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Electron-hole-correlation effects in generation-recombination noise

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The current-noise spectral density in a semiconductor with recombination centers in the band gap is shown to be dependent on the variances of both charge-carrier ensembles: The cross-correlation term between electrons and holes cannot be neglected and plays a decisive role under nonequilibrium conditions. We present a model for generation-recombination noise in the stationary nonequilibrium state. Negative contributions to the noise spectra producing a maximum in the frequency dependence of the noise power can arise, depending on the defect parameters. Good agreement is found when comparing our theory with experimental results on heterostructure $In_{0.53}Ga_{0.47}As/InP$ photodiodes.

I. INTRODUCTION

Fluctuation phenomena have been studied since the beginning of the century from the different but complementary perspectives of statistics, thermodynamics, and solid-state physics.[1-3] In the early 1950s and 1960s, the main theoretical framework was set by the Shockley-Read-Hall generation-recombination model,[4] by Burgess's variance theorem,[5-7] and by van Vliet, Lax, van der Ziel, and others[8-15] for the case of steady state at or close to thermal equilibrium. The technological explosion of the last two decades has produced very-high-quality materials and, simultaneously, started the age of highly structured devices. These complex structures, such as avalanche photodiodes and semiconductor lasers, are often designed to work far from thermal equilibrium. Therefore effects due to nonequilibrium conditions need to be taken into account.

It was soon recognized that normal thermodynamical approaches did not apply in this case, and a different theoretical work was required to explain experimental results. Furthermore high-quality semiconductor materials have made it possible to investigate intrinsic or structural effects that in the past were obscured by material defects. As a result, specific interest arose in the noise behavior[16-18] of complex structures working far from thermal equilibrium. More recently, noise-measurement techniques which are sensitive to small fluctuations in the presence of large steady-state background values of currents have become very important investigative tools for the characterization of semiconductor devices,[19-24] since greater physical insights may be accomplished.

Generation and recombination processes cause random fluctuations in the concentration of carriers in a semiconductor and this can be detected as current noise. The measured current-noise spectral density is known to be proportional to the Fourier transform of the correlation function of the carrier ensemble. If the system can be described by a single variable interacting with two energy levels (e.g., for electrons the conduction band and trap level), the correlation function shows the typical exponential shape with time constant τ and the current-noise spectrum is the well-known Lorentz curve. On the other hand, as will be shown in this paper, recombination centers within the band gap of the semiconductor make the system essentially two dimensional. A simple superposition of the contributions of electrons and holes taken separately is no longer adequate. In this case and especially under nonequilibrium conditions a decisive role is played by the cross-correlation function between the positively and negatively charged carrier ensembles.

In this paper we present a model for generation-recombination noise in the stationary nonequilibrium state. In Sec. II we briefly outline the assumptions underlying the model. Time reversibility is shown not to hold in our case. In Sec. III the general equations are presented. The theoretical (analytical) results of the model will be applied to a two-dimensional system in Sec. IV and compared with measurements on heterostructure $In_{0.53}Ga_{0.47}As/InP$ photodiodes in Sec. V.

II. BASIC ASSUMPTIONS

Since we apply our model to a system in a steady state far from equilibrium, we cannot use a thermodynamical description but must instead use a statistical approach. We follow the Lax and van Vliet treatment and make the following generalizations.

(1) The system is Markovian, i.e., there exists a small set of variables $\underline{a}$, whose knowledge at time 0 determines the whole future behavior of the system for $t > 0$ (mean values, distribution function) *and* makes any previous ($t < 0$) information superfluous.

(2) The system is stationary, i.e., it is invariant with respect to time shifts such as the translation of the origin of time.

(3) Fluctuations $\underline{\alpha}$ are calculated from a nonequilibrium steady state $\underline{a}^{SS}$ since our system is "driven" by electric bias and/or external carrier generation.

(4) The system is quasilinear. This means that we can treat the fluctuations $\underline{\alpha}$ as linear variables. It should be noted that mean values and probability functions are linear functions of $\underline{\alpha}$.

(5) Within the small time interval dt only one transition

between the energy levels is allowed to occur (the probability of two or more generation or recombination steps is negligible as dt approaches zero).

A further assumption that is usually made is time reversibility,[25–28] which states that the mean square of $\underline{\alpha}$ is invariant when we reverse the time arrow ($t \to -t$). It holds in thermal equilibrium systems and it is equivalent to the condition in Eq. (35) for bivariate systems as we shall see in Sec. IV. Onsager[25] assumed it held in steady states without magnetic fields and Coriolis forces. We find that it does not hold in our model of a bivariate semiconductor in nonequilibrium and therefore present a general calculation without the time-reversibility assumption.

From a physical point of view we can argue that carrier generation from illumination and electron-hole recombination via deep levels in the band gap are nonreversible processes. Looking backward on a time scale according to the reversibility principle would mean having light emission through recombination and carrier generation via a deep level. However, the probabilities (and therefore the time constants) of the two pairs of processes differ by several orders of magnitude. Light absorption and nonradiative recombination therefore do not satisfy time reversibility: the entropy increase due to the capture of the charge carriers into the trap (*multiphonon* process) cannot be sufficiently compensated for by the *photon* annihilation in the electron-hole generation.

III. THE EQUATIONS

We are now ready to set up the equations to describe a multilevel system with a_i carriers in the energy state E_i, a_i^{SS} being the steady-state value of a_i and p_{ij} the transition probability per unit time that a carrier will go from E_i to E_j as a function of the occupancies $\{a_i\}$ before the transition occurred, for $i=1,\ldots,s$. The condition of charge neutrality can be written as

$$\sum_{i=1}^{s} a_i = C \ , \tag{1}$$

where C is an appropriate sum over all space charges and compensating centers. In this case one of the carrier variables can be directly eliminated. Nevertheless, it can also happen that the sum of all carriers is a position-dependent quantity, such as in space-charge regions whose value cannot be determined *a priori*:

$$\sum_{i=1}^{s} a_i = C(x) \ . \tag{2}$$

Since we are interested in the carrier fluctuations and not primarily in their absolute values, we can still calculate the partial derivatives of the sth variable with respect to the other $(s-1)$ ones and work with the $(s-1)$-dimensional fluctuation set:

$$a_i = C(x) = \sum_{\substack{j=1 \\ (j\neq i)}} a_j \ , \tag{3a}$$

$$\frac{\partial a_i}{\partial a_j} = -1 \quad \text{for } i \neq j \ . \tag{3b}$$

It is important to note that C now plays the role of an external parameter.

