k}\mathbf{k}'} a^\dagger_{\mathbf{k}\uparrow} a^\dagger_{-\mathbf{k}\downarrow} a_{-\mathbf{k}'\downarrow} a_{\mathbf{k}'\uparrow} \ , \tag{8}$$

with

$$V_{\mathbf{k}\mathbf{k}'}=V\cos(2\theta_{\mathbf{k}\uparrow})\cos(2\theta_{\mathbf{k}'\uparrow}) \ . \tag{9}$$

In the mean-field approximation, H can be written as

$$H_{\text{MF}}=\sum_{\mathbf{k},\sigma} E_{\mathbf{k}\sigma} a^\dagger_{\mathbf{k}\sigma} a_{\mathbf{k}\sigma} - \sum_{\mathbf{k}} \Delta_{\mathbf{k}} (a^\dagger_{\mathbf{k}\uparrow} a^\dagger_{-\mathbf{k}\downarrow} + \text{H.c.}) + \sum_{\mathbf{k}} \Delta_{\mathbf{k}} \langle a^\dagger_{\mathbf{k}\uparrow} a^\dagger_{-\mathbf{k}\downarrow} \rangle \ , \tag{10}$$

where the superconducting gap is

$$\Delta_{\mathbf{k}}=\sum_{\mathbf{k}} V_{\mathbf{k}\mathbf{k}'} \langle a_{-\mathbf{k}'\downarrow} a_{\mathbf{k}'\uparrow} \rangle \ . \tag{11}$$

H_{MF} is diagonalized using the following Bogoliubov transformation:

$$\begin{aligned} \gamma_{\mathbf{k}\uparrow} &= u_{\mathbf{k}} a_{\mathbf{k}\uparrow} - V_{\mathbf{k}} a^\dagger_{-\mathbf{k}\downarrow} \ , \\ \gamma^\dagger_{-\mathbf{k}\downarrow} &= V_{\mathbf{k}} a_{\mathbf{k}\uparrow} + u_{\mathbf{k}} a^\dagger_{-\mathbf{k}\downarrow} \ , \end{aligned} \tag{12}$$

where $u_{\mathbf{k}}^2+V_{\mathbf{k}}^2=1$ and

$$u_{\mathbf{k}}^2=\frac{1}{2}\left[1+\frac{E_{\mathbf{k}\uparrow}-\mu}{\lambda_{\mathbf{k}}}\right] \ . \tag{13}$$

μ is the chemical potential. $\lambda_{\mathbf{k}}$ is the energy of the quasiparticles described by the operators $\gamma_{\mathbf{k}\uparrow}$ and $\gamma_{-\mathbf{k}\downarrow}$ and is given by

$$\lambda_{\mathbf{k}}=[(E_{\mathbf{k}\uparrow}-\mu)^2+\Delta_{\mathbf{k}}^2]^{1/2} \ . \tag{14}$$

Using Eqs. (12)–(14), one has

$$\langle a_{-\mathbf{k}\downarrow} a_{\mathbf{k}\uparrow} \rangle = \langle a^\dagger_{\mathbf{k}\uparrow} a^\dagger_{-\mathbf{k}\downarrow} \rangle = \frac{\Delta_{\mathbf{k}}}{2\lambda_{\mathbf{k}}}[1-2f(\lambda_{\mathbf{k}})] \ , \tag{15}$$

where $f(\varepsilon)$ is the Fermi function. Replacing this in the second member of Eq. (11), one obtains an integral equation for $\Delta_{\mathbf{k}}$. The fact that the dependence of $V_{\mathbf{k}\mathbf{k}'}$ on $\mathbf{k}$ and $\mathbf{k}'$ appears in a factorized form makes it possible to reduce this problem to an ordinary equation in one variable.[12,15] One gets

$$\Delta_{\mathbf{k}}=\cos(2\theta_{\mathbf{k}\uparrow})\Delta_0 \ , \tag{16}$$

where, for each temperature T, Δ_0 is given by the equation

$$1=V\sum_{\mathbf{k}} \cos^2(2\theta_{\mathbf{k}}) \frac{1-2f(\lambda_{\mathbf{k}})}{2\lambda_{\mathbf{k}}} \ . \tag{17}$$

$\lambda_{\mathbf{k}}(\Delta_0)$ is obtained replacing Eqs. (16), (6), and (7), in Eq. (14). For weak coupling the dependence of the superconducting critical temperature on G is given in Ref. 12.

