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X-ray phase plate

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An x-ray analog of the classic Fresnel rhomb can be devised using multiple Bragg reflections from a perfect crystal. A single, channel-cut crystal can provide phase shifts between coherent σ and π polarizations ranging from 0° to 200°. Rapid alterations between left and right circular polarizations can be obtained through a simple angular shift provided by a piezoelectric drive.

Recent advances in synchrotron-radiation research have led to an increased interest in the creation of various states of polarization of x radiation emitted from storage rings. In particular, recent studies of magnetic[1] and nuclear[2] Bragg scattering have enhanced the desire of the research community to have available to it sources of circular polarization. Toward this end, two approaches have been taken. One is to design insertion devices for storage rings[3] with couplings of periodic magnetic devices to create phase shifts producing, in general, elliptically polarized light. A second approach is to use crystal optics[4] in which the phase shifts of different polarization states upon Bragg scattering can be utilized to create similar nonlinear degrees of polarization.

In this paper[5] we describe a method, using Bragg reflections, to create a variable phase shift, which, under appropriate conditions, will produce elliptically or circularly polarized x rays. The device described here is the x-ray analog of the classical Fresnel rhomb. The Fresnel rhomb produces circularly polarized radiation by taking advantage of the phase shift produced by total internal reflection. The σ and π polarization components of the radiation within the total-reflection range have phase shifts which depend upon the angle of reflection. In the Fresnel rhomb a linearly polarized beam is incident normally on one surface of a rhombohedral prism. Its polarization vector is inclined 45° to the optic axis. The ray then reflects internally from an adjacent surface such that the relative phase shifts between σ and π is 45°. This is then repeated from the opposite parallel face, giving a net shift of 90° and producing a circularly polarized beam.

A similar situation exists in dynamical x-ray reflection from perfect crystals. As is well known,[6] the Darwin curve in symmetric Bragg reflection from a perfect crystal produces a flat-topped region of total reflection whose angular range is the order of arcseconds. Throughout the range of total reflection, the relative phase shift of the diffracted beam ranges from 0° to 180°. In the wings of the Darwin curve, the relative phase shift is precisely 0° to 180° for the nonabsorbing case.

The angular ranges of total reflection for the σ and π polarizations differ by a factor $\cos 2\theta$, where θ is the Bragg angle. Thus, at any angle of reflection within the total-reflection range of the σ polarization, different degrees of phase shift occur because the 0° to 180° ranges for σ and π polarizations do not exactly overlap. We show in Fig. 1 the Darwin-Prins curves as well as the phase shifts upon diffraction from a perfect germanium (220) crystal as a function of Bragg angle within the range of total reflection. The angle scale η is normalized such that the range of total reflection for the σ case is -1 to

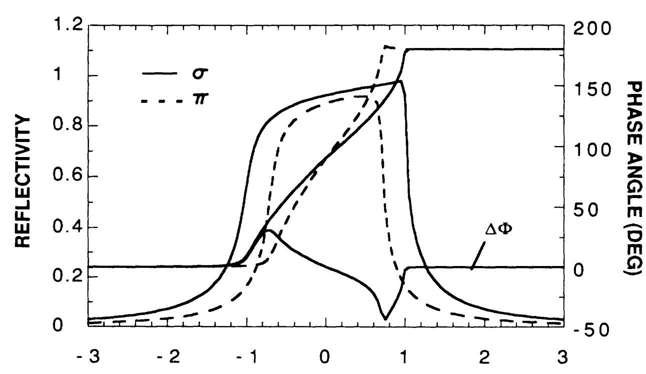

FIG. 1. Theoretical plots for σ and π polarization of reflectivity and phase angle for the Ge(220) reflection using 8-keV photons. $\Delta\phi$ represents the difference in phase between the σ and π components as a function of η, which is proportional to the deviation $\Delta\theta$ from the Bragg angle.