Under our assumptions the equations for $\langle \alpha \rangle$ and $\langle \alpha\alpha \rangle$ are found to be

$$\frac{\partial}{\partial t}\langle a \rangle = -\underline{M}\langle \alpha \rangle \ , \tag{4}$$

where $\underline{M}$, the phenomenological relaxation matrix, given by

$$M_{ij} = \sum_{k=1}^{s} \left[\frac{\partial p_{ik}}{\partial a_i} - \frac{\partial p_{ki}}{\partial a_j} \right]_{\{a_i\}=\{a_i^{\mathrm{SS}}\}} \ , \tag{5}$$

is positive definite and its eigenvalues λ_M are inversely proportional to the time constants of the generation-recombination processes (in the case of one variable $\lambda = 1/\tau = \omega_0$ represents the characteristic frequency of the Lorentz curve); and

$$\langle \alpha\alpha \rangle \underline{M}^t + \underline{M}\langle \alpha\alpha \rangle = \underline{B} = 2\underline{D} \ , \tag{6}$$

where $\underline{D}$ is the diffusion matrix and $\underline{M}^t$ is the transpose of matrix $\underline{M}$. This latter equation represents a generalization of the Einstein relation between mobilities and diffusion constant which is valid also for multivariate systems far from thermal equilibrium.[11] The elements of the matrix $\underline{B}$ can be readily calculated as

$$B_{ij} = B_{ji} = -p_{ij} - p_{ji} \ ,$$

$$B_{ii} = \sum_{k=1}^{s} p_{ik} + \sum_{k=1}^{s} p_{ki}, \quad k \neq i \ . \tag{7}$$

Once we have a relationship for the variances we can, at least formally, calculate the correlation function; its Fourier transform yields the noise spectra we require. The correlation function given by

$$\underline{\Phi}(t) = \langle \alpha(t)\alpha(0) \rangle \tag{8}$$

satisfies the following time equation:

$$\frac{d\underline{\Phi}}{dt} = -\underline{M}\underline{\Phi} \ , \tag{9}$$

with the solution

$$\underline{\Phi}(t) = \exp(-\underline{M}t)\underline{\Phi}(0) \ , \tag{10}$$

where $\underline{\Phi}(0) = \langle \alpha\alpha \rangle$.

The complex noise spectrum $\underline{S}$ is by definition the Fourier transform of $\underline{\Phi}$:

$$\underline{S}(\alpha,\omega) = 2\int_{-\infty}^{+\infty} e^{-i\omega t}\underline{\Phi}(t)\,dt \ . \tag{11}$$

Since it is preferable to work with positive time values ($t > 0$), we can transform Eq. (11),

$$\underline{S}(\alpha,\omega) = 2\int_{0}^{+\infty} [e^{-i\omega t}\underline{\Phi}(t) + \underline{\Phi}(-t)e^{+i\omega t}]\,dt \ , \tag{12}$$

where

$$\underline{\Phi}(-t) = \langle \alpha(-t)\alpha(0) \rangle = \langle \alpha(0)\alpha(t) \rangle = \underline{\Phi}^t(t) \tag{13}$$

and

$$\underline{\Phi}^t(t) = [\exp(-\underline{M}t)\langle \alpha\alpha \rangle]^t = \langle \alpha\alpha \rangle \exp(-\underline{M}^t t) \ . \tag{14}$$

The real noise spectrum $\underline{G}$ of the variable α (representing the fluctuations of particle numbers) is then

$$\underline{G}(\alpha,\omega)=\mathrm{Re}\underline{S}(\alpha,\omega) \ . \tag{15}$$

By substitution of Eq. (14) into Eq. (12) we obtain

$$\underline{S}(\alpha,\omega)=2\int_0^{+\infty}(e^{-(\underline{M}+i\omega\underline{I})t}\langle\alpha\alpha\rangle+\langle\alpha\alpha\rangle e^{-(\underline{M}^t-i\omega\underline{I})t})dt \tag{16a}$$

$$=2(\underline{M}+i\omega\underline{I})^{-1}\langle\alpha\alpha\rangle+2\langle\alpha\alpha\rangle(\underline{M}^t-i\omega\underline{I})^{-1} \ , \tag{16b}$$

where $\underline{I}$ represents the unit matrix.

It should be noted that $\underline{S}$ is always real (and symmetrical) if

$$\langle\alpha\alpha\rangle\underline{M}^t=\underline{M}\langle\alpha\alpha\rangle \ , \tag{17}$$

which holds only for time-reversible systems and especially in thermal equilibrium. In this case

$$\underline{G}(\alpha,\omega)=\underline{S}(\alpha,\omega)=2\omega^{-2}\mathrm{Re}\left[\left[\underline{I}+\frac{1}{i\omega}\underline{M}\right]^{-1}\underline{B}\right] \ . \tag{18}$$

On the other hand, no extra condition needs to be imposed on $\underline{G}$ to make it symmetrical since $\underline{B}$ (or $\underline{D}$) is symmetrical by definition [see Eq. (6)]: this means that *time reversibility is neither a condition nor a consequence of $\underline{G}$'s symmetry.*

Since we are interested in the general case without time reversibility, we are only allowed the next step [from Eqs. (16) and (6)]:

$$(\underline{M}+i\omega\underline{I})\underline{S}(\underline{M}^t-i\omega\underline{I})=2(\langle\alpha\alpha\rangle\underline{M}^t+\underline{M}\langle\alpha\alpha\rangle)=2\underline{B} \ . \tag{19}$$

By inversion we obtain

$$\underline{G}=\mathrm{Re}\underline{S}=2\,\mathrm{Re}[(\underline{M}+i\omega\underline{I})^{-1}\underline{B}(\underline{M}^t-i\omega\underline{I})^{-1}] \ , \tag{20}$$

and for $M=1/\tau$ (one-variable system)

$$G=\frac{2B\tau^2}{1+\omega^2\tau^2} \quad \text{(Lorentz curve)} \ , \tag{21}$$

which is often seen in the form

$$G=\frac{4\langle(\Delta N)^2\rangle\tau}{1+\omega^2\tau^2} \ , \tag{22}$$

where $\langle(\Delta N)^2\rangle$ represents the variance of the number of charge carriers. In the next section we present the extended calculation for the bivariate steady state.