From Eqs. (16) and (6), one can see that $\Delta_{\mathbf{k}}$ is independent of k_x and k_y and vanishes for $k_z=-k_F$. On the Fermi surface, $\Delta_{\mathbf{k}}$ depends only on the angle θ between $\mathbf{k}$ and the z direction. This dependence is illustrated in Fig. 2. $\Delta_{\mathbf{k}}$ vanishes on a line defined by $\theta=\theta_c$.

The density of states of the superconducting system can be obtained using cylindrical coordinates in reciprocal space with axis k_z and changing the variable normal to k_z by $\lambda_{\mathbf{k}}$. The result can be written in the form

$$\rho(\lambda)=\frac{2\Omega m}{h^2}\int dk_z \frac{\lambda}{(\lambda^2-\Delta_{\mathbf{k}}^2)^{1/2}}\theta(\lambda^2-\Delta_{\mathbf{k}}^2) \times[\theta((\lambda^2-\Delta_{\mathbf{k}}^2)^{1/2}+\mu-F_{\mathbf{k}\uparrow}) + \theta(-(\lambda^2-\Delta_{\mathbf{k}}^2)^{1/2}+\mu-F_{\mathbf{k}\uparrow})] \ , \tag{18}$$

where Ω is the volume of the system, $\theta(x)$ is the step

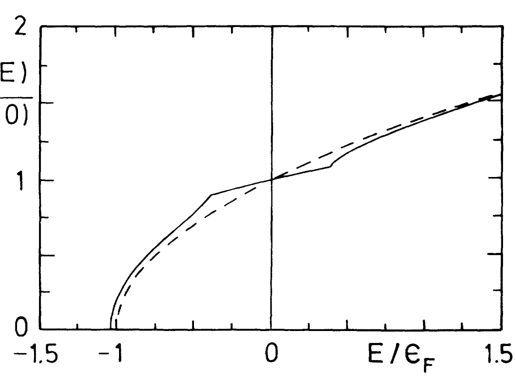

FIG. 1. Density of states as a function of the energy for a normal system with a spiral spin-density wave (solid line) compared with that of a free-electron gas (dashed line) with the same Fermi wave vector k_F and mass m. The energies are measured from the chemical potential and $\varepsilon_F=(\hbar k_F)^2/2m$. The parameter is $G=0.4\varepsilon_F$.

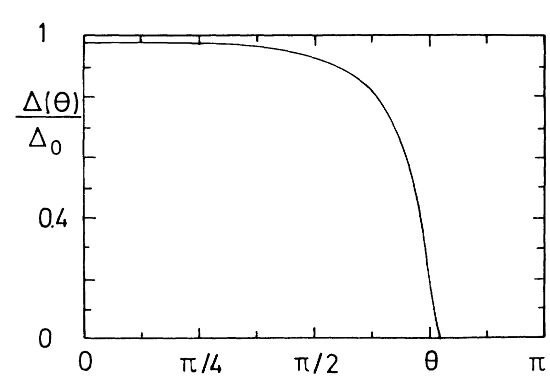

FIG. 2. Superconducting gap as a function of the angle between $\mathbf{k}$ on the Fermi surface and k_z [$\theta \cong \arccos(k_z/k_F)$]. Parameters are $G=0.8\varepsilon_F$ and $k_B T_c=10^{-4}\varepsilon_F$.

