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Phase transitions in layered superconductors

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A system of superconducting layers with Josephson coupling J between them is studied. When the in-layer penetration depth λ_e is larger than the spacing d between layers, as in CuO_2-based superconductors, there is a single *three-dimensional* transition temperature T_c. The ratio T_c/τ, where $\tau=\phi_0^2/4\pi^2\lambda_e$, is found to vary from $\sim\frac{1}{8}$ to ~ 1, being near $\frac{1}{8}$ when J/T_c is exponentially small. When $\lambda_e \lesssim d$ a *two-dimensional* (2D) behavior is possible; in particular, bulk superconductors separated by 2D junctions exhibit a 2D transition, below the bulk transition, in which the junctions become ordered. This 2D behavior is due to the gauge coupling and is absent in an XY model where $\lambda_e \to \infty$.

Phase transitions in layered superconductors are of considerable interest since most of the high-temperature superconductors have a layered structure. In particular, the separation between conducting CuO_2 layers can be controlled by preparing $YBa_2Cu_3O_7/PrBa_2Cu_3O_7$ superlattices.[1–3] T_c is found to drop from $\sim$90 to $\sim$20 K, indicating that T_c of $YBa_2Cu_3O_7$ is far above that of an isolated layer. Data on multilayers of $Bi_2Sr_2CaCu_2O_8$ with semiconducting $Bi_2Sr_2CuO_6$ have also shown a decrease of T_c from 59 to $\sim$30 K.[4] These experiments focus our attention on the crossover from a two-dimensional (2D) phase transition of an isolated layer to a 3D transition of coupled layers.

The model for layered superconductors[5,6] involves a Ginzburg-Landau continuum model for each layer and a Josephson coupling J between neighboring layers. This model defines two types of topological excitations: (i) vortices, which are point singularities in each plane, and (ii) fluxons, which are lines parallel to the layers across which the relative phase of neighboring layers changes by 2π.

The system with $J=0$ has been studied by several authors.[7–10] It was found that although the planes are coupled via the 3D magnetic field, the vortex-vortex interaction is logarithmic in distance, similar to the case of an isolated layer. It is expected then that a Kosterlitz-Thouless (KT)-type phase transition[11–13] will occur at a temperature T_v, although an explicit renormalization-group (RG) study has not been given so far.

When $J\neq 0$ fluctuations of fluxon loops compete with the vortex transition. Assuming that vortices are absent, the system has a phase transition at T_f; at $T>T_f$ fluctuations of fluxon loops destroy the correlation between layers allowing for an independent 2D behavior of each layer, while for $T<T_f$ the layers are correlated resulting in a 3D long-range order. The neglect of vortices is consistent for isolated or widely separated junctions,[14] e.g., junctions on twin boundaries in $YBa_2Cu_3O_7$, or if $T_f<T_v$ (Ref. 15) so that vortices are not excited thermally.

A discrete-Gaussian version for the free energy of layered superconductors has recently been studied by Korshunov.[16] He found that $T_v<T_f$ for all model parameters, thus eliminating the possibility of a KT transition. The 3D transition T_c was then claimed to be near T_v with $\ln(T_c-T_v)\sim T_c/J$. A closely related model is that of an anisotropic layered XY system,[17] where it was argued that $T_c-T_v\sim \ln^{-2}(J/T_c)$.

In the present work I follow the brief outline in Ref. 18 and study the phase transition in three steps: (i) Solution of the $J=0$ system by second-order RG, showing that a KT-type transition occurs at a temperature T_v. (ii) Solution of the $J\neq 0$ system, assuming no vortices, by RG and showing a KT-type transition at $T_f>T_v$. (iii) Finding the 3D transition temperature T_c by comparing the vortex and fluxon correlation lengths. It is also shown here that when the layer spacing d becomes larger than the effective penetration depth λ_e, a finite-size transition at $T_v^{\text{eff}}>T_f$ is possible. In particular, a 2D junction separating bulk superconductors can be thermally disordered;[14] this feature is absent in a corresponding XY model, for which $\lambda_e\to\infty$.