IV. NOISE IN TWO-VARIABLE SYSTEMS

We now describe the noise behavior of the (two-variable) three-level system. The conduction band and the valence band can be represented as energy E_C and E_V, respectively. We can equally interpret the a's introduced above as representing densities without any change in notation or formulas. In this case the Shockley-Read-Hall rate equations keep their usual and well-known form. Furthermore we consider a density N_T of recombination centers in the band gap at the energy level E_t. For consistency with the previous section we label the energy levels E_1 (E_C), E_2 (E_V), E_3 (E_t). The density of the electronic population in each state is then given by $a_1=n$ (density of free electrons in the conduction band), $a_2=-p$ (density of free holes in the valence band), and $a_3=i$ (density of occupied traps). In our case a position-dependent charge balance condition holds and therefore we can write

$$a_1+a_2+a_3=n-p+i=C(x) \ . \tag{23}$$

Since we are only interested in fluctuations, we reduce the number of independent variables to *two* $(n,-p)$, treating i as a dependent variable. In the following we need the derivatives of i. According to Eq. (3b) we obtain

$$\frac{\partial i}{\partial n}=-1, \quad \frac{\partial i}{\partial(-p)}=-1 \ , \tag{24}$$

i itself can be calculated from the Shockley-Read-Hall formula

$$i=N_T\frac{c_n n+c_p p_1}{c_n(n+n_1)+c_p(p+p_1)} \ , \tag{25}$$

with n and p taken in steady-state conditions and where

$$n_1=N_C g\exp[(E_t-E_C)/kT] \ , \tag{26a}$$

$$p_1=N_V\frac{1}{g}\exp[(E_V-E_t)/kT] \ , \tag{26b}$$

are the densities of carriers which would be found in the conduction or valence band if the Fermi level coincides with the energy of the recombination center (g denotes the degeneracy of the trap); c_n and c_p are the capture coefficients of the traps for electrons and holes, respectively.

For transition probabilities p_{ij} between the energy states we assume a quadratic mass action law:[12]

$$p_{12}=\beta np \tag{27a}$$

(we set $\beta=0$, negligible direct recombination);

$$p_{21}=\gamma \tag{27b}$$

(external light-generation rate);

$$p_{13}=c_n n(N_T-i) \ , \tag{27c}$$

$$p_{31}=c_n n_1 i \ , \tag{27d}$$

$$p_{23}=c_p p_1(N_T-i) \ , \tag{27e}$$

$$p_{32}=c_p p i \tag{27f}$$

(Shockley-Read-Hall relations). In thermal equilibrium we can put $p_{ij}^0=p_{ji}^0$. However under steady-state conditions this is no longer true. n,p (and i) have only their steady-state values.

Following the outline of Sec. III we now need to determine the 2×2 matrices $\underline{M}$ and $\underline{B}$ [Eqs. (5) and (7)]. According to the definition

$$M_{11}=c_n(N_T-i+n+n_1)\big|_{\{a\}=\{a^{\mathrm{SS}}\}}\ , \tag{28a}$$

$$M_{12}=c_n(n+n_1)\big|_{\{a\}=\{a^{\mathrm{SS}}\}}\ , \tag{28b}$$

$$M_{21}=c_p(p+p_1)\big|_{\{a\}=\{a^{\mathrm{SS}}\}}\ , \tag{28c}$$

$$M_{22}=c_p(i+p+p_1)\big|_{\{a\}=\{a^{\mathrm{SS}}\}}\ . \tag{28d}$$

For the $\underline{B}$ matrix we obtain

$$B_{11}=c_n[(N_T-i)n+in_1]+\gamma\ , \tag{29a}$$

$$B_{22}=c_p[(N_T-i)p_1+ip]+\gamma\ , \tag{29b}$$

$$B_{12}=B_{21}=-\gamma\ . \tag{29c}$$

A. The reversibility condition

In the case of time-reversible systems we have

$$\langle\alpha\alpha\rangle\underline{M}^t=\underline{M}\langle\alpha\alpha\rangle\ ,$$

which can be substituted into Eq. (6). With

$$\langle\alpha\alpha\rangle=\begin{bmatrix}x_{11} & x_{12}\\ x_{21} & x_{22}\end{bmatrix}=\langle\alpha\alpha\rangle^t\ \ (x_{12}=x_{21})\ , \tag{30a}$$

$$\underline{M}=\begin{bmatrix}M_{11} & M_{12}\\ M_{21} & M_{22}\end{bmatrix}\quad \underline{M}^t=\begin{bmatrix}M_{11} & M_{21}\\ M_{12} & M_{22}\end{bmatrix}\ , \tag{30b}$$

$$\underline{B}=\begin{bmatrix}B_{11} & B_{12}\\ B_{21} & B_{22}\end{bmatrix}\ , \tag{30c}$$

