# Nonlinear screening of a totally occupied Landau level

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The quadratic-response function of the particle density to a static harmonic potential is calculated for a totally occupied Landau level. Then, with use of a variational approach, it is shown that the linear component of the density response is dominant for any wave vector $q$ of the external potential and for a large range of amplitudes, including those which remove the gap up to the next Landau level. The contribution of the nonlinear components does not exceed a few percent if $q < l_B^{-1}$ and may be statisfactorily described by the quadratic component if $q < 2l_B^{-1}$, where $l_B = (\hbar c/eB)^{1/2}$ is the magnetic length.

## I. INTRODUCTION

The two-dimensional electron gas in a perpendicular magnetic field has peculiar screening properties. Its linear longitudinal dynamic dielectric response was extensively analyzed for all magnetic-field strengths and temperatures in the middle 1970s.[1] In particular, in the static limit, for high magnetic fields and low temperatures, the screening length has an oscillatory dependence on the chemical potential; for large wavelengths, the screening is perfect when the highest Landau level is partially occupied and the screening is absent when this Landau level is totally occupied.[1] The two-dimensional (2D) electron gas has been considered as a perfect planar system[1] as well as in a slab model,[2] their static screening properties being essentially the same.

Another peculiarity of the screening consists in its nonlinear behavior. Since the electrons involved in the long-wavelength screening are only limited to those contained in the partially filled Landau level (if there is one), a potential which is linearly screened by a half-filled level might be nonlinearly screened if the level becomes almost empty or full, since the number of available electrons or holes might become too small for a linear response. Consequently, an external nonuniform electrostatic potential could induce, in the system, metalliclike regions (with strong screening) and dielectriclike regions (with weak screening).[3,4] This leads to a special interpretation[3] of the percolative description of the quantum Hall effect.[5]

Let us consider the model of strict two-dimensional electron gas, the electric neutrality being ensured by a uniform background of positive charges. In order to define the response functions, let us suppose the external electrostatic potential[6]

$$V^{\text{ext}}(r) = V\cos(\mathbf{q}\cdot\mathbf{r}) \tag{1.1}$$

is small enough so that the electron density be given by

$$\rho(r) = \rho_0 + V\chi(q)\cos(\mathbf{q}\cdot\mathbf{r}) + V^2\Theta(q)\cos(2\mathbf{q}\cdot\mathbf{r}) + \dots , \tag{1.2}$$

where $\rho_0$ is the unperturbed density and $\mathbf{r}=(x,y)$; $\chi(q)$ is the usual linear-response function and $\Theta(q)$ is the quadratic-response function. The expansion (1.2) holds independently of how the electron-electron interaction is included in the response functions.

Neglecting completely the electron-electron interaction, the linear-response function of the two-dimensional electron gas partially filling the lowest Landau level has the simple form[1,7,8]

$$\chi_0^{(0)}(q) = -\frac{\nu(1-\nu)}{2\pi T} e^{-q^2/2} . \tag{1.3}$$

Here the upper label refers to the absence of the interaction and the lower one to the lowest Landau level, $\nu$ being the filling factor of the level and $T$ the temperature (in energy units) which is assumed to be small.

Throughout this paper the magnetic units are used: the cyclotronic energy $\hbar\omega_c \equiv \hbar eB/mc = 1$ and the magnetic length $l_B \equiv (\hbar c/eB)^{1/2} = 1$. The electronic charge $e$ and Boltzmann's constant are also taken equal to unity.

With the same assumptions as for the linear-response function, the quadratic-response function may be given by[7]

$$\Theta_0^{(0)}(q) = \frac{\nu(1-\nu)(1-2\nu)}{8\pi T^2} e^{-3q^2/2} . \tag{1.4}$$

The low-temperature singularities of the response functions (1.3) and (1.4) support the arguments for the nonlinear behavior of the screening. Taking the electron-electron interaction into account, the linear screening breaks down if the amplitude $V$ of the external potential is of order $T + 2\nu(1-\nu)/q$ for an electron gas strongly confined in the plane.[4] If a weaker confinement is considered, as given by a triangular well, the breakdown begins for a $V$ of order $T$, which means any external potential is nonlinearly screened at low enough temperatures.[9]

If the electrons totally fill a Landau level, their screening ability diminishes. For the lowest Landau level the linear-density-response function of noninteracting electrons is[1,8,10,11]

$$\chi_0^{(0)}(q) = -\frac{q^2}{2\pi} e^{-q^2/2} \sum_{m\geq 1} \frac{1}{m!\,m} \left[\frac{q^2}{2}\right]^{m-1} . \tag{1.5}$$

It gives no screening in the $q \to 0$ limit,

 

$$\epsilon^{(\mathrm{RPA})}=\lim_{q\to 0}\left[1-\frac{2\pi}{q}\chi^{(0)}(q)\right]=1\ , \tag{1.6}$$

$\epsilon$ being the dielectric constant in the random-phase approximation (RPA) (this result being valid for any Landau level). This is in strong contrast with the perfect screening behavior in the case of partial filling, i.e., $\epsilon^{(\mathrm{RPA})}=\infty$.[1]

However, for shorter wavelengths of the external potential the density modulation (1.2) may be comparable for the two types of filling. This is the case of the modulation produced by a periodic microstructured gate obtained by microlithographic techniques in small devices.[4,12]