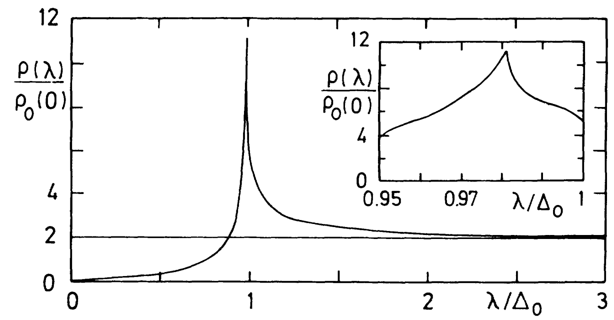

FIG. 3. Density of states as a function of the energy in the superconducting phase normalized to that in the normal phase at the Fermi level. Parameters as in Fig. 2.

function, and $F_{\mathbf{k}\uparrow}$ is the part of $E_{\mathbf{k}\uparrow}$ that depends on k_z only:

$$F_{\mathbf{k}\uparrow}=E_{\mathbf{k}\uparrow}-(\hbar^2k_x^2+\hbar^2k_y^2)/2m \ . \tag{19}$$

The result of the numerical evaluation of Eq. (18) is represented in Fig. 3. The main differences with the standard result for an isotropic superconducting gap is that $\rho(\lambda)$ is different from either zero or infinity for all values of λ. In particular, $\rho(\lambda)$ turns out to be linear in λ for $\lambda\to 0$. In this limit Eq. (18) can be evaluated analytically: For $\lambda \ll \Delta_0$, $\Delta_{\mathbf{k}}<\lambda$ implies that k_z is near $-k_F$ [see Eqs. (16) and (6)]. For $k_z\to -k_F$, from Eq. (6), $\cos 2\theta_{\mathbf{k}}\to 2\varepsilon_F x/G$, where $x=1+k_z/k_F$. Thus, for $\lambda\ll\Delta_0$, using Eq. (16), we obtain, calling $x_0=G\lambda/2\Delta_0\varepsilon_F$,

$$\rho(\lambda)\cong\frac{2\Omega m k_F}{h^2}\int_0^{x_0}\frac{2}{[1-(x/x_0)^2]^{1/2}}dx=\frac{\Omega k_F^3 G}{8\pi\varepsilon_F^2\Delta_0}\lambda \ . \tag{20}$$

The linear dependence of the density of states with energy for low energy seems to be a general feature of superconductors with a gap vanishing along a line on the Fermi surface.[1] As a consequence, the specific heat for $T\ll T_c$ has a T^2 dependence.[12] However, as we show in the next section, the temperature dependence of the ultrasonic attenuation cannot be inferred from the energy dependence of the density of states.

III. LONGITUDINAL ULTRASONIC ATTENUATION

In this section we calculate the linear response of the system to a compressibility wave of wave vector $\mathbf{q}\to 0$. The interaction of electronic plane-wave states with the sound wave has the form[14]

$$H_{e\text{-ph}}^{\mathbf{q}}=C\sum_{\mathbf{k},\sigma}\sqrt{\omega_{\mathbf{q}}}c_{\mathbf{k}+\mathbf{q},\sigma}^{\dagger}c_{\mathbf{k}\sigma}(b_{\mathbf{q}}-b_{-\mathbf{q}}^{\dagger}) \ , \tag{21}$$

where C is a constant, $\omega_{\mathbf{q}}$ is the frequency of the sound wave, and $b_{\mathbf{q}}$ annihilates a longitudinal phonon of wave vector $\mathbf{q}$.[14] When Eq. (21) is written in terms of the eigenstates of H_0 using Eqs. (5) and (6), it retains the same form replacing the operators $c_{\mathbf{k}\sigma}$ by $a_{\mathbf{k}\sigma}$ in the limit $\mathbf{q}\to\mathbf{0}$. Using Eqs. (12), $H_{e\text{-ph}}^{\mathbf{q}}$ can be written in terms of the operators that diagonalize H_{MF}. The result for $\mathbf{q}\to 0$ is

