Oscillation of the transport lifetime due to LO phonon scattering in quasi-one-dimensional channels

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The transport properties of quantum wires (QW's), mainly under the action of strong magnetic field, have been studied both theoretically and experimentally. In this report we use the memory-function projection-operator theory to obtain the resistivity of a QW built by structural and electric confinement and show that, in the absence of a magnetic field, oscillations in the transport lifetime appear due to scattering by LO phonons. These oscillations describe the harmonic confinement and are weakly dependent on the carrier concentration in the quantum limit. We suggest that this could be an easy way to characterize that structure using dc transport.

During the past few years the advances on microlithography techniques brought attention to the investigation of quasi-one-dimensional systems. Microstructures on Si metal-oxide semiconductors (MOS) and $Ga_{1-x}Al_x/GaAs$ heterostructures[1] have been used to confine laterally the quasi-two-dimensional electron gas into a quasi-one-dimensional channel.[2] The crossover to the 1D behavior occurs when the channel width become comparable with the length scales, defined by the Fermi wavelength, the phase-coherence length, the thermal-diffusion length, and the elastic mean free path [for a two-dimensional electron gas (2DEG) with a carrier density of 10^{11} cm^{-2}, $\lambda_F \sim 100$ nm]. Several techniques have been used to produce the so-called quantum wires (QW), ranging from electron beam[3] to holographic[4] lithographies. In the latter case an applied gate voltage creates the lateral confinement necessary to reach the quasi-one-dimensional signature. When this occurs, the energy states for noninteracting electrons are given by

$$E_{m,n}(k)=\varepsilon_{m,n}+\frac{\hbar^2k^2}{2m^*}, \tag{1}$$

where the first term requires two quantum numbers (n refers to the 2D subband, m to the bound state due to subsequent lateral confinement). It has been shown[5] that for low carrier density the low-lying states become equally separated by an energy $\hbar\omega_0$ and the confining potential (assumed to be in the x direction) is approximated by a harmonic well

$$V(x)=\tfrac{1}{2}m\omega_0^2x^2 . \tag{2}$$

In that case, assuming the confinement in the growth direction (z axis) is given by an infinite well of width L_z, the eigenenergies are

$$E_{m,n}(k_y)=\hbar\omega_0(m+\tfrac{1}{2})+\frac{\pi\hbar^2}{2\mu^*L_z^2}n^2+\frac{\hbar^2k_y^2}{2\mu^*} \tag{3}$$

and the wave functions describing the electron states are

$$\psi_{l,\mathbf{k}}=\psi_{m,n,\mathbf{k}}(\mathbf{r})=\left[\frac{2}{L_z}\right]^{1/2}\phi_m(x)e^{ik_yy}\sin\frac{n\pi z}{L_z}, \tag{4}$$

where $\phi_m(x)$ is the harmonic-oscillator function. Then, the density of states, carrying the 1D signature, is

$$D(E)=\frac{2}{\pi\hbar}\left[\frac{\mu^*}{2}\right]^{1/2}\sum_{m,n}(E-E_{m,n})^{-1/2} . \tag{5}$$

Although, as already mentioned, quasi-one-dimensional channels can be realized in semiconductor heterostructures using several techniques, in the present work we treat the interesting case of the electric confinement due to an applied gate voltage V_G generating the harmonic potential.[4] In the past few years several works have appeared studying the transport properties of that system under the action of a strong magnetic field.[6] Measurements have been performed of the derivative of the magnetoresistance with respect to the gate voltage as a function of the applied magnetic field.[7] Here we will study, in the absence of magnetic field, the dc transport relaxation time, due to scattering by LO phonons. We assume the phonons to be bulklike vibrations interacting with quasi-1D electrons. This is not a totally correct approach, since it is well known that the lattice dynamics is perturbed by the interfaces generating confined and interface modes that differ considerably from the bulk vibrations, mainly in their interaction with the electrons.[8] However in this calculation, using a more realistic electron-phonon coupling will not change our conclusion.