Eq. (17) becomes

$$\begin{bmatrix}x_{11} & x_{12}\\ x_{21} & x_{22}\end{bmatrix}\begin{bmatrix}M_{11} & M_{21}\\ M_{12} & M_{22}\end{bmatrix}=\begin{bmatrix}M_{11} & M_{12}\\ M_{21} & M_{22}\end{bmatrix}\begin{bmatrix}x_{11} & x_{12}\\ x_{21} & x_{22}\end{bmatrix}\ , \tag{31}$$

which is always true for the diagonal elements. The off-diagonal products yield

$$x_{11}M_{21}+x_{12}M_{22}=x_{12}M_{11}+x_{22}M_{12}\ . \tag{32}$$

From Eq. (6) we obtain three equations in the three unknowns x_{11},x_{12},x_{22}:

$$\begin{bmatrix}2M_{11} & 2M_{12} & 0\\ M_{21} & M_{11}+M_{22} & M_{12}\\ 0 & 2M_{21} & 2M_{22}\end{bmatrix}\begin{bmatrix}x_{11}\\ x_{12}\\ x_{22}\end{bmatrix}=\begin{bmatrix}B_{11}\\ B_{12}\\ B_{22}\end{bmatrix}\ , \tag{33a}$$

which can be put in the short form

$$\underline{\tilde{M}}\,\underline{X}=\underline{\tilde{B}}\ . \tag{33b}$$

The determinant of the matrix $\underline{\tilde{M}}$ is given by $\det(\underline{\tilde{M}})=4\det(\underline{M})\times\mathrm{Tr}(\underline{M})$ and is nonvanishing since $\underline{M}$ is positive definite. $\mathrm{Tr}(\underline{M})$ represents $M_{11}+M_{22}$. This means that

$$\underline{X}=\underline{\tilde{M}}^{-1}\underline{\tilde{B}}\ . \tag{34}$$

We now substitute the solutions x_{ij} in the reversibility condition given by Eq. (32). The following results:

$$M_{21}B_{11}-M_{12}B_{22}+(M_{22}-M_{11})B_{12}=0\ . \tag{35}$$

We will discuss this relationship in Sec. IV C.

B. The noise spectral density

In the next step we determine the noise spectral density $\underline{G}(\alpha,\omega)$ according to Eq. (20). First we modify the right-hand side of Eq. (20) as follows:

$$\begin{aligned}\underline{G}&=2\,\mathrm{Re}[(\underline{M}+i\omega\underline{I})^{-1}\underline{B}(\underline{M}^t-i\omega\underline{I})^{-1}]\\ &=2\omega^{-2}\,\mathrm{Re}\left[\left[\frac{\underline{M}}{i\omega}+\underline{I}\right]^{-1}\underline{B}\left[\underline{I}-\frac{\underline{M}^t}{i\omega}\right]^{-1}\right]\\ &=2\omega^{2}\,\mathrm{Re}\left\{\left[\omega^{-2}\left[\frac{\underline{M}}{i\omega}+\underline{I}\right]\right]^{-1}\underline{B}\left[\omega^{-2}\left[\underline{I}-\frac{\underline{M}^t}{i\omega}\right]^{-1}\right]\right\}.\end{aligned} \tag{36}$$

The contents of the first set of square brackets on the right-hand side of Eq. (36), called for convenience $(\underline{M}_1)^{-1}$, is now

$$(\underline{M}_1)^{-1}=\begin{bmatrix}\omega^2-i\omega M_{22} & i\omega M_{12}\\ i\omega M_{21} & \omega^2-i\omega M_{11}\end{bmatrix}\frac{1}{\det(\underline{M}_1)} \tag{37}$$

and

$$\det(\underline{M}_1)=\omega^2[\omega^2-i\omega\,\mathrm{Tr}(\underline{M})-\det(\underline{M})]\ . \tag{38}$$

The expression in square brackets of Eq. (38) is very similar to the eigenvalue equation of $\underline{M}$:

$$\lambda^2-\lambda\,\mathrm{Tr}(\underline{M})+\det(\underline{M})=0\ , \tag{39a}$$

$$\mathrm{Tr}(\underline{M})=\lambda_1+\lambda_2=\tau_1^{-1}+\tau_2^{-1}\ ,$$

$$\det(\underline{M})=\lambda_1\lambda_2=(\tau_1\tau_2)^{-1}\ , \tag{39b}$$

$$\tau_{1,2}^{-1}=\tfrac{1}{2}[(M_{11}+M_{22})\pm\sqrt{(M_{11}+M_{22})^2-4\det(\underline{M})}]\ , \tag{39c}$$

where τ_1 and τ_2 (the time constants we are interested in) are the reciprocal eigenvalues of $\underline{M}$. In the case of Eq. (38) we have as solutions

$$i\omega=\tau_1^{-1},\tau_2^{-1}\ , \tag{40}$$

and Eq. (38) becomes

$$\det(\underline{M}_1)=-\omega^2(i\omega-\tau_1^{-1})(i\omega-\tau_2^{-1})\ , \tag{41a}$$

$$[\det(\underline{M}_1)]^{-1}=\frac{-\tau_1\tau_2}{\omega^2(1+i\omega\tau_1)(1+i\omega\tau_2)}\ . \tag{41b}$$

Similarly, we get for the contents of the second set of square brackets on the right-hand side of Eq. (36), called for convenience $(\underline{M}_2)^{-1}$,

$$(\underline{M}_2)^{-1}=\begin{bmatrix}\omega^2+i\omega M_{22} & -i\omega M_{12}\\ -i\omega M_{21} & \omega^2+i\omega M_{11}\end{bmatrix}\frac{1}{\det(\underline{M}_2)}\ , \tag{42}$$

where $(\det M_2)^{-1}$ is now

$$[\det(\underline{M}_2)]^{-1}=\frac{-\tau_1\tau_2}{\omega^2(1-i\omega\tau_1)(1-i\omega\tau_2)}\ . \tag{43}$$

Substituting into Eq. (36):

$$\underline{G}=2\omega^2\mathrm{Re}\left\{\begin{bmatrix}\omega^2-i\omega M_{22} & i\omega M_{12}\\ i\omega M_{21} & \omega^2-i\omega M_{11}\end{bmatrix}\underline{B}\begin{bmatrix}\omega^2+i\omega M_{22} & -i\omega M_{12}\\ -i\omega M_{21} & \omega^2+i\omega M_{11}\end{bmatrix}\frac{\tau_1^2\tau_2^2}{\omega^4(1+\omega^2\tau_1^2)(1+\omega^2\tau_2^2)}\right\}. \tag{44}$$