The conventional effective free energy F for layered superconductors[5–10] is derived[5,6] by assuming a constant amplitude for the order parameter (i.e., T is not too close to the mean-field transition temperature) and a weak cosine-type Josephson coupling between neighboring layers. (In fact, higher powers of the cosine can be shown to be irrelevant near the fluxon transition T_f.) Hence,

$$F=\frac{1}{8\pi}\int d^2r\,dz\left\{(\nabla\times\mathbf{A})^2+\frac{1}{\lambda_e}\sum_n\left[\frac{\phi_0}{2\pi}\nabla\varphi_n(\mathbf{r})-\mathbf{A}(\mathbf{r},z)\right]^2\delta(z-nd)\right\}$$
$$-\frac{J}{\xi_0^2}\sum_n\int d^2r\cos\left[\varphi_n(\mathbf{r})-\varphi_{n-1}(\mathbf{r})-(2\pi/\phi_0)\int_{(n-1)d}^{nd}A_z(\mathbf{r},z')dz'\right]-E_c\sum_{\mathbf{r},n}s_n^2(\mathbf{r})\,, \qquad (1)$$

where $\varphi_n(\mathbf{r})$ is the superconducting phase on the nth layer, $\mathbf{r}$ is the position vector in the layer, $\mathbf{A}(\mathbf{r},z)$ is the vector potential, $\phi_0=hc/2e$ is the flux quantum, E_c is the loss of condensation energy in a volume $\xi_0^2d_0$, and $s_n=\pm1$ at the vortex sites while $s_n=0$ otherwise. The length scales are $\lambda_e=\lambda^2/d_0$ with λ the London penetration depth parallel to the layers, d_0 the thickness of each layer, d ($>d_0$) the separation between layers, and ξ_0 the in-plane correlation length; typically

$\lambda_e \approx 10^6$–10^7 Å$\gg d \approx 10$ Å for the CuO_2 systems. The four terms of (1) describe the 3D magnetic energy, the 2D supercurrents, the Josephson coupling, and the vortex core energies, respectively.

It is useful to rewrite Eq. (1) in terms of the natural variables $s_n(\mathbf{r})$ and $\theta_n(\mathbf{r})$, where

$$\varphi_n(\mathbf{r}) = \sum_{\mathbf{r}'} s_n(\mathbf{r}')\alpha(\mathbf{r}-\mathbf{r}') + \varphi_n^0(\mathbf{r}) , \quad \theta_n(\mathbf{r}) = \varphi_n^0(\mathbf{r}) - \varphi_{n-1}^0(\mathbf{r}) - (2\pi/\phi_0)\int_{(n-1)d}^{nd} A_z(\mathbf{r},z')dz' , \tag{2}$$

with $\alpha(\mathbf{r}) = \tan^{-1}(y/x)$, $\varphi_n^0(\mathbf{r})$ is the nonsingular part of $\varphi_n(\mathbf{r})$, and the sum on $\mathbf{r}$ is defined on a lattice of spacing ξ_0. The vector potential $\mathbf{A}(\mathbf{r},z)$ is a Gaussian variable and can be integrated out by first solving its equation of motion in terms of $s_n(\mathbf{r})$ and $\theta_n(\mathbf{r})$ and resubstituting in (1). [Note that A_z in (1) is determined by a gauge condition.] The gauge invariant result is

$$F' = \tfrac{1}{2} T \sum_{n,\mathbf{r};n',\mathbf{r}'} s_n(\mathbf{r}) G_v(\mathbf{r}-\mathbf{r}', n-n') s_{n'}(\mathbf{r}') + E_c \sum_{n,\mathbf{r}} s_n^2(\mathbf{r}) + \tfrac{1}{2} T \sum_{\mathbf{q},k} G_f^{-1}(\mathbf{q},k) |\theta(\mathbf{q},k)|^2$$
$$- (J/\xi_0^2) \sum_n \int d^2 r \cos\left\{ \theta_n(\mathbf{r}) + \sum_{\mathbf{r}'} [s_n(\mathbf{r}') - s_{n-1}(\mathbf{r}')] \alpha(\mathbf{r}-\mathbf{r}') \right\} , \tag{3}$$

where $\mathbf{q},k$ are the Fourier transform variables of $\mathbf{r},n$, respectively, and

$$G_v(\mathbf{q},k) = \pi d(\tau/T)\{[1+f(\mathbf{q},k)]q^2\}^{-1} , \tag{4a}$$

$$G_f(\mathbf{q},k) = 4\pi(T/\tau)(d^2/\lambda_e)[1+(4\lambda_e/d)\times\sin^2(kd/2)]/q^2 , \tag{4b}$$

with $f(\mathbf{q},k) = \sinh(qd)/\{2\lambda_e q[\cosh(qd) - \cos(kd)]\}$ and $\tau = \phi_0^2/(4\pi^2\lambda_e)$. Equation (3) is the effective free energy written in terms of the natural excitations of the system —vortex singular points $s_n(\mathbf{r})$ and the phase $\theta_n(\mathbf{r})$ which defines fluxons; the latter are topological line excitations such that $\theta_n(\mathbf{r})$ varies by 2π across a line in between one pair $(n,n-1)$ of planes.