Our calculation is based on the memory-function projection-operator formalism, extensively used in other systems.[9] We briefly recall the basic equations of this formalism, since its detailed version can be obtained elsewhere.[9] The transport lifetime is given by

$$\tau^{-1}=\lim_{\omega\to 0}\left[-\frac{1}{N\mu^*\omega}\mathrm{Im}[\Pi^R(\omega)-\Pi^R(0)]\right], \tag{6}$$

where $\Pi^R(\omega)$ is the yy component of the retarded force-force correlation function,

$$\Pi^R(\omega)=-i\int_{-\infty}^{\infty}\Theta(t)\langle[U_y(t),U_y(0)]\rangle e^{i\omega t}dt , \tag{7}$$

and U is the frictional force acting on the center of mass of the electron system due to scattering by phonons; N is the carrier density. In the lowest order of approximation and using the Matsubara representation,[10] we have

$$\Pi(i\omega)=-\sum_{\mathbf{q}} q_y^2|D(q)|^2\frac{1}{\beta}\sum_{ip_n} S^0(\mathbf{q},ip_n)D^0(\mathbf{q},i\omega-ip_n)\ , \tag{8}$$

where $D^0(\mathbf{q},i\omega-ip_n)$ is the bare phonon propagator and $S^0(\mathbf{q},ip_n)$ is the density-density correlation function. In view of Eq. (4) the density operator becomes

$$\rho(\mathbf{q})=\sum_{l',l}\sum_{\mathbf{k}} c^{\dagger}_{l',\mathbf{k}+\mathbf{q}_y}c_{l,\mathbf{k}}F_{l',l}(q_x,q_z)\ , \tag{9}$$

where

$$F_{l',l}(q_x,q_z)=\frac{4\pi}{L_z}J_{m'm}(q_x)I_{n'n}(q_z)\ , \tag{10}$$

and

$$J_{m'm}(q_x)=\int_{-\infty}^{+\infty}\phi^*_{m'}(x)e^{iq_x x}\phi_m(x)dx\ , \tag{11}$$

$$I_{n'n}(q_z)=\int_0^{L_z}\sin\frac{n'\pi z}{L_z}e^{iq_z z}\sin\frac{n\pi z}{L_z}dz\ . \tag{12}$$

Then,

$$S^0(\mathbf{q},ip_n)=\frac{1}{\beta}\sum_{l',l}\sum_{\mathbf{k}}\sum_{ik_n}|F_{l',l}(q_x,q_z)|^2 \times\mathcal{G}^0(l';\mathbf{k}+\mathbf{q}_y,ik_n-ip_n) \times\mathcal{G}^0(l;\mathbf{k},ik_n)\ , \tag{13}$$

with $\mathcal{G}^0$ representing the bare electron propagator:

$$\mathcal{G}^0(l;\mathbf{k},ik_n)=\frac{1}{ik_n-\epsilon_{l,\mathbf{k}}}\ . \tag{14}$$

The present case is similar to that under strong magnetic field, except that here the levels (m,n) are nondegenerate and time-reversal invariance is not broken.

After performing the ususal Matsubara frequency summation we obtain

$$\tau^{-1}=\frac{24\alpha}{N\hbar}\frac{\beta e^{\hbar\beta\omega_{\mathrm{LO}}}}{(e^{\hbar\beta\omega_{\mathrm{LO}}}-1)^2}\frac{(\hbar\omega_{\mathrm{LO}})^{3/2}}{(\hbar\omega_0)^{-1/2}} \times\left[G^{00}\ln\frac{y_b^{00}}{y_a^{00}}+\sum_{m'=1}^{m'<\mathrm{int}(\hbar\omega_{\mathrm{LO}}/\hbar\omega_0)}G^{0m'}\ln\frac{y_b^{0m'}}{y_a^{0m'}}\right]\ , \tag{15}$$

where

$$G^{mm'}=\sum_{i=0}^{m}\frac{\Gamma(1+\frac{1}{2})\Gamma(m'-m+i+\frac{1}{2})\Gamma(m-i+\frac{1}{2})}{\Gamma(i+1)\Gamma(m'-m+i+1)\Gamma(m-i+1)}\ , \tag{16}$$

$$y_a^{0m}=1-\frac{\Delta m}{2\Delta_F}-1-\left[\frac{\Delta m}{\Delta_F}\right]^{1/2}\ , \tag{17}$$