In this paper the quadratic-response function $\Theta_n^{(0)}(q)$ for the $n$th Landau level totally occupied is calculated using the second-order perturbation theory (Sec. II). Then, in Sec. III, an attempt is made to estimate the conditions in which the density modulation produced by the external potential (1.1) may be described satisfactorily by the linear- and quadratic-response functions. To this end the local particle density is calculated for the lowest Landau band totally occupied using variational wave functions. The linear component of the density response is dominant in a large range of external potential amplitudes $V$, including those which remove the gap between the first two Landau bands, and for any wave vector $q$; for $q<1$ the contribution of the nonlinear components is at least one order of magnitude smaller; for $q<2$ the linear- and the quadratic-response functions are sufficient to describe the full density response. If the gap is removed, a certain filling factor of the next band is necessary in order to prevent the depopulation of the first one, whose contribution to the total density response may be described as above. The electronic spin is ignored throughout the paper.

## II. DENSITY RESPONSE-PERTURBATIVE APPROACH

### A. Response functions

Let us consider a two-dimensional system of noninteracting electrons in a perpendicular magnetic field plus the external potential (1.1). The two axes in the plane may be chosen so that the external potential varies only in the $x$ direction, as long as the boundary conditions can be neglected. In the Landau gauge and in magnetic units the vector potential is then $\mathbf{A}=(0,x)$ and the electrons' wave functions are

$$\psi(x,y)=L_y^{-1/2}e^{iky}\varphi(x)\ , \tag{2.1}$$

with the system being considered enclosed in a large rectangular box, i.e., $|x|<L_x/2$ and $|y|<L_y/2$. The unidimensional wave functions $\varphi(x)$ are the eigenfunctions of the perturbed-oscillator Hamiltonian (in magnetic units)

$$H_x=-\frac{1}{2}\frac{d^2}{dx^2}+\frac{1}{2}(x-k)^2+V\cos(qx)\ , \tag{2.2}$$

where $k=(2\pi/L_y)\times(\text{integer})$ from periodic boundary conditions on the $y$ axis and $|k|<L_y$ from box confinement.

In the unperturbed basis, i.e.,

$$L_y^{-1/2}e^{iky}f_n(x-k)\ ,$$

in which

$$f_n(x)=\frac{1}{(n^{1/2}2^n n!)^{1/2}}e^{-x^2/2}H_n(x)\ , \tag{2.3}$$

$H_n(x)$ being the Hermite polynomials, due to the particular choice of the coordinate axes, the matrix elements of $V^{\mathrm{ext}}(x)$ are diagonal in $k$:

$$(V^{\mathrm{ext}})_{nk,n'k'}=V^{\mathrm{ext}}_{nn'}(k)\delta_{kk'}\ .$$

This means nondegenerate perturbation theory may be used and, consequently, the eigenfunctions of the reduced Hamiltonian (2.2) are given, up to the second order in $V$, by

$$\varphi_{nk}(x)=f_n(x-k)+\sum_{m(\neq n)}\frac{V^{\mathrm{ext}}_{nm}(k)}{n-m}f_m(x-k)-\left[\frac{1}{2}\sum_{m(\neq n)}\left[\frac{V^{\mathrm{ext}}_{nm}(k)}{n-m}\right]^2\right]f_n(x-k)$$
$$-\sum_{m(\neq n)}\left[\frac{V^{\mathrm{ext}}_{nn}(k)V^{\mathrm{ext}}_{mn}(k)}{(n-m)^2}-\sum_{p(\neq n)}\frac{V^{\mathrm{ext}}_{mp}(k)V^{\mathrm{ext}}_{pn}(k)}{(n-m)(n-p)}\right]f_m(x-k)\ , \tag{2.4}$$

where the unperturbed energies are $\varepsilon_n=n+\frac{1}{2}$.

The density of noninteracting electrons for a totally occupied Landau band $E_{nk}$ is

$$\rho_n(x)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}dk\,|\varphi_{nk}(x)|^2\ , \tag{2.5}$$

in the thermodynamic limit (as $L_x,L_y\to\infty$). Using (2.4) in (2.5) and keeping only the term linear in $V$, the density fluctuation is obtained as

$$\delta\rho_n(x)=V\chi_n^{(0)}(q)\cos(qx)\ , \tag{2.6}$$

where the susceptibility $\chi_n^{(0)}$ is[1,8,10,11]

TABLE I. Coefficients [see Eqs. (2.7) and (2.9)] in the power series for the linear-response function $\chi_n(q)$ and for the quadratic-response function $\Theta_n(q)$ of the first three Landau levels. For the coefficients $a_0^m$, with $m=0,1,\ldots,$ and 7, etc., see Eq. (1.5).