+1. η is related to the relative incident Bragg angle $\Delta\theta$ by

$$\eta = \frac{b\Delta\theta \sin\theta + \frac{1}{2}\Gamma F_0(1-b)}{\Gamma |P| \sqrt{|b|} [F_H F_{\bar{H}}]^{1/2}} ,$$

where $\Gamma = r_e \lambda^2/\pi V$, r_e = classical electron radius, λ the wavelength, and V the unit-cell volume. P is the polarization factor (1 or $\cos 2\theta$) for σ and π, respectively, and F_H is the structure factor of the cell of volume V. b is the asymmetry factor and equals the ratio of direction cosines of the incident and diffracted wave vectors with respect to the inward surface normal. It is straightforward to show that $b = -\sin(\theta-\alpha)/\sin(\theta+\alpha)$ (α is the angle between the crystal surface and the Bragg planes) and equals -1 for the symmetric Bragg case. Figure 1 also shows the relative phase difference $\Delta\phi$ of the two components as a function of incident angle. As can be seen from the figure, $\Delta\phi$ follows a rather complex curve, which changes monotonically within the range of total reflection. The two beams are precisely in phase at the center of the total reflection region ($\eta=0$). The curve shows the particular case of a Ge(220) reflection, whose Darwin width is about 13 arc sec for the σ polarization using 8-keV x-ray photons.

The basic principle of the present device is as follows: A plane linear polarized beam is incident upon a crystal at the Bragg angle with its plane of polarization inclined at 45° with respect to the plan of incidence. The resolved σ and π components will, upon diffraction, be phase shifted with respect to one another according to $\Delta\phi$ of Fig. 1. Since our goal is to produce phase shifts of order 90° leading to circular polarization, we see from the available $\Delta\phi$ that, as is the case with the Fresnel rhomb, more than one reflection must be involved.

For our particular case, we choose a relatively simple channel-cut crystal used extensively in the monochromatization of synchrotron radiation. At the 8 keV photon energy in this experiment, the beam will be reflected 5 times before leaving the channel-cut crystal. The channel-cut monochromator is made by simply cutting a slot in a single crystal of germanium such that, at the Bragg angle, the beam will bounce through five reflections before coming out. From Fig. 1 we see that at incident angles corresponding approximately to $\eta = \pm 0.8$ ($\Delta\theta \approx 6$ sec) a phase shift of $\pm 18°$ is created for each reflection, giving $\pm 90°$ after five reflections. The experimental problem in defining a particular phase shift is to have an incident beam whose intrinsic angular width is narrow compared to the Darwin curve.

In our setup the channel cut (henceforth called the wave plate) acts as the second crystal in a standard parallel double-crystal nondispersive diffractometer. The first crystal is an asymmetrically cut Ge(220) crystal whose surface makes an angle α with respect to the Bragg planes. In our case $\alpha = 19.2°$ $\theta = 22.63°$, giving $b = -0.09$. The Darwin width of this reflection is $|b|^{1/2}$ of the symmetric value. When convoluted with the channel-cut reflection, it produces a reflection curve close to the single unconvoluted Darwin curve. Thus we are able to select a narrow incident range of η and hence a narrow range of phase shifts.

The experimental arrangement is shown in plan view in Fig. 2 using a conventional sealed copper target x-ray tube. The wave plate is approximately 2 cm long and the channel width is 2 mm. The output of the wave plate is monitored for polarization using the analyzer consisting of a Ge(333) reflection whose 2θ is very close to 90°. In the experimental arrangement described up to this point (without the polarizing crystal shown in Fig. 2), the Ge(333) analyzer merely acts as a deflecting device, recording an intensity proportional to the output of the wave plate. The wave plate is mounted on a stage whose angular rotation is produced by a piezoelectric drive. The beam incident on the monochromator is unpolarized, and the beam entering the wave plate is partially polarized.