$$\begin{aligned}H_{e\text{-ph}}^{\mathbf{q}}=C\sum_{\mathbf{k},\sigma}\sqrt{\omega_{\mathbf{q}}}[&(u_{\mathbf{k}}^2-V_{\mathbf{k}}^2)(\gamma_{\mathbf{k}+\mathbf{q},\uparrow}^{\dagger}\gamma_{\mathbf{k}\uparrow}+\gamma_{-\mathbf{k}\downarrow}^{\dagger}\gamma_{-\mathbf{k}-\mathbf{q},\downarrow})\\&+2u_{\mathbf{k}}V_{\mathbf{k}}(\gamma_{\mathbf{k}+\mathbf{q},\uparrow}^{\dagger}\gamma_{-\mathbf{k}\downarrow}^{\dagger}+\gamma_{-\mathbf{k}-\mathbf{q},\downarrow}\gamma_{\mathbf{k}\uparrow})]\\&\times(b_{\mathbf{q}}-b_{-\mathbf{q}}^{\dagger}) \ .\end{aligned} \tag{22}$$

The energy attenuation coefficient $\alpha_{\mathbf{q}}$ of the sound wave can be calculated from the difference between the probabilities of phonon absorption and emission.[14] The contribution of the terms in Eq. (22), in which two γ quasiparticles are created or destroyed simultaneously, is much smaller than the rest because these processes are ruled out by the condition of conservation of energy if $\hbar\omega_{\mathbf{q}}<2\Delta_{\mathbf{k}}$. This condition is satisfied over most of the Fermi surface except very near T_c for realistic frequencies and superconducting transition temperatures. Neglecting these terms, the attenuation is

$$\alpha_{\mathbf{q}}=\frac{\Omega}{4\pi^2 v_s}\omega_{\mathbf{q}}^2C^2\sum_{\sigma}\int d\varphi\, dk_\rho k_\rho dk_z\left[\frac{E_{\mathbf{k}\uparrow}}{\lambda_{\mathbf{k}}}\right]^2\frac{\partial f(\lambda_{\mathbf{k}})}{\partial\lambda_{\mathbf{k}}}\delta((\nabla_{\mathbf{k}}\lambda_{\mathbf{k}})\cdot\mathbf{q}-\sigma\hbar w_{\mathbf{q}}) \ , \tag{23}$$

where $f(\varepsilon)$ is the Fermi function, v_s the sound velocity, $\varphi=\arctan(k_y/k_x)$, and $k_\rho=(k_x^2+k_y^2)^{1/2}$. Changing the variable of integration k_ρ to $\lambda_{\mathbf{k}}$, we can write

$$\alpha_{\mathbf{q}}=\int\frac{\partial f(\lambda)}{\partial\lambda}g_{\mathbf{q}}\lambda d\lambda \ , \tag{24}$$

where the density $g_{\mathbf{q}}(\lambda)$ for $\mathbf{q}=(0,0,q)$ is given by

$$g_{\parallel}(\lambda)=\frac{\Omega\omega_{\mathbf{q}}C^2k_F^2}{4\pi\varepsilon_F}\sum_{\sigma}\int dk_z\frac{(\lambda^2-\Delta_{\mathbf{k}}^2)^{1/2}}{\lambda}\times\delta\left[\frac{\partial\lambda_{\mathbf{k}}}{\partial k_z}-\sigma\hbar v_s\right] \ , \tag{25}$$

while for $\mathbf{q}$ normal to the z direction we obtain

$$g_{\perp}(\lambda)=\frac{\Omega k_F^3C^2\omega_{\mathbf{q}}^2}{2\pi^2\varepsilon_F}\times\int dk_z\frac{1}{[\hbar^2v_F^2k_\rho^2(\lambda)-\hbar^2v_s^2k_F^2\lambda^2/E^2]^{1/2}} \ . \tag{26}$$