| $m$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $a_1^m$ | 1 | $-5/8$ | $5.56\times10^{-2}$ | $3.04\times10^{-3}$ | $1.91\times10^{-4}$ | $1.16\times10^{-5}$ | $6.50\times10^{-7}$ | $3.34\times10^{-8}$ |
| $a_2^m$ | 1 | $-11/8$ | $5.14\times10^{-1}$ | $-6.03\times10^{-2}$ | $1.32\times10^{-3}$ | $3.76\times10^{-5}$ | $1.48\times10^{-6}$ | $6.20\times10^{-8}$ |
| $b_0^m$ | 3 | $5/6$ | $2.19\times10^{-1}$ | $4.33\times10^{-2}$ | $7.53\times10^{-3}$ | $1.14\times10^{-3}$ | $1.56\times10^{-4}$ | $1.93\times10^{-5}$ |
| $b_1^m$ | 9 | $-61/6$ | 1.34 | $-1.62\times10^{-1}$ | $6.81\times10^{-3}$ | $-1.05\times10^{-3}$ | $6.07\times10^{-5}$ | $5.02\times10^{-6}$ |
| $b_2^m$ | 15 | $-247/6$ | $3.15\times10^{1}$ | $-9.96$ | 1.57 | $-1.49\times10^{-1}$ | $1.14\times10^{-2}$ | $-8.11\times10^{-4}$ |

$$\chi_n^{(0)}(q)=-\frac{1}{\pi}e^{-q^2/2}\sum_{\substack{m(\geq 0)\\(m\neq n)}}\frac{n!}{(m-n)\,m!}\left[\frac{q^2}{2}\right]^{m-n}\left[L_n^{m-n}\left[\frac{q^2}{2}\right]\right]^2$$

$$\equiv-\frac{q^2}{2\pi}e^{-q^2/2}\sum_{\substack{m(\geq 0)\\(m\neq n)}}a_n^m q^{2m}\,, \tag{2.7}$$

$L_n^p$ being Laguerre's polynomials. Some coefficients $a_n^m$ of the power series are given in Table I. An integral representation of Eq. (2.7) may be found in Ref. 13.

The four terms proportional to $V^2$ given by (2.4) in (2.5) are expressed as infinite sums of terms each containing an integral of the form

$$\int dk\int dx_1\int dx_2\cos(qx_1)\cos(qx_2)f_\alpha(x_1-k)f_\beta(x_1-k)f_\beta(x_2-k)f_\gamma(x_2-k)f_\gamma(x-k)f_\alpha(x-k)\,.$$

After a lengthy but straightforward calculation,[14] the quadratic-response function [see Eq. (1.2)] is obtained as

$$\Theta_n^{(0)}(q)=\frac{n!}{4\pi}e^{-3z}\left[\sum_{k,m}\frac{(-1)^{n+m}2^{k-m}}{(k-n)(m-n)\,k!}z^{k-n}L_n^{k-n}(z)L_n^{m-n}(z)L_m^{k-m}(4z)\right.$$
$$+2\sum_{k,m}\frac{2^{k-n}}{(k-n)(m-n)\,k!}z^{k-n}L_m^{k-m}(z)L_n^{m-n}(z)L_n^{k-n}(4z)$$
$$\left.-2\sum_k\frac{2^{k-n}}{(k-n)^2\,k!}z^{k-n}L_n^{k-n}(z)L_n(z)L_n^{k-n}(4z)-\sum_k\frac{(-1)^{k+n}}{(k-n)^2\,k!}z^{k-n}(L_n^{k-n}(z))^2L_n(4z)\right]\,, \tag{2.8}$$

where $z\equiv q^2/2$ and $k,m\geq 0$, $k,m\neq n$. The rearrangement of the terms in the power series of Eq. (2.8) transforms it into

$$\Theta_n^{(0)}(q)=\frac{q^6}{32\pi}e^{-3q^2/2}\sum_{m(\geq 0)}b_n^m q^{2m}\,, \tag{2.9}$$

where $b_n^m$ are constants which may be found in Table I.

The question which arises now is how large the external potential may be for the density fluctuation (of interacting electrons) to be described only by its first two harmonics in the form

$$\delta\rho_n(x)=V\chi_n\cos(qx)+V^2\Theta_n\cos(2qx) \tag{2.10}$$

and the other higher-order response functions to be neglected. The answer to this question, at least in the case of the lowest Landau level, will be given in Sec. III.

For the moment, let us consider the influence of the electron-electron interaction on the response functions $\chi$ and $\Theta$ in the random-phase approximation.

### B. Random-phase approximation (RPA)

Let us consider the density response to the external potential (1.1) approximated by its first two harmonics, each one in the leading order in $V$,

$$\delta\rho(r)=V\chi(q)\cos(\mathbf{q}\cdot\mathbf{r})+V^2\Theta(q)\cos(2\mathbf{q}\cdot\mathbf{r})\,, \tag{2.11}$$

the response functions $\chi$ and $\Theta$ including the electron-electron interaction.

The interaction will be taken into account in RPA. This means considering in the density response the noninteracting response functions ($\chi^{(0)}$ and $\Theta^{(0)}$) instead of the interacting ones ($\chi$ and $\Theta$), but replacing the external potential (1.1) with the total (external plus induced) one which, in our case, must have the form

$$V^{\rm tot}(r)=V_1\cos(\mathbf{q}\cdot\mathbf{r})+V_2\cos(2\mathbf{q}\cdot\mathbf{r})\,. \tag{2.12}$$

Following this prescription, the density fluctuation restricted to the first two harmonics is obtained as

$$\delta\rho(r)=V_1\chi^{(0)}(q)\cos(\mathbf{q}\cdot\mathbf{r})+V_1^2\Theta^{(0)}(q)\cos(2\mathbf{q}\cdot\mathbf{r})$$
$$+V_2\chi^{(0)}(2q)\cos(2\mathbf{q}\cdot\mathbf{r})\,, \tag{2.13}$$

where the first two terms represent the linear and quadratic responses to the first harmonic of the "external" potential (2.12) and the third term represents the linear response to the second harmonic.