In Figs. 3(a) and 3(b) we show the theoretically expect-

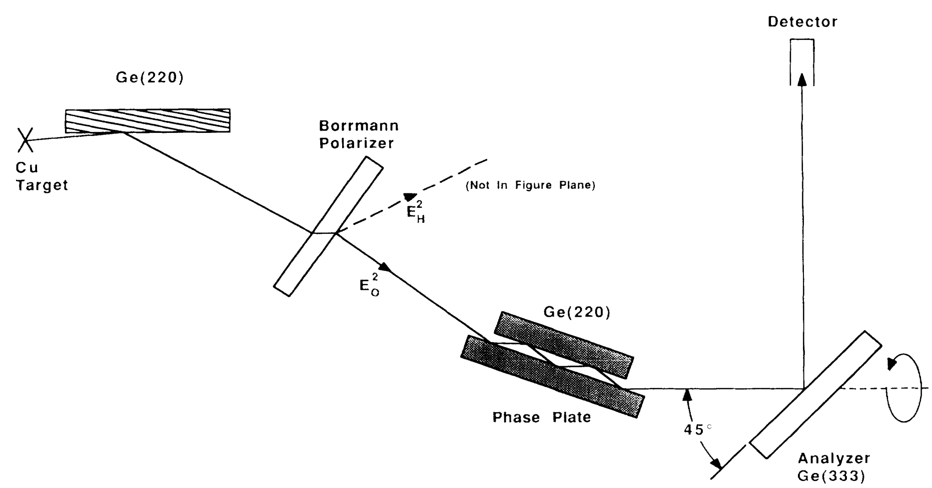

FIG. 2. Plan view of the experimental arrangement. The monochromator is an asymmetrically cut Ge(220) crystal. The Bormann polarizer is a silicon crystal slab cut for (111) symmetrical Laue transmission. The diffracted (111) beam from the polarizer (shown as a dashed line) diffracts out of the plane of the figure and is not used in the experiment.

ed Darwin curve (labeled "no polarizer") after five bounces using the asymmetric monochromator as a convolution (broadening) function. The curve is no longer flat topped because of the five reflections, but the reflectivity is still high.

In order to create an incident photon beam whose polarization vector is inclined 45° with respect to the σ and π components of the wave plate, we used the phenomenon of anomalous transmission of x rays, the Borrmann effect (see Ref. 6). The polarizer is a single crystal of silicon symmetrically cut for the (111) transmission Laue reflection case (see Fig. 2). The thickness is picked such that the product $\mu t = 10$, where μ is the linear absorption coefficient for 8-keV photons and t is the thickness. In this case the only surviving dynamical wave field is one whose polarization is parallel to the diffracting planes of the polarizer. When this wave field emerges from the crystal, it breaks up into two plane waves, one traveling in the diffracted 2θ direction and the other in the forward direction.

As shown in Fig. 2, the forward-diffracted beam of the

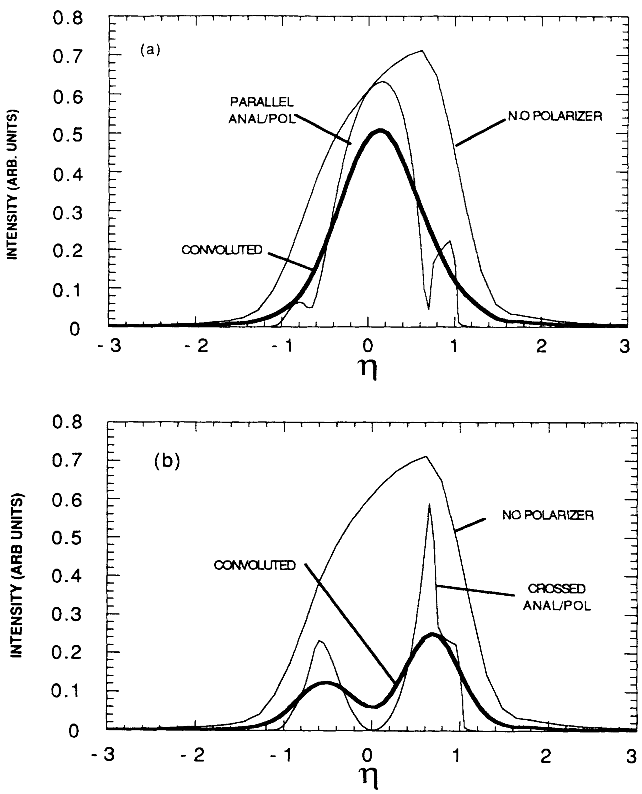

FIG. 3. All are theoretical curves for a Ge(220) reflection using 8-keV photons. (a) Polarizer and analyzer have parallel polarization planes. The "no-polarizer" curve is the Darwin curve of Fig. 1 after five reflections and convoluted with the output of the asymmetric monochromator. The "parallel anal/pol" curve is the expected output of the wave plate for an incident plane polarized wave included at 45° with respect to the plane of Fig. 2 as a function of incident angle η. The remaining curve shows this output after convolution with the monochromator function. (b) is the corresponding set of curves when the polarizer and analyzer are crossed.

polarizer is used with its Bragg planes inclined 45° to the plane of the figure. This Borrmann polarizer is critical to the functioning of the wave plate. The parallel-slab polarizer, as with any other refractive optical element with parallel faces, ensures that the forward-diffracted beam leaves the element undeviated from the incident plane wave producing it. In this manner the nondispersive aspect of the parallel double-crystal diffractometer determined by the monochromator and wave plate is unaffected by insertion of the polarizer.