Comparing Eqs. (18) with (25) or (26), one realizes that the density that enters the attenuation $g_{\mathbf{q}}(\lambda)$ is quite different from the density of states $\rho(\lambda)$. We discuss first the case of $\mathbf{q}$ parallel to the SSDW axis. $\rho(\lambda)$ and $g_{\parallel}(\lambda)$ are displayed in Figs. 3 and 4, respectively. It is clear

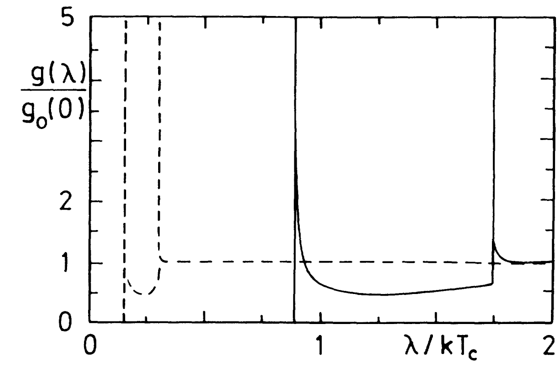

FIG. 4. Density that defines the longitudinal ultrasonic attenuation parallel to the SSDW axis [see Eq. (24)] as a function of the energy normalized to that in the normal phase at the Fermi energy. Solid line $T=0$ and dashed line $T=0.99T_c$. Parameters are $\varepsilon_F/k_B=10^4$ K, $G=0.8\varepsilon_F$, $T_c=1$ K, $k_F=1.5\times10^8$ cm^{-1}, and $v_F=4\times10^5$ cm/s.

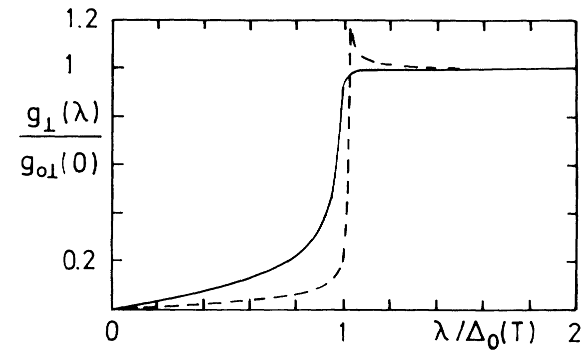

FIG. 6. Same as in Fig. 4 for attenuation normal to the SSDW axis. Solid line $G=0.8\varepsilon_F$ and dashed line $G=0.08\varepsilon_F$. Other parameters are $\varepsilon_F/k_B=10^4$ K, $T=0$, $T_c=1$ K, $k_F=1.5\times10^8$ cm^{-1}, and $v_s=4\times10^5$ cm/s. The result is very weakly dependent on temperature. The usual BCS temperature dependence was assumed for $\Delta_0(T)$.

that one cannot infer in general the temperature dependence of $\alpha_{\mathbf{q}}$ from the energy dependence of $\rho(\lambda)$, as was suggested in Ref. 1. In our particular case, we found essential singularities in $g_{\parallel}(\lambda)$ due to the fact that the argument of the δ function in Eq. (25) vanishes together with its derivative with respect to k_z for certain values of λ. Thus, actually, $\alpha_{\parallel}$ diverges and we cannot assume that $H^{\mathbf{q}}_{e\text{-ph}}$ is small (as we have done) for a perfectly clean system. However, for a real system the quasiparticles have a finite lifetime and the δ function of conservation of energy should be replaced by a Lorentzian function.

To simulate the effect of a finite lifetime, we have truncated the function $g_{\parallel}(\lambda)$, replacing all values larger than a certain cutoff value by the latter. We found that the results are rather insensitive to this cutoff if it is chosen less than 100 times the value of $g_{\parallel}(\lambda)$ in the normal phase. This value $g_{0\parallel}(\lambda)$ is practically independent of temperature for $T<T_c$.