The self-consistency of Eqs. (2.12) and (2.13) is ensured by Poisson's equation

$$V^{\text{tot}}(\mathbf{r})=V^{\text{ext}}(\mathbf{r})+\int \frac{\delta\rho(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}d\mathbf{r}' , \tag{2.14}$$

which leads to

$$V_1=V+\frac{2\pi}{q}V_1\chi^{(0)}(q) ,$$
$$V_2=\frac{\pi}{q}[V_1^2\Theta^{(0)}(q)+V_2\chi^{(0)}(2q)] . \tag{2.15}$$

Introducing Eq. (2.15) in Eq. (2.13) and comparing with Eq. (2.11), we obtain

$$\chi^{(\text{RPA})}(q)=\frac{\chi^{(0)}(q)}{1-\frac{2\pi}{q}\chi^{(0)}(q)} , \tag{2.16}$$

i.e., the usual random-phase approximation of the linear-response function and

$$\Theta^{(\text{RPA})}(q)=\frac{\Theta^{(0)}(q)}{\left[1-\frac{2\pi}{q}\chi^{(0)}(q)\right]^2\left[1-\frac{\pi}{q}\chi^{(0)}(2q)\right]} \tag{2.17}$$

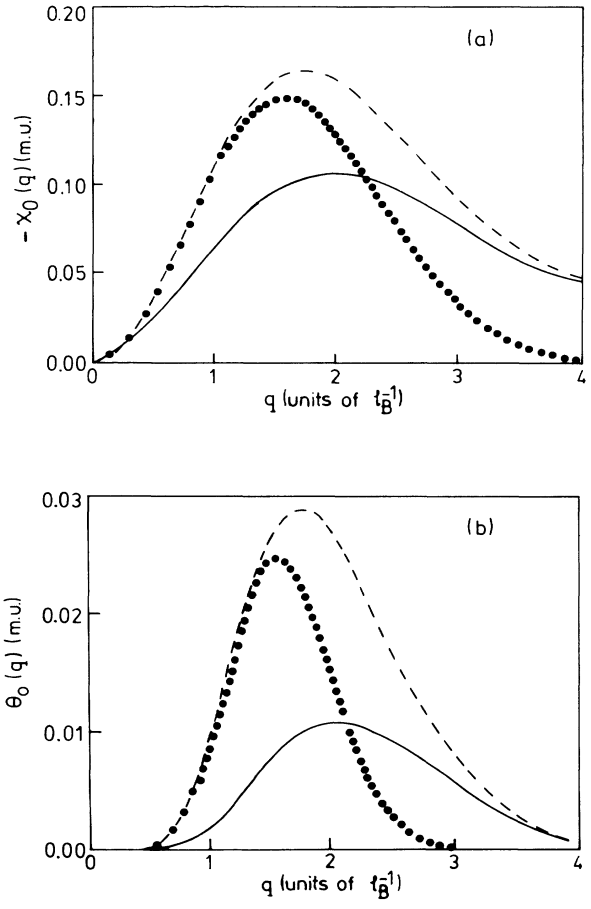


FIG. 1(a) The linear-response function and (b) the quadratic-response function of the lowest Landau level for noninteracting electrons (dashed lines) and in random-phase approximation (solid lines), together with the variational approximation (dotted lines), versus the wave vector $q$; m.u. means "magnetic units."

as the RPA for the less usual quadratic-response function (see also Ref. 7).

In Figs. 1(a) and 1(b) the response functions for the lowest Landau level $\chi_0(q)$ and $\Theta_0(q)$ in the noninteracting and random-phase approximations [Eqs. (1.5), (2.7), (2.9), (2.16), and (2.17)] are plotted together with the response functions given in the variational approach in Sec. III.

## III. DENSITY RESPONSE-VARIATIONAL APPROACH

### A. Variational wave functions and energies

If the amplitude $V$ and the wave vector $q$ of the external potential are small enough, the eigenstates of the reduced Hamiltonian (2.2) may be considered to be of the oscillatory type. Hence a variational wave function for the electrons in the lowest Landau band ($n=0$) may be

$$\varphi_{0k}(x)=\left[\frac{\omega_0}{\pi}\right]^{1/4}\exp[-\omega_0(x-k_0)^2/2] , \tag{3.1}$$

where the frequency $\omega_0=\omega_0(k)>0$ and the equilibrium position $k_0=k_0(k)$ will be given by the minimum-energy condition. (In the absence of the perturbation, $\omega_0\equiv 1$ and $k_0\equiv k$.)