Consider now the $J=0$ system. The energy of a vortex of length l (i.e., l singularities on the planes $n=0, 1, \ldots, l-1$ at $\mathbf{r}=0$) for $\lambda_e \gg d$ and $l \ll (\lambda_e/d)^{1/2}$ is $E(l) = (\tau/4) l \ln(R/\xi_0)$, where R is the in-plane size. The lowest-energy excitation corresponds to $l=1$; following the KT (Ref. 11) argument by adding the entropy $-T\ln R^2$ indicates an instability at $T_v = \tau/8$ above which $l=1$ vortices are thermally excited.

The proper description of this transition is via RG equations.[11,19] A duality transformation to a sine-Gordon system with a field $\chi_n(\mathbf{r})$ and integrating its in-plane momentum from $1/\xi$ to $1/\xi - d(1/\xi)$ renormalizes (i) the fugacity $y_0 = 2\exp(-E_c/T) = y(\xi_0)$ to $y(\xi)$ and (ii) the coupling τ for the on-plane terms $(n=n')$ of $\nabla\chi_n(\mathbf{r})\nabla\chi_{n'}(\mathbf{r})$ to $\tau(\xi)$ via

$$dy = y\{2 - (\tau/4T)[1-u(\xi)]\} d\xi/\xi , \tag{5a}$$

$$d\tau = -2\gamma^2 y^2 \tau(\tau/8T)^2 [1-u(\xi)] d\xi/\xi , \tag{5b}$$

where

$$u(\xi) = \frac{\sinh(d/\xi) b(\xi) \xi}{2\lambda_e [1-b^2(\xi)]^{1/2}} ,$$

$$b(\xi) = 2\lambda_e / [2\lambda_e \cosh(d/\xi) + \xi \sinh(d/\xi)] ;$$

γ depends on the procedure for smoothing the cutoff[19,20] (a cutoff represented by a mass insertion yields $\gamma = 8\pi$).

To first order in y_0 [Eq. (5a)] the fixed point is determined by $u(\infty) = (1+4\lambda_e/d)^{-1/2}$ so that the phase transition temperature is

$$T_v = (\tau/8)[1-(1+4\lambda_e/d)^{-1/2}] + O(y_0) . \tag{6}$$

To second order in y_0, the fixed point depends on $u(\xi)$. However, for typical cases of Cu-O_2 layers with $d \lesssim \xi_0 \ll \lambda_e$, $u(\xi)$ is fairly close to $u(0)$ for all $\xi > \xi_0$ and Eq. (5) yields the standard KT trajectories[11,17] (dashed lines in Fig. 1); T_v is then $T_v = (\tau/8)(1-\gamma y_0)$ for $\lambda_e \gg d$.

Note that $\tau = \tau(T)$ since the effective free energy (1) involves a temperature dependent λ. Defining T_c^0 as the transition temperature of a corresponding isotropic 3D system and assuming that the relevant temperatures are not too close to T_c^0, the mean field form $\lambda(T) = \lambda_0(1-T/T_c^0)^{-1/2}$ can be used. Hence $\tau(T) = \tau_0(1-T/T_c^0)$ where $\tau_0 = \phi_0^2 d_0/4\pi^2\lambda_0^2$ (typically $\tau_0 \approx 10^3$ K) and from Eq. (6) $T_v = T_c^0[1+8T_c^0/\tau_0]^{-1}$ for $d \ll \lambda_e$.