The last term is always real and can be put in front of the real-part operator. After multiplication and noting that M_{ij} and B_{ij} are real, we have

$$\underline{G}=\begin{bmatrix}G_{nn} & G_{np}\\ G_{pn} & G_{pp}\end{bmatrix}$$

$$=\mathrm{Re}\begin{bmatrix}S_{nn} & S_{np}\\ S_{pn} & S_{pp}\end{bmatrix} \tag{45a}$$

$$=2\frac{\tau_1^2\tau_2^2}{(1+\omega^2\tau_1^2)(1+\omega^2\tau_2^2)}\begin{bmatrix}g_{nn} & g_{np}\\ g_{pn} & g_{pp}\end{bmatrix} \tag{45b}$$

$$=2\frac{\tau_1^2\tau_2^2}{(1+\omega^2\tau_1^2)(1+\omega^2\tau_2^2)}\mathrm{Re}\begin{bmatrix}s_{nn} & s_{np}\\ s_{pn} & s_{pp}\end{bmatrix}, \tag{45c}$$

where

$$g_{nn}=\omega^2B_{11}+M_{22}^2B_{11}+M_{12}^2B_{22}-2M_{22}M_{12}B_{12}, \tag{46a}$$

$$g_{np}=g_{pn}=\omega^2B_{12}+(M_{11}M_{22}+M_{12}M_{21})B_{12}-(M_{22}M_{21}B_{11}+M_{11}M_{12}B_{22}), \tag{46b}$$

$$g_{pp}=\omega^2B_{22}+M_{11}^2B_{22}+M_{21}^2B_{11}-2M_{11}M_{21}B_{12}. \tag{46c}$$

C. Time relationships and reversibility

It is interesting to see what we neglect by taking the real value in the definition of $\underline{G}$. The diagonal elements s_{nn} and s_{pp} are *real* so that in this case *(autocorrelation)* we do not make any approximations. Furthermore, it is plausible that autocorrelation terms yield a real spectrum. (We should recall the meaning of an imaginary spectrum or quantity. We find imaginary power only in the presence of *time relationships,* time delays, or phase shifts due, for example, to capacitive or inductive elements in a circuit. These time "shifts" do not dissipate power, they just transfer it to different positions in space or time.)

Even more interesting is that the nondiagonal elements (related to the *cross-correlation* between electrons and holes) have an imaginary part but *only* if the system is *not time reversible:*

$$\mathrm{Im}s_{np}=i\omega[(M_{11}-M_{22})B_{12}+M_{12}B_{22}-M_{21}B_{11}] \tag{47}$$

[see Eq. (35)]. $s_{np}=s_{pn}^*$ have imaginary parts which are equal and opposite.

This confirms our interpretation of the imaginary part being related to time relationships, to an "arrow of time." It does not matter here whether it points to the future or to the past. It points somewhere and it cannot be reversed. The imaginary part of the cross-correlation term stands for the flow of time in our model. It is time spent without noise-power dissipation, i.e., time spent waiting for a causal relationship to happen. For example, the trap needs to capture an electron which only then can recombine with a hole. Again, the fact that the imaginary parts of s_{np} and s_{pn} are equal and opposite supports this time model. There is an (captured) electron in the past of a (recombining) hole, just as there is a (to-be-captured) hole in the future of a (to-be-recombined) electron. The overlap of their existences is equal in magnitude and opposite in sign. (We have assumed so far that the short time constant be associated with a fast capture-emission interaction between the deep level and the conduction band, but everything would still hold exchanging the role of holes and electrons.)

In thermal equilibrium the carrier populations adjust exactly in the way given by detailed balance; time does not play a role anymore. Every process can be reversed and is actually *defined* by the fact that it can be reversed. No imaginary part appears in the fluctuation spectra in this case since Eqs. (47) and (35) are identically zero.

D. Frequency dependence of the noise spectrum

The current noise spectral density is given by

$$G_I(\omega)=\left[\frac{I}{bn+p}\right]^2[b^2G_{nn}(\omega)+2bG_{np}(\omega)+G_{pp}(\omega)], \tag{48}$$

where n and p are the electron and hole densities, respectively, I the steady-state current value and b (in most cases) the ratio of the electron and hole mobilities μ_n/μ_p. [For a derivation of Eq. (48) see the Appendix.] By substituting Eqs. (45) ad (46) into (48) we get

$$G_I(\omega)=\frac{Z(\omega)}{(1+\omega^2\tau_1^2)(1+\omega^2\tau_2^2)}G_{I0}, \tag{49}$$

where Z is a second-order polynomial with no linear term,

$$Z(\omega)=c_2\omega^2+c_0 \tag{50a}$$

and

$$G_{I0}=\left[\frac{I}{bn+p}\right]^2\tau_1^2\tau_2^2. \tag{50b}$$

The form of the noise spectrum depends on the coefficients c_0 and c_2 and their relationship to the time constants τ_1 and τ_2. In the following we assume $\tau_1<\tau_2$. We can also write $G_I(\omega)$ as

$$G_I(\omega)=\frac{K_1}{1+\omega^2\tau_1^2}+\frac{K_2}{1+\omega^2\tau_2^2}\ , \tag{51}$$

where K_1 and K_2 satisfy

$$K_1+K_2=c_0G_{I0}\ , \tag{52a}$$

$$K_1\tau_2^2+K_2\tau_1^2=c_2G_{I0}\ . \tag{52b}$$

Usually $K_2>0$, but there is a possibility (given by the cross correlation as we shall see) that K_2 may be *negative* depending on the defect parameters

$$K_1=\frac{c_2-c_0\tau_1^2}{\tau_2^2-\tau_1^2}G_{I0}\ , \tag{53a}$$

$$K_2=\frac{c_0\tau_2^2-c_2}{\tau_2^2-\tau_1^2}G_{I0}\ , \tag{53b}$$

$$K_2<0 \Longrightarrow \tau_2^2<\frac{c_2}{c_0}\ , \tag{53c}$$

where

$$c_2=b^2B_{11}+2bB_{12}+B_{22}\ , \tag{54a}$$

$$\begin{aligned}c_0=&\,b^2(M_{22}^2B_{11}+M_{12}^2B_{22}-2M_{22}M_{12}B_{12})+2b\,[(M_{11}M_{22}+M_{12}M_{21})B_{12}-(M_{22}M_{21}B_{11}+M_{11}M_{12}B_{22})]\\&+(M_{11}^2B_{22}+M_{21}^2B_{11}-2M_{11}M_{21}B_{12})\ .\end{aligned} \tag{54b}$$

All M_{ij} and B_{ij} are positive except for B_{12}. As we can see, the cross-correlation terms (coefficients of b) are negative. As in Ref. 9 we can argue that an increase in the number of minority carriers causes a decrease in the number of majority carriers because recombination becomes more effective: $G_{np}<0$.