The lowest Landau band appears as

$$E_{0k}=\frac{1}{4}\left[\omega_0+\frac{1}{\omega_0}\right]+\frac{1}{2}(k_0-k)^2 + V\cos(qk_0)\exp\left[-\frac{q^2}{4\omega_0}\right] , \tag{3.2}$$

whose stationary points $(\omega_0,k_0)$ satisfy the nonlinear system

$$\omega_0^2=1-q^2V\cos(qk_0)\exp\left[-\frac{q^2}{4\omega_0}\right] , \tag{3.3a}$$

$$k_0=k+qV\sin(qk_0)\exp\left[-\frac{q^2}{4\omega_0}\right] . \tag{3.3b}$$

If the external potential is smooth enough one may perform a parabolic approximation of the potential energy terms in the reduced Hamiltonian (2.2). This means neglecting all powers higher than 2 in the local expansion of $V^{\text{ext}}(x)$ around a shifted oscillation center $k_0$, which, together with the shifted frequency $\omega_0$, will appear as solutions of Eqs. (3.3) with the exponential factors dropping out; this could be anticipated by the smoothness condition $q\ll 1$. The eigenfunctions of the approximate Hamiltonian will then be obviously (3.1), but the corresponding eigenvalues will differ from Eq. (3.2) in the first term which will be replaced by $\omega_0/2$. Thus the eigenvalues will coincide only if $q^2V\ll 1$ which may be interpreted as another smoothness condition. Therefore, by choosing variationally both the shifted oscillation center and frequency, the Gaussian wave functions (3.1) are pushed beyond the local parabolic approximation in a larger domain of $q$ and $q^2V$. As we shall see, the response functions $\chi_0^{(0)}$ and $\Theta_0^{(0)}$ will be reproduced by the varia-

TABLE II. The largest perturbation $V_{max}$ in the variational approach and the amplitude $V^*$ that removes the gap between the first two Landau bands (Fig. 2).

| $q$ | 0.1 | 0.4 | 0.7 | 1.0 | 1.3 | 1.5 | 1.8 |
|---|---|---|---|---|---|---|---|
| $V_{max}$ | 100.0 | 7.2 | 2.7 | 1.6 | 1.2 | 1.0 | 0.91 |
| $V^*$ | 0.50 | 0.54 | 0.64 | 0.84 | 1.3 | 1.6 | 1.6 |

tional wave functions with good accuracy for $q<1.5$.

Let us concentrate now on the solutions of Eqs. (3.3). A brief inspection of Eq. (3.2) shows at least one minimum for any fixed $k$, and only one if $V$ and $q$ are small enough so that the oscillating term is not too large. In the following we shall be interested only in the case when $E_{0k}(\omega_0,k_0)$ has a single minimum. This means the system (3.3) has only one solution $(\omega_0(k), k_0(k))$. $\omega_0$ and $(k_0-k)^2$ being even and periodic functions of $k$, with a period $2\pi/q$, their relevant domain is $[0,\pi/q]$. Considering in Eq. (3.3a) that $\omega_0=\omega_0(k_0)$ and combining it with (3.3b), we may express $k=k(k_0)$, the relevant interval being again $[0,\pi/q]$. Because $k_0(0)=0$ and $k(\pi/q)=\pi/q$, the condition in which (3.3) has only one solution for fixed $k$ is

$$\frac{d}{dk_0}k(k_0)>0\ , \tag{3.4}$$

which means there is a one-to-one correspondence between $k$ and $k_0$. The maximum amplitude $V_{max}$ that still satisfies (3.4) is given in Table II for different wave vectors $q$.

The wave functions in the second Landau level will be chosen in the form

$$\varphi_{1k}(x)=\left[\frac{\omega_1}{4\pi}\right]^{1/4}\exp[-\omega_1(x-k_1)^2/2] \times H_1(\omega_1^{1/2}(x-k_1))\ , \tag{3.5}$$

where $\omega_1=\omega_1(k)>0$ and $k_1=k_1(k)$. The orthogonality of the full wave functions (2.1) of the second Landau band on those of the first one is ensured by the integral over $y$ for different values of $k$, but for the same $k$ it requires

$$k_1=k_0\ . \tag{3.6}$$

The energy band is obtained as

$$E_{1k}=\frac{3}{4}\left[\omega_1+\frac{1}{\omega_1}\right]+\frac{1}{2}(k_0-k)^2 +V\cos(qk_0)\left[1-\frac{q^2}{2\omega_1}\right]\exp\left[-\frac{q^2}{4\omega_1}\right] \tag{3.7}$$

and the frequency corresponding to a minimum energy is given by the equation

$$\omega_1^2=1-q^2V\cos(qk_0)\left[1-\frac{q^2}{6\omega_1}\right]\exp\left[-\frac{q^2}{4\omega_1}\right]\ , \tag{3.6'}$$

which has a single $\omega_1(k_0)$ solution for all the values of $V$ and $q$ which will be used below.

The first two energy bands are plotted in Fig. 2 for three different $(q,V)$ values. They have the same symmetry properties in $k$ space as the external potential (1.1) has in $r$ space. Figure 2 yields information on the size of the gap between the Landau bands as a function of the external potential parameters. The gap vanishes for $V=V^*$, which is shown in Table II. If $V^*>V_{max}$ it may be estimated by extrapolation. The accuracy of the energy bands is not very important here. It depends, of course, on the trial wave functions whose validity will be established through the density response in Sec. III B.

The influence of the electron-electron interaction upon the energy-band structure may be considered in RPA. In the linear approximation, this means considering $V_1=V/\epsilon^{(RPA)}$ [Eq. (1.6)] instead of $V$ in the energy-band expressions (3.2) and (3.7).