The insertion of the polarizer reduces the analyzer intensity by about a factor of 10^4 because of its narrow acceptance of the vertical divergence available. Because of this, we pick two special directions for analysis: one where the polarizer and analyzer are parallel (with respect to their polarization directions) and the other where they are crossed. We measure the analyzer output in each case as we sweep the wave plate through its reflection range. In this manner we sample sequentially all the phase shifts (times 5) shown in Fig. 1. The expected output for the two cases is shown in Fig. 3, including the effect of convolution of the asymmetric monochromator.

Figure 3(a) shows three curves for the parallel case. The curve labeled "no polarizer" refers to the output of the wave plate for a substantially unpolarized incident beam and is basically the Darwin curves of Fig. 1 raised to the fifth power and convoluted with the instrumental width of the asymmetrically cut monochromator. The "parallel" curve is the theoretical output expected with the polarizer inserted and results from the changing phase shift of the wave plate as a function of angle η across the range of reflection. Note the discontinuous nature of the curve and the narrowing of the central peak compared with the unpolarized case. The intensity is maximum at $\eta = 0$, where there is zero phase shift and the polarization output of the wave plate is the same as the input. The remaining curve shows the convolution of this theory curve with the monochromator function, which can be seen to mask some of the more intricate aspects of the curve.

Fig. 3(b) shows the corresponding case when the polarizer and analyzer are crossed. Note that the central portion of the reflection curve is removed at $\eta = 0$, as expected, since the output beam will be crossed with respect to the analyzer. What remains are two peaks where the phase shifts lie between 90° and 180°, corresponding to a substantial polarization component rotated 90° with respect to that of the incident beam. The convoluted curve shows that the experimental output is an asymmetrically broadened doublet.

Finally, we show in Fig. 4 the experimental results, together with relevant theoretical expectations. The experimental angle scale used in all curves which relates the piezoelectric voltage to η is determined from the single normalization of the "no-polarizer" curve of Fig. 4(a). The "parallel" case theory is normalized only in intensity to match the data. Note that the predicted narrowing and its displacement with respect to the un-phase-shifted curve matches the data very well.

In Fig. 4(b) we show theory and experiment in the crossed polarizer-analyzer combination. The η scales are determined from the fit in Fig. 4(a). Theory and experiment are in reasonable agreement and show the dramatic change in the diffraction curve due to the wave plate.

The results show in a direct way that the x-ray optical arrangement of Fig. 2 is a variable-phase-shift wave plate. The intensity for the regime near $\eta=\pm0.8$ is close to circularly polarized radiation. This is strongly inferred because of the very good fit between theory and experiment. Alternation between left and right circular polarization is easily achieved by shifting the voltage on the wave-plate piezoelectric drive between the values corresponding to $\eta=\pm0.8$.

The count rate available with this device is very low because of the large vertical divergence necessitated by the low flux of the conventional sealed tube source. Synchrotron sources offer several possibilities for enhancing the throughput of the wave plate. The natural vertical collimation and horizontal polarization are ideally suited for perfect crystal optics and could suggest modifications of the present setup to produce beams of variable polarization of considerably higher intensity than estimated above.

We have used the phase-plate concept (see Ref. 7) to test the recently created bend-magnet beam line D-1 at CHESS, the Cornell High Energy Synchrotron Source. The geometrical arrangement of Fig. 2 was modified somewhat to adapt to a synchrotron-radiation environment. The first asymmetric crystal was replaced by a quasiparallel crystal pair consisting of a symmetric (220) germanium reflection followed by an asymmetric (220) silicon crystal. The Borrmann polarizer and five-bounce channel cut were the same as before. As a result of the high flux available, we were able to scan the analyzing crystal through its rocking curve for each η value of the phase plate. The integrated counts for each rocking curve comprised the corresponding datum point of the analyzed polarized beam.

The results, shown in Fig. 5, are presented to correspond to Fig. 4. The theory curves take into account the convolution of the new monochromator broadening function.