$$y_b^{0m}=1-\frac{\Delta m}{2\Delta_F}+1-\left[\frac{\Delta m}{\Delta_F}\right]^{1/2}\ , \tag{18}$$

with

$$\Delta_F=E_F-\epsilon_{0,1}\ , \tag{19}$$

$$\Delta_m=m\hbar\omega_0-\hbar\omega_{\mathrm{LO}}\ . \tag{20}$$

Equation (15) is plotted in Fig. 1 as a function of $\hbar\omega_0$. We observe several peaks corresponding to $\omega_0=\omega_{\mathrm{LO}}/m$, where m is an integer. Since we assumed the Fermi energy between the first and second subbands, $\hbar\omega_0$ is limited by $\hbar\omega_0>\Delta_F$. In fact, Fig. 1 should be an expected result. The force-force correlation function is also a convenient way to express the absorption (real part of the dynamic conductivity). Coupling the quasi-one-dimensional electron gas to an external field of frequency ω, we will observe divergencies in the absorption when $\omega=m\omega_0$, mapping the divergencies in the quasi-1D joint density of states. In our case the LO phonons play the role of the incident excitation. The fact that we are assuming low temperature makes the absorption coefficient weak. Equivalently, those peaks may be broadened by other scattering mechanisms. The peaks however, can provide a very good mechanism for characterization of the QW. In fact, let us suppose that we have reached the formation of the quasi-one-dimensional channel and that it is in a region of V_G, the gate voltage, in which the relation between V_G and $\hbar\omega_0$ can be approximated as linear. In that case a plot of τ^{-1} (or ρ) as a function of V_G would reproduce Fig. 1. Three consecutive peaks, separated by ΔV_G and $\Delta V'_G$, will correspond to values of $\hbar\omega_0$: $\hbar\omega_{\mathrm{LO}}/(m-1)$, $\hbar\omega_{\mathrm{LO}}/m$, and $\hbar\omega_{\mathrm{LO}}/(m+1)$, respectively. Then, the peak in the middle corresponds to a value of ω_0 given by

$$\omega_0=\omega_{\mathrm{LO}}\left|\frac{\Delta V_G-\Delta V'_G}{\Delta V_G+\Delta V'_G}\right|\ . \tag{21}$$

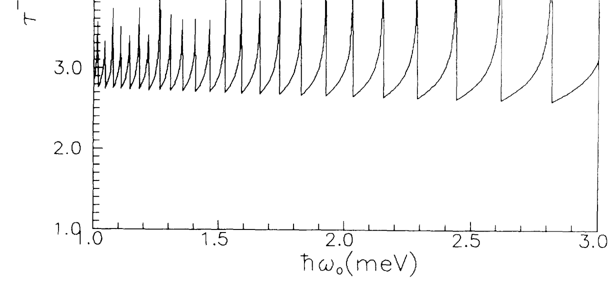

FIG. 1. Plot of τ^{-1}/σ_0, as a function of the intersubband separation $\hbar\omega_0$, where $\sigma_0=24\alpha\beta\exp(\hbar\beta\omega_{\mathrm{LO}})(\hbar\omega_{\mathrm{LO}})^{3/2}/N\hbar[\exp(\hbar\beta\omega_{\mathrm{LO}})-1]^2$. The peaks correspond to $\omega_0=\omega_{\mathrm{LO}}/m$ with m an integer such that $\hbar\omega_0>E_F-E_{1,0}$.

This would be a dc measurement of the intersubband separation usually done by infrared absorption. In GaAs, $\hbar\omega_{LO}$ is 36.6 meV. Quantum confinement has been reported with $\hbar\omega_0$ of the order of 1.8 meV,[4] i.e., m in the range 18–22. Then this kind of measurement would be able to detect changes in ω_0 of the order of 5%. However, for that range of ω_0 the harmonic approximation needs to hold up to levels in the range 18–21. On the other hand, if a quantum confinement can be made, even for other materials, in such a way that the realizable ratio $\omega_0/\leq\omega_{LO}$ is higher, not only does the harmonic approximation become more plausible for the number of levels involved, but also this kind of measurement will be much more sensitive to changes in ω_0.

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