### B. Electron density

The one-to-one relationship $k\leftrightarrow k_0$ allows one to express the electron density of the lowest Landau level with all the states occupied as [see Eq. (2.5)]

$$\rho_0(x)=\frac{1}{2\pi}\int dk_0\frac{dk}{dk_0}|\varphi_{0k}(x)|^2\ , \tag{3.8}$$

i.e., from (3.1) and (3.3),

$$\rho_0(x)=\frac{1}{2\pi^{3/2}}\int dk_0\,\omega_0^{1/2} \times\left[\omega_0^2-\frac{q^6V^2\sin^2(qk_0)e^{-q^2/2\omega_0}}{8\omega_0^3+q^2(1-\omega_0^2)}\right] \times e^{-\omega_0(x-k_0)^2}\ . \tag{3.9}$$

This being a periodic function of $x$, a Fourier expansion is valid:

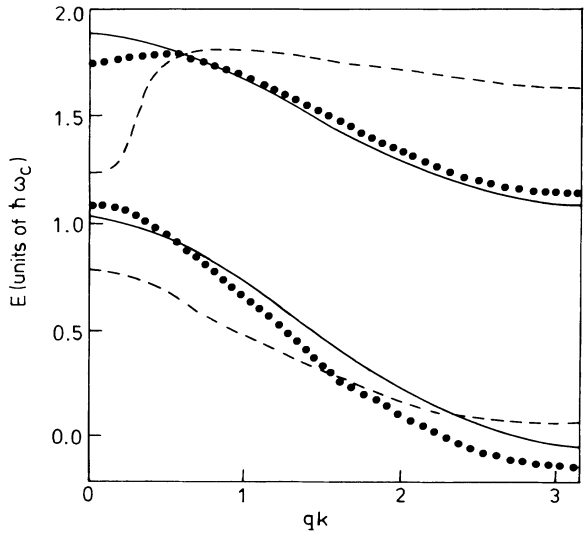


FIG. 2. The first two Landau bands generated by a harmonic perturbation in the variational approximation for $q=0.7$ and $V=0.6$ (solid lines), $q=1.0$ and $V=0.8$ (dotted lines), and $q=1.8$ and $V=0.8$ (dashed lines).

$$\rho_0(x)=\frac{1}{2\pi}+\sum_{l\geq 1}\rho_l\cos(lqx)\ , \tag{3.10}$$

where other terms are excluded by the parity condition imposed by the external potential, the Fourier coefficients being

$$\rho_l=\frac{q}{\pi^2}\int_0^{\pi/q}dk_0\cos(lqk_0)\times\left[\omega_0^2-\frac{q^6V^2\sin^2(qk_0)e^{-q^2/2\omega_0}}{8\omega_0^3+q^2(1-\omega_0^2)}\right]\times e^{-l^2q^2/4\omega_0}\ . \tag{3.11}$$

The last expression is obtained by integrating $\rho_0(x)\cos(lqx)$ from Eq. (3.10) over an infinite number of periods in order to obtain the Fourier transform of $\exp[-\omega_0(x-k_0)^2]$.

The response functions $\chi_0^{(0)}(q)$ and $\Theta_0^{(0)}(q)$ may now be derived from Eq. (3.11) by making a perturbative expansion. It may be seen from (3.11) and (3.3a) that the appropriate small parameter for the particle density is $u=q^2V$. According to the discussion in Sec. III A, this is because a small perturbation should be characterized by $u\ll 1$ rather than by $V\ll 1$.

After a direct power expansion we get for the amplitude of the first harmonic

$$\rho_1=V\chi_0^{(0)}(q)+V^3\sigma_0^{(0)}(q)+O(V^5)\ , \tag{3.12}$$

where

$$\chi_0^{(0)}(q)=-\frac{q^2}{2\pi}e^{-q^2/2}\left[1+\frac{q^2}{8}\right]\ , \tag{3.13}$$

$$\sigma_0^{(0)}(q)=\frac{q^8}{512\pi}e^{-q^2}(9+q^2-q^4)\ , \tag{3.14}$$

and for the amplitude of the second one

$$\rho_2=V^2\Theta^{(0)}(q)+O(V^4)\ , \tag{3.15}$$

where

$$\Theta_0^{(0)}(q)=\frac{q^6}{32\pi}e^{-3q^2/2}(3+\tfrac{3}{2}q^2)\ . \tag{3.16}$$

As a general rule, each term of the power series is proportional to

$$V^m q^{2(m+1)}\exp[-(l^2+m)q^2/4]\ , \tag{3.17}$$

where $m=l,\ l+2,\ldots$, except the leading one, i.e., for $m=l=1$ [Eqs. (3.12) and (3.13)]. The factor (3.17) is followed by a polynomial in $q^2$. Formula (3.17) can be guessed directly from Eq. (3.11) as follows: The power expansion of the integrand without the factor $\cos(lqk_0)$ with respect to $u\equiv q^2V$ gives for any fixed $l$ a series of terms proportional with $u^m[\cos(qk_0)]^n$ $(n=m-2,m)$ each one having a Fourier expansion in $\cos(pqk_0)$ with $p\leq n$ even or odd if $m$ is even or odd. Therefore, the factor $\cos(lqk_0)$ will select the terms with $m=l,l+2,\ldots$. To any $u$ a factor $\exp(-q^2/4)$ must be associated apart from the global factor $\exp(-l^2q^2/4)$. In the $q\to 0$ limit the second term in the square brackets of (3.11) may be neglected and expanding the $l$-dependent exponential we can see that each harmonic has a global $q^2$ factor not included in $u$, except the first harmonic given by $u\cos(qk_0)$ contained in $\omega_0^2$.