It is interesting to consider the case of $d \gtrsim \lambda_e$ which applies to isolated or well-separated 2D junctions, e.g., junctions on twin boundaries in $YBa_2Cu_3O_7$.[14] For $\xi \ll \lambda_e$, $u(\xi) \ll 1$ and the scaling (5) acts as if $T_v = \tau/8$, while in the final integration range $\xi \gg d$, $u(\xi) \approx 1 - 2\lambda_e/d$ and scaling proceeds as appropriate for the thermodynamic limit transition $T_v = \lambda_e\tau/4d \lesssim \tau/8$. Thus for $\lambda_e\tau/4d \lesssim T$

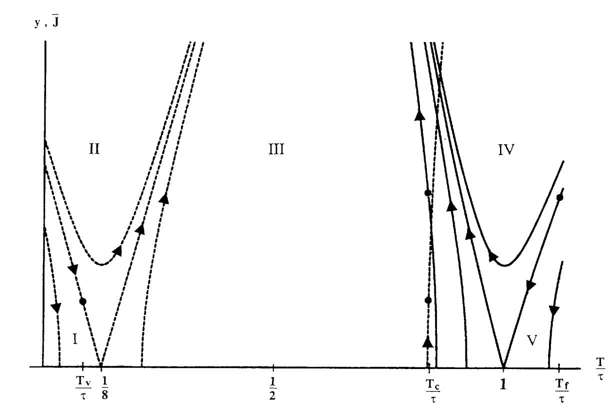

FIG. 1. Scaling trajectories of vortex fugacity y (dashed lines) and of Josephson coupling $\bar{J}$ (solid lines) for $d \ll \lambda_e$. Regions II and IV are in between the straight lines $\gamma y_0 = \pm|8(T/\tau)-1|$ and $\tilde{\gamma}\bar{J}_0 = \pm|\tau/T-1|$, respectively; regions I, III, and V cover the other regions, as indicated. Circles mark the initial values of y or $\bar{J}$ on the trajectories defining T_v and T_f, and on the crossing trajectories (for which $\xi_v \approx \xi_f$) which determine T_c.

$\lesssim \tau/8$, $y(\xi)$ first decreases, as if the system were ordered, but eventually $y(\xi)$ increases when $\xi > \lambda_e$ to signal the actual disordered phase. It is possible, however, that the latter increase is smaller than the former decrease due to a finite size R. This defines a finite-size transition temperature $T_v^{\rm eff}$ which can be much higher than T_v if $d \gg \lambda_e$,

$$T_v^{\rm eff} = (\tau/8)\ln(2\lambda_e/\xi_0)/\ln(R/\xi_0) + O(y_0),\ \ R \gg \lambda_e\ , \qquad (7a)$$

$$T_v^{\rm eff} = \tau/8 + O(y_0),\ \ R \ll \lambda_e\ . \qquad (7b)$$

Note that the existence of $T_v^{\rm eff}$ is due to the gauge coupling e; for the corresponding XY model ($e \rightarrow 0$ and $\lambda_e \sim e^{-2} \rightarrow \infty$) the condition $d \gtrsim \lambda_e$ for Eqs. (7) cannot apply. Consider two interesting limiting examples where the effect of $e \neq 0$ is significant. First, the well-known case of an isolated superconducting film,[13] i.e., $d \rightarrow \infty$. Unlike the XY system, there is no strict phase transition in this case ($T_v \sim d^{-1} \rightarrow 0$) and just the finite-size transition (7) survives. This is due to the screening of the vortex singularity beyond the distance λ_e, a screening which is absent in an XY model. The second example is the case of thick layers, i.e., a system of bulk superconductors joined by 2D junctions (*both* d and d_0 become large). Although the starting model equation (1) is not applicable when $d_0 \gg \lambda_e$, it is instructive to see the formal result of the model in this limit, i.e., $T_v \rightarrow 0$ while $T_v^{\rm eff} \rightarrow T_c^0$; the latter is the correct T_c, since the layer is now a bulk system.

I proceed next to study a second limit of Eq. (1), namely $J \neq 0$, but vortices are assumed to be absent, i.e., $s_n(\mathbf{r}) = 0$ in (3). This is a 2D sine-Gordon-type problem which can be studied with the same RG procedure as above. Since the q dependence of $G_f(\mathbf{q},k)$ is just $1/q^2$, there are no correction terms like $u(\xi)$ above; instead, however, there is an essential k dependence which in order J^2 is renormalized. Define therefore a scaling function $g(k,\xi)$ via $G_f(\mathbf{q},k;\xi) = 8\pi g(k,\xi)d/q^2$, with the initial form $G_f(\mathbf{q},k;\xi_0) = G_f(\mathbf{q},k)$ given by Eq. (4b). To second order in J, increasing the scale ξ from ξ_0 renormalizes $\bar{J}_0 = J/T$ to $\bar{J}(\xi)$ and $x_0 = (1 + d/2\lambda_e)T/\tau$ to $x(\xi) = (d/2\pi)\int dk\, g(k,\xi)$ via the recursion relations

$$d \ln\bar{J} = 2[1 - x]d \ln\xi\ , \qquad (8a)$$

$$dx = -2\tilde{\gamma}^2\bar{J}^2x^3 d\ln\xi\ , \qquad (8b)$$

where

$$\tilde{\gamma}^2 = \tfrac{3}{2}\gamma^2[1 + 2d/3\lambda_e + d^2/6\lambda_e^2](1 + d/2\lambda_e)^{-2}\ .$$