The c's and the K's depend in a complicated way on the steady-state carrier densities [see definition of M_{ij} and B_{ij} in Eqs. (28) and (29)] which are in turn a function of the defect parameters and of external experimental conditions such as temperature, voltage, and illumination intensity, amongst others; i.e.,

$$K_1,K_2=K_1,K_2(c_n,c_p,n_{\mathrm{SS}},p_{\mathrm{SS}},E_t,N_T,\mu_n/\mu_p;T,P_{\mathrm{light}},I_{ph})\ . \tag{55}$$

What do we expect as a noise spectrum on a double logarithmic scale? For this it is easier to look at G as a product of three functions and a constant:

$$G_I(\omega)=Z(\omega)P_1(\omega)P_2(\omega)G_{I0}\ , \tag{56a}$$

$$Z(\omega)=c_2\omega^2+c_0=c_2(\omega^2+f_0^2)=c_2Z^0(\omega)\ , \tag{56b}$$

$$P_{1,2}(\omega)=\frac{1}{1+\omega^2\tau_{1,2}^2}=\frac{1}{\tau_{1,2}^2}\frac{1}{\omega^2+f_{1,2}^2}=\frac{1}{\tau_{1,2}^2}P_{1,2}^0(\omega)\ , \tag{56c}$$

where $f_0^2=c_0/c_2$ and $f_{1,2}^2=1/\tau_{1,2}^2$.

G_I can then be written as

$$G_I(\omega)=G_I(0)Z^0(\omega)P_1^0(\omega)P_2^0(\omega)\ , \tag{56d}$$

where $G_I(0)$ represents the amplitude of $G_I(\omega)$ at zero frequency and the other three factors determine its frequency behavior. Since we already chose $f_1>f_2$ there are three possibilities of ordering f_0 (see Fig. 1):

Case (a),

$$f_0>f_1>f_2\ . \tag{57a}$$

Case (b),

$$f_1>f_0>f_2\ . \tag{57b}$$

Case (c),

$$f_1>f_2>f_0\ . \tag{57c}$$

Case (b) has been observed in different physical systems, in photoconductive cells such as PbS, PbSe, PbTe,[9] semiconducting materials such as CdS,[21] and in $\mathrm{Al}_y\mathrm{Ga}_{1-y}\mathrm{As/GaAs}$ as well as $\mathrm{In}_{1-x}\mathrm{Ga}_x\mathrm{As/InP}$ heterostructures.[22,23] Case (c) shows the results of a negative superposition of Lorentz spectra as given by Eq. (53c). In Eq. (57c) f_0 is the smallest of the frequencies appearing in $G_I(\omega)$. In this case Eq. (53c) is satisfied and K_2 is negative.

V. NUMERICAL SIMULATIONS AND COMPARISON WITH EXPERIMENTS

We see from Eq. (55) that the coefficients K_1 and K_2 depend on several defect parameters and on the experimental conditions. Once these values are set it is possible to calculate the noise spectral density according to Eqs. (28), (29), and (51)–(54). Through proper choice of the parameters in our simulation program we were able to observe the appearance of a maximum in the current noise spectra as shown in Fig. 2 (the values of the parameters are reported in Table I). All parameters were left unchanged except for the electron concentration n which was varied between 10^6 and $10^8\ \mathrm{cm}^{-3}$. For the lower n values the common Lorentz superposition is found [see Fig. 1(b)]; the higher n range shows a simple Lorentz spectrum. Very uncommon, however, is the behavior in the region around $10^7\ \mathrm{cm}^{-3}$ where a broad *maximum* arises.

The electron concentration in a semiconductor can vary over *orders of magnitude* depending on doping and voltage, especially within space-charge regions. The

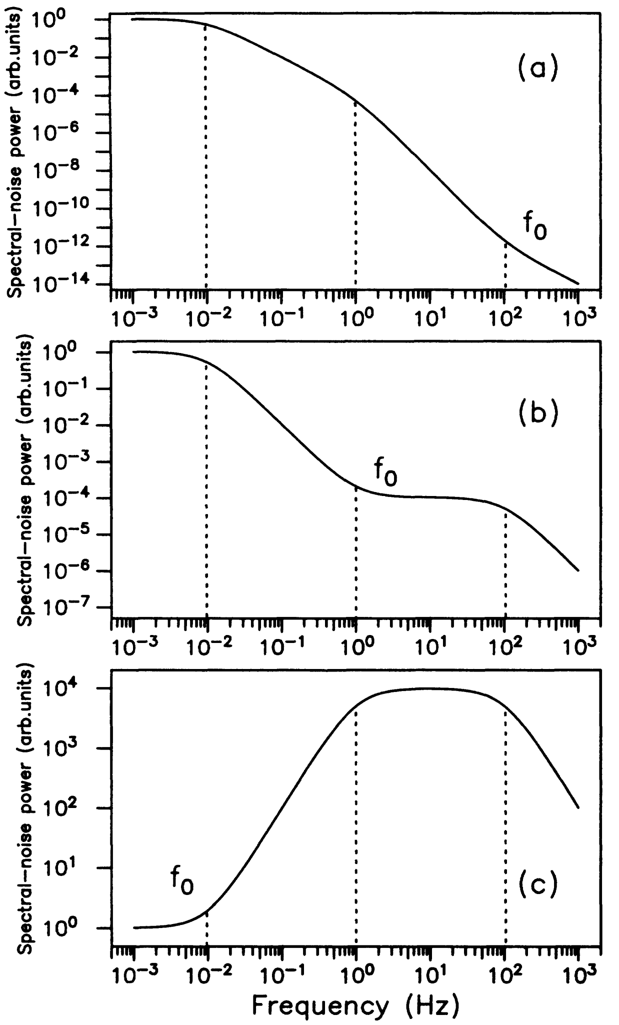

FIG. 1. Double logarithmic representation of the normalized current-noise spectra $G_I(\omega)/G_I(0)$ for different values of the zero frequency f_0: (a) $f_0>f_1>f_2$, (b) $f_1>f_0>f_2$, (c) $f_1>f_2>f_0$.

presence of a noise maximum within a very narrow range of n values shows how sensitive correlation effects can be. In this region K_2 is *negative* and we find a *subtraction* of the Lorentz spectra. We also point out that time reversibility does not hold in this case. Equation (35) does not yield zero for the chosen set of parameters, in agreement with our assumptions.