Thus, the two response functions given by the variational approach, i.e., Eqs. (3.13) and (3.16) differ significantly from the "exact" ones obtained perturbatively, i.e., Eqs. (1.5) or (2.9) (for $n=0$), only for wave vectors larger than $\approx 1.5$, see Figs. 1(a) and 1(b). This is the effect of the special choice of the variational wave functions (3.1) which, as we can see, are satisfactory not only if $q^2V$ is small enough, but also if $q$ is small enough. According to the discussion in Sec. III A, they include only to some extent the corrections to the local parabolic approximation.

In Figs. 3(a) and 3(b) the amplitudes $\rho_1$ and $\rho_2$ of the first two harmonics are plotted versus $V$ and $V^2$, respectively, for three values of the wave vector $q$. An almost perfect linearity is obtained for both families of curves, for any $V<V_{\max}$. The choice $q=1.8$ was made because it provides the greatest ratio $\Theta_0^{(0)}(q)/\chi_0^{(0)}(q)$; for larger wave vectors this ratio decreases exponentially.

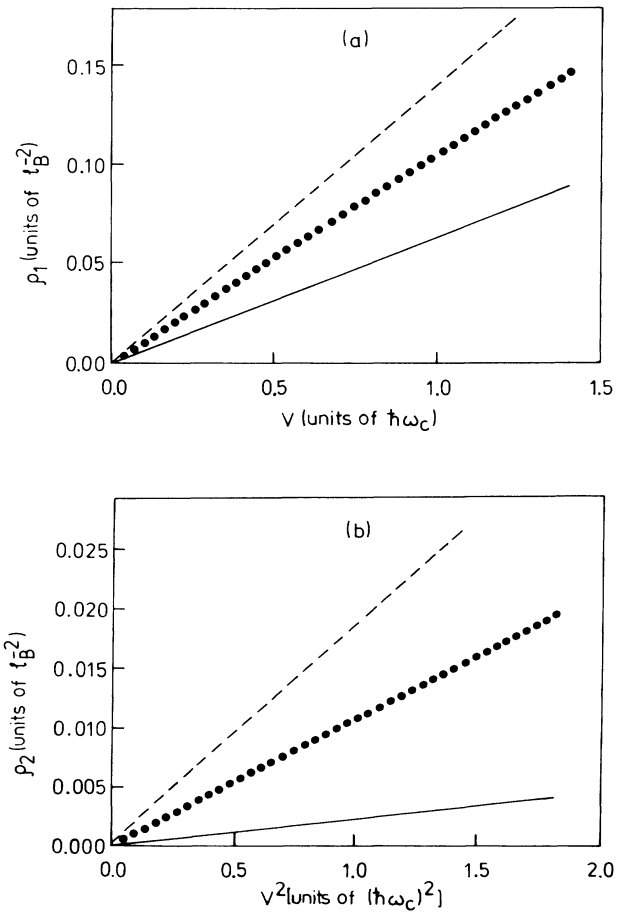


FIG. 3. The amplitude of (a) the first harmonic and (b) the second harmonic of the density modulation given by the external potential $V\cos qx$ versus (a) $V$ and (b) $V^2$ for $q=0.7$ (solid lines), $q=1.0$ (dotted lines), and $q=1.8$ (dashed lines).

The third harmonic amplitude $\rho_3$ is one order of magnitude (or even more) less than $\rho_2$ for any $V < V_{max}$, at least when $\rho_2$ is significant compared to $\rho_1$, and it becomes exponentially small faster than $\rho_2$ when $q$ is increased [see (3.17)].

Therefore, we may conclude that in the variational formula (3.9) for the particle density of the lowest Landau band totally occupied, the linear component [the first term of (3.12)] is dominant for any wave vector $q$ within the whole range of external potential amplitudes $V$ considered, i.e., $0 < V < V_{max}$ (see Table II). Of course, for $V > V^*$ the second Landau level must have an appropriate filling factor in order to keep the first one totally occupied. For $q < 1$ the correction due to the higher-order components is at least one order of magnitude smaller than the linear response, and for $q < 2$ this correction may be reduced to the quadratic response even if $V_{max} < V < V^*$ (see Table II). For $q > 2$ it would be hazardous to estimate the contribution of the nonlinear terms as long as the variational approach yields large errors in the response functions (Fig. 1), but the linear component should still dominate since all the other ones have shorter exponential tails [see (3.17)].