Equations (8) yield KT-type trajectories (solid lines in Fig. 1) and a phase transition at the temperature

$$T_f = \tau/[(1 + d/2\lambda_e)(1 - \tilde{\gamma}\bar{J}_0)]\ . \qquad (9)$$

For $T > T_f$, J is irrelevant (region V in Fig. 1), i.e., thermally excited fluxon loops cause J to renormalize to zero. For $T < T_f$ (regions I–IV) J is relevant and long-range order between layers is established (assuming of course no vortices, $s_n = 0$). Note that not too close to T_f (below region IV) the $x(\xi)$ renormalization can be neglected and Eq. (8a) yields

$$\bar{J}(\xi) = \bar{J}_0(\xi/\xi_0)^{2(1-x_0)}\ . \qquad (10)$$

The correlation length ξ_f is then identified by $\bar{J}(\xi_f) \approx 1$.

For $d \ll \lambda_e$ and y_0, $\tilde{\gamma}\bar{J}_0 \ll 1$, Eq. (9) is a factor of 8 larger than Eq. (6), or in terms of τ_0, $T_v \approx T_f/(1 + 7T_f/\tau_0)$, so that T_v can be considerably smaller than T_f; in fact, for all values of d/λ_e we have $T_v < T_f$. The scheme above is therefore inconsistent in regions II–IV where both y and J are relevant variables. This confirms Korshunov's result[16]—there is no 2D regime and the transition is intrisically 3D with $T_v < T_c < T_f$.

Finite-size effects change the latter conclusion when $\lambda_e \ll d$. The vortex system has a finite-size transition given by Eq. (7) as well as a strict thermodynamic transition at $T_v = \phi_0^2/16\pi^2 d + O(y_0)$. On the other hand, the fluxon system has just the strict transition at $T_f = \phi_0^2/2\pi^2 d + O(J)$; thus $T_f \approx 8T_v$, but $T_f \ll T_v^{\rm eff}$. Finite-size effects in $\lambda_e \ll d$ systems can therefore provide a temperature range $T_f < T < T_v^{\rm eff}$ in which 2D behavior is effectively valid. In particular, 2D junctions between bulk superconductors where $T_v^{\rm eff} \rightarrow T_c^0$ have the range $T_f < T < T_c^0$ in which the junctions are thermally disordered. [T_f formally vanishes as $d \rightarrow \infty$; however, modifying the model equation (1) for this case leads to a finite T_f (Ref. 14).]

The latter result is remarkable: a boundary such as a junction can be thermally disordered while the bulk has long-range order. This result is due to $e \neq 0$—the finite screening length λ_e allows fluxon fluctuations in the junction while the bulk remains ordered. For the XY model, $\lambda_e \sim e^{-2} \rightarrow \infty$ and the junction orders as soon as the bulk does.

I proceed now to determine T_c by comparing the correlation length ξ_v and ξ_f. Interpreting ξ_v^{-2} as the mean density of vortices,[11,19,21] vortices are absent in the cosine term of (3) for $\xi < \xi_v$ and the scaling of the fluxon system is valid up to $\min(\xi_v,\xi_f)$. Now if $\xi_f < \xi_v$, i.e., $\bar{J}(\xi) \approx 1$ for $\xi = \xi_f < \xi_v$, the system becomes isotropic and individual vortex fluctuations are suppressed, i.e., $\xi_f < \xi_v$ corresponds to a 3D-ordered phase. On the other hand, if $\xi_v < \xi_f$, vortices on a scale ξ_v interfere in the cosine term of (3) and prevent $\bar{J}(\xi)$ to fully renormalize. The system remains anisotropic and disordered; hence the criterion for T_c is $\xi_v \approx \xi_f$. Note that if renormalizations of ξ_f by x_0 [Eq. (10)] and of ξ_v by J (see below) are neglected, the criterion becomes that of Ref 17, i.e., the interlayer coupling in area ξ_v^2 satisfies $J\xi_v^2/\xi_0^2 \approx T$; this obviously misses the effect of T_f on the transition.