The function $g_{\parallel}(\lambda)$ vanishes for λ less than a certain critical value λ_c for finite $\omega_{\mathbf{q}}$, because it is not possible to satisfy the δ function of conservation of energy [see Eq. (25)]. λ_c decreases for decreasing ratio of sound and Fermi velocities. For example, if this ratio is decreased 10 times compared with the value of Fig. 4 (increasing 10 times ε_F and keeping the other values of the parameters), λ_c decreases to 8.5×10^{-2} K at $T=0$. It is interesting to note that this behavior is similar for a polar triplet superconductor.[1] In this case we obtain, assuming $\Delta_0 \ll \hbar v_s k_F$, that $\lambda_c=\Delta_0 v_s/(2v_F)$.

The resulting longitudinal sound attenuation as a function of temperature for $\mathbf{q}\parallel\hat{\mathbf{z}}$ is shown in Fig. 5 for two different effective masses. For lower values of the quotient between sound and Fermi velocities, the attenuation for low temperatures grows in comparison with that near T_c. As shown in Fig. 5, the decrease of $\alpha_{\mathbf{q}}(T)$ for decreasing temperature is not so fast as for the usual case of an isotropic BCS superconductor. However, neither the overall temperature behavior nor that for low T can be fitted by a power law.

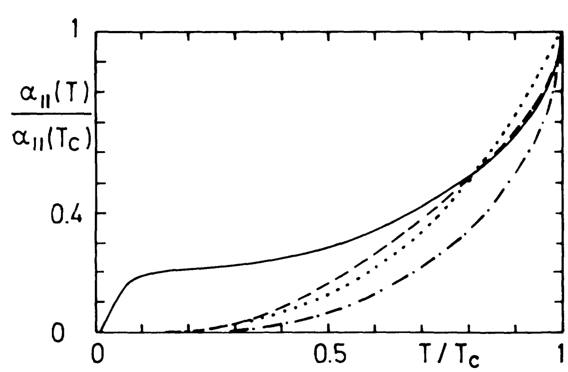

FIG. 5. Longitudinal ultrasonic attenuation as a function of temperature for wave vector parallel to the SSDW axis. Solid line $\varepsilon_F/k_B=10^5$ K and dashed line $\varepsilon_F/k_B=7000$ K. Other parameters as in Fig. 4. For comparison the results for an isotropic gap (dot-dashed line) and function $(T/T_c)^3$ (dotted line) are also shown.

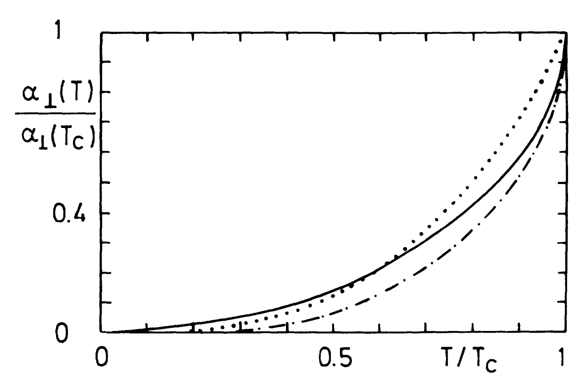

FIG. 7. Longitudinal ultrasonic attenuation for wave vector normal to the SSDW axis as a function of temperature for $G=0.8\varepsilon_F$ and $\varepsilon_F/k_B=10^5$ K. Other parameters as in Fig. 6. The results for an isotropic gap (dot-dashed line) and $(T/T_c)^3$ (dotted line) are also shown for comparison.