So far, only the case of the lowest Landau level was considered. If many Landau bands are totally occupied, the contribution of each band to the local particle density may be calculated using variational wave functions describing higher excited oscillator states $\varphi_{nk}(x)$ as in (3.1) and (3.5). The orthogonality conditions yield the shifted equilibrium positions $k_n = k_0$ for any $n$, and the shifted frequencies $\omega_0 = \omega_2 = \omega_4 = \cdots$, and $\omega_1 = \omega_3 = \omega_5 = \cdots$. Hence, for any fixed $k$, the only equations that need to be solved are those obtained minimizing the energy of the first two levels, i.e., (3.3a) and (3.3b) and (3.6'). Therefore, the perturbative response functions will be reproduced only for wave vectors $q$ lower than those that occur in the case when only the first Landau level is considered [as may be expected since the strongest localized states ($n = 0$) are the least affected by the external potential]. This is the range of $q$ for which in any case $\Theta_n^{(0)}(q) \ll \chi_n^{(0)}(q)$. For larger $q$ both oscillate around zero and may become comparable [Eqs. (2.7) and (2.9) for $n \geq 1$]. Nevertheless, the resultant quadratic-response function $\Theta^{(0)}$ (i.e., the sum of all $\Theta_n^{(0)}$ corresponding to occupied levels) is at least one order of magnitude smaller than the resultant linear one $\chi^{(0)}$. While $\Theta^{(0)}$ has several changes of sign, $\chi^{(0)}$ is oscillating but negative definite (see Fig. 1 of Ref. 10). The exponential tails start for wave vectors which increase with the number of Landau levels. Therefore, if many levels are considered, the estimations of the linear and nonlinear contributions to the density response are qualitatively the same as those already done for the lowest Landau level.

In Fig. 4 the density $\rho_0(x)$ is shown using the exact perturbative expressions for the response functions [Eqs. (1.5) and (2.9) with $n = 0$] renormalized in RPA [Eqs. (2.16) and (2.17)]. The wave vector $q = 2.0$ corresponds to the maximum value of $\Theta_0^{(RPA)}/\chi_0^{(RPA)}$ (see Fig. 1). The amplitude $V = 2.3$ of the external potential yields a total linearly screened potential $V_1 = V/\epsilon^{(RPA)} = 1.5$. Considering it as the external potential in Eqs. (3.2) and (3.7) for the energy bands, still we may suppose a gap persisting in the energy spectrum; this means $V_1 \lesssim V^*(q = 2)$ (see Table II). For these values of $q$ and $V$, the largest deviation of $\rho(x)$ from the first-harmonic approximation is obtained without populating the second Landau band, i.e., at very low temperatures.

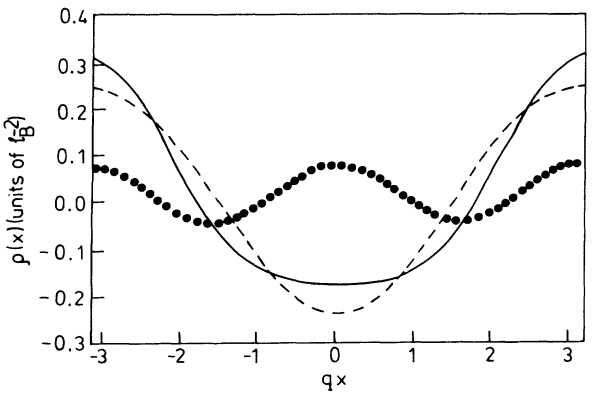


FIG. 4. Density modulation of the electrons totally occupying the first Landau level, produced by a harmonic external potential with $V = 2.3$ and $q = 2.0$, in random-phase approximation (solid line); the dashed line represents the first harmonic, the dotted line represents the second harmonic.

## IV. CONCLUSIONS AND DISCUSSION

The density response of a totally occupied Landau band produced by a harmonic external potential has a dominant linear component for any wavelength and for a large range of amplitudes, including those yielding an overlap with the next Landau band (as long as the former band remains totally occupied).

For small wave vectors, the quadratic response is very small because the quadratic-response function is proportional to $q^6$. Any higher-order response is much smaller, going like a higher power of $q$ [see (3.17)]. Therefore, for any $q < 1$, the total contribution of the nonlinear components amounts to at most a few percent. For wave vectors $q < 2$, the full response may be satisfactorily described by the first two harmonics, in the form (2.12), within errors, no larger than a few percent, at least in the case of the lowest Landau level.

If the lowest band is totally occupied and the next band is totally empty, the density modulation has the second-harmonic amplitude not exceeding 25% that of the first one, a value obtained when the external potential gives an almost vanishing gap, and considering the electron-electron interaction in the random-phase approximation.

When overlapping takes place, the lower level being full and the upper one empty in the unperturbed state, a redistribution of the electrons produces partial filled bands. Their strong and nonlinear response discussed in Sec. I enhances the nonlinear effects which may be seen in Fig. 4.[15]

The temperature may prevent a Landau band from

remaining totally occupied. Its influence may be estimated by considering the Fermi level in the middle of the gap

$$\varepsilon_F = \frac{E_n^{\max} + E_{n+1}^{\min}}{2} , \tag{4.1}$$

$E_n^{\max}$ and $E_{n+1}^{\min}$ being the highest and the lowest energies of the $n$th and $(n+1)$th bands, and the Fermi function

$$\mathcal{F}(E_n^{\max}) = \frac{1}{e^{\Delta_n/2T}+1} = 1 - \mathcal{F}(E_{n+1}^{\min}) , \tag{4.2}$$

where $\Delta_n = E_{n+1}^{\min} - E_n^{\max}$. If, e.g., $\Delta_n = 0.1$, the contribution of the $(n+1)$th band to the density response will be within at most a few percent for $T = 0.01$ (which in SI units means, e.g., $T \approx 1$ K and $B \approx 5$ T for an effective mass as in GaAs).

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[15]See Figs. 2 and 6 of Ref. 4; for integer filling factor the linear-response approximation (in RPA) is violated only in the space regions corresponding to overlapping bands.