The fluxon scaling is valid for $\xi < \min(\xi_v,\xi_f)$ and Eq. (10) can be used. In contrast, the vortex scaling equation (5) is never correct for $J \neq 0$ since nonlinearity due to vortices is present in both their direct interaction [first term of (3)] and in the cosine term of (3). This asymmetry is related to the fact that Eq. (1) is not self-dual. To find ξ_v I construct a variational free-energy density $f(\xi_v)$ which includes in addition to the usual interaction and entropy terms[21] the free-energy gain from integrating the J^2 term[19,20] up to ξ_v,

$$f(\xi_v) = \frac{(T - \tau/8)\ln[\xi_0^2/e\xi_v^2] + E_c - T\ln 2}{b(T)\xi_v^2} - 2\pi T\int_{\xi_0}^{\xi_v} x(\xi)\bar{J}^2(\xi)\xi^{-3}d\xi\ , \qquad (11)$$

where $b(T)$ is small for $T < \tau/4$ and increases to ~ 0.8 at $T=\tau$. The vortex density is found by minimizing (11), which together with $J(\xi_v \approx \xi_f) \approx T$ and Eq. (10) yields for T_c in regime III

$$T_c \approx \tau [E_c' + \tfrac{1}{8} \tau \ln T_c/J]/[E_c' + \tau \ln T_c/J] \qquad (12)$$

where

$$E_c' = [E_c - T \ln 2 + \pi b(T) T x(\xi_f) \bar{J}^2(\xi_f)]$$

is enhanced by J.

Very near T_v (region II) $\ln \xi_v \sim (T - T_v)^{-1/2}$ and $T_c - T_v \sim \ln^{-2}(T_c/J)$ as in Ref. 17. Equation (12) yields the more generic behavior of T_c in the wide regime III. In particular, $T_c \rightarrow T_v$ for $\ln(T/J) \gtrsim 8E_c'/\tau$, i.e., J is exponentially small, while $T_c \rightarrow \tau \approx T_f$ for $\ln(T/J) \lesssim E_c'/\tau$, i.e., $J \approx T$. (In fact $T_c/\tau > 1$ in region IV is possible if E_c'/τ is large). Equation (12) thus shows the crossover from fluxon-dominated to vortex-dominated transitions; unless J is exponentially small the transition is near T_f. Note also that the possibility of variations of order 8 in T_c/τ is due to the parameter E_c, which is absent in an XY model.

The data on superlattices[1–3] are consistent with $T_c \approx T_f$ for $YBa_2Cu_3O_7$, dropping to $T_c \approx T_v$ when $PrBa_2Cu_3O_7$ layers of $d \approx 200$ Å are added. The results for the sharper transitions[3] of well-separated 3, 4, and 8 Cu-O bilayers with $T_c^0 \approx T_c = 92$ K, and T_v of Eq. (6) fit $\tau_0 = 1200$ K $\pm$ 30%, consistent with a direct estimate of τ_0. From Eq. (9), $T_f = T_c^0/[1 + 0.08(1 - \tilde{\gamma}\bar{J})]$ which is indeed very close to T_c^0.

I have shown here that even if T_c is near T_v, e.g., the above $d \approx 200$-Å system, T_c is a 3D transition since $d \ll \lambda_e$. Thus the hallmark of 2D superconductivity—the power-law current-voltage relation[13]—should not hold. However, extremely weak magnetic fields parallel to the layers can decouple the layers and lead to strict 2D phases.[18] Thus the $YBa_2Cu_3O_7/PrBa_2Cu_3O_7$ or the $Bi_2Sr_2CaCu_2O_8/Bi_2Sr_2CuO_6$ superlattices are ideal for studying the interplay between 2D and 3D transitions.

To conclude, the present work has shown insight into the nature of phase transitions in anisotropic systems with competing topological excitations. The transition is 3D when $d \lesssim \lambda_e$; furthermore, based on the present analysis and that of current-voltage relations,[18] I propose that T_c in $YBa_2Cu_3O_7$, $Bi_2Sr_2CaCu_2O_x$, and $Tl_2Ba_2CaCu_2O_8$ is dominated by fluxon fluctuations and is near T_f. The role of the gauge coupling e is significant only when $d \gtrsim \lambda_e$; in particular, a 2D junction between bulk superconductors has a regime below T_c in which the junction has 2D correlations, an impossible situation for an XY system.

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