Now we consider the case $\mathbf{q}\perp\hat{\mathbf{z}}$. $g_\perp(\lambda)$ is shown in Fig. 6. Comparing this with Fig. 3, we see that, although $g_\perp(\lambda)$ and the density of states $\rho(\lambda)$ are different, both functions are linear in the energy λ for $\lambda \ll \Delta_0(T)$. We obtain the same result for a polar triplet superconductor.[1] $g_\perp(\lambda)$ [and also $g_\parallel(\lambda)$] depends on temperature through the temperature dependence of $\Delta_0(T)$. For simplicity we have taken for $\Delta_0(T)$ the usual temperature dependence of an isotropic BCS gap. This is a good approximation.[12] We obtain that with high accuracy $g_\perp$ depends on λ and T through the quotient $\lambda/\Delta_0(T)$ only. For $G\to 0$, $g_\perp(\lambda)$ is proportional to the step function $\theta(\lambda-\Delta_0(T))$. Inserting this in Eq. (24), one obtains the standard result for an isotropic gap, $\alpha=2f(\Delta_0(T))$ for $G\to 0$.

The attenuation for $\mathbf{q}\perp\hat{\mathbf{z}}$ is shown in Fig. 7. As a consequence of the low-λ behavior of $g_\perp(\lambda)$, $\alpha_\perp(T)$ is linear in T for $T\to 0$ [see Eq. (24)]. This should also be the case of a triplet polar superconductor (in spite of what is said in Ref. 1). For small or moderate values of G, the temperature behavior near T_c is not quite different from the standard BCS result.

IV. DISCUSSION

We have calculated the longitudinal ultrasonic attenuation of a superconductor with a line of zeros of the superconducting gap. This property might be shared with several heavy-fermion superconductors. Recent muon-spin-relaxation experiments suggest that the gap in UPt_3 has in fact polar lines of nodes and also axial point nodes.[16] There is also evidence from Raman measurements that the superconducting gap in high-T_c systems is anisotropic, compatible with *p*- or *d*-wave superconductivity,[17] although this in contradiction with measurements of the penetration depth, lower critical field, and spin susceptibility, which suggest a constant superfluid density at $T \ll T_c$.[18] Recently, the possibility of an isotropic gap changing sign at ε_F has been proposed.[19]

The low-temperature T^2 dependence of the specific heat[12] is shared with other superconductors with a gap vanishing along a line on the Fermi surface, as a consequence of the linear low-energy dependence of the density of states.[1] Instead, we find that the ultrasonic attenuation cannot be directly related to the density of states and depends on the detailed wave-vector dependence of the superconducting gap near the line of zeros. For $T\to 0$ the temperature dependence of $\alpha_\mathbf{q}$ is exponential (linear) for $\mathbf{q}$ parallel (normal) to the SSDW axis. We find the same behaviors for $T\to 0$ in a polar triplet superconductor, in agreement with the results obtained by Rodriguez for $\alpha_\perp$ in the hydrodynamic limit,[20] but contrary to the T^2 dependence reported in Ref. 1. The overall T dependence has a certain similarity to the T^3 dependence measured in UPt_3 by Müller *et al.*[12]

Our results do not show the peak below T_c in the ultrasonic attenuation found in several heavy-fermion[1–4] and high-T_c systems.[10,11] This peak can be explained if the symmetry of our system is broken in such a way that no operation containing time reversal leaves the system invariant.[9,12] This is the case of anyon superconductivity[22] or excitonic polarons,[9] for example. For our system, although as stressed in Ref. 12 the states described by $a_{\mathbf{k}\uparrow}$ and $a_{-\mathbf{k}\downarrow}$ are not time-reversal partners, they are connected by time reversal followed by a translation of magnitude π/Q in the z direction. This is a symmetry operation of the system in both normal and superconducting phases, since no symmetry [except the usual global U(1) symmetry related with the charge] is broken at the superconducting phase transition.

Note added in proof. After submission of our paper we learned about the work of Coppersmith and Klemm.[23] They obtained that for clean systems as $T\to 0$, $\alpha_\perp(T)\sim T$ for a polarlike state (in agreement with our results) and $\alpha_\perp(T)\sim T^2$ for an axial-like state.

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