TABLE I. Parameter values for the numerical simulations shown in Fig. 2.

Electron capture coefficient c_n	10^{-11} cm^3/s
Hole capture coefficient c_p	10^{-9} cm^3/s
Light power γ	10^{13} s^{-1}
Electron steady-state concentration n_{SS}	10^6–10^8 cm^{-3}
Hole steady-state concentration p_{SS}	10^{10} cm^{-3}
Temperature T	172 K
Recombination center energy level E_C-E_t	300 meV
Recombination center concentration N_T	10^{16} cm^{-3}
Mobility ratio μ_n/μ_p	10^3

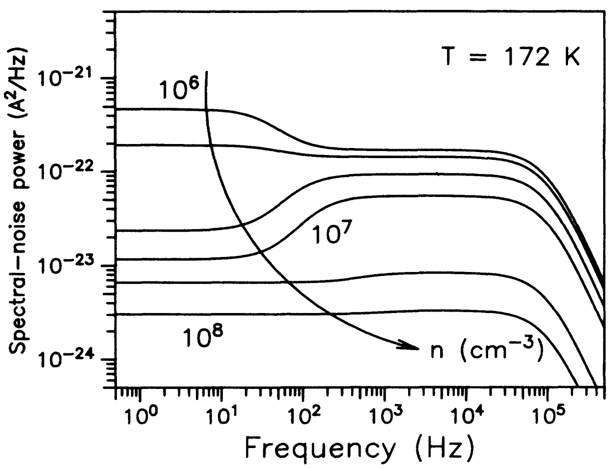

FIG. 2. Numerical simulation of $G_I(\omega)$ at 172 K (parameter values in Table I) using Eq. (48).

In Fig. 3 the numerical fit of photocurrent noise measurements[29] on $\mathrm{In}_{0.53}\mathrm{Ga}_{0.47}\mathrm{As/InP}$ heterostructures under illumination with 1.3 μm monochromatic light is shown. The voltage range corresponds to the region of onset of the photocurrent when the border of the space-charge region crosses the heterointerface and the electric field reaches into the light-absorbing material ($\mathrm{In}_{0.53}\mathrm{Ga}_{0.47}\mathrm{As}$). The fit parameters are the time constants τ_1,τ_2 [Eq. (40)] and the amplitudes K_1,K_2 [Eq. (53)] as they appear in Eq. (51) with a negative K_2 value (i.e., we assume a *negative* superposition of Lorentz curves). Very good agreement is found with the proposed theory. Moreover the experimental conditions correspond to the theoretical assumptions of a steady state far from thermal equilibrium without time reversibility. Even the low values of n found in the above calculation can be satisfied in the space-charge region around the $\mathrm{In}_{0.53}\mathrm{Ga}_{0.47}\mathrm{As/InP}$ interface. Nevertheless the knowledge of just four empirical fit parameters does not allow a unique determination of the seven physical pa-

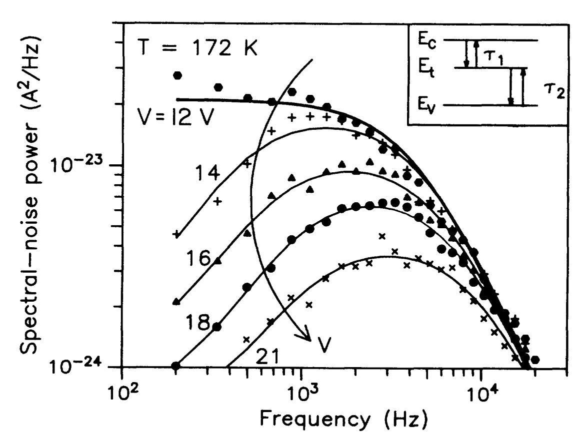

FIG. 3. Maxima in current-noise spectral density as function of reverse bias. Points: experimental results; solid lines: fit of the experiment with Eq. (51) (bold line as noise reference just before saturation); inset: interface state as electron trap (τ_1) and hole trap (τ_2) in the band gap.

rameters of the system. Independent measurements are needed in order to reliably determine the defect parameters.

We note that in our work, a negative superposition of Lorentz spectra has been theoretically predicted and experimentally observed in generation-recombination current noise in a nonreversible steady state far from thermal equilibrium. The *cross-correlation* term is responsible for this effect and offers interesting insights into charge-carrier behavior in semiconductors with energy states within the band gap, including the determination of defect parameters and a better understanding of device characteristics.

VI. CONCLUSIONS

We have presented a model for the determination of the current-noise spectral density in a two-variable system in nonequilibrium steady state *without* the assumption of time reversibility. While it is common to calculate the noise power just from the autocorrelation function of the system through a Fourier transform, in this case the cross-correlation term between electrons and holes plays a decisive role and cannot be neglected. The carrier reservoirs do not fluctuate independently of one another if the presence of defects in the forbidden gap offers a connection path between them. The electron-hole recombination process via deep levels is the physical realization of the connection *(cross correlation)* shown in the mathematical model. Moreover it justifies the assumption of no time reversibility, since it is a process with a definite time direction. *Negative* contributions to the overall noise power were found depending on the defect parameters and lead to an anomalous *maximum* in the noise spectra. Experimental results from $\mathrm{In}_{0.53}\mathrm{Ga}_{0.47}\mathrm{As/InP}$ heterostructures show good agreement with our theory. We then conclude that a negative superposition of Lorentz spectra has been theoretically predicted and experimentally observed in generation-recombination current noise.