The intensity through the five-bounce crystal at the peak of the reflection curve was 1×10^5 cps for storage-ring operation at 5.2 GeV and 45 mA. At the η angle corresponding to a phase shift of 90° ($\eta\sim0.8$), the intensity is about 5×10^4 cps. A wiggler source would have increased this by an order of magnitude, and although

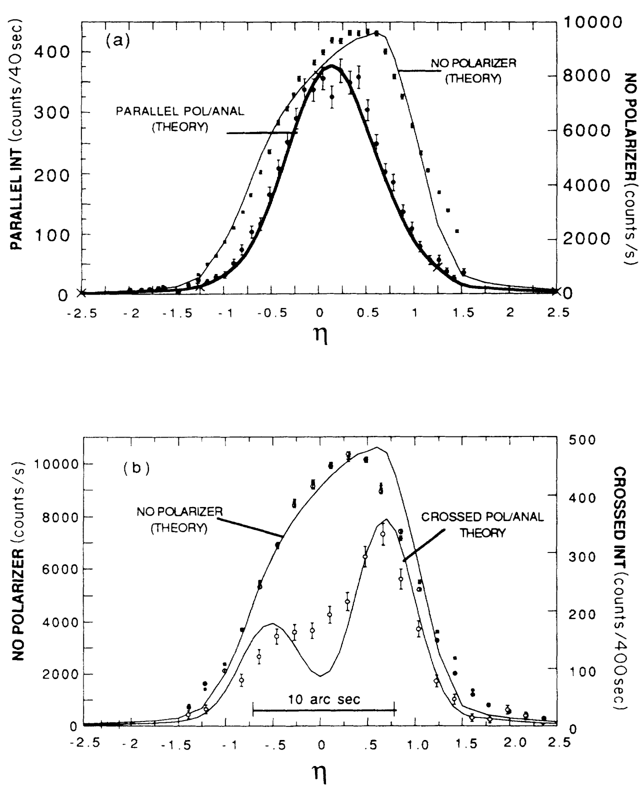

FIG. 4. Comparison of experimental with the theoretical curves of Fig. 3. Data are the experimental points shown with error bars. The drawn curves are theory. The η scale for all data is determined from the single fit to theory for the curve labeled "no polarizer" in (a). The second set of data points in (a) is the output of the analyzer in the uncrossed (parallel) case. (b) shows theory and experiment for the "crossed" case, but with the η scale normalized only from the "no-polarizer curve" of (a).

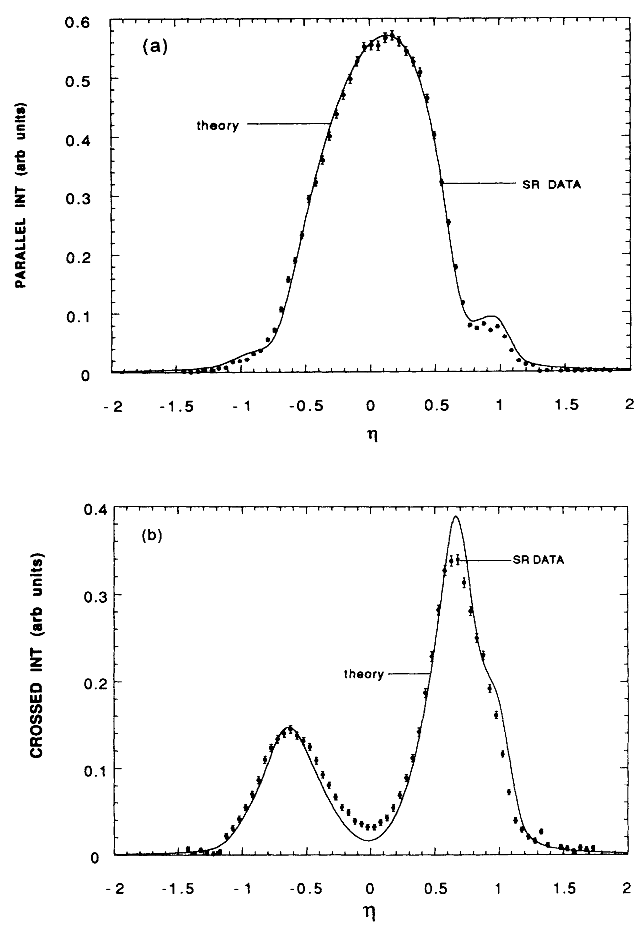

FIG. 5. These results (see Ref. 7) taken with synchrotron radiation at CHESS correspond to Fig. 4. Details are given in the text.

difficult to estimate, an undulator would probably add another factor of 10.

A phase plate using transmission through a very thin Bragg crystal has recently been reported[8] by Hirano *et al.*, showing that variable polarization can be inferred from analyzer data of the transmitted beam. This phase shift is produced by an interference between forward-diffracted σ and π waves whose net phase difference will depend upon crystal thickness and departure $\Delta\theta$, from the exact Bragg condition. In this case the transmitted beams correspond to diffraction outside the range of total reflection.

One disadvantage of the present wave plate is the limitation to a relatively narrow energy for a given crystal pair. On the other hand, the device is simple to construct and offers a means of changing the degree of polarization from left circular to linear to right circular in a rapid periodic manner.

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[1]G. Schütz, W. Wanner, W. Wilhelm, P. Kienle, R. Zeller, R. Frahm, and G. Materlik, Phys. Rev. Lett. **58**, 737 (1987); M. J. Cooper, D. Laundy, D. A. Cardwell, D. N. Timms, and R. S. Holt, Phys. Rev. B **34**, 5984 (1986); D. Mills, *ibid.* **36**, 6178 (1987).

[2]E. Gerdau, R. Ruffer, H. Winkler, W. Tolksdorf, C. P. Klager, and J. P. Hannon, Phys. Rev. Lett. **54**, 835 (1985); E. Gerdau, R. Ruffer, R. Hollatz, and J. P. Hannon, *ibid.* **57**, 1141 (1986); D. P. Siddons, J. B. Hastings, G. Faigel, L. E. Berman, P. E. Houston, and J. R. Grover, *ibid.* **62**, 1384 (1989).

[3]S. Yamamoto, H. Kawata, H. Kitamura, M. Ando, N. Saki, and N. Shiotani, Phys. Rev. Lett. **62**, 2672 (1989); B. M. Kincaid, J. Appl. Phys. **48**, 2684 (1977).

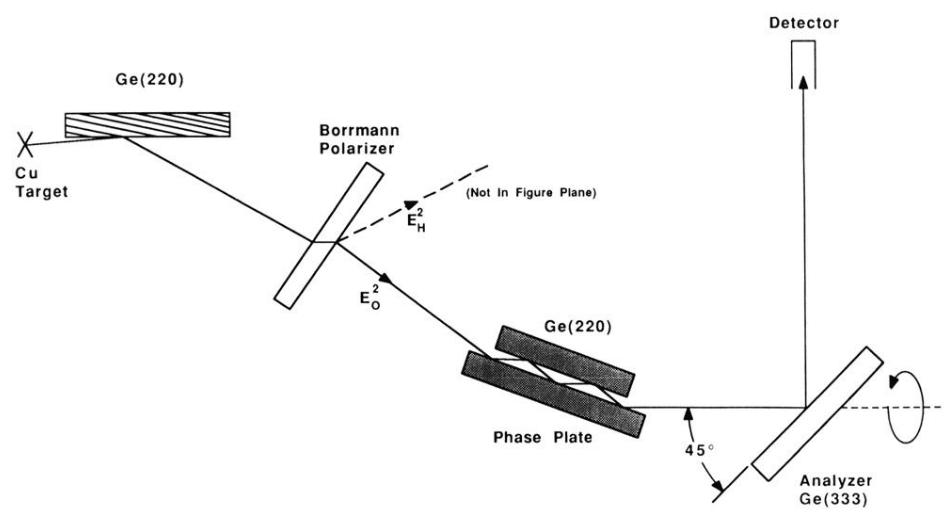
[4]M. Hart, Philos. Mag. B **38**, 41 (1978); D. Mills, Nucl. Instrum. Methods A **266**, 531 (1988); J. A. Golovchneko, B. M. Kincaid, R. A. Levesque, A. E. Meixner, and D. R. Kaplan, Phys. Rev. Lett. **57**, 202 (1986).

[5]B. W. Batterman, Preliminary results of this work were presented at the Thirteenth Congress of the International Union of Crystallography [Acta. Crystallogr. Suppl. A **40**, (1984). Independent work by Brümmer and his colleagues described a similar approach; O. Brümmer, Ch. Eisenschmidt, and R. Höche, Acta Crystallogr. A **40**, 394 (1984).

[6]B. W. Batterman and H. Cole, Rev. Mod. Phys. **36**, 681 (1964).

[7]These preliminary results, unpublished, were obtained in collaboration with M. Bedzyk, K. Finkelstein, and S. Shastri.

[8]K. Hirano, K. Izumi, T. Ishikawa, S. Annaka, and S. Kikuta, Jpn. J. Appl. Phys. **30**, 407 (1991).



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