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APPENDIX

To derive Eq. (48) it is important to make clear the sources of fluctuations that affect the current $I(t)$. Nyquist[1] showed that in the absence of external bias, detectable fluctuations are only thermal in origin. Since we ignore these fluctuations and apply no external voltage in the y and z directions we need only consider fluctuations in the x direction, as we only apply a bias in this direction. We shall also assume in the following a constant cross section A of the sample. The current density can then be considered as a constant across A and may be written as a time-dependent quantity only:

$$\operatorname{div}\left[j+\epsilon_0\epsilon_r\frac{\partial F(x,t)}{\partial t}\right]=\frac{\partial}{\partial x}\left[j+\epsilon_0\epsilon_r\frac{\partial F(x,t)}{\partial t}\right]=0\ , \tag{A1a}$$

$$j+\epsilon_0\epsilon_r\frac{\partial F(x,t)}{\partial t}=\frac{I(t)}{A}\ . \tag{A1b}$$

According to the generalized Ramo-Shockley theorem[30] and to the derivation of Kuhn *et al.* (Appendix A of Ref. 31) it is possible to show that the displacement current does not appear in the outside circuit if, as in our case, the voltage V is kept constant. For the case of the electron current

$$\frac{I_n(t)}{A}=\frac{1}{L}\int_0^L\left[j_n(x,t)+\epsilon_0\epsilon_r\frac{\partial F(x,t)}{\partial t}\right]dx \tag{A2a}$$

$$=\frac{1}{L}\int_0^L\left[en(x,t)v_d^n(x,t)+\epsilon_0\epsilon_r\frac{\partial F(x,t)}{\partial t}\right]dx \tag{A2b}$$

$$=\frac{1}{L}\int_0^L en(x,t)v_d^n(x,t)dx+\frac{1}{L}\epsilon_0\epsilon_r\frac{\partial}{\partial t}\int_0^L\left[-\frac{\partial V}{\partial x}\right]dx\ , \tag{A2c}$$

we can see that

$$\frac{\partial}{\partial t}\int_0^L dx\,F(x,t)=\frac{\partial}{\partial t}V(t)=0\ . \tag{A2d}$$

Here v_d^n is the electron drift velocity, F the electric field, ϵ_0 and ϵ_r the vacuum permittivity and the relative static dielectric constant, respectively, and L the length of the sample under consideration.

Equation (A2d) shows that the current $I_n(t)$ can be written as

$$I_n(t)=\frac{eA}{L}\int_0^L n(x,t)v_d^n(x,t)dx\ , \tag{A3}$$

which is only a function of the *number* and *velocity* of the particles.

The fluctuation of the current I can therefore be written as

$$\Delta_n I(t)=\frac{eA}{L}[N(t)v_d^n(t)-\langle N(t)v_d^n(t)\rangle] \tag{A4}$$

$$\cong\frac{eA}{L}[\Delta N(t)\langle v_d^n\rangle+\langle N\rangle\Delta v_d^n(t)]\ , \tag{A5}$$

where we have neglected second-order fluctuations (Appendix B of Ref. 31). For the description of generation-recombination noise, we can neglect the velocity fluctuations since their relaxation time is much faster than the processes under consideration (generation-recombination processes take place in milliseconds to microseconds whereas particle moments relax within picoseconds). For the electron current we can therefore write, keeping only the first term in the square brackets of Eq. (A5),

$$\Delta I_n(t)=\frac{eA}{L}[\Delta N(t)\langle v_d^n\rangle]\ , \tag{A6a}$$

and similarly for the hole current

$$\Delta I_p(t)=\frac{eA}{L}[\Delta P(t)\langle v_d^p\rangle]\ . \tag{A6b}$$

For a total current $I(t)$

$$I(t)=\frac{eA}{L}[N(t)v_d^n(t)+P(t)v_d^p(t)]\ , \tag{A7}$$

the total fluctuation is then

$$\Delta I(t)=\frac{eA}{L}[\Delta N(t)\langle v_d^n\rangle+\Delta P(t)\langle v_d^p\rangle]\ . \tag{A8}$$

This means we can write I and ΔI in the forms

$$I=a(bN+P)\ , \tag{A9a}$$

$$\Delta I=a(b\Delta N+\Delta P)\ , \tag{A9b}$$

where

$$a=\frac{eA}{L}\langle v_d^p\rangle \tag{A10a}$$

and

$$b=\frac{\langle v_d^n\rangle}{\langle v_d^p\rangle}\ . \tag{A10b}$$

The variance is given by

$$\langle(\Delta I)^2\rangle=a^2[b^2\langle(\Delta N)^2\rangle+2b\langle\Delta N\,\Delta P\rangle+\langle(\Delta P)^2\rangle]\ , \tag{A11}$$

where

$$a^2=\frac{I^2}{(bN+P)^2}\ . \tag{A12}$$

Since the Fourier transform is a linear operator, the current-noise spectral density maintains the same form of a relationship as given in Eq. (A11) and we write

$$G_I(\omega)=a^2[b^2G_{nn}(\omega)+2bG_{np}(\omega)+G_{pp}(\omega)] \tag{A13a}$$

$$=\frac{I^2}{(bN+P)^2}[b^2G_{nn}(\omega)+2bG_{np}(\omega)+G_{pp}(\omega)]\ . \tag{A13b}$$

Equation (A13b) is identical to Eq. (48) used in Sec. IV, except for the use of particle numbers instead of densities. This difference is only important when comparing amplitudes of simulations and experimental data, but is not important in a general consideration on *frequency dependence* as is the case in Sec. IV D.

The general definition of b in Eq. (A10b) can be easily shown to be

$$b=\frac{\mu_n}{\mu_p}\frac{dE_n/dx}{dE_p/dx}\quad\text{or}\quad b=\frac{\mu_n}{\mu_p}\ , \tag{A14}$$

depending on the Fermi energies as in Refs. 10 and 12 to which we refer in